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Fuzzy Blackholes

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Fuzzy Blackholes

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Chapter 1

Introduction

The fuzzball model of a black hole is an attempt to resolve the many paradoxes and puzzles of black hole physics that have revealed themselves over the last century. These badly behaved solutions of general relativity have given physicists one of the few laboratories to test candidate quantum theories of gravity. Though little is known about exactly what lies beyond the event horizon, and what the ultimate fate of matter that falls in to a black hole is, we know a few intriguing and elegant semi-classical results that have kept physicists occupied. Among these are the known black hole entropy and the Hawking radiation process.

In chapter 2, we provide background on an assortment of topics that will be used to describe the fuzzball model. Since this model is based on string theory, these topics include some of the basic ideas of string theory and its various subfields like non-commutative field theory. In chapter 3, we describe some results regarding the actual formation process of a fuzzy black hole. We argue for a phase transition of infalling matter when the horizon is approached. Some characterstics of the new phase are described and the stability of such a system is argued for. In chapter 4, a brief description of the interaction of two fuzzy membranes is given.

Chapter 2

Background

2.1 String Theory And Her Mothers, Sisters and Daughters

String theory is an attempt at a unified quantum theory of all known interactions including gravity. The basic assumptions of the theory are that the fundamental degrees of freedom are quantum strings. These strings can be closed or open, i.e. they can be loops or have free endpoints. Different modes of the string are identified with different elementary particles. For example, the graviton is a mode of the closed string while electrons and quarks are modes of open strings.

Starting with these ideas and requiring mathematical consistency has taken string theorists a long way in understanding the structure of the theory. To begin with, string theory requires that the number of space-time dimensions is 10. To connect with our observed 4 dimensional universe, it is assumed that 6 of the spatial dimensions will have to be made compact and small and hence presently undetectable. Secondly, since there are an infinite number of modes for a string, there are infinite towers of as-yet-undetected elementary particles. Presumably, only the lightest ones are accessible to

our accelerators. Besides these predictions, string theory also requires the presence of a new symmetry that relates fermions and bosons. This is called supersymmetry and is absent in our observed universe, calling for explanations of how this symmetry is broken.

Initially, there were five different consistent superstring theories that had been constructed by physicists based on different choices of symmetries and characteristics of the strings. These were labelled Type I, IIA, IIB, Heterotic $SO(32)$ and Heterotic $E8 \times E8$. In the mid-nineties, it was realized that these five theories were actually related to each other by dualities that provided a dictionary to translate statements and objects in each theory to the others. Moreover, all the five theories were now seen as different limits of one unique 11-dimensional theory, called M-theory.

Also, the significance of higher dimensional objects, called branes, to the theory was realized. D-branes were initially introduced in string theory as objects that open string endpoints are constrained to. The world volume of these branes then provided the ideal stage to generate realistic gauge theories. The behavior of open strings constrained to N coincident D3-branes, for instance, produces a $U(N)$ Yang-Mills theory on the 4-dimensional world volume of the branes at low energies.

The low energy limit of the different string theories produces supergravity where only the graviton and a few other particles survive. In this limit, D-branes become black holes. Thus, a D0-brane, which is a pointlike object, becomes a point black hole, while a D2-brane becomes a black sheet and so on. Using D-branes, the entropy of certain black holes has been calculated and found to agree precisely with the Bekenstein-Hawking prescription (see [1]). This is one of the most significant results of string theory.

Though none of the predictions of string theory have any direct experimental support today, interest in string theory has been sustained over the years due to periodic advances in the mathematical understanding of the theory that have produced un-

expected and elegant ideas that stand independent of the veracity of string theory itself. One of these is the holographic duality (see [12]) which builds a surprising bridge between quantum field theory and gravity, saying that they are in fact different descriptions of the same physics. This duality will be a principal tool in our analysis. For a more detailed introduction to string theory, see [2], [3].

