

2011

# Ganas and the Swan: American Materialism in Mathematics Education

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## Recommended Citation

Berliant, Taylor, "Ganas and the Swan: American Materialism in Mathematics Education" (2011). *CMC Senior Theses*. Paper 167.  
[http://scholarship.claremont.edu/cmc\\_theses/167](http://scholarship.claremont.edu/cmc_theses/167)

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**CLAREMONT McKENNA COLLEGE**  
**GANAS AND THE SWAN:**  
**AMERICAN MATERIALISM IN MATHEMATICS EDUCATION**

SUBMITTED TO  
PROFESSOR ROBERT VALENZA  
AND  
DEAN GREGORY HESS  
BY  
TAYLOR R. BERLIANT

FOR SENIOR THESIS  
FALL 2010 - SPRING 2011  
APRIL 25, 2011



## **Acknowledgements**

I do not have the words to express how much the love, support, guidance, and encouragement meant to me that made this possible. In breaks during my writing I have literally sat in silence just thinking about how lucky I am to have so many wonderful people in my life.

To Professor Valenza, the time spent in discussion was indispensable in giving this thesis life. You guided me when I needed direction, and listened to me as I found my voice.

To my mom, dad, and sister, you have given me so much love and support over the years and during the time writing my thesis.

I hope you all know how appreciative I am beyond the words I can find to express it. Thank you.

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*I remember doing timed multiplication tables in the fourth grade. That is my earliest recollection of being good at math. I never finished first – Jon Nichols always did – but I was top 5 every time. My teacher advised me that the problems were in columns and it was most time efficient to go top to bottom for the odd columns and bottom to top for the even columns so that time was not wasted refocusing from the bottom of the page to the top.*

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Math is a unique subject. It is a different type of thinking for many people. Even though topics in math are separate, math is seen as a whole unlike many other subjects. Science, literature, history, art, languages, and physical education, the subjects, in addition to math, that make up the typical high school curriculum, are hardly ever referred to without additional labeling. Many of the subjects even contain topics with nearly incomparable material.

The idea of math as a whole, at least in high school, is not completely untrue. A lot of the topics rely on the knowledge of previous material. Also, quantitative thinking, which is the primary way of thinking in math, is believed to be a genetic predisposition, thus those people who are considered math people, possess that quality in any topic. While the subject material builds off earlier topics, the concepts at each level tend to be their own. And even though there is a more apparent range of inherent math ability, it does not need to have a bearing on mathematical success.

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*I remember on the last day of class in fifth grade, Teddy Meeks put up an algebra problem on the board that his older sister had taught him. I was not able to solve the equation  $(x - 1 = 0)$  and was confused why there were letters in my math. It was frustrating because I had thought of myself as one of the best math students in my class. Even when Ms. W. explained that you add 1 to both sides, it still did not make sense conceptually to me.*

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The focus of this paper is algebra. The range of math ability is noticeable in prior classes, but algebra is when separation occurs that can define the student. Algebra begins the conversation that math is more than just numbers and generally more than just the rules, theorems, and formulas that pertain to a topic. Algebra is the gateway to the idea that education is an organism, dependent on each of its organs. But even when it does not directly serve the larger organism, it possesses its own beauty.

There are two aspects to algebra. There is course material. These are the formulas and theorems that are classified by the use of variables to solve math equations. There are inequalities, systems of equations, and specific formulas that students learn to solve the variable equations. But algebra offers more: it is a way of looking at the world. The way letters generalize numbers can be applied to the world. Algebra takes a problem and breaks it down to a form that can be manipulated because the student has the necessary information and the appropriate tools. The problem is solved in its new form and that answer is put back into the initial problem. This describes solving problems in algebra,

but it also describes a way of approaching situations in other subjects of education and the world in general.

