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How to Teach Your Own Liberal Arts Mathematics Course

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Synopsis

The article encourages Mathematics faculty members to design their own Liberal Arts Mathematics courses by using their own interests and expertise to link mathematics to the world of their students. The author argues that any such course should be guided by these five principles: Draw on the interests of each individual student; teach important mathematics; go slowly enough so students have a sense of mastery; encourage the students to use the mathematics they already know; and let students create projects on topics they choose and then share their projects with the class. The author describes how she implements these principles in two of her own Liberal Arts courses, "Mathematics, Philosophy, and the Real World" and "Mathematics in Many Cultures." The article includes examples of the materials used in these courses, and provides an extensive bibliography. It also lists a set of actual student projects from each course. It concludes that courses designed according to its principles result in students being able and willing to do mathematics, and knowledgeable and enthusiastic about the role mathematics plays in the wider world.

1. Introduction

Many of us are asked to teach liberal arts mathematics courses. The students in such courses generally are taking mathematics only because it is required. These students have already had 12 years of mathematics, yet they do not know that mathematics is beautiful and that mathematics has applications to all sorts of important things they care about. A course that reviews high school mathematics certainly will not inspire these students. So, what to do?

There are, of course, good textbooks designed for liberal arts mathematics courses. An instructor can adopt one of these books and use it as the basis of a course. But another alternative is to design one's own course, and that is what I have chosen to do. This article describes the principles I believe should inform this type of course, and shows how I have implemented them in my own classes. My hope is that this discussion will aid readers in designing and teaching their own liberal arts mathematics courses.

I first wondered about how to teach such a course when I was in graduate school. To gather inspiration, I audited a "Mathematics for Non-Mathematicians" course that began by analyzing the game of NIM. The topic was chosen because at the time there was a highly-regarded film, *Last Year at Marienbad*, in which there was one character who always won at NIM. The instructor started the course with an explanation of the game and of the winning strategy.

To be sure, this was interesting, but it was interesting only to those people who had seen and cared about that movie. And this is the problem in motivating students who define themselves as non-mathematicians: Whether the topic is NIM, or baseball, or cooking, yes, these subjects use mathematics, and some students find them fascinating, but others do not care about them at all.

So what can one do instead? This question leads to the first of my general principles:

(1) Work from the individual student's own interest.

Later on in this essay, I will describe some of the ways the instructor can make this happen.

A second reason this professor chose to begin the course with NIM was that the mathematics underlying the analysis of the game is accessible to liberal arts students. Indeed, accessibility is necessary, but it is not sufficient. This leads to the second general principle:

(2) Make the mathematics important mathematics.

By *important* I do not mean merely that the mathematics should be recognizable by mathematicians as central to the mathematical enterprise. To motivate liberal arts students, the importance should extend to the world of thought and society at large. Later on, I will discuss examples of accessible topics that have both these types of importance.

Still, the emphasis on accessibility is instructive, since it highlights the fact that liberal arts students do know some mathematics. It is crucial to get them to recognize this, to use what they already know, and help them build on their current competence. But many liberal arts students lack the confidence to do these things – which leads me to the next principle:

> (3) Learning mathematics should empower the students, not overwhelm them.

My slogan in my liberal arts courses is, “We’re not going anywhere.” A general-education mathematics course is not a prerequisite to anything. The instructor does not need to cover every conceivable topic. I tell my students that my goal is to go slowly enough so that 90% of the students will get 90% of the mathematics, and that I will take an extra day or two if the class needs the time to master a particular set of ideas even if this requires omitting another topic. This frees the students from the lock-step pace caused by the prerequisite structure of traditional courses. A teacher is not at liberty to leave integration out of an introductory calculus course just because part of the class has not mastered derivatives by the predetermined date. But nobody is going to say to my students, “What? You took a course called ‘Mathematics in Many Cultures’ and did not learn how to use Pythagorean triples to construct ancient Hindu altars?”

The need to give the students the experience of mastery is related to my next principle:

> (4) All students have expertise.