2.2 Black Holes And Their Properties

Black holes are solutions to general relativity that have a curvature singularity. The most well-known black hole is the 4-dimensional Schwarzschild solution. This solution is spherically symmetric and has a singularity at the center, which is shielded by an event horizon from the external space-time. In D dimensions, the metric is

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2 \tag{2.1}$$

$$f = 1 - \left(\frac{r_0}{r}\right)^{D-3}. \tag{2.2}$$

The mass M of the black hole is related to the horizon radius r_0 by

$$r_0^{D-3} = 2M \tag{2.3}$$

where the D dimensional Newton constant and the speed of light have been set to one. The horizon is a closed null surface, which classically acts as a one way gate to the singularity. Anything that passes the horizon is bound to hit the singularity in finite proper time while nothing, not even light, can reach the external space-time from within the horizon.

Physicists assume that the singularity in the solution is a sign that general relativity breaks down at such high curvatures, presumably the Planck scale. A quantum theory of gravity would be needed to describe physics beyond such scales. The horizon, however, is not a region of strong gravity and we would expect that general relativity can be applied here without fear.

Some classical results about black holes suggest that the area of the horizon of a black hole never decreases during processes like the merger of two black holes. It was also noticed that the relationship between the energy and area of horizon had a very similar form to the thermodynamic relationship between energy and entropy (see [4]). For example, the horizon radius of a charged and spinning black hole with charge Q and angular momentum J is given by

$$r_0 = M + \sqrt{M^2 - (Q^2 + \frac{J^2}{M^2})}. \tag{2.4}$$

Putting this in differential form,

$$dM = \frac{\kappa}{8\pi} dA + \Omega dJ + \Phi dQ \tag{2.5}$$

where

$$\kappa = \frac{1}{4M} \tag{2.6}$$

is the surface gravity at the horizon, Ω is the angular velocity, and Φ is the electrostatic potential at the horizon. Note the similarity to the familiar thermodynamic relation

$$dU = \tau dS - pdV + \mu dN. \tag{2.7}$$

These admittedly weak hints prompted Bekenstein to identify the area of the horizon (or a multiple of it) with the entropy of the black hole and the surface gravity with the temperature.

This is an intriguing proposal since classically the black hole has only one state. It is completely characterized by its mass, angular momentum and charge. Hence the entropy we would associate with it is zero. However, Hawking (see [5]) showed that the behavior of quantum fields in the vicinity of the horizon results in radiation that is precisely thermal from the black hole. He calculated the temperature and entropy

to be

$$T = \frac{\kappa}{2\pi} \tag{2.8}$$

$$S = \frac{A}{4} \tag{2.9}$$

in units where the gravitational constant is one. The black hole thus radiates away its energy and eventually disappears.

These results are strikingly elegant and hence demand an explanation. The source of this entropy, i.e. the explicit microstates, need to be identified. Additionally, the Hawking radiation process also poses a few puzzles. If we assume that general relativity is valid near the horizon due to the low gravity there, Hawking's derivation produces perfectly thermal radiation that cannot carry away information about the matter that formed the black hole, leading to a loss of information. The conservation of information however is a fundamental principle of physics. This leads us to believe that the horizon may not be as innocent as it seems. The fuzzball model to be discussed later builds on this idea.

2.3 Holography

Gravity is a geometric theory, dealing with concepts like the metric and curvature. Quantum Field Theory (QFT), on the other hand, describes the behavior of matter fields on flat space-times, successfully accounting for elementary particles like the electron, quarks and the gauge bosons. Attempts to make a quantum field theory out of gravity have failed due to the non-renormalizability of the theory. However, recently, a surprising connection between gravity and quantum field theories has been uncovered by string theorists. This is the AdS/CFT correspondence.

Imagine a D-brane with open strings living on it. In the surrounding bulk space, closed strings generate gravity among other fields. These two sectors interact by

processes like the emission of strings from the brane into the bulk and vice versa. We wish to focus on a low energy regime where the two sectors are decoupled. That is, the closed strings and the open strings no longer interact and we are left with two independent theories. The limit is a delicate one which is done by sending the string scale l_s to zero while enforcing the correct scaling of various physical quantities. When the decoupling limit is taken, we are left with a QFT on the brane and supergravity in the bulk.

