Book Review: *What's Luck Got to Do with It? The History, Mathematics, and Psychology of the Gambler's Illusion* by Joseph Mazur

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Synopsis

This review of Joseph Mazur's book on the history of gambling, for a general audience, is in three parts, paralleling the structure of the book. The first part briefly outlines Mazur's coverage of the history of probability from prehistory to the present day, with a focus on gambling. The second part examines the relationship between the mathematics of gambling and probability theory, and summarizes classical problems in probability arising from gambling such as Galileo's dice and the Pascal-Fermat problem of points. The third part, on psychology, discusses the gambler's illusion and psychological motivations for gambling.

> **What's Luck Got to Do with It? The History, Mathematics, and Psychology of the Gambler's Illusion.** By Joseph Mazur, Princeton University Press, Princeton, 2010. (US$29.95, 296 pages. ISBN-10: 0691138907.)

I am not a gambler by nature. I have never been to Las Vegas; I have only been to Atlantic City because it is the closest place on the ocean to my childhood home in southern New Jersey. But the number 244/495 means something to me; it is the probability of winning the pass bet at craps. Why do I know this number? Because I am a probabilist – a discrete probabilist, at that – and so I have walked several classes, often full of students not of legal gambling age, through computing this probability. The history of my subject is intimately tied up with the history of gambling. It is this connection that Joseph Mazur explores in his new book, *What's Luck Got to Do with It: The History, Mathematics, and Psychology of the Gambler's Illusion.*

Mazur's book opens with the story of his uncles gathering around the table "telling jokes while accounting their week's gambling wins and losses" (page *xi*). From the beginning – in fact, from the subtitle – we know that this will not be a straight-up math book. The book has three parts of roughly equal length. The first is an overview of the history of gambling, from prehistoric relics through the 2008 economic crisis. The second is a brief introduction to probability theory, through the lens of gambling, covering basic counting, the weak law of large numbers and the binomial theorem, and applications to games such as blackjack, craps, and slot machines. The third part is psychological, focusing on the "illusion of control" and the psychology of gambling addiction.

Mazur decided to write this book when his car broke down at a convenience store and he watched people spend money they did not have on lottery tickets. He "toyed with the idea that the book should be sermonizing the consequences of addictive behavior" (page *xv*). The reader should be glad that Mazur does not sermonize. But at the same time we often hear that gambling is a "tax on stupidity"[1] – perhaps interpreted as a tax on the uneducated. Perhaps then in introductory courses in probability and statistics, we should make sure that students learn the house always wins. Too often students come away from such courses knowing how to compute probabilities of complicated events but with very little sense of what these probabilities mean. This is not their fault. Humans did not evolve in a setting where it was necessary to understand probability. But at the same time do we not owe them at least that much?

History. The historical section which opens the book covers history that will be unfamiliar to many mathematical readers, not only because we mathematicians tend to be ignorant of our history but also because gambling is embedded in a social context and much of its history is not the history of mathematics at all. Mazur tells us that his uncles believed they had "supernatural personal control" (page *xii*) over what happened to them, and this is a theme that is carried through the earliest parts of the history of probability. The first chapter of the book, "Pits, Pebbles, and Bones", brings us from prehistory to the Renaissance, and the second chapter, "The Professionals",

[1]This quip is often attributed to Samuel Johnson. Paulos [12, page 120] attributes it to Voltaire. It seems to be a condensation of the opening lines of Henry Fielding's farce "The lottery": "A lottery is a Taxation / Upon all the Fools in creation".

shows the emergence of probability theory in the 17th century through problems arising in gambling. "Gambling is about odds, the chances of things going one way or another" (4)[2]; thus wherever there is uncertainty, there is gambling. But uncertainty is also associated with the divine – we still see this in arguments concerning the "god of the gaps", in which a supernatural cause is attributed to those events that we observe but do not (yet) understand. A predecessor of the die was the *astragalus*, the irregularly shaped ankle-bone of a sheep, which was used for divination purposes and later for gambling. Mazur gives the example of a shaman using the astragalus to decide whether to go hunting the next day; in light of what we now know, this almost looks like a very early example of a mixed strategy in game theory.

The third and fourth chapters, "From Coffee Houses to Casinos" and "There's No Stopping It Now", trace the institutionalization of gambling, and its prohibition around the turn of the twentieth century coupled with its recent resurgence in the form of lotteries and widespread casinos. These chapters are purely historical, fitting well in a popular history of gambling. This section of the book is well-illustrated with contemporary paintings and engravings, showing how gambling was embedded in the culture. We also see that gambling was a pastime both of the rich and the poor in nineteenth-century England, continental Europe, and America, despite being widely illegal and officially frowned upon. But what is the gambling industry selling? Not riches – the law of large numbers ensures that the riches generally flow away from the player, and the continued existence of that industry is a testament to this fact – but entertainment. By placing a bet one purchases the right to fantasize about riches.