It may not be in mathematics, but they have expertise in their major and in many outside interests. Let them build a project for their mathematics course that incorporates that expertise, linking mathematics and their own interests, so that they can teach and impress everybody else. This will be a new experience for many of them in a mathematics course.

This suggestion leads to the last of my stated principles:

> (5) Have the students share their course projects with each other in class through giving reports.

If the students start with what they are personally interested in, a class of 20-30 students will produce many more applications of mathematics than could any individual instructor.

There are many ways one could teach a course based on these principles. Let me now describe how I have done this in the two liberal arts courses that I currently teach: "Mathematics, Philosophy, and the 'Real World'" and "Mathematics in Many Cultures."[1]

2. Mathematics, Philosophy, and the "Real World"

"Mathematics, Philosophy, and the 'Real World'" focuses on the relationship between significant mathematics and philosophy. There are two important types of mathematics – defining "important" as having influenced the thought and lives of citizens of all industrialized countries – that are accessible to liberal arts students: Euclidean geometry, and elementary probability and statistics.

First, let me make the case for geometry. Throughout the history of Western civilization, geometry has influenced philosophers, both in seeming to have achieved truth, and in having found a method of proving truths. In order to understand this influence, students need to understand the logical structure of geometry in depth, and to study examples of the philosophy influenced by geometry's logical structure and by the truth claims made for it.

In my course, students read the first book of Euclid's *Elements*, concentrating on learning the nature of proof, and especially emphasizing the logical structure of the theory of parallels contained in theorems 27 - 32. I also include some readings and exercises in logic, since I have found over the years that my students rarely have been taught logic. Euclid's indirect proofs make no sense unless the students thoroughly understand the distinction between the valid inference "If **p**, then **q**; not **q**, therefore not **p**" and its evil twin "If **p**, then **q**; not **p**, therefore not **q**."

In this part of the course, the students also read works by philosophers who have been influenced by the ideal of geometry, including Plato, Aristotle, René Descartes, Baruch Spinoza, Immanuel Kant, and Thomas Jefferson, whose Declaration of Independence starts with "We hold these truths to be self-evident" and concludes "We, *therefore*, ... declare, that these United Colonies are, and of Right ought to be Free and Independent States" [italics added]. This part of the course ends with an introduction to non-Euclidean

[1] A bibliography of background reading for these two courses will be found in Section 5 of this paper.

geometry, and some philosophical reflections on this geometry and on how it is related to the general theory of relativity.

The readings for the geometry part of the course include part or all of these works:

- Euclid, *Elements*, Book I;
- Plato, *Meno*;
- Handouts on logic, some by the instructor, others selected from an introductory text like Wesley Salmon, *Logic*, Prentice-Hall (1984);
- A collection of readings, compiled and sold by our campus bookstore which has arranged for the required copyright permissions, that includes selections from Plato's *Republic*, Aristotle's *Posterior Analytics*, Voltaire's *Philosophical Dictionary*, Descartes's *Discourse on Method*, *The Declaration of Independence*, Kant's *Prolegomena* and *Critique of Pure Reason*, Spinoza's *Ethics Demonstrated in Geometrical Order*, and, for non-Euclidean geometry, George Gamow's *One, Two, Three... Infinity* and Hermann von Helmholtz's essay "On the Origin and Significance of Geometrical Axioms."

Now let us turn to the other part of the course, elementary probability and statistics. Statistics, and reasoning about them, pervade modern society. As an introduction to the issues in this part of course, the students read Blaise Pascal's "Wager" from his *Pensées*, the first cost-benefit argument in history, and Stephen Jay Gould's essay "The Median Is Not the Message" (Gould 1992). I teach enough elementary combinatorics and probability for the students to understand the combinatorics underlying the bell curve and to be able to calculate various probabilities. We look at the graphical presentation of data, examining both good and bad examples. We then turn to the applications of probability to social questions. I teach the course in the fall semester so that there will be significant elections and interesting polls for the class to analyze. The course also considers some of the philosophical implications of statistical and probabilistic reasoning, including the ethics of cost-benefit analysis, and the question of free will versus determinism.