For the purpose of this paper, I am going to use the term *algebraic thought* to describe this mode of thinking. It can also be thought of as an umbrella term to define logic, problem solving, and pattern recognition. It has a similar flavor to critical thinking, but critical thinking is popularly associated with the humanities. I thought a different term was worth using, and even though I believe the two are almost the same, it is not the intent of this thesis to explore that similarity in greater depth than with what will be addressed.

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*I remember math journals in my eighth grade math class. I do not remember the math I learned, or anything I wrote in my math journals. Once I wrote a math journal entry at my desk in the five minutes before class started. I do know I thought the math journals were a stupid waste of time.*

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This paper will explore the nature of math and math education. True understanding of how it is taught and how it is learned cannot come without the necessary aspects of math. The clichés of math are not literally true, but are founded in truth; and these truths need to be revealed. These notions blend with the focus of education and the faculty of the students to help understand the state of math education.

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*I remember taking an Algebra II class during my sophomore year of high school. I think back to all the kids in my class who struggled with the material, hated the teacher, hated math, and had conceded a life of mathematical mediocrity because they were not math people. I had a mind for math, and I cannot count the hours I spent tutoring classmates in random basements and bedrooms the night before tests. They were grateful that I was spending so much time helping them, but I would always reassure them that it helped me study to make sure I knew and communicate the different topics.*

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Math education has not reached its full potential. For the most part, the focus of math education is in the material. In all subjects, the students “have got to be made to feel that they are studying something, and are not merely executing intellectual minuets.”<sup>1</sup> In the current standing of other subjects, this is more often practiced and achieved – the notion of presenting material without further inquiry is even considered unacceptable, but is condoned in math. It is believed that only learning the material in math will suffice to serve its purpose in the current education system, but “The goal should be, not an aimless accumulation of special mathematical theorems, but the final recognition that the preceding years of work have illustrated those relations of number, and of quantity, and of space, which are of fundamental importance.”<sup>2</sup>

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<sup>1</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 15.

<sup>2</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 121.

**""You know, Kurt; there's nothing like a visit from the Inspector General once in a while to keep things in line.""  
-Colonel Harris, *The Spectre General*, Theodore Cogswell**

The first question that must be asked is: why is education important for society? The Department of Education is career focused. As stated on their website, "The mission of the Department of Education is to promote student achievement and preparation for global competitiveness by fostering educational excellence and ensuring equal access."<sup>3</sup> Students go to school to prepare themselves for life beyond school. But according to the Department of Education, life beyond school is about global competitiveness. This competitiveness is in the job market and in the quantity and quality of discoveries and production that can be associated with a specific country. It is an arms race of professionals. Obama introduced his discussion of education during his 2011 State of the Union address with the idea that "Maintaining our leadership in research and technology is crucial to America's success. But if we want to win the future – if we want innovation to produce jobs in America and not overseas – then we also have to win the race to educate our kids."<sup>4</sup> I do not think Barack Obama is the source of America's global competitiveness career focus, but I would have rather heard him address the unemployment rate or the average salary of high school dropouts. These are the results of education that affect the individual in society. The government should serve the people

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<sup>3</sup> "What We Do -- ED.gov." (U.S. Department of Education), <<http://www2.ed.gov/about/what-we-do.html>>. 22 Apr. 2011.

<sup>4</sup> "State of the Union: Education Excerpts – ED.gov Blog." (U.S. Department of Education), <<http://www.ed.gov/blog/2011/01/state-of-the-union-education-excerpts/>>. 22 Apr. 2011.

beyond simply providing a society to live it. The government should benefit the individual's personal gain. Career choices should benefit society, but as a consequence of benefitting the individual. Obama does continue to say that "over the next 10 years, nearly half of all new jobs will require education that goes beyond a high school education. And yet, as many as a quarter of our students aren't even finishing high school. The quality of our math and science education lags behind many other nations."<sup>5</sup> But again, this does not address the quality of life of the individuals, but rather, the competitive success as a country.