In Chapter 5, "Betting with Trillions", Mazur takes a strong stance on the recent financial crisis: stock exchanges are gambling. Socially necessary gambling, in order to raise capital for valuable enterprises, but still gambling. He writes of the recent wave of financial innovation: "The banking industry's extensive risks are another story. They were reckless ventures goaded by unrestrained greed" (61). "Mathematicians" (more accurately, "quants," or quantitative analysts working for financial institutions) have taken some portion of the blame for the financial crisis in the popular media. But it is perhaps a bit harsh to blame the quants; some have suggested that the blame lies on those who uncritically used the models they produced. Mazur's

[2]Numbers following quotes are for page numbers from the book under review.

stance is that "we can blame much of [the recent crisis] on gambling under the influence of reckless greed – the deliberate courting of danger" (71). This reckless greed is coupled with the fact that financial markets had grown increasingly more complicated and brittle; minor perturbations in one part of the market, instead of being dampened, were amplified and brought the market to its knees.

Mathematics. Probability, of course, is not *about* gambling. Most of the problems considered in probabilistic analysis of gambling are discrete – but many real-world applications are continuous. And in gambling problems we "play God" and know all the rules of the game; in the "real world" we are ignorant of the rules. In his column for *Bernoulli News*, David Aldous writes that "[t]o caricature a little, we have built a vast intellectual edifice upon the observation that there's a mathematical model for the results of throwing dice" [1]. And dice are a potent *symbol* of gambling. Surely Einstein knew that quantum mechanics does not literally involve cubes with numbers on them; yet in his famous letter to Born he still said that "I am at any rate convinced that He does not play dice".[3]

So why gambling? Perhaps because gambling provides an "in" to mathematics for some people. In Mazur's previous book *Euclid and the Rainforest* [9], he tells of sitting in on a course at Columbia, called "Chance", from a professor whom he nicknames "Lenin" (this professor resurfaces in the book under review). Another student sitting in on this class, Uriah Brown, had come to probability directly from an "addiction to poker and gambling" [9, page 197]; in that book's conclusion, Mazur finds himself decades later on a commuter train sitting next to a statistician who consults on gambling matters, with a briefcase monogrammed U. B. [9, pages 263–264]. With the current surge of interest in poker, those of us who teach probability get our fair share of questions about the game. As far as students brought to math by gambling, Persi Diaconis comes to mind. Diaconis dropped out of high school to become a magician, supported himself by playing (and cheating at) poker [6], and found his way back into the academic fold because he was determined to understand Feller's classic text [4]. Indeed Diaconis is known to the wider public largely for his results on shuffling cards, such as the paper [2], often paraphrased as "seven shuffles suffice". And as Aldous suggests,

[3]In German, "Jedenfalls bin ich überzeugt davon, dass der nicht würfelt." This is often shortened to "Gott würfelt nicht;" "würfel" is German for "die" or "cube."

perhaps we teach about gambling because it is philosophically easy to teach; there is no question of *why* probabilities should be what they are.

Many of the standard introductory problems in probability are really counting problems. For example, Mazur discusses (on pages 24-25) the problem of "Galileo's dice" – why, when rolling three fair dice, do the sums 10 or 11 occur more frequently than the sums 9 or 12? After all, there are six ways to get a sum of 9: $(6,2,1)$, $(5,3,1)$, $(5,2,2)$, $(4,4,1)$, $(4,3,2)$, $(3,3,3)$. And there are six ways to get a sum of 10: $(6,3,1)$, $(6,2,2)$, $(5,4,1)$, $(5,3,2)$, $(4,4,2)$, $(4,3,3)$. But Galileo wrote "it is known that long observation[4] has made dice-players consider 10 and 11 to be more advantageous than 9 and 12" [5]. The resolution of this problem, of course, is that order matters – so there are 25 ways to roll a 9, and 27 ways to roll a 10. Students of probability still struggle today with when order matters and when it does not.

Another such early problem is the *problem of points*, usually associated with the Pascal-Fermat correspondence; these letters have been translated into English [11] and Devlin has written a popular book on this correspondence [3]. Two gamblers wager on the results of flipping a coin, one taking heads and the other tails; the winner will be the first to win n flips. But they are interrupted after r heads and s tails; how should the stakes be divided? An old rule was to give $r/(r+s)$ of the stake to the heads player, and $s/(r+s)$ of the stake to the tails player. But this rule is obviously flawed; just consider the case $r = 1, s = 0$. Furthermore, n does not enter at all. Any "common-sense" rule should give a function of $n-r$ and $n-s$; a team leading $1-0$ in a best-of-five series has the same chance of winning as a team leading $2-1$ in a best-of-seven series. Fermat solves this as a brute-force problem in combinatorics. Say that $n-r = p$ and $n-s = q$; so the heads player needs p heads to win, and the tails player needs q tails. Then the game will surely be over after $p+q-1$ flips; just write out the 2^{p+q-1} possible futures and see in how many of them each player wins. This method works, but if $p+q-1$ is large it is quite cumbersome, and it is not easy to see how to extend it to more players; in fact Fermat tripped over the three-player generalization. Pascal, perhaps thinking of the triangle that bears his name, finds a recursive solution. Call the state where p heads or q tails are needed to win (p,q). If $p = 0$ or $q = 0$ the game is over, and the

