Motivating Question
A restaurant receives a reservation request, should the restaurant accept or reject this request?

Reasons for Accepting the Request
• Immediate realized revenue.

Reasons for Rejecting the Request
• The restaurant might only be able to accommodate the pending request at a table that is too big, resulting in unused seats; or
• The restaurant might be forced to place the pending request at a time-slot/table combination that causes a significant reduction in the turn count of the table.

Turn Count
Turn Count: Maximum number of remaining reservations that a table can host.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>slot 1</td>
<td>slot 1</td>
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<tr>
<td>slot 2</td>
<td>slot 2</td>
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<tr>
<td>slot 3</td>
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<td>slot 4</td>
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<td>slot 5</td>
<td>slot 5</td>
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<tr>
<td>slot 6</td>
<td>slot 6</td>
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</tbody>
</table>

Assuming a constant meal duration of two time slots, Table 1 has a turn count of three, and Table 2 has a turn count of two.

Motivating Example
To illustrate the necessity for a mathematical approach to the reservation-scheduling problem consider a simple example where the only remaining open table in a restaurant is a table for six at 7 p.m. If the restaurant assigns a reservation request for four to this table, then it has lost the revenue associated with the two empty seats. If the next incoming request is for a party of six, it would have been more profitable to reject the original request for four and accept the request for six. This example demonstrates that the decision at each step should only be made after a careful forecast and analysis of future requests.

Reservation Models
Both models use the dynamic programming framework shown below.

1. Consider the scenario where the reservation of size \( k \) is accepted. Tentatively assign pending request to time-slot/table combination.
2. Generate a stream of theoretical future requests.
3. Employ a heuristic to estimate the potential future seat utilization and call this value \( Seats_{To\ Go\ Accept} \).
4. Consider the scenario where reservation is rejected.
5. Repeat steps 3 and 4 to count \( Seats_{To\ Go\ Reject} \).
6. Repeat steps 1 through 6 for another Monte Carlo simulation.
7. Accept the reservation if \( Seats_{To\ Go\ Accept} + k \geq Seats_{To\ Go\ Reject} \).

Bossert's Model
Bossert (2009) considered assigning requests to the time slot that has been requested. The heuristic used to estimate the future seat utilization is shown below.

Two-Pass Heuristic
First Pass: Only make time-slot/table assignments that are an exact fit. In order for an assignment to be characterized as an exact fit, the following two qualities must be satisfied:
1. Party size = Table Size;
2. Turn count of the table is only reduced by one.
Second Pass: If not accepted during First Pass, accept if a table is available at requested time.

Flexibility Model
The new model incorporates an element of flexibility that is absent in the accept/reject model proposed by Bossert. The flexibility element allows requests to be assigned to a time slot that differs from the initially requested time. In adding this flexibility element to the model, the likelihood that a party will accept an arrival time differing from the one they have requested must be considered.

Two-Pass Heuristic
First Pass: As in Bossert (2009), only make time-slot/table assignments that are an exact fit.
Second Pass: For each request, the open time slot closest to the customer’s request is found at each request. The expected number of unused seats is calculated for each of these time slot/table combinations, and the combination with the smallest expected gap is offered to the customer.

Simulations
To test the proposed model, a set of simulations was conducted at Trattoria Toscana, a very small Italian restaurant in the Boston Fenway neighborhood. Trattoria Toscana has 30 different tables, and subsets of these tables are grouped to form eight configurations. Each of the configurations has at most 10 tables, and the restaurant can accommodate at most 90 people per day. The model was simulated on four different types of customers.

Results
To compare our model to actual restaurant operations, we examine three naive seating models. In most restaurants it is unlikely that the hostess or maize d’ uses any sort of mathematical analysis to assess the value of a reservation request. These reservation schedulers generally have some rule of thumb for trying to place the request at their desired time slot without having too many unused seats.

Conclusion
The proposed Flexibility Model appears to be a successful approach to optimizing restaurant-reservation scheduling. Not only is this element of flexibility important for an accurate representation of the restaurant-reservation scheduling process, but its addition also allows the restaurant to seat more people throughout the dining service. The flexibility element yielded seat utilization gains ranging from 15 to 35%.

References

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For Further Information
jacob_feldman@hmc.edu