1 Introduction

One topic in circuit complexity theory is the minimum circuit (made up of some set of gates, such as AND, NOT, and OR gates) which computes a particular boolean function (i.e., a function from \(\{0,1\}^n\) to \(\{0,1\}\).

Livnat and Pippenger([1]) discusses some applications of circuit complexity theory to theoretical biology. Specifically, they consider theoretical model organisms with some computational limitation, which they model as some limit on the number of gates in a boolean circuit describing the organism’s binary choices. They are interested in systematic mistakes - mistakes due to limitations in the circuit. They show that, for most functions, the best circuits (i.e., the smallest ones which are correct on a certain fixed fraction \(\epsilon\) of input bitstrings) of a certain size depend on all their inputs, and conclude from this that most such circuits make systematic mistakes.

Sholomov ([3]) considers incompletely-defined boolean functions and bounds on their circuit sizes. A boolean function is said to be incompletely defined if it is only defined on a subset of \(\{0,1\}^n\), i.e., there are rows of its truth table which are marked by a “don’t care” symbol. The same paper also shows that, as long as the number of specified rows \(N_n\) of the truth table is at least

\[ n \log_{2^{1+\delta}} n, \]

for some positive \(\delta\), then as \(n\) grows the minimum number \(L\) of gates in a circuit for a function, for most circuits, goes to

\[ \rho \frac{N_n}{\log_2 N_n}, \]

where \(\rho\) is a constant depending on the basis.

Sholomov ([3]) does not discuss the effects of allowable errors (the fraction \(\epsilon\) in the paper of Livnat and Pippenger([1])) on the complexity of circuit sizes. This is considered by Pippenger([2]), where the size of a circuit

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realizing a function having some fixed fraction of its inputs specified and al-
lowed to make some fixed fraction of errors is asymptotically given. Unlike
Sholomov([3]), Pippenger([2]) considers only circuits with a fixed fraction
(i.e., growing exponentially) of specified rows in the truth table.

2 Proposed Research

I will try to find bounds on minimum circuits to realize a function which
is specified on something smaller than a fixed fraction of its possible input
strings, for example, one which is specified on polynomially many of them,
and which are allowed a fixed fraction of errors. This is a sort of combi-
nation of the papers of Pippenger([2]) and Sholomov([3]), the first of which
considers fixed fractions of allowed errors and the second which considers
almost any polynomial number of input strings.

References

[1] A. Livnat and N. Pippenger. Systematic mistakes are likely in bounded
optimal decision-making systems. *Journal of Theoretical Biology*,

[2] N. Pippenger. Information theory and the complexity of Boolean func-

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