We Could All Be Having So Much More Fun! A Case For The History Of Mathematics In Education

Louise Anderton

The School of Education, Communication and Language Sciences, Newcastle University, Newcastle upon Tyne, UK

louiseanderton@hotmail.co.uk

David Wright

The School of Education, Communication and Language Sciences, Newcastle University, Newcastle upon Tyne, UK

d.g.wright@ncl.ac.uk

Synopsis

Many students experience mathematics as ahistorical and acultural. We review the philosophical roots of this experience and pose alternatives. We argue that there is evidence that the inclusion of a historical dimension into the teaching of mathematics courses at all levels, combined with an *active* approach to learning, will improve motivation and achievement.

1. Introduction

For most students, it appears that mathematics is still experienced as an ahistorical, fixed body of knowledge. Few have the opportunity to appreciate the "other faces" of mathematics as a creative medium and a cultural artefact with its own history. We argue that the inclusion of these dimensions of mathematics at all levels of mathematical education, combined with an *active* approach to learning, should have a positive motivational impact and improve the *image* of mathematics. This is an opportunity to break the cycle of underperformance in mathematics education.

2. The Absolutist View of Mathematics

A common perception of mathematics is that it is the *purest* of all sciences; that it is an objective, logical and a certain body of knowledge [6]. This

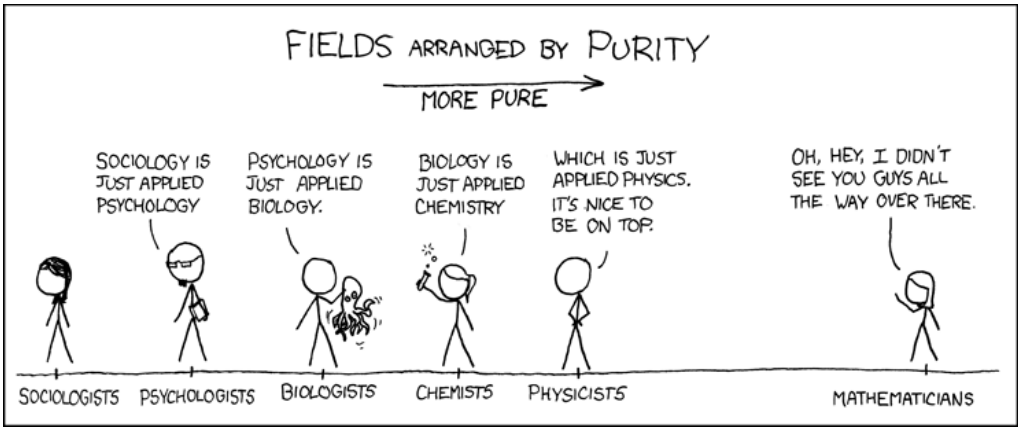

Figure 1: A common perception of mathematics. Image from `http://xkcd.com/435/` This and other "pearls of wisdom" may be found in [29].

is the absolutist perspective, and many people are drawn to mathematics for these precise qualities [9].

Ernest [8] provides an account of the absolutist view of mathematical knowledge. According to absolutism, (arguably the dominant philosophy of mathematics all through the twentieth century) mathematical knowledge is timeless, culture-free and ahistorical. The belief is that although new theories and findings may be added to our core understanding, the history of mathematics is extrinsic to mathematical knowledge. Ernest goes on to say

> Although it exceeds its intended scope, absolutism suggests a philosophically sanctioned image of mathematics as rigid, fixed, logical, absolute, inhuman, cold, objective, pure, abstract, remote and ultra-rational.

———o———0———o———

Sam Square: *That sounds about spot on. I would definitely agree with the fact that maths is inhuman and remote – so much of it seems totally irrelevant to real life.*

Edith Acute: *Woah – hold your horses Sam! Nobody said it's a fact that mathematics is inhuman and remote; that is simply one perspective on the nature of mathematics. A dominant and perhaps overshadowing one, but a perspective nonetheless.*

———o———0———o———

3. Students' Experiences and Perceptions of Mathematics

> The impression we are given is of something very cold and highly technical, that no one could possibly understand – a self-fulfilling prophesy if there ever was one [21].