The readings for the probability and statistics part of the course include two books: Darrell Huff's *How to Lie with Statistics* (Huff 1993), and Warren Weaver's *Lady Luck: The Theory of Probability* (Weaver 1982).

There is also more material from the collection of readings mentioned above, including selections from Pierre-Simon Laplace's *Analytical Essay on Probabilities* and James Clerk Maxwell's *Kinetic Theory of Gases*. The students also read articles about public opinion polls, current elections, and contemporary public-policy issues, as these appear in the press or online during the semester.

What has been said so far explains the "important mathematics" part of the course.[2] Now, let us turn to helping the students apply mathematics to what is important to them. Each week, every student does what I call a real-world problem, using the mathematics that they already know to solve a problem whose answer somebody – hopefully, they themselves – might care about. The idea here is that even elementary mathematics can tell one useful things about the real world. Uri Treisman has said that pushing students through algebra and calculus too fast leads to what he calls "the low-level use of high-level skills." In solving real-world problems, I encourage the students to practice the reverse: the high-level use of low-level skills.

I hand out sample problems to give them the idea. Here is one of these examples:

> The situation: You are a school principal. Your school has a contract with a bus company that can supply only one sort of bus. Each bus holds 100 children. You need to transport 250 children on a field trip. How many buses do you order?
>
> Mathematical description and solution: 250 children / 100 children per bus = 2.5 buses.
>
> Real-world solution: You order three buses. (The answer 2.5 is incorrect. This is a famous problem, which caused controversy when it was on a California statewide mathematics test that was multiple-choice. Students chose 2.5 and were distressed to be marked "wrong.")

The mathematics itself is too simple for a college-level course, but this example illustrates very well the difference between a formulaic mathematical solution and a real-world one. Claudia Zaslavsky, in her wonderful book on teaching mathematics to non-traditional students, *Fear of Math: How to*

[2] For a DVD course based on what has been described so far, see (Grabiner 2009).

Get Over It and Get On with Your Life, tells of an inner-city school whose students answered, "Order two buses." The children could sit three to a seat instead of two to a seat and the school can use the money saved for needed supplies (Zaslavsky 1994, p. 143). This is another (unfortunately) real-world solution to the same problem.

Another, more pedestrian example, asks them to imagine laying out a baseball diamond. The problem asks whether the pitcher's rubber in baseball, 60 feet 6 inches from home plate, is at the intersection of the diagonals of the baseball diamond (a square whose side is 90 feet), making it easy to locate, or whether the distance has to be specially measured. Yet another sample problem, using figures from a college publication, asks students to choose which of the available meal plans is the cheapest over the course of a semester.

These weekly "real-world problem" assignments empower the students to use their own mathematical skills, even if these are elementary, and to encourage them to see the mathematics in their own world. (To illustrate the real-world value of this, I tell my classes about a former student whose parents were about to buy a car, seduced by the promise of a low monthly payment for 36 months. The student chose for a real-world problem to multiply the monthly payment by 36. The parents then realized that the car was not worth this much. There was nothing arcane about the mathematics involved here; instead, the lesson is "I know how, I don't need to rely on others or on some web page; just do the math!") As the semester proceeds, I specify that the topics for the week's real-world problems must come from what we are studying: this week geometry, next week analyze the logic in an advertisement or op-ed article, next week analyze a public opinion poll. Since I grade these assignments myself, I can assist individual students with their mathematical understanding, at whatever level the individual student may be.

At the end of the course, each student does a final project. The students report on their projects to the whole class, including a required visual aid, which can be a poster, a set of transparencies, handouts, or a PowerPoint presentation. (To make this work, the instructor needs to bring a stopwatch and enforce a strict time limit.) The students also are required to submit a written version of their report, on which I comment extensively.

