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Book Review: Across the Board: The Mathematics of Chessboard Problems by John J. Watkins

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Review of *Across the Board: The Mathematics of Chessboard Problems* by John J. Watkins, Princeton University Press, 2004.

I think I became a mathematician because I loved to play games as a child. I learned about probability and expectation by playing games like backgammon, bridge, and Risk. But I experienced the greater thrill of careful deductive reasoning through games like Mastermind and chess. In fact, for many years I took the game of chess quite seriously and played in many tournaments. But I gave up the game when I started college and turned my attention to more serious pursuits, like learning real mathematics.

So when I first picked up a copy of *Across the Board: The Mathematics of Chessboard Problems,* I was pleasantly surprised. I think I expected to find mathematical analysis of questions like "How should White play to mate as quickly as possible?" or "What sequence of moves produced this sequence of positions?", as were effectively described in the recent *Intelligencer article by Elkies and Stanley [1]. Instead, what I found was a delightfully written book on "real mathematics," loaded with theorems with elegant proofs, directed at problems that arise on the chessboard. Aside from knowing how each chess piece moves, the reader does not need to know how to play chess nor any of its notation.*

The book begins with a discussion of knight's tours whereby a chess knight visits every square on a chessboard exactly once, beginning and ending the tour at the same square. Although knight's tours exist on an 8*-by-*8 *chessboard, they do not exist for boards of all sizes. For instance, it is easy to see that a knight's tour is impossible when* m *and* n *are both odd, since the color of the square alternates at each move; the knight could not return to its original square in* mn *moves. Knight's tours are clearly impossible for* 1*-by-*n *and* 2*-by-*n *boards and by colorful arguments can be proved impossible for boards of size* 4*-by-*n*,* 3*-by-*6*, and* 3*-by-*8*. The book then outlines a proof of "Schwenk's Theorem" which says that all rectangular chessboards have a knight's tour, except for the aforementioned cases.*

Since knight moves alternate colors, then on an 8*-by-*8 *chessboard, we could place* 32 *knights on all the black squares or all of the white squares, and no two knights will attack each other. No other arrangement of* 32 *or more knights is possible, for such an arrangement would necessarily use two adjacent squares in a knight's tour. In general, it can be shown that the maximum number of "independent" knights on an* n*-by-*n *chessboard is* $\lceil n^2/2 \rceil$ *(where* $\lceil x \rceil$ *denotes the ceiling of* x*).*

How many non-attacking bishops can be placed on an n*-by-*n *board? Since there are* $2n-1$ *positively sloping diagonals, including the length-one diagonals of the lower right and upper left square, there can be at most* $2n-2$ *such bishops. One way to achieve this is by placing bishops on every square of the first and last row, except for the rightmost squares. Watkins goes on to prove that there are exactly* 2^n *ways to accomplish this, and all of them must have the bishops only occupy the outer ring of the chessboard.*

In a similarly thorough fashion, the book also addresses the related problems

of "domination," such as how many kings can be placed on an n-by-n board so that every square is either occupied or attacked. (Here the answer is $\lceil n/3 \rceil^2$.) As with the independence questions, the book provides satisfying answers to domination questions for all pieces, and on a variety of surfaces, including toroidal chessboards and "Klein bottle" boards.

I found Watkins's style of writing very engaging, as one would expect for a book on recreational mathematics. Each chapter begins with some easy results and builds gradually in sophistication, much like a Martin Gardner article, culminating in numerous open problems suitable for exploration by undergraduates. Indeed, many of the theorems in the book arose from joint work of the author and several undergraduates. I frequently found myself earmarking theorems and problems that could be used as interesting homework questions for the next time I teach discrete mathematics or graph theory.

So at the risk of offending "rank and file" mathematicians, I would say that if you are looKING for a book that can capture your imagination or igKNIGHT a passion for discrete mathematics, then buying a copy of Across the Board *would be a great move. I guarantee that these "board problems" will not lead to "bored readers."*

References

[1] N. D. Elkies and R. P. Stanley, The Mathematical Knight, *Math. Intelligencer,* **25** *(2003), 22–34.*

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