Students' impressions of mathematics in school often correspond with the absolutist position – perhaps due to the manner in which mathematics is taught [8, 24]. For example, inspection reports from England [25] show that it is still common practice for teachers to ask students to complete repetitive mathematical tasks often emphasizing that each problem must be solved using the appropriate, memorized approach. Some learners enjoy this approach, but for many others this experience is so unpleasant that the outcome can be the development of a phobia of mathematics. It is clear that underachievement in mathematics is damaging for students' career prospects, and the attached anxiety is disastrous for the subject itself.

So what about the high-achievers then? There must be happier story here, surely? Unfortunately, the answer is *no, not really* [25]. Even though they achieve success, many experience an approach where "mathematics can feel like something to be memorised rather than understood" [11]. Skemp [32] argues that many learners adopt an *instrumentalist* approach, believing that success in mathematics consists of memorising a formula for a solution. Surprisingly, this approach is quite common even among students studying advanced mathematics at university level. It is common for learners to see mathematics as "a game with arbitrary rules [...] that is unconnected to anything" [20].

There is evidence that the mere thought of mathematics can induce in some students a state of panic [2]. We believe that teaching methods that reinforce the "cold, hard and rigid" impression of mathematics need to loosen up and adopt a more humane approach if there is any hope of improving the image of mathematics and students' motivation and attainment.

———o———0———o———

Sam Square: *Glad to hear I'm not the only one, then!*

Edith Acute: *Oh Sam! Don't you think it's a shame that you're studying mathematics and yet have no idea how it even came into existence? The history of mathematics is a fascinating story that begins way back four thousand years ago!*

Sam Square: *Maths has a* history*??? I had no idea...*

———o———0———o———

4. Mathematics Does Have a History, Yes!

Some commentators argue that year upon year, young learners have studied a subject called mathematics, which bears only a minor resemblance to the mathematics on the other side of the school gates [4]. For example, the idea that mathematics has always existed (and is the same everywhere) in an unchanged form is false, even with such a basic concept as counting.

4.1. Peoples' Ages before Zero...

It was a very long time before the concept of zero was accepted in the West, in part due to the fact that medieval scholars associated zero with the void – and the void with Satan [30]. Up until the time when zero was introduced, numbers always had to start with one. Using the one-based counting system, a new born baby was described as being "aged one" for the first year of her life. A year on from the birth date and the baby girl turned two [38]. To our modern minds, this seems a terribly strange concept, as we would customarily express babies' ages in terms of days, weeks and months (essentially, we assign a newborn an age of zero). Only after a birthday would we call the child a one-year-old.

Surprisingly, the transition to a zero-based counting system occurred just 150 years ago [38], and it is easy to make mistakes while calculating people's ages. For example, say you encounter a 150 year-old gravestone and notice that the occupier died "aged 34". If you don't know about the one-based counting system, you would naturally assume that he passed away during his 34th year – but you would be wrong! Using the modern convention of age calculation, the poor fellow would have been 33 years old when he breathed his last.

Mathematical innovations have influenced almost every man-made object in the modern world; yet in mathematics education most students have seen only a "seemingly petrified structure" [20], with not even a hint of the imaginative and inspiring human roots of mathematics. "By concentrating on what, and leaving out why, mathematics is reduced to an empty shell" [21].