We now step down the energy ladder. The string theory in the bulk becomes supergravity in the bulk while the brane becomes a black hole. In this scenario, we can again take a corresponding decoupling limit that separates the bulk from the black hole. As a result, we are left with supergravity in the bulk and supergravity in the near horizon region. To be more specific, in the best understood example of holography, the near horizon geometry of the black hole is an Anti-deSitter (AdS) space which is a space of uniform negative curvature. Thus, we have two independent, non-interacting gravitational theories.

Looking at the theories that we are left with upon taking the decoupling limit at the two energy scales, we see that supergravity in AdS space is a low energy approximation of QFT on the brane. Remembering that supergravity is a low energy approximation of string theory, Maldacena and others (see [12], [13]) conjectured that the QFT, which in this case is a Conformal Field Theory (CFT), is exactly equivalent to the full blown string theory in AdS space.

This conjecture has been extensively tested in certain cases and the circumstantial evidence for its validity is great. The reason it is called the holographic duality is that the brane world volume can be considered to be the boundary of the bulk geometry. Thus string theory (or supergravity) in the bulk space is equivalent to a QFT on the boundary of this space. This comprehensive encoding of information on a lower dimensional space from the bulk is reminiscent of a hologram.

Part of the magic of the holographic duality lies in the way it maps strongly coupled QFTs to weakly coupled string theory and vice versa. The coupling strength of a theory decides the strength of interaction of the objects in the theory. Since calculations at strong coupling are virtually impossible in both theories, the duality allows us to perform the corresponding calculations in the weakly coupled dual and translate the results back to our theory of interest. Thus besides being an elegant and remarkable mathematical fact, it is a powerful computational tool.

As stated earlier, the entropy of a black hole scales with the area of its horizon. This somewhat odd dependence seems less perplexing in the light of the holographic principle. The information contained within the horizon can presumably be coded on the horizon.

2.4 NCSYMs

We will be looking at a particular class of Quantum Field Theories in the next chapter called Non-Commutative Super Yang-Mills (NCSYM) theories. A Yang-Mills theory or a gauge theory is a field theory whose action is dictated by invariance under a particular group of symmetries. The gauge field itself transforms as an affine representation, i.e. a non-linear representation, of the gauge group.

The simplest Yang-Mills theory is electromagnetism. The gauge group in this case is $U(1)$ and the familiar gauge transformations leave the action invariant. Explicitly, the action is

$$S = -\frac{1}{4}\int F^{\mu\nu}F_{\mu\nu} \tag{2.10}$$

where

$$F = dA \tag{2.11}$$

$$\Rightarrow F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} \tag{2.12}$$

is the exterior derivative of the real valued gauge field A. A gauge transformation of A can be written as

$$A \to A + d\lambda \tag{2.13}$$

where λ is a scalar. Remembering that $d^2 = 0$, we see that F is invariant since

$$F \to dA + d^2\lambda = dA. \tag{2.14}$$

Since $U(1)$ is abelian, this is a particularly simple theory. For more complex groups, where the group elements don't commute, the gauge field takes its values in the Lie algebra of the gauge group and is represented by matrices. The general Yang-Mills action is given by

$$S = -\frac{1}{4}\int TrF^{\mu\nu}F_{\mu\nu} \tag{2.15}$$

where the trace is taken over the gauge index.

For conceptual purposes, it is best to simply think of these theories as more complicated versions of electromagnetism. The strong and the weak interactions are both described by gauge theories. The gauge groups in these cases are SU(3) and SU(2) respectively.

In string theory, Yang-Mills theories occur as the low energy description of open strings that live on coincident D-branes. Since the Standard Model is entirely based on Yang-Mills theories, attempts to reproduce it from string theory have centered on such systems.

A non-commutative field theory (see [11]) is one where the space-time coordinates do not commute. That is, the coordinates satisfy an algebra of the form

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} \tag{2.16}$$

where $\theta^{\mu\nu}$ are numbers that decide the scale of non-commutativity.