While the government seems to focus on career development for the purpose of global competitiveness, career development is also beneficial to the individual. Beyond the idea that living in a more successful society leads to more happiness, the individual pursuit of happiness is aided by education. Without digressing into a philosophical discussion of happiness, for the purposes of this paper, happiness is the pursuing of passions and the feeling of satisfaction with one's lifework. Education aids this happiness in many ways.

The first step of pursuing passions is the discovery of passions. In a well-rounded education, students are exposed to a multitude of subjects. If not initially enamored with one, they are given the opportunity to explore those subjects deeper until they choose to specialize. Once specialization occurs, the education system gives students the

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<sup>5</sup> "What We Do -- ED.gov." (U.S. Department of Education), <<http://www2.ed.gov/about/what-we-do.html>>. 22 Apr. 2011.

opportunity to explore even deeper. The student will be able to go as far as they can in a given subject and will go through the proper preparation to pursue their passions beyond academia in the professional world.

Through education, students are also relieved of burdens that restrict the ability to pursue their passions. The main blockade is the financial restriction. In an ideal world, everyone would be able to find careers that they are passionate about and that came with comfortable salaries. Sadly, this is not the case. There are some passions, (e.g. music, art), where salaries are not livable or not guaranteed, and there are some passions, (e.g. family, fantasy sports), where careers are not available under normal circumstances. In those instances, education offers career opportunities in other fields to support someone financially that chooses to pursue those passions.

In a similar way, education helps to create satisfaction in one's lifework. A way to define success is by the amount of time and effort one was able to put in to pursuing their passions, which we just determined is facilitated by education. Again without going into a philosophical discussion of satisfaction, helping one's family or bettering society through a career can be satisfying. This satisfaction includes creative expression. Education aids career placement, but it also gives the student a sense of discovery to properly express themselves.

The next question to ask is: how does math aid the educational goal of career development? Math-based professions are relatively few in the spectrum of careers, which creates an initial concern of wasted education because there are more students than jobs; however, those jobs that involve math are the most important for moving society

forward. It is important to note that math-based professions are broader than mathematicians and math professors. The end result of mathematicians is not to drive society forward. While mathematic advances can be applied to other fields where the knowledge can be used for innovation, the mathematicians do not only engage in discovery for that purpose. Moreover, not all math-based professions have monumental impact. But curing cancer, advancing technology, managing the economy, sending men further into space, these will be achieved in math-based professions. These kinds of discoveries improve quality of life, create jobs, and lead to even more innovation on a large scale. Even when discoveries of that quality are not made, these math-based professions are still of major value to society. Below, I will attempt to explain which careers are, in my opinion, math-based, and justify their importance to society.

The first math-based profession to consider is a teacher. I believe that all teaching positions through secondary education should be math-based, by which I mean that those teachers should be proficient through high school math. Clearly, math teachers need to be proficient in the math they teach and the math leading up to their subject, but because of the emphasis on advancement, teachers in subjects before calculus cannot stop at their subject. Teachers need to excite students about future math so students want to continue in math. The continuation of math education throughout different topics and years will be covered in greater depth in a later section.

It is more abstract to think that a non-math teacher needs to be mathematically proficient. But a teacher should be more than the translator of a textbook. Along with sharing the knowledge of their subjects with students, teachers are role models. Teachers

are (sometimes the only) liaisons between students and the academic world and even the intellectual world as a whole. A teacher should aim to inspire a student to fully explore those worlds. By assuming these roles, a teacher has the power and influence to tell his students that all subjects are important. That there is value in the material, and there is also value in the struggle. A teacher who has not explored and succeeded in all subjects himself does not have the credibility to encourage his students to do the same. By the same logic, non-math teachers will have more power to encourage students to succeed in their own classrooms. An English teacher who is not proficient in math, as previously defined, has not succeeded in all areas of his education. As a role model, he cannot completely expect his students to do the same. If he confronts a student who is struggling or has given up in his class, the student is partly justified to believe that because his teacher did not finish his math education, it is not necessary for the student to complete his English education. I support this claim in all areas of the high school curriculum, that is, math teachers need a full proficiency of other subjects as well; however, for the nature of math (which will be explained in a later section) and the purposes of the paper, the proficiency in math needs to be emphasized.