———o———0———o———

Sam Square: *Hey, that's pretty cool – I'd never thought about maths' impact on mankind before. I guess it is true, without maths we would still be living in the dark ages!*

Edith Acute: *The main point is that most students aren't given the opportunity to learn about the history of mathematics, yet it has so much to offer. It's important that students develop an appreciation of mathematics, ideally through learning about its development.*

Sam Square: *Hmmm, I'm not so sure. At the end of the day we learn maths in school and university so that we can do sums and equations.*

Edith Acute: *But don't you know? There's a whole philosophy of mathematics – different peoples' views about it, debates about the nature of mathematics. Did you know that some people think maths is an art?*

Sam Square: *Come again?*

———o———0———o———

5. Alternative Views of Mathematics

The antithesis to absolutism is fallibilism; a philosophy of mathematics which has developed since its initial formulation in the latter part of the twentieth century [6]. Building on Popper's philosophy of science, Lakatos in *Proofs and Refutations* [17], one of the seminal documents of fallibilism, argues that mathematics is "human, corrigible, historical and changing" [5]. In essence, the idea is that mathematical knowledge is receptive to revision and amendments.

Fallibilism does not deny the value of logic and order in mathematics; it simply rejects the idea that there is a singular, inflexible, and unrivalled hierarchy of mathematical knowledge. It takes the standpoint that mathematics consists of "many overlapping structures which, over the course of history, grow, dissolve, and then grow anew, like trees in a forest" ([35], cited in [8]). This approach values mathematicians' efforts to develop our knowledge, and regards mathematics' history, culture, ideals and education as key issues for philosophical attention [8].

In 1997 the International Commission on Mathematics Instruction (ICMI) carried out a review of the current state of history in mathematics education. They asserted that an appreciation for the true nature of mathematics – that it can be a "study of twists, turns, false paths and dead-ends" – makes the subject more accessible and helps students "towards a more realistic appreciation of their own endeavours" [13]. Cuoco *et al* [4] agree, and Lockhart further explains:

> To do mathematics is to engage in an act of discovery and conjecture, intuition and inspiration; to be in a state of confusion – not because it makes no sense to you, but because you gave it sense and you still don't understand what your creation is up to; to have a breakthrough idea; to be frustrated as an artist; to be awed and overwhelmed by an almost painful beauty; to be alive [21].

Alternatively, some argue that mathematics is in fact an art. For example, Laubenbacher *et al* [20] declare that "we have sold mathematics short by presenting it only as the *language of science*". They make the point that just because a subject is useful does not automatically make it stimulating. It is the imaginative, artistic side of mathematics that makes mathematicians tick, so it's ironic that creative dimensions of the subject are frequently ignored. We believe that students need to be freed from the frequently uninspiring, inflexible mathematics taught in school (and even at university!), and given the chance to learn from the masters themselves. Learners rarely get a glimpse of the creative aspect of mathematics – they are missing out on one of the oldest, most insightful and abstract of arts.

> If anyone were exposed to mathematics in its natural state, with all the challenging fun and surprises that it entails, I think we would see a dramatic change both in the attitude of students towards mathematics, and in our conception of what it means to be good at maths [21].

———o———0———o———

Sam Square: *Why have I never heard of this before? How come none of my maths teachers ever mentioned this perspective of maths? This is all very new to me.*

Edith Acute: *You're right – the philosophy and history of mathematics have been completely neglected in the curriculum. In some countries a* *different relationship between mathematics and its history is projected; for example in parts of Scandinavia the history of mathematics is considered fundamental to the subject itself (Fauvel and Maanen, 2002). What a pity the rest of the world hasn't caught on yet!*

———o———0———o———

6. History of Mathematics in Teaching (or The Neglect Thereof!)

> What other subject is routinely taught without any mention of its history, philosophy, thematic development, aesthetic criteria, and current status? What other subject shuns its primary sources – beautiful works of art by some of the most creative minds in history – in favour of third-rate textbook bastardizations? [21]

Ernest [7] believes that despite mathematicians' and educators' apparent love for their subject, they frequently fail to foster an appreciation of mathematics in their students. The majority of mathematics students are unaware that there is a conflict over the true nature of mathematics; that mathematics has a rich and diverse history; that it is at once a science and an art. By failing to raise awareness of its international origins, topical development, unique aesthetic and irreplaceable contribution to human culture, teachers are failing their students. However, it is hardly surprising that this dimension is missing from teaching, since the teachers themselves will not have had the opportunity to experience this aspect of mathematics.