The reason for interest in such structures is that they occur in string theory under certain conditions. When a D-brane has a magnetic field on it, generated by closed strings, the coordinates of the end points of the open strings that live on the brane no longer commute and satisfy the above algebra. The resulting field theory that describes the low energy behavior of these open strings is then a NCSYM.

This non-commutativity can be understood qualitatively using the following analogy. Consider the ordinary non-relativistic quantum mechanics of a charged point particle in a uniform magnetic field (in the $\hat{z}$ direction, say). The vector potential in this case is

$$\vec{A} \sim -\vec{r} \times \vec{B} \tag{2.17}$$

$$\sim (-yB, xB, 0) \tag{2.18}$$

As is well known, the commutation relations for a point particle in the presence of an electromagnetic field are given by

$$[x^\mu, p^\mu - eA^\mu] = i\hbar\delta^{\mu\nu} \tag{2.19}$$

where $p_\mu = -i\hbar\frac{\partial}{\partial x^\mu}$ is the momentum operator. When the magnetic field is very strong, the p term can be dropped in comparison to the eA term. Substituting the known vector potential, we get

$$[x, eyB] = i\hbar \tag{2.20}$$

$$\Rightarrow [x, y] = \frac{i\hbar}{eB}. \tag{2.21}$$

Thus the x and y coordinates of a charged particle in a uniform magnetic field do not commute. Something very similar happens to open strings on a D-brane with a B field.

Chapter 3

Fuzzy Black Holes

The fuzzball model posits that the horizon is a region of non-traditional physics. Samir Mathur and others (see [7], [8], [9]) suggest that stringy degrees of freedom start displaying their personality in this region. To provide evidence for this, string-inspired families of solutions of general relativity have been found that match the Schwarzschild metric far away from the horizon, but end in a smooth cap within the horizon instead of featuring a singularity at the center. These solutions are sourced by string theory objects like strings and branes and their excitations. The entropy of such stringy black holes has also been calculated and match the Bekenstein-Hawking prescription (see [1]).

3.1 Formation of the Fuzzy Horizon

Consider a shell of N strongly interacting D0-branes in D space-time dimensions starting from rest with initial radius R_0 much greater than the horizon radius, r_0, associated with the (ADM) mass of the shell M. We wish to examine the evolution of this shell as it collapses due to gravity. We expect that the final result of such a

collapse is a finite temperature D0 black hole.

The properties of the matter in the shell are governed by a strongly coupled quantum field theory to be described below. To understand the phase structure of this matter, we use the holographic duality to move to weakly coupled gravity where a phase transition is easily deduced. This information is then translated back as a thermodynamic statement about the D0-branes. The details of the following section can be found in [16].

3.2 The NCSYM

The behavior of a D0-D2 brane (fuzzy membrane) is described by a 2+1 dimensional Non-Commutative Super Yang-Mills theory (NCSYM) at strong coupling, derived from type IIA string theory. The presence of D0-branes is indicated by a magnetic field on the D2-brane world volume. If N_2 is the number of D2-branes, the resulting Yang-Mills theory is a $U(N_2)$ theory. Let the world volume coordinates be x_0, x_1, x_2.

The coupling of the theory is given by (see [11])

$$g_{YM}^2 = \frac{G_s}{l_s} \tag{3.1}$$

with

$$G_s = \frac{g_s B_{x_1 x_2}}{g_{x_1 x_1}} \quad , \quad G_{x_1 x_1} = G_{x_2 x_2} = \frac{B_{x_1 x_2}^2}{g_{x_1 x_1}} \quad , \quad \theta^{x_1 x_2} = \frac{2\pi\alpha'}{B_{x_1 x_2}}. \tag{3.2}$$

Here l_s is the string scale, $\sqrt{G_s}$ is the open string coupling of the parent theory, g_s is the closed string coupling, $B_{x_1 x_2}$ is the magnetic field, g_{ab} is the closed string metric (in the bulk), G_{ab} is the world volume metric on the brane and $\theta^{x_1 x_2}$ is the non-commutativity scale. The brane is assumed to be in the shape of a sphere, so the coordinates x_1, x_2 are compact. Let the surface area of the brane be V_2.