The key fact about the project is that it must stem from the individual student's interest. I encourage them to challenge me: "Tell me what you're interested in, and I'll help you find the mathematics." Over the years, individual students have said to me, "My chief interest is high-performance cars (or photography, or helping the homeless), and there's no math in that." It is a revelation to them that mathematics, be it in designing braking systems, the geometrical optics of camera lenses, or the statistical study of homelessness, is indeed relevant to what they care about. Because of the diversity of their interests, the students collectively come up with a much wider range of applications than I could ever do on my own.

Here is a list of what my students did in a recent semester, arranged in the order that they volunteered to give their reports. (Every year I make such a list, prepared for the students' reference for the final examination. And every year I marvel at what is on it.)

- Chuck: Set theory and Russell's paradox (His example of a set which is a member of itself was the set of all soft pillows, which is itself a very soft pillow.)
- Alia: Misuse of mathematics in movies.
- Link: Photography.
- Georgia: High-altitude cooking (She explained the formulas that tell the cook things like how long to boil an egg on top of Mt. McKinley.)
- Lindsey: Probability, applied to the game of "Twister."
- Whitney: Don DeLillo's novel *Ratner's Star* (This was a gifted English major who wanted to report on a work of literature that used mathematics in a deep and essential way. There is a great deal of history and philosophy of mathematics in DeLillo's novel.)
- William: Computer animation.
- Chris: Car leasing from the bank's point of view (This student worked for a bank. He explained what banks do to minimize their losses.)
- Marc: Baseball arbitration: how to evaluate player quality (This student, a talented ballplayer, hopes someday have his own talent evaluated by a major-league team according to the formulas he reported on.)

- Jack: Fractals: “irregularity that repeats itself” was a well-researched report that went beyond gee-whiz graphics to explain what was going on.)
- Brittany: Economics of moviemaking.
- Derek: Waves and surfing: speed and height of waves, and predictions about them.
- Kristin: Oxy-acetylene welding (This student competently fielded a question about whether this kind of welding torch is how a well-liked local restaurant flames its crème brulée – her answer was “No, they use a butane torch.”)
- Betsy: How musical instruments are tuned.
- Kate: The “Living Wage” and America’s working poor.
- Will: Visualizations: From sound to visual images (computer demonstration).
- Chelsea: Mathematics and music: Rhythm, measures, and beats.

For their oral reports, the students come into class as experts in, say, philosophy, or welding, even though they may not be experts in mathematics. During the brief question period, each student has the experience of being the best-informed person in the room about his or her own topic. Class attendance is nearly perfect. Partly this is because after the 25 years I have been teaching this course, other students will have passed the word: “The reports will be cool.” (I often have visitors, including the faculty secretary.) Also, the students are required to fill out daily response sheets for homework credit, on which they are asked to summarize the report that most interested them, to make a generalization about the day’s reports, and to ask a question about some report on which they would like more information; I comment on these response sheets at length. Of course, it is also relevant that there will be questions about the student reports on the final examination.

A word on measuring the outcomes: My final examinations are about one-third philosophy, the rest mathematics. The mathematics part requires them to prove theorems in geometry, some directly, others indirectly, with examples drawn both from Euclidean and non-Euclidean geometries. It requires them also to distinguish between valid and invalid logical arguments, to calculate probabilities, to critique a survey, and to evaluate probabilistic arguments about topics in medicine and public policy. The last question on the final examination asks them to identify and explain what they consider the most interesting thing they learned in the course. The most common answers, supported with examples, are some version of "Math is actually about something, it's useful, it's interesting, it's cool." The fact that different students give me such a diversity of examples in answering this question shows that they have learned that mathematics speaks to them personally. (Some students even send me real-world problems long after the course is over.)

3. Mathematics in Many Cultures

Three events that coincided led me to design this course. First, I attended a meeting of the Canadian Society for the History and Philosophy of Mathematics at which a number of the talks reported on recent scholarship about mathematics in China, India, and the medieval Islamic world. Second, in the same year, Marcia Ascher, author of the acclaimed book *Ethnomathematics* (Ascher 1991), came to Claremont, at Alvin White's invitation, to speak at Harvey Mudd College about her work on the mathematics done by non-literate peoples. These two events convinced me that there were, for the first time, excellent and easily accessible materials with which to teach about mathematics outside the western tradition. Third, my institution, Pitzer College, was in the process of adding a multicultural dimension to its curriculum, and there was great student interest in this new approach. Accordingly, I decided to try out a new liberal arts course called "Mathematics in Many Cultures."