Lockhart argues that part of the problem lies with a school syllabus which consists of a collection of detached topics and monotonous problem-solving exercises, with no historical grounding or unifying themes [21]. The curriculum's unrelated topics are associated only through their shared readiness to be simplified down to "step-by-step" formulas. At university level there is generally little improvement: undergraduate mathematics courses are traditionally made up of a variety of independent modules, which collectively present mathematics as "a fixed and formalised body of knowledge" and often cause "disjoint understanding among students" ([15, 23, 26, 27, 31], cited in [34]).

Another weakness in the current teaching approach is that it fails to demonstrate to students that mathematics is a process; that a mathematical theorem is the "tip of an iceberg" which conceals the thought processes involved in its discovery. The real story about how mathematicians actually think is inadvertently neglected, yet this is precisely what pupils need to grasp [20].

An alternative approach is advocated by Wigley [43] where each topic is supplemented with a narrative of its historical background and cultural context. Swetz ([36] cited in [34]) also suggests that the history of mathematics "can serve as a major pedagogical tool for the teaching of higher mathematics", while elsewhere it is said that, through gaining a historical understanding, learners will better appreciate "the meaning of mathematical concepts, methods, theories, and proofs" ([37] cited in [34]). As 19$^{\text{th}}$ century Norwegian mathematician Niels Henrik Abel is often quoted to have said:

> It appears to me that if one wants to make progress in mathematics, one should study the masters and not the pupils [1].

———o———0———o———

Sam Square: *I feel like I'm missing out. Maybe if we'd been shown what maths really is – its creative side and its long history, then people would be more encouraged to get stuck in. I feel like creating my own maths, making something new.*

Edith Acute: *Fantastic! There's a lot of evidence that learning about the nature and history of mathematics has a motivational effect on students...*

———o———0———o———

7. Recovering Motivation With the History of Mathematics

Siu [33] believes that history "can make learning mathematics a lively and meaningful experience, so that learning will come easier and will go deeper". It is thought that one can inspire enthusiasm in learners by asking them to examine mathematical problems from the past and consider how mathematicians of the day worked through them to find a solution [14].

There are a range of findings that do indeed suggest using history in teaching gives students a renewed motivation for mathematics. Laubenbacher and Pengelley [19] have used original sources with their students since 1987 – yielding "extraordinary" results. However, the ICMI Study Volume (2002) [10] declares that studying original sources is more demanding on students than research using secondary sources only. Laubenbacher and Pengelley do not deny this, but claim that the experience of learning history of mathematics through *first-hand accounts* is very effective [18].

Laubenbacher *et al* [20] found that when students are challenged to confront original sources, they enthusiastically respond with a heightened intensity of study. They begin to see mathematics in a new way – it is "no longer a collection of arcana, unrelated within and unconnected to anything without, but becomes a whole, an art form". It may be inferred that learners' enhanced motivation for mathematics arises from the challenge and opportunity to take *possession* of their own learning. It is thus arguable that all students should be given the opportunity to do this in all their mathematics courses.

An alternative type of motivation has been found when teaching students about the multi-cultural history of mathematics. The ICMI discussion document [13] cites the finding that minority-group children are known to obtain *pride and strength* from learning about the mathematical contributions of their culture. In this sense, mathematics is the heritage of all nations.

———o———0———o———

Sam Square: *Well, that's interesting and there's probably some truth in it, but what's the point? So – I'll agree – I've seen Maths in a new light today, but why bother showing everyone – teaching kids in schools and uni students about all of this – what do they stand to gain?*

Edith Acute: *Not only does it show students a new perspective, but the "history of mathematics can empower the students and valuably support the learning process."* [13]

———o———0———o———

8. Deepening Understanding With the History of Mathematics

Research at the University of North Carolina Wilmington shows strong evidence that learning about the history of mathematics deepens students' mathematical understanding [34].

The study investigated the growth of students' understanding when they explored the historical development of a mathematical topic. The belief was that researching, reflecting on, and writing about a particular concept would allow students to review and deepen their understanding.