[4]The probability of rolling a 9 is 25/216, and the probability of rolling a 10 is 27/216. To tell these apart empirically requires thousands of rolls.

division of stakes is obvious. Otherwise the (p, q)-state is equally likely to be followed by the $(p-1, q)$ or $(p, q-1)$-states, and so Pascal argues that the division of stakes in the (p, q)-state should be midway between those in each of its possible successors. This turns out to give a sum of binomial coefficients. Thus begins the "counting without counting" view of combinatorics, and by extension discrete probability.

The mathematical section of this book opens with Mazur's professor "Lenin" developing the theory of statistical sampling by having his students pick handfuls of stones out of a large bowl of black and white Go stones, determining if dice are fair by rolling them, and so on. This, Mazur tells us, was transformational: "Until then, I thought mathematical questions had definite answers, if they had any answers at all. *Nearly* was not a word in my mathematics vocabulary" (78). Our poor intuition for probability theory has formed the basis for the Nobel Prize-winning work of Daniel Kahneman and Amos Tversky [7]. For example, people believe a coin is more likely to show the sequence $THTHHT$ than $TTTHHH$, because the prior sequence "looks more random" and is often taken as a representative for a larger class of "random-looking sequences". And any baseball fan has seen players batting 0.000 or 1.000 at the beginning of a season and hopefully knows not to extrapolate from this. But we are quick to generalize, perhaps because we are uncomfortable with uncertainty.

The most mathematics-intensive chapters of the book are Chapter 8, "Someone Has To Win", on the binomial distribution and the central limit theorem, and Chapter 9, "A Truly Astonishing Result", on the weak law of large numbers. Mazur introduces the binomial distribution as the distribution of the number of successes in n independent trials each with success probability p; the probability of k successes is $\binom{n}{k}p^k(1-p)^{n-k}$. The central limit theorem is then introduced as a device for approximately computing $\sum_{k=r}^{s}\binom{n}{k}p^k(1-p)^{n-k}$. A histogram of the number of successes in n flips of a biased coin forms a smooth curve; by appropriate scaling and shifting this curve can be made to nearly coincide with the normal curve. Since Mazur's main interest in this book is in gambling, he uses this to compute the probability of being ahead after making many bets on red or black at roulette. (This is the case of large n and $p = 18/38$.) There are some minor numerical errors in these computations but the main point, that one is very likely to be behind after a large number of spins, stands. I would have liked to see some acknowledgement that the normal distribution is ubiquitous precisely because it does not *only* occur as the limit of binomial distributions, but under

many other conditions in which a random quantity can be viewed as a sum of many small, not necessarily independent or identically distributed contributions. In particular this makes it clear that it is not possible to win in the long run at negative-expectation games – that is, "life is a supermartingale".

In Chapter 9 Mazur states the weak law of large numbers (WLLN): "the probability that the success ratio [after N independent trials each with probability p] differs from p is as close to zero as one wishes, provided that N can be taken as large as needed to force that condition" (119). This is somewhat difficult to parse. Let k be the random number of successes; does this mean that

$$\text{for every } \epsilon > 0,\ P[|k/N - p| < \epsilon] \to 1 \text{ as } N \to \infty, \text{ or}$$

$$\text{for every } n > 0,\ P[|k - pN| < n] \to 1 \text{ as } N \to \infty?$$

The confusion is unsurprising; the first of these is the weak law of large numbers (WLLN), and the second is some form of the gambler's fallacy. (Mazur does state, in modern notation, that he means the first of these; but the lay reader should not have to understand epsilons and deltas.) For the second of these to be true, some sort of "restoring force" would be necessary – after a run of tails, heads would have to be more likely, and vice versa. Indeed the *ratio* of heads to tails after many flips of a fair coin almost surely goes to 1, but the *difference* grows arbitrarily large. These two are often conflated by gamblers, with disastrous results. In an appendix, Mazur proves the WLLN (for Bernoulli trials only, the original historical context) from the Chebyshev inequality. This he leaves unproven, but the point is that we can leverage a weak result (like Chebyshev) into a strong one (like the WLLN).