The effective coupling of the theory is a function of the temperature, T, and is given by the dimensionless quantity

$$g_{eff}^2 = \frac{g_{YM}^2}{T}. \tag{3.3}$$

We are interested in the regime where the coupling is strong, i.e. $g_{eff}^2 \gg 1$. In this regime, calculations in the Yang-Mills theory are impossible using perturbative methods and hence we need to use the holographic duality and shift to the gravitational theory in the background.

3.3 The Gravitational Dual

The gravitational dual of the above NCSYM is the background geometry around the same D0-D2 system. A spherical D0-D2 brane, however, is not stable gravitationally and collapses as we would expect. Hence, we instead look at the background cast by D0-D2 branes wrapped around a torus which is a stable configuration. The thermodynamic conclusions drawn from this toroidal setup are then extended to the spherical case. We first do the analysis in 10 dimensions.

The metric around the D0-D2 brane is given by (see [15])

$$ds_{str}^2 = H^{-1/2}\left(-hdt^2 + D\left(dx_1^2 + dx_2^2\right)\right) + H^{1/2}\left(h^{-1}dr^2 + r^2 d\Omega_6^2\right) \ , \tag{3.4}$$

with

$$H = 1 + \frac{q^5}{r^5} \quad , \quad h = 1 - \frac{r_0^5}{r^5} \quad , \quad D = \frac{Q_0^2 + Q_2^2}{H^{-1}Q_0^2 + Q_2^2} \ . \tag{3.5}$$

Assuming N_0 and N_2 are the number of D0 and D2 branes respectively, the D0 and D2 charges are

$$Q_0 = N_0 T_0 = \frac{N_0}{g_s l_s} \quad , \quad Q_2 = N_2 T_2 V_2 = \frac{N_2 V_2}{g_s (2\pi)^2 l_s^3} \ ; \tag{3.6}$$

with q^5 defined through

$$Q = \sqrt{Q_0^2 + Q_2^2} = \frac{5V_2\Omega_6}{2\kappa^2} r_0^{5/2} q^{5/2} \sqrt{1 + \frac{q^5}{r_0^5}} \; ; \tag{3.7}$$

and

$$2\kappa^2 = (2\pi)^7 \, g_s^2 \alpha'^4 \quad , \quad \Omega_6 = \frac{16}{15}\pi^3 \; . \tag{3.8}$$

The directions parallel to the two dimensional torus are described by x_1 and x_2. The compact nature of these coordinates implies that

$$x_1 \simeq x_1 + \sqrt{V_2} \quad , \quad x_2 \simeq x_2 + \sqrt{V_2} \; . \tag{3.9}$$

Besides the metric, the closed strings generate a dilaton field, ϕ, and the NSNS B field given by

$$e^{2\phi} = g_s^2 H^{1/2} D \quad , \quad B_{x_1 x_2} = \frac{Q_0}{Q_2}\frac{D}{H} \; . \tag{3.10}$$

The D0 and D2 branes also generate RR fields

$$A_t = -\frac{r_0^{5/2}}{q^{5/2}} \left(1 + \frac{q^5}{r_0^5}\right)^{1/2} \frac{Q_0}{QH} \tag{3.11}$$

$$A_{t x_1 x_2} = -\frac{r_0^{5/2}}{q^{5/2}} \left(1 + \frac{q^5}{r_0^5}\right)^{1/2} \frac{D}{H}\frac{Q}{Q_2}. \tag{3.12}$$

The ADM mass of the metric is calculated to be

$$M = \frac{V_2\Omega_6}{2\kappa^2}\left(6r_0^5 + 5q^5\right) \tag{3.13}$$

and the Hawking-Bekenstein entropy, calculated from the area of the horizon, is

$$S = \frac{4\pi V_2 \Omega_6}{2\kappa^2} r_0^6 \sqrt{1 + \frac{q^5}{r_0^5}} \; . \tag{3.14}$$

The asymptotic values of $g_{x_1 x_2}$ and $B_{x_1 x_2}$ as $r \to \infty$ in the bulk geometry are the ones that appear in the NCSYM described earlier. Explicitly, their values are

$$g^{\infty}_{x1x1} \to 1 \quad , \quad B^{\infty}_{x_1 x_2} = -(2\pi)^2 \alpha' \frac{N_0}{N_2 V_2} \; . \tag{3.15}$$