Providing a background for her study, Slaten believes that learning occurs when we connect new information to what we already know, so giving students the chance to communicate about mathematics can be considered an instrument for creating such connections. Self-awareness is required when students consider how to "transform their mathematical knowledge into written communication", and when they re-interpret what is offered and make sense of it in their own terms, mathematical concepts become easier to understand and remember ([16] cited in [34]). Put in another way "[t]he learning mechanism is fuelled not by energy but by awareness" ([3] cited in [12]).

Slaten analysed the students' written reflections to assess their self-awareness of their learning. She found that the students relished the opportunity to "reconnect, re-examine and enrich" their initial awareness and understanding of a concept. Several students in the study described how they now think differently about the mathematical topics they explored. This can be linked to Martin's definition of deep learning: "learning as seeing something in a different way" [22]. Others have noted that "[e]xploring historical origins often opens a window to the interconnectedness of mathematics itself" [28].

It should be noted that this indicates that teaching students about the history of mathematics also needs to adopt an *active* approach which involves more than straightforward surface learning (i.e. superficial or rote learning), and reaches learners on a deeper level. A course which required students to memorise facts about the history of mathematics would be unlikely to have the same impact.

The ICMI study [10] of history in mathematics reports that history of mathematics can benefit learning processes, and to quote Laubenbacher *et al* [20], "students discover the roots of modern problems, ideas, concepts, even whole subjects. They also see the obstacles that earlier thinkers had to clear in order to move ahead, and thereby gain insight into current problems and how to approach them."

Slaten concludes that students do not learn to communicate mathematically if they are only taught to answer generic step-by-step questions in class. If students are to develop an appreciation for mathematics, a more complete knowledge of its concepts, and become skilled at communicating mathematically, then an approach involving the history of mathematics is recommended.

———o———0———o———

Sam Square: *I suppose it really does seem worth learning about. I can't believe so many mathematics educators haven't yet accepted the benefits of the history of maths.*

Edith Acute: *And remember, in a history of mathematics course you'll also get the chance to develop your critical reading and writing skills - skills that have probably been neglected in your mathematics education up until now.*

———o———0———o———

9. Studying the History of Mathematics: Learning Outcomes

It is recognised that many undergraduate mathematics courses neglect to develop students' research, reading and essay writing skills [13]. However, integrating the history of mathematics into student courses opens the possibility for more varied assessments, which can be designed to test a far wider range of skills. This enables students to develop their personal abilities in many different areas, including group projects, oral presentations and written communication. Many students relish the opportunity to develop themselves in new directions, with an observed effect on their enthusiasm and satisfaction with the subject [13].

Some existing undergraduate modules in the history of mathematics already provide evidence of numerous learning outcomes for students. Students develop an appreciation for the nature of mathematics [42, 41] and how mathematics and culture have interacted over history [40, 39]; an enriched understanding of mathematical concepts and the ability to take part in informed discussions on these; and a critical sense of what is important and exciting about mathematics and its evolution [40, 39, 42]. Additionally, students analyse primary and secondary sources, improve their note-taking and essay writing skills, and engage in oral discussions and presentations [40].

Students need to be able to talk about mathematics with the eloquence, detail, accuracy, and enthusiasm it deserves. An understanding of the history of mathematics could make this possible.

———o———0———o———

Sam Square: *Ok, I'm convinced – learning about the history of maths will give me a renewed interest in the subject, give me a deeper understanding and appreciation of maths in human culture, I'll see how the separate strands in maths link together, I'll be developing my personal skills along the way, and I've already seen maths in a new light today! There's so much more to it than I ever thought!*

Edith Acute: *Indeed. In the words of Lockhart: "We could all be having so much more fun."*

———o———0———o———

References

[1] Abel, Niels Henrik, *Memoirs de Mathematique*, Oslo University Library - Confirming admiration of the work of Simon Laplace, in a note, dated 6 August 1826, scribbled in the margin on page 79 of volume 2 of Abel's "Paris Notebook" diary.

