On The Occasion Of Your Graduation

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Dear Madison:

I hardly know whether to start with congratulations on your news that you and Jeremy are expecting a baby, or on your successfully defended thesis. As you see, I have chosen modesty and placed first the accomplishment that I had nothing to do with, though I am fairly certain that your thesis represents far more effort. So far, anyway: my colleagues who have children tell me that they can be a lot of work too.

As your advisor, I feel guilty that my sabbatical leave has made me miss both your defense and your upcoming graduation. Twenty years ago, when air travel was affordable, I would have flown back from Paris for both occasions; today, even one round trip ticket in a year is ruinously expensive. By the same token, with gasoline at 20 euros per liter, I haven't seen as much of France outside of Paris as I had planned. I have seen Versailles, which is something to be sure, but not Avignon, Brittany, or Carcassonne. At seventy, I don't suppose that I ever shall, now.

Anyhow, Madison, I have something important to ask. I hesitate to lay this burden on you, but I feel that this decision should be made by a mathematician with her (or his) career still ahead – and by somebody with children, somebody whose interest in the future of the human race is less theoretical than mine.

As you may remember, three years ago, around the time that you started your doctorate (and also, if I recall, about the time you met Jeremy), I began a series of lectures on a new idea that I called "homotopical logic." In the first few seminars, I mentioned Gödel's second incompleteness theorem, the one that shows that the consistency of mathematics cannot be proved; and I showed how to extend Gödel's work by constructing a sequence of abstract homotopy groups representing the interactions between the underlying arithmetic and the encoded statements. At the end of the fourth seminar, I said that I expected a significant result within the week.

I never gave the next seminar, though, because my results seemed so absurd. At the level of the eighth homotopy group there was an obstruction, corresponding to an inconsistency in the fabric of mathematics. It was impossible to check directly; with eight levels of Gödel encoding involved, the inconsistent statement in question, expanded out in terms of elementary operations, would have been more than ten to the trillionth characters long! Indeed, I have since discovered a heuristic argument that suggests that without such an encoding trick, any sequence of deductions less than about ten thousand pages long should be consistent. Nobody's checkbook will fail to balance.

Over the next year, I checked everything in painful detail. Eventually I realized that I was right. This would be the mathematical result of the twenty-first century, maybe even of the third millennium. You can imagine my excitement as I started to type it up.

Halfway through the introduction, I paused. If I was indeed right, practically all mathematics since Pythagoras has been built upon sand. Constructivism would not save it; my proof avoids both the axiom of choice and the law of the excluded middle. It would be one of the few stones left standing in the ruins of the vast temple of mathematics.

No doubt somebody would eventually find a way of weakening the rules of inference, to stop the mathematical snake just short of choking on its own tail; but how many decades would we all have to spend taping the foundations of mathematics back together? And what beautiful results might not survive? Suddenly I felt as if I were poised to throw a brick through one of the stained glass windows of Chartres Cathedral. (I did get there, by the way; it's only eighty kilometers away, half a day by the new *miniélectriques interurbains*. The French call them *trains de grande paresse*, but at least they still run.)

The more I thought about it, the worse the situation seemed. The public may not realize it, but the human race has never needed mathematics more urgently than today. Solar energy, wind farms, tidal power: none of these has done more than blunt the edge of the world's energy famine. The only chance your children have (forgive me for mentioning this at what should be a joyful time for you, but I must be honest) of lives anything like the one you enjoy – let alone what I grew up with – lies in the solution of very difficult problems such as controlled fusion, steady-state economics, climate control, and global ecology.

Perhaps, with rigorous proof temporarily unattainable, the mathematical community might leave abstraction to a less desperate generation, and throw its efforts into applied problems where the only "proof" is what works in practice. But is it not just as likely that there could be a massive loss of heart, with the bright young minds we need so badly deciding not to waste their lives on a broken system?

And what about the public? Most people today are poorer, by any reasonable criterion, than their parents were; and they face the unpleasant prospect that their children may be poorer still. They are in no mood to waste money. The average voter won't follow the subtleties of my construction; but it can be *vulgarisée* (as they say here) easily, and Internet headlines saying "Mathematics Proved Wrong" would not be quickly forgotten. And what chance of funding would any mathematical research, no matter how vital to our survival, have then?

My dear Madison – my last doctoral student – in three years I have reached no answer to these questions. Perhaps they are not mine to answer. I have little research left ahead of me, and no children, no hostages to fortune. Attached you will find the manuscript of my still-unpublished paper, and a cover letter to the editor of the *Annals*. If, after deliberation, you think that it should be published, please submit it on my behalf. Otherwise delete it (I have not kept a copy), and try very hard to forget everything I've told you. You may speak to Jeremy about this – his background in economics and engineering should prove valuable – but, please, to nobody else.

Avec toute mon affection to you, Jeremy, (and Epsilon!) from

Your proud advisor,

Kenneth