Students who are alienated from mathematics sometimes say that mathematics, besides not being useful or relevant to their lives, is inhuman and Eurocentric. This course disarms those who hold these views, because mathematics is found in virtually all cultures. Other cultures may not care about microeconomics or software design, but they still find value in mathematical processes and ideas. They use mathematics to solve problems that are important to their cultures, just as modern western societies do. This course lets the students see how.

Since a key goal of a liberal arts course is to use the students' interests to teach them important mathematics, the topics in "Mathematics in Many

Cultures" are traditional ones, though appearing in unusual settings. For instance, many students are unaware that there are other ways of representing numbers than base-10 place-value. To be sure, they have seen Roman numerals and used base-60 Babylonian fractions in converting hours or degrees to minutes and seconds; they just have never reflected on this. So number systems are a good multicultural leadoff topic. Then I follow Ascher's *Ethnomathematics* to address graph theory. Several cultures in Africa and in the South Pacific have knowledge of what we now call Eulerian paths and of classifications of symmetry, using this knowledge for storytelling, for representing myths, and for art. Then, we turn to elementary group theory. André Weil showed, in a chapter he contributed to Lévi-Strauss's famous book on kinship systems,[3] that the structure of the kinship systems of several cultures of the South Pacific are representable by groups. Some of these systems are described in Chapter 3 of Ascher's *Ethnomathematics*. We look at other examples of groups, and the students learn to prove a few theorems about groups as well as to appreciate the power and generality of abstract mathematical ideas.

The mathematical heart of the course, though, is the study of elementary combinatorics and Pascal's triangle (hundreds of years before Pascal). We encounter the triangle and the binomial coefficients in early classical Indian calculations of the number of different combinations of long and short vowels in an n-syllable line of Sanskrit poetry, in the medieval Rabbi ben Ezra's calculation of the number of possible conjunctions of the seven then-known planets, in working out the probability of outcomes in a Native American game called "Dish," and in techniques used to approximate solutions of polynomial equations in medieval China.

We also look at right-triangle theory in a famous Chinese mathematical work, learning the Chinese cut-and-paste proof of the Pythagorean Theorem.[4] I take this occasion to compare Chinese methods of geometric proof, which are visual, with the more abstract, and *indirect*, Greek proof that the square root of 2 is irrational. We then study the Islamic geometric approach to solving quadratic equations by completing actual squares, a visual approach to something the students have seen derived, if at all, only algebraically. Many students say that they now understand completing the

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[3] (Lévi-Strauss 1969)

[4] See, e.g., `http://www-users.math.umd.edu/~gfleming/JIM/PtPww/PtPwwFrame.html`.

square, and the resulting quadratic formula, for the first time; previously, nobody had ever drawn them a picture. We also look at plane trigonometry in Indian sources, since the sine function was central to trigonometry in classical India (and first received its infinite-series representation in the late fourteenth century in southern India), and in Islamic sources, where, in the ninth century, all six trigonometric functions first appeared. Finally, we consider spherical trigonometry, which, in Islamic culture, was strongly encouraged by the Quranic injunction to pray in the direction of Mecca from any place on the globe. And, although it was not my conscious intention, I am not displeased when students tell me that the combinatorics, algebra, and trigonometry in this course have helped them on the Graduate Record examination.