[2] Allen, B (2004). "Pupils' Perspectives on Learning Mathematics". In: Allen, B. and Johnstone-Wilder, S. (eds) *Mathematics Education: Exploring the Culture of Learning.* Oxford: Routledge. pages 233–242.

[3] Claxton, G. (1984). *Live and learn: An introduction to the psychology of growth and change in everyday life.* London: Harpercollins College Div (Harper & Row).

[4] Cuoco, A., Goldenberg, E. P., and Mark, J. (1996). "Habits of Mind: An Organizing Principle for Mathematics Curricula", *Journal of Mathematical Behavior*, **15**, pages 375–402.

[5] Davis, P. and Hersh, R. (1981). *The Mathematical Experience.* Boston: Birkhäuser.

[6] Ernest, P. (1995). *The Philosophy of Mathematics Education.* Hampshire: The Falmer Press.

[7] Ernest, P. (2000). "Why Teach Mathematics?" In J. White, S. Bramall (eds.) *Why Learn Maths?* London: London University Institute of Education. Page numbers not available.

[8] Ernest, P. (2004). "What is the Philosophy of Mathematics Education?" Available at: http://people.exeter.ac.uk/PErnest/pome18/PhoM_%20for_ICME_04.htm [Accessed 5 Oct 2010].

[9] Fauvel, J. (1991). "Using History in Mathematics Education", *For the Learning of Mathematics*, **11**, no. 2, pages 3–6.

[10] Fauvel, J. and Maanen, J. V. (eds) (2002). *History in Mathematics: The ICMI Study*. London: Kluwer Academic Publishers.

[11] Hewitt, D. (2001a). "Arbitrary and Necessary: Part 2 Assisting Memory", *For the Learning of Mathematics*, **21** no.1, pages 44–51.

[12] Hewitt, D. (2001b). "Arbitrary and Necessary: Part 3 Educating Awareness", *For the Learning of Mathematics*, **21**, no.2, pages 37–49.

[13] International Commission on Mathematics Instruction (ICMI) (1997) *ICMI Study on the Role of History of Mathematics in the Teaching and Learning of Mathematics: Discussion Document*. Available at: http://www.mat.uc.pt/~jaimecs/icmihm.html [Accessed 2 Mar 2011].

[14] Katz, V. J. (1986). "Using History in Teaching Mathematics", *For the Learning of Mathematics*, **6**, no.3, pages 13–19.

[15] Katz, V. J. (2007). "Stages in the history of algebra with implications for teaching", *Educational Studies in Mathematics*, **66**, pages 185–201.

[16] Keith, S. Z. (1990). "Writing for educational objectives in a calculus course". In A. Sterrett (Ed.), *Using Writing to Teach Mathematics* (pages 6–10). Washington, D.C.: Mathematical Association of America.

[17] Lakatos, I. (1976). *Proofs and Refutations*. Cambridge: Cambridge University Press.

[18] Laubenbacher, R. and Pengelley, D. (1999). *Mathematical Expeditions: Chronicles by the Explorers*. New York: Springer-Verlag.

[19] Laubenbacher, R. and Pengelley, D. (2010). "Teaching with Original Historical Sources in Mathematics". Available at: http://isis.nmsu.edu/~history/ [Accessed 2 Mar 2011].

[20] Laubenbacher, R., Pengelley, D. and Siddoway, M. (1994). "Recovering Motivation in Mathematics: Teaching with Original Sources", *UME Trends*, **6** (September): page numbers not available.

[21] Lockhart, P. (2002). "A Mathematician's Lament". Available at: `http://www.maa.org/devlin/LockhartsLament.pdf` [Accessed 5 Oct 2010].

[22] Martin, E. (1998). *Changing academic work*. Buckingham, SRHE/Open University Press.

[23] Martin, L. C. (2008). "Folding back and the dynamical growth of mathematical understanding: Elaborating the Pirie-Kieren Theory", *Journal of Mathematical Behavior*, **27**, pages 64–85.