While proficiency is necessary for lower level teachers, the same is not so for collegiate and graduate level professors. Part of the job for younger age level teachers is to foster an environment of learning, which is supposed to carry on into the collegiate world. For many, college is when specialization occurs. For these students, the priority is exploration of a single subject in a higher volume and intensity. Professors are not expected to encourage general excitement of learning, but rather, are supposed to help

students excel in a specific area of their education. This is not to say that professors are incapable or unwilling to do the former, but it is not necessary in order to be a successful professor.

Another field that is math-based is science. Math material is not necessary for all science, nor is it necessary for all scientific success. Excellence in math is not required to be good at science either – I know many science majors who fit this description – but it helps. The highest tier of scientists will have the ability to be proficient in math, even in the disciplines without direct connection to mathematical material – Freud and Darwin were both outstanding in math.<sup>67</sup> And the material can assist the highest level of scientists in fields that require it. And, accidental discoveries aside, it is the top scientists who are going to make those discoveries that will change the world in a significant way. Albert Einstein is not a mathematician. It was not his profession and he personally denied it. He declared, “Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.”<sup>8</sup> His lack of mathematical proficiency is relative, but in absolute terms, he was quite advanced. In order to complete his work on theories of gravity, he needed to familiarize himself with non-Euclidean geometry. More specifically, “geometry studied by Riemann, in which all triangles have angle sums *greater* than 180 degrees. This geometry is called, ‘elliptic,’ or the geometry of a surface with ‘positive’ curvature, like a

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<sup>6</sup> Anthony Storr. Freud (Oxford: Oxford UP, 1989).

<sup>7</sup> Adrian J. Desmond and James R. Moore. Darwin (New York: Warner, 1991).

<sup>8</sup> "Your Quotes - BrainyQuote." (Famous Quotes at BrainyQuote), <<http://www.brainyquote.com/quotes/keywords/your.html>>. 22 Apr. 2011.

sphere.”<sup>9</sup> This is rather advanced for a non-mathematician, but through some combination of intelligence and perseverance, Einstein recognized it as an indispensable part of his ideas, and was able to master it to a point of use.

To his old gravitational equation he added a new term – called the “cosmological term” – characterized by a new small constant (in addition to the gravitational constant). This ‘cosmological constant’ does not modify the old predictions of the theory that have to do with “local” phenomena involving the solar system. Einstein was able, with the help of mathematician J. Grommer, to find a solution to these equations.<sup>10</sup>

It is nothing sort of extraordinary that Einstein was able to thrive mathematically, but that is what is necessary for extraordinary results. Elite scientists will know math or have the mental capacity to learn it, even if they do not consider themselves mathematicians.

The same is true for economists. Economics is a field that has the power to significantly alter society. While not the sole variable, I imagine that miscalculating equations and statistics undoubtedly played a role in the most recent financial crisis. It is apparent how many lives it affected, and how much impact the economic state has on the day-to-day life of an individual in our country. It is a field that is math-based. Like scientists, economists do not need to be brilliant mathematicians to be successful, some may even argue against it, but the major developments in economics, which have the power to significant affect society, will require mathematics.

Not much needs to be said in regards to engineering. It is perhaps the easiest field to see innovation on a day-to-day scale, and also the easiest to see its application of math.

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<sup>9</sup> Jeremy Bernstein. Einstein( New York: Viking, 1973), 148.

<sup>10</sup> Jeremy Bernstein. Einstein( New York: Viking, 1973), 156.

Just consider the shower and conventional oven. Engineering has value in agriculture, which is perhaps the most important field for societal development, in addition to its value as a catalyst of innovations in infrastructure and the space program.