Psychology. I began this review by saying that I am not a gambler by nature. Perhaps I am unusual in this regard; Mazur writes that "[i]n my boyhood years it seemed that everyone in my neighborhood gambled at something" (159), taking us back to the Bronx of the late forties and early fifties, where the boys gambled on marbles and the men on horse racing and the "numbers" game. Or does gambling flourish in some places and times but not in others? After all, "we kids were smart enough to know that gambling was not a privilege reserved for grownups" (160). Or perhaps the gambling urge is still with us – but institutionalized in the form of widespread casinos and lotteries, and televised game shows that essentially consist of watching someone else gamble.

The "gambler's illusion" of the book's subtitle is never explicitly defined, which is unfortunate since this term at first seems to be one of Mazur's own

invention. "Illusion" here most likely stands for Langer's concept of "illusion of control" [8], defined as "an expectancy of a personal success probability inappropriately higher than the objective probability would warrant". For example, lottery players prefer to choose numbers that have some special significance for them, rather than allowing lottery computers to choose their numbers at random.[5] This seems quite curious from a "rational" point of view, at least in games of pure luck – why should a player have any control over the balls selected from a lottery machine hundreds of miles away? But as Mazur points out in Chapter 10, we can position games on a "skill-luck spectrum". On one end of this spectrum are games such as slot machines, lotteries, and roulette, where the only strategic element is deciding how much to bet; blackjack is perhaps in the middle; poker is perhaps the common casino game involving the most skill. (Mazur also mentions horse racing and sports betting as gambling games, but these seem fundamentally different to me; here one is not wagering on the outcome of a random number generator but on the performance of human beings. Perhaps an athlete may be viewed as "a weighted random number generator" [10] but in wagering on sports the weights are unknown.)

One of these games that amounts to watching someone else gamble is the TV show "Deal or No Deal". In this show, 26 numbered briefcases contain amounts of money ranging from one cent to one million dollars. The player selects one briefcase to keep, which they may open only at the end of the game. An invisible "banker" offers to buy the briefcase, for an amount usually less than its expected value. The player can choose "Deal" and go home with the banker's offer. Or they can choose "No deal", in which case they are allowed to pick some predetermined number of cases to be removed from the game; the process is then repeated. The rules by which the banker's offers are determined are not publicized but the offers are highly predictable [13]; usually early in the game the offers are below the expected value of the amount in the hidden case but they get close to and sometimes surpass the expected value towards the end of the game. But players on the show seem to take some big risks, and this may be explained by the "house money effect" – people are more likely to gamble with money they have recently won than

[5]There is one strategy for the lottery, if the jackpot can be shared – play numbers that nobody else plays! Since people play their "lucky numbers", this means avoiding lucky numbers, such as birthdays. Of course this only works if word doesn't get out.

with their "own" money, even though, purely rationally, money is fungible. This may be connected to whether a given gamble is seen as a chance to win money or a chance to lose money. Actual subjects show an asymmetry between gains and losses – many people prefer a sure gain of $1,000 to a 50% chance of winning $2,500, but simultaneously prefer a 50% chance of losing $2,500 to a sure loss of $1,000. And when life-changing amounts of money are at stake, utility functions may become complicated; although one generally assumes that utility functions are convex (that is, an extra dollar is worth more to a person with a smaller fortune), a 45% chance at enough money to pay for a new house might be worth more than the certainty of half that much money. Much analysis of gambling rests on the assumption that expected value is to be maximized – but the justification here lies in the Law of Large Numbers, and if you only get a few big gambles in your life the numbers simply are not large enough for what would happen in the long run to really matter.

Finally, why gamble? What makes a gambler? Some suggest that gambling is simply an addiction like any other – tobacco, alcohol, illegal drugs – and indeed recent research shows that addicts with different addictions have similar patterns of brain activity. One paradox, though, is that those who quit one addiction may compensate by picking up another – is there some hard-wired susceptibility to addiction that latches on to whatever is culturally available? And in gambling, one does not always win, nor does one even win on some regular schedule, but one wins *randomly*, by the very nature of the process; this is what behavioral psychologists call "intermittent reinforcement" and is particularly effective in forming long-lasting habits. Recent research has even shown that rats, like humans, "play the odds" and there is some hope that this may help in the development of treatments for gambling addiction [14].

Conclusion. I recommend this book enthusiastically. Mathematical readers will not learn much new mathematics but they will learn quite a bit about history, both of probability and of the social context in which it emerged, and will have a glimpse of the psychological issues arising in the translation of theoretical results to the real world. Non-mathematical readers, conversely, may find in the historical and psychological aspects of the subject a "hook" that will pull them into the mathematics. Finally, Mazur is most in his element when telling stories from his own life; interweaving these stories with related mathematics is a hallmark of Mazur's work, and a most enjoyable one.

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