[24] Nardi, E., and Steward, S. (2003). "Is Mathematics T.I.R.E.D.? A Profile of Quiet Disaffection in the Secondary Mathematics Classroom", *British Educational Research Journal*, **29**, no.3, pages 345–367.

[25] Office for Standards in Education (2008). *Mathematics: Understanding the score*. HMI: 070063. London.

[26] Pirie, S., Kieren, T. (1994a). "Beyond metaphor: Formalising in mathematical understanding within constructivist environments", *For the Learning of Mathematics*, **14** no.1, pages 39–43.

[27] Pirie, S., Kieren, T. (1994b). "Growth in mathematical understanding: How can we characterize it and how can we represent it?", *Educational Studies in Mathematics*, **26**, pages 165–190.

[28] Reimer, L. and Reimer, W. (1995). "Connecting Mathematics with its History: A Powerful, Practical Linkage". In: P. A. House and A. F. Coxford (eds) (1995) *Connecting Mathematics across the Curriculum* (pages 104–114), Yearbook of the National Council of Teachers of Mathematics, Reston, VA: National Council of Teachers of Mathematics.

[29] San Francisco State University Department of Mathematics. *Mathematical Quotes*. Available at: `http://math.sfsu.edu/beck/quotes.html` [Accessed 25 Mar 2011].

[30] Seife, C. (2000). *Zero: The Biography of a Dangerous Idea.* London: Penguin Books.

[31] Sfard, A. (1991). "On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin", *Educational Studies in Mathematics*, **22**, pages 1–36.

[32] Skemp, R. R. (1976). "Relational Understanding and Instrumental Understanding", *Mathematics Teaching*, **77**, pages 20–26.

[33] Siu, M. K. (1997). "The ABCD of Using History of Mathematics in the (Undergraduate) Classroom". Available at: `http://hkumath.hku.hk/~mks/ABCD.pdf` [Accessed 21 Feb 2011].

[34] Slaten, K. M. (2010). "Effective Folding Back via Student Research of the History of Mathematics". Available at: `http://sigmaa.maa.org/rume/crume2010/Archive/Slaten.pdf` [Accessed 18 Feb 2011].

[35] Steen, L. A. (1988). "The Science of Patterns", *Science*, **240**, no.4852, pages 611–616.

[36] Swetz, F., Fauvel, J., Bekken, O., Johansson, B., Katz, V. J. (Eds.). (1995). *Learn From the Masters*. Washington, DC: Mathematical Association of America.

[37] Tzanakis, C., Arcavi, A. (2000). "Integrating history of mathematics in the classroom: an analytic survey". In J. Fauvel, J. Van Maanen (Eds.), *History in mathematics education: the ICMI study* (pages 201–240). Dordrecht, The Netherlands: Kluwer Academic Publishers.

[38] The University of Arkansas (2011). *One Based Math – Bible Mathematics*. Available at: `http://130.184.5.79/wiki/index.php/One_Based_Math--Bible_Mathematics`. [Accessed 27 Mar 2011].

[39] The University of Manchester Faculty of Life Sciences (2010). *History of Mathematics*. Available at: `http://www.ls.manchester.ac.uk/undergraduate/courses/modules/module/?id=2099` [Accessed 22 Nov 2010].

[40] University of Oxford Mathematical Institute (2010). *Synopsis of History of Mathematics*. Available at: `http://www.maths.ox.ac.uk/courses/course/12552/synopsis` [Accessed 22 Nov 2010].

[41] University of St Andrews School of Mathematics and Statistics (2010). *Topics in the History of Mathematics*. Available at: http://www-maths.mcs.st-andrews.ac.uk/ug/hon4/MT4501.shtml [Accessed 22 Nov 2010].

[42] University of Warwick Department of Physics (2008). *Module Outlines*. Available at: http://www2.warwick.ac.uk/fac/sci/physics/archive/teach0708/mathsphys/year3b/modules/ [Accessed 22 Nov 2010].

[43] Wigley, A. (1992). "Models for Teaching Mathematics", *Mathematics Teaching*, **141**, December, pages 4–7.