From this we can calculate the other parameters of the NCSYM like the world volume metric G_{ab} and the non-commutativity parameter $\theta^{x_1x_2}$.

$$G_{x_1x_1} = G_{x_2x_2} = (2\pi)^4 \frac{N_0^2}{N_2^2}\frac{\alpha'^2}{V_2^2} \quad , \quad \theta^{x_1x_2} = \frac{1}{2\pi}\frac{N_2}{N_0}V_2 \quad , \quad G_s = (2\pi)^2 \frac{g_s}{V_2}\frac{N_0}{N_2}\alpha' \ . \tag{3.16}$$

Hence the effective coupling of the theory is

$$g_{eff}^2 \equiv \frac{g_Y^2}{T} = \frac{G_s}{l_s T} = \frac{N_0}{N_2}(2\pi)^2 \frac{g_s l_s}{V_2 T} \ . \tag{3.17}$$

3.4 The Decoupling Limit

To use the holographic duality, we have to take the decoupling limit where the closed and open string sectors decouple. This is done by sending the string scale $\alpha'(= l_s^2) \to 0$ while ensuring that certain quantities scale the right way. We need

$$g_s \sim l_s^3 \quad , \quad e^{\phi} \sim \text{finite} \quad , \quad ds_{str}^2 \sim \alpha' \quad , \quad B_{12} \sim 1/\alpha' \tag{3.18}$$

The scaling of the closed string coupling, g_s, ensures that the effective Yang-Mills coupling is finite in the limit. The metric is kept proportional to α' to decouple gravity from the NCSYM, and the B field scaling ensures non-commutativity of x_1 and x_2 in the NCSYM.

$$H^{1/2} D g_s^2 \sim 1 \quad , \quad H^{1/2} r^2 \sim \alpha' \quad , \quad H^{-1/2} D V_2 \sim \alpha' \tag{3.19}$$

These set of conditions have one solution of interest given by

$$g_s \sim l_s^3 \quad , \quad r \sim \alpha' \quad , \quad V_2 \sim \alpha'^2 \ . \tag{3.20}$$

We define the following rescaled parameters that are finite in the decoupling limit.

$$g \equiv \frac{g_s}{l_s^3} \quad , \quad v_2 \equiv \frac{V_2}{\alpha'^2} \quad , \quad u \equiv \frac{r}{\alpha'} \tag{3.21}$$

The quantities of interest describing the background are then expressed in terms of these new finite parameters.

$$q^5 = \frac{3}{2}(2\pi)^4 \frac{N_0 g}{v_2} \alpha'^3 \tag{3.22}$$

$$H = \frac{6Q_0}{5E} = \frac{6N_0}{5gE} \frac{1}{\alpha'^2} \tag{3.23}$$

The energy above extremality $E \equiv M - M_{BPS}$ is

$$E = \frac{4}{5(2\pi)^4} \frac{v_2}{g^2} u_0^5 \tag{3.24}$$

with $u_0 \equiv r_0/\alpha'$. Note also that in the decoupling limit M_{BPS} of the D0-D2 system is dominated by the BPS mass of the D0 branes

$$M_{BPS} = \sqrt{M_{D0}^2 + M_{D2}^2} \to M_{D0} = \frac{1}{\alpha'^2} \frac{N_0}{g} \to \infty \ . \tag{3.25}$$

The entropy is given by

$$S^2 = \frac{8}{75(2\pi)^2} \frac{v_2 N_0}{g^3} u_0^7 \tag{3.26}$$

Eliminating u_0 between equations 3.26 and 3.24,we get the equation of state for a D0-D2 system.

$$E_{D0D2} \simeq \frac{v_2^{2/7} g^{1/7}}{N_0^{5/7}} S^{10/7} \tag{3.27}$$

The equation of state of the D0 matter, derived from the D0-brane black hole metric, is given by

$$E_{D0} \simeq N^{-7/9} g^{1/3} S^{14/9}. \tag{3.28}$$

3.5 Phase Structure

We now argue that in the gravitational dual, a phase transition, known as the Gregory-Laflamme transition (see [14], [15]), occurs when the volume of the torus is decreased and becomes comparable to the horizon radius squared.