For this course too, I give reading assignments from a collection produced by our campus bookstore, which secures the appropriate copyright permission. Here are the sources from which the reading selections are chosen:

- Marcia Ascher, *Ethnomathematics: A Multicultural View of Mathematical Ideas*;
- Frank Swetz, *Was Pythagoras Chinese? Right-Triangle Theory in Ancient China*;
- Aasger Aaboe, *Episodes from the Early History of Mathematics*;
- J. L. Berggren, *Episodes in the Mathematics of Medieval Islam*;
- R. Gillings, *Mathematics in the Time of the Pharaohs*;
- N. L. Rabinovitz, *Probability and Statistical Inference in Ancient and Medieval Jewish Literature*;
- W. Feldman, *Rabbinic Mathematics*;
- D. E. Smith and Yoshio Mikami, *A History of Japanese Mathematics*;
- T. Crump, *The Japanese Numbers Game*;
- G. Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics*;
- Claudia Zaslavsky, *Africa Counts: Number and Pattern in African Culture.*

One of the virtues often claimed for multicultural education is that it helps students look at things in new and fruitful ways that their own culture does not provide. This is certainly true in mathematics: the emphasis on the visual in Islamic algebra, the visualization of lines of verse in studying combinatoric questions in classical India, and the use of graph theory in many cultures in constructing designs, lead my students, as they repeatedly tell me, to understand topics that they have seen but never understood.

At the end of this course each student does a report on a topic in mathematics and its cultural setting. Some students choose a subject from their own cultural background, some from a culture they are studying or from a part of the world they have visited, and some just try something entirely new. As in my other course, I find that the students select topics I would not have imagined and sometimes know nothing about. As in my other course, a visual aid is mandatory, a time limit enforced, response sheets for credit are required, a written version of the report is handed in and graded, and reports are asked about on the final examination.

Here are the report topics from a recent semester:

- Quinn: Ancient Greek astronomy
- Chris: The French Republican Calendar
- Zachary: Maori strip patterns
- Stephanie: Religious associations of the pentagon
- Natari: The Japanese lottery
- Hannah: Fibonacci series, the golden ratio, and geometry in art
- Junho: Yut-nol-i, a traditional Korean game
- Jordan: Women in Mathematics
- Nelson: Rubik's cube: from Hungary to the U. S.
- Alia: The Navajo "Code Talkers": Are there unbreakable codes?
- Kate: Eratosthenes and the circumference of the earth
- Kathryn: The art of M. C. Escher

- Rebecca: The Aztec tax system and its measurements
- Nikki: The Ishango bone: the oldest mathematical artifact
- Alexandra: Mathematics in Mayan culture
- Emily: Gematria in Jewish culture
- Alicia: Inca Strip patterns
- Melissa: The Cambodian calendar
- Steven: An example of Native American Mathematics: the Texcoco
- Raven: The ancient Egyptian game of Senat
- Zach: Spread betting: who does it and why
- Mattie: Navajo ideas about space
- James: Blackjack: theory and practice in Las Vegas
- Denyse: The number system used in Yoruba culture

The students, as in my other course, are building on their own strengths and interests. So, they come in confidently to give their reports. The overall effect of listening to all the reports is educational for me as well as for the class.

4. Conclusion

The general principles I use to design these courses are accessible to every teacher of mathematics for non-mathematicians. Draw on the interests of the individual student; teach important mathematics; emphasize understanding, going as slowly as you need to go rather than trying to cover one more topic, so the students have a sense of mastery; encourage them to use the mathematics they already know and build on it; let the students create a project of their own choosing and share it with everybody else.

The specific courses I teach stem from my being a historian of mathematics with a background in philosophy. My colleagues at Pitzer College have used their own expertise to design other liberal arts mathematics courses. For instance, Professor Jim Hoste has taught courses on mathematics and art, on architecture, on fractals and chaos, on elementary topology, and on the mathematics of gambling. Professor David Bachman has taught courses on cartography, on game theory, and on the mathematics and strategy of poker. I hope that this article's general principles, and the examples of individual courses, can suggest ways that readers can draw on their own educational background and non-mathematical interests to find ways to link mathematics to the world of their students, and then send these students out into the larger world able and willing to do some mathematics, and able and eager to explain the major role mathematics plays everywhere.

5. Further Reading about Topics in "Mathematics, Philosophy, and the 'Real World"' and "Mathematics in Many Cultures"

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