The government focuses on career development in mathematics because these professions benefit society in a large way. It is important to have an end goal in mind, but it is potentially harmful to be too future oriented if it results in presenting a subject as a merely a stepping-stone to further education or career. By stepping-stone, I mean that the objective of the class is to acquire knowledge of the necessary material to get to the next step. Upper-level mathematics and the professions above require some knowledge of previous material to be successful. Truly, the material and thought are both necessary for the future, but generally, if an education system is future oriented it focuses on the material needed. It is actually when a system focuses on both the material and the thought that it puts the student in the best position for career development and overall wellbeing. The idea of mathematics providing something more than just career development for math-based professions will be addressed in a later section.

America is on the edge. The focus of the 2011 State of Union, in regards to education, was on advancing in global competitiveness. Obama remarked, “If we take these steps – if we raise expectations for every child, and give them the best possible chance at an education, from the day they are born until the last job they take – we will reach the goal that I set two years ago: By the end of the decade, America will once again

have the highest proportion of college graduates in the world.”<sup>11</sup> I do not think this is a bad goal, but it highlights America’s focus for education. The end goal is college graduation, but as a comparative measure against other countries. We need to learn algebra because if we do not learn algebra we will not be able to graduate college. However, advancing in math as a society will come when it becomes more than advancement to satisfy global competitiveness. When algebra is reduced to a stepping-stone for career development, it deters that development. Whitehead notes, “Education is the acquisition of the art of the utilisation of knowledge,” and when math is treated simply as material for future math, it will not be utilized in academia or in professional life.<sup>12</sup>

The current Secretary of Education, Arne Duncan, does have a more effective idea of career development. He believes, “We have to do more than read and math, we have to give our children access to a well-rounded education. And when we do that, I think we give teachers much more room to innovate and be creative.”<sup>13</sup> He understands that mastery of the material is not enough to excel in that subject. Education needs to revolve around the application of material. It is equally if not more important to know when to use the quadratic formula as it is to know the quadratic formula. What is taught

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<sup>11</sup> "State of the Union: Education Excerpts – ED.gov Blog." (U.S. Department of Education), <<http://www.ed.gov/blog/2011/01/state-of-the-union-education-excerpts/>>. 22 Apr. 2011.

<sup>12</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 6.

<sup>13</sup> "Secretary Duncan on Supporting Teachers, Improving Teacher Evaluation, and Providing a Well-Rounded Education – ED.gov Blog." (U.S. Department of Education), <<http://www.ed.gov/blog/2011/01/secretary-duncan-on-supporting-teachers-improving-teacher-evaluation-and-providing-a-well-rounded-education/>>. 22 Apr. 2011.

in schools should be how knowledge is applied, instead of knowledge in itself. This is especially true for math, and in particular, it is part of creating a proper approach to math education.

**“People who talk in metaphors oughta shampoo my crotch.”**  
**-Melvin Udall, *As Good As It Gets***

Algebra is often applied to word problems. Nearly just as often, these word problems are inapplicable to students' lives. Furthermore, if the problem does have real world value, where it can be applied, it will not often be applied with the methods learned in class. Consider the classic word problem of a train leaving the station: If a train leaves Station A for Station B, which is  $x$  miles away and is traveling at  $y$  miles per hour, when will it arrive at Station B? A student will try to translate this sentence into the language of math in equation form. The equation will look something like  $T = I + x/y$ , where  $I$  is the initial time and  $T$  is the final time. Then, since  $I$ ,  $x$ , and  $y$  are given in the question prompt, the student is left with a single-variable equation and can solve for  $T$ . There are two questions that need to be asked. First, why not give the students a single-variable equation to solve without having to evaluate an expression? Second, why use this subject material in the problem.

The first question has a clear answer. Application of math is important. Regardless of the level of mastery one has in a math discipline, if students are unable to recognize the situation when it is needed, they will not be able to take advantage of it. It goes back to the Department of Education's mission to prepare students for the future;

students do not need to learn to be good students, students need to learn to be good after that. It is not important to excel at solving single-variable equations, it is important to excel at recognizing real world problems and then being able to solve them.

The second question is a bit more complicated. It has already been established that real world application is important, but why a train? Even though it is an ultimately inadequate answer to understand why trains were used, it is worth mentioning that, in general, children like trains so the problem engages them on a recreational level. Beyond that, trains are common, so the teacher will not have to waste time explaining the specifics that are unimportant to the subject material. Finally, and perhaps most valuably, trains suit the material. Trains commonly travel at constant speed in a single direction, at least more so than other vehicles, and since conductors operate trains, children will not miss the subject material in the problem by adding individual nuances. Also, this abstraction from the subject of the word problem maintains the abstract nature of algebra. I would not remember reading *The Trumpet of the Swan* in fourth grade if not for one passage:

“Sam, if a man can walk three miles in one hour, how many miles can he walk in four hours?”

“It would depend on how tired he got after the first hour,” replied Sam.

The other pupils roared. Miss Snug rapped for order.

“Sam is quite right,” she said. “I never looked at the problem that way before. I always supposed that man could walk twelve miles in four hours, but Sam may be right: that man may not feel so spunky after the first hour. He may drag his feet. He may slow up.”

Albert Bigelow raised his hand. “My father knew a man who tried to walk twelve miles, and he died of heart failure,” said Albert.

“Goodness!” said the teacher. “I suppose that could happen, too.”

“Anything can happen in four hours,” said Sam. “A man might develop a blister on his heel. Or he might find some berries growing along

the road and stop to pick them. That would slow him up even if he wasn't tired or didn't have a blister."

"It would indeed," agreed the teacher. "Well, children, I think we have all learned a great deal about arithmetic this morning, thanks to Sam Beaver. And now, here is a problem for one of the girls in the room. If you are feeding a baby from a bottle, and you give the baby eight ounces of milk in one feeding, how many ounces of milk would the baby drink in two feedings?"

Linda Staples raised her hand.

"About fifteen ounces," she said.

"Why is that?" asked Miss Snug. "Why wouldn't the baby drink sixteen ounces?"

"Because he spills a little each time," said Linda. "It runs out of the corners of his mouth and gets on his mother's apron."

By this time the class was howling so loudly the arithmetic lesson had to be abandoned. But everyone had learned how careful you have to be when dealing with figures.<sup>14</sup>

The line that stands out to me the most is when Miss Snug says, "I think we have all learned a great deal about arithmetic this morning."<sup>15</sup> No they have not. This passage and Sam's answer has nothing to do with arithmetic, but everything to do with the shortcomings of the word problem. The arithmetic and algebraic form of the problem is: if  $3/1=x/4$ , solve for  $x$ . The answer is 12. The answer is not: "It would depend..."<sup>16</sup> Sam is avoiding the subject material by adding a personal nuance to the problem. Because everybody walks and has the experience of walking for a long distance, they can imagine the word problem's scenario without the subject material. Because no student has ever conducted a train, they are unable to add unnecessary details to the problem.

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<sup>14</sup> E.B. White, *Trumpet of the Swan* (New York: HarperCollinsPublishers, 2000), 75.

<sup>15</sup> E.B. White, *Trumpet of the Swan* (New York: HarperCollinsPublishers, 2000), 76.

<sup>16</sup> E.B. White, *Trumpet of the Swan* (New York: HarperCollinsPublishers, 2000), 76.

While there is certainly a place to consider confounding variables and individual circumstances, a simple applied math problem is not it. So the more variables that are controlled in the problem, like riding a train instead of some form of self-transportation, the better suited it is to teach the intended concepts. If the situation is vague and the student is trying to avoid the question or express himself creatively, the student will not learn the material. In the second word problem, Linda has clearly gone out of her way to try to match Sam's creativity. She understands that the "correct" answer to the problem is sixteen ounces, but decides to say fifteen ounces and add her own personal touch to the prompt. It seems as though Linda understands the material, but she is causing unnecessary confusion for those students who do not understand the algebraic concepts as well as wasting class time to pursue the course material.

It is important to mention again that what is gained from the modes of thought used in word problems should not be substituted for the mathematical subject matter. There are examples of algebraic thought that neglect the subject matter of algebra. One such example is the Rubik's cube, which is known as an example in group theory, which is a topic in abstract algebra. The way a Rubik's cube can be solved is algebraic thought because the restoration of the front face (real life problem) is solved on the back face (useable tool) and then implemented back onto the front face. It is the same ideology of solving a word problem by translating it into a mathematical equation and then reapplying it to the specific problem. It is not algebra, however, because the Rubik's cube is not solved with variable equations and thus would not be seen in an algebra curriculum.

Now consider the train problem. When initially introduced, the useable information was the speed of the train and the distance from Station A to Station B. The problem was then translated into a variable equation where the information could be applied. Thus, algebraic subject matter was used. The train problem could be prompted differently to avoid the need of algebraic subject matter; that is, it can be solved with algebraic thought that does not consider variable equations. The obvious way to find out when the train arrives at Station B is to look at the train schedule. That is algebraic thought because I have a problem, assess the problem using the tools I know, and reapply the problem.

This is a weak example of algebraic thought in the idea of translation and reapplication, but it highlights the importance of knowing what tools and information are available. When a student analyzes a problem, they need to distinguish the useful information and tools. In the initial prompt, the useful information is the speed, distance, and their relationship to time. Without those three things, the problem is unsolvable. Although looking at a train schedule might be considered common sense and not algebraic thought, knowing what information is available and useful is just as much a part of algebraic thought as translating and reapplying problems. Certainly algebra is not trying to replace train schedules, but the problem sets up a nice prompt to learn the material and thought of algebra.

Another classic word problem and one that is most effectively solved with variable equations is computing interest on an initial payment. There is an equation, given information, and a variable that can be found. But there is a problem in its actual

application. Even though in the real world I could, I would not try to solve an interest problem by hand. I would plug in my given information into Microsoft Excel and it would produce the answer faster and with less error than I could by hand. But again, that is not algebra. If I saw a student in algebra class open up a computer and give the answer by plugging it in to Excel, I would not give them full credit on the question because they did not use the subject material.

There is still value in word problems that are not applicable. It relates to the initial idea that creating a fun or interesting situation for students to use math in more stimulating than just serving them equations to solve. While it is not the most efficient method, if I want to know how long it takes a train to go from Station A to Station B using algebra, I would first need to establish what information and tools I need to solve the problem. What is mathematical time? Distance is equal to rate multiplied by time, so time is equal to distance divided by rate. That means I need to know the distance and the rate, which are given in the problem. I have been able to dissect the word problem and break it down into tools given in the subject of algebra. I have used those tools and the information given to solve the problem and then reapply it into the form of the initial problem. While that may not be the most efficient means of solving the problem, the student uses the appropriate subject material to solve the problem.

Amongst the classes at Harvey Mudd College, there are a few that have a reputation for their bizarre word problems. In first-semester relativity class, there is an exam question that is prompted with: The sun explodes. Two ships leave Earth traveling in the same direction at two different speeds. The slower ship loses its engines and has a

given amount of time before life support runs out. They send a message ahead to the faster ship, which sends back a rescue shuttle at a given speed. The students had to solve whether the rescue shuttle got to the slow ship before life support ran out. In second-semester mechanics, there is a problem that has a prompt involving a pig firing a gun in outer space. There is another from the class in which a greased pig slides down the outside surface of a dome.

These are inapplicable because they are impossible situations, but still are effective prompts to word problems. The first reason is that they balance generality and specificity. Because they are specific problems, they are easier to visualize. If the first problem were just to say: two objects moving in the same direction, it would be harder to grasp the idea of what is going on. Similarly with the pigs, it is easy to visualize what is happening in the problem. It is important to make sure that a problem is not too farfetched that it is inapplicable. If it is too general, the effectiveness of the word problem to relate the material to real life is lost. Whitehead remarks, “Passing now to the scientific and logical side of education, we remember that here also ideas which are not utilised are positively harmful. By utilising an idea, I mean relating it to that stream, compounded of sense perceptions, feelings, hopes, desires, and of mental activities adjusting thought to thought, which forms our life.”<sup>17</sup> It is important to note that Whitehead does not say the only way to utilize an idea is through word problems. Word problems are effective for another reason, which was mentioned earlier: they are fun, or at least more fun than straight equations. Word problems can engage students in a way that does feel like

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<sup>17</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 4.

traditional learning. If more methods of learning are presented and disguised as fun, the more likely the material will be successfully understood by the student. However, it is not guaranteed that a student will find word problems, or any type of learning, fun. The Harvey Mudd prompts do this very well. That stated, word problems can be effective at utilizing an idea, but when they serve other purposes, it is necessary for that utilization to occur in other elements of the course.

One way that material can be made real for the students is with appropriate conversation. Despite its misrepresentation of arithmetic, the passage from *The Trumpet of the Swan* does offer a lot along the lines of education. The teacher is doing a good job of getting people engaged. Both Albert and Linda were excited to talk in class, which is an important objective in cultivating an effective learning environment.

I have been riffling through the textbook, *Beginning Algebra: Seventh Edition*, written for college students who have not had a sufficient introduction to algebra. It prefaces itself explaining that it is for students who “require further review before taking additional courses in mathematics, science, business, or computer science.”<sup>18</sup> The textbook uses algebra for its application to the real world and professional development. There are little sections in each chapter called “Connections” that show real world application for algebra. These are worth exploring.

Because the first chapter is a review section, our first “Connection” comes from the title page of the second chapter, *Solving Equations and Inequalities*: “The use of

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<sup>18</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), vii.

algebra to solve equations and applied problems is very old. The 3600-year-old Rhind Papyrus includes the following ‘world problem.’ ‘Aha, its whole, its seventh, it makes 19.’ This brief sentence describes the equation  $x + x/7 = 19$ .<sup>19</sup>

It goes on to talk about the origin of the word algebra. Certainly the origins of algebra are worth knowing, but the first “connection” to the subject that these students need for their future is an amusing word problem. These students do not care that algebra was originally ancient riddles. For whatever reason, the students’ secondary education failed to give them the proper knowledge, but because they understand the importance of algebra for their development, they are trying again. They are there for the sole purpose of advancing to other subjects that require algebra. This “connection” fails to encourage that purpose.

The next connection is not much better. It reads, “After completing this section you will be able to solve linear equations algebraically.”<sup>20</sup> This is not quite a connection to an application in life; it just restates what the student is supposed to learn in the section. A proper connection would explain the value of the material. It would tell why linear equations are important for professional life as well as personal enrichment. Obviously, in the section on linear equations, students would assume that they were going to learn linear equations. This shows the disconnect between the material and its

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<sup>19</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 81.

<sup>20</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 88.

application. A student wants to know why they are learning what they are learning. Or, at least have faith that there is a reason. With this “connection,” not only is the student without a reason why he is learning linear equations, he is to assume there is no reason because the book, the authority on the subject, cannot give an example of a real world application in the allotted space for real world applications. Especially for the student using this specific textbook, whose sole purpose for learning algebra is its application to other subjects, there is a need to know why the material is worth learning other than that it exists and can be categorized in the subject of algebra.

Finally, on the section of applications of linear equations, the “connection” gives a sense of application and introduced algebraic thought