To understand the Gregory-Laflamme transition, let us look at it in the case of a simpler black hole. Consider a point black hole with horizon radius r_0 in a compact dimension of size R and a 'smeared' black string. When R is large, the entropy of the point black hole is proportional to r_0^{D-2} and hence as a function of its energy M, is

$$S_p \propto M^{\frac{D-2}{D-3}} \tag{3.29}$$

where $D = 10$ is the total number of space-time dimensions. For the black string, however, the horizon area is proportional to $r_0^{D-3}R$ which means that

$$S_{str} \propto M^{\frac{D-3}{D-4}} \tag{3.30}$$

Clearly the point black hole has greater entropy and is more stable in the large R regime. Conversely, when R is decreased and becomes comparable to the horizon radius r_0, we can imagine the horizon fusing with its images to form a smeared toroidal horizon, transitioning to the black string.

In our case, we start with a certain number of D0-branes which translate in the gravitational side to point black holes. As the volume of the torus decreases the horizons of the point black holes fuse to form a D0-D2 toroidal black hole. This is translated back to the matter side as a D0-D2 brane. Thus the gravitational Gregory-Laflamme transition indicates that the collapsing D0-brane shell undergoes a phase transition where the D0-branes condense into D0-D2 branes.

Since the transition occurs when the size of the torus becomes comparable to the horizon radius, we find the condition to be

$$g_{x_1x_1}v_2 < g_{\Omega_6}r_0^2. \tag{3.31}$$

Simplifying, we find that

$$v_2 \simeq g^{4/7} E_{D0D2}^{2/7}. \tag{3.32}$$

Since this is stated in terms of the energy of the resulting phase rather than the initial phase, we rewrite it in terms of entropy using the equation of state (3.27) to get

$$v_2 \simeq N_0^{-2/9} g^{2/3} S^{4/9}. \tag{3.33}$$

Note that this is precisely the scaling relation between the entropy and energy of a black hole in 10 dimensions. Using the equation of state of D0 matter, we can write the condition in terms of the energy of the D0 matter. We find

$$v_2 \simeq g^{4/7} E_{D0}^{2/7} \tag{3.34}$$

which is also the right scaling of horizon area with energy for a black hole. Thus the phase transition occurs at the same time as when the horizon radius is approached by the collapsing shell. And the entropy and the energy of the resulting phase (D0-D2 branes) scales with volume as it should for a black hole.

The above results can be easily generalized to d non-compact dimensions by smearing the metric along the compact ones. The results are still found to be valid (see [16]). The scaling laws for d-dimensional black holes are obeyed by the transition point and the resulting phase.

For a detailed derivation of the phase structure, see [16].

3.6 Stabilization

We have argued that when the shell of D0 branes approaches the horizon radius, it undergoes a phase transition and forms D0-D2 membranes, called fuzzy spheres. It is

reasonable to assume that these fuzzy spheres are evenly spread over the $D-d-2$-dimensional sphere which would be the classical horizon. The actual dynamics of this system is very complicated but we can make an argument for the stability of the same.

The Myers dielectric effect (see [10]) is a well known mechanism by which a D0-D2 brane is stabilized in the presence of an external $F^{(4)}$ field, i.e. the 4-form RR field produced by D2 brane charge. The Myers effect is analogous to the effect of an electric field on a molecule or atom. The molecule is polarized and a dipole moment develops. Similarly, under the F field, a multipole D2 brane charge is created, represented by a spherical D2 brane, and is stable.

If the strength of the field is f, and N_0 is the number of D0-branes, the size of the stabilized D2 brane is

$$b \simeq \alpha' f N_0 \tag{3.35}$$

and the non-commutativity scale on the brane is

$$[x^1, x^2] \sim i\alpha' f x^3. \tag{3.36}$$

Since a number of fuzzy membranes are formed in our case, we expect that each of them produces an $F^{(4)}$ field and is affected by the field of all the other membranes. In [16], it is argued using some dimensional analysis that the mean field produced by the membranes is of the right order to satisfy the above relations.

More detailed study of the stabilization process is needed to make a convincing argument. In this direction, we computed the $F^{(4)}$ field of a single fuzzy membrane along its axes of symmetry to be

$$F^{0123} = \frac{4\pi r^3 c_2}{3c_1\Omega_8(r^2 + x^i x^i)^{9/2}} \tag{3.37}$$

where c_1 and c_2 are the coefficients of the Yang-Mills self-term and the interaction-through-the-potential term in the action of the brane.

Chapter 4

Brane Interaction

4.1 Setup of Problem

We wish to investigate the interaction of 2 D0-D2 branes. To do this, we determine the background due to one of them and analyze the dynamics of the other brane in this background. Again, since a spherical D0-D2 brane is not stable, we use the toroidal case as a laboratory.

The action of the probe has two terms - the Dirac-Born-Infeld (DBI) term and the Chern-Simons (CS) term. Explicitly,

$$S_{DBI} = -T_2 \int d^3x \left(e^{-\phi} \sqrt{-det(P[G+B]_{ab} + 2\pi l_s^2 F_{ab})} \right) \tag{4.1}$$

and

$$S_{CS} = \mu_2 \int P[C1 \wedge B + C3] + \lambda P[C1] \wedge F. \tag{4.2}$$

We look at three different configurations of the branes. They can be parallel to each other, meaning both their extended dimensions are parallel, or only one of them can be parallel while the other is perpendicular, or both can be perpendicular. Only for the parallel case is the CS term non-zero.

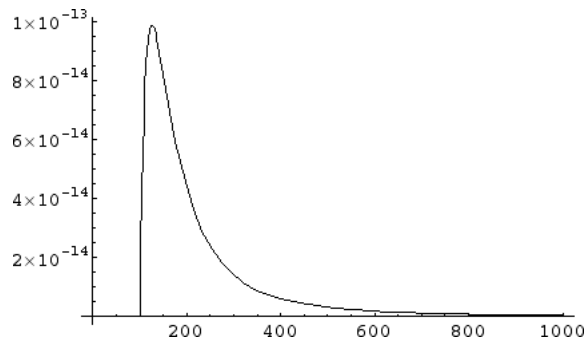

Figure 4.1: A plot of the potential $V(r)$ versus r for the mixed case. The bump is near the horizon which is at $r = 200$.

4.2 Analysis

After setting up appropriate coordinate systems, the action and the background were fed into Mathematica and the equation of motion of the probe brane was evaluated and written in the form

$$\dot{r}^2 + V(r) = 0 \tag{4.3}$$

where r describes the separation between the probe and the source in a spherical coordinate system.

The potential was then plotted by Mathematica for various cases. A sample plot for the mixed alignment case is shown in figure 4.1. There is a bump in the potential near the horizon radius. The same qualitative behavior is seen for the parallel case and the perpendicular case in certain regimes of the parameter space. More detailed study of the parameter space is needed before any definite conclusions can be drawn. This is an area for future work.

Chapter 5

Conclusions

The fuzzball model of a black hole has many appealing features, and it promises to resolve many apparent paradoxes in the traditional view of black holes. If this model is taken seriously, the horizon is no longer a region where general relativity is applicable despite the fact that gravity is weak here. The fuzzy membranes would sustain themselves at the horizon radius and demand stringy physics to be applied.

We have looked at the collapse of a shell of D0-branes and identified a phase transition near the horizon. The D0-branes apparently bind together to form fuzzy membranes. This was deduced using the holographic duality to translate the Gregory-Laflamme transition of black holes into the binding of D0-branes.

The stability of such a system was also argued for using the Myers dielectric effect. The mean F field produced by the membranes is of the right order of magnitude to stabilize the individual membranes. The exact details of the interaction of fuzzy membranes are difficult to work out. More analysis is necessary before a convincing argument can be made for the model (see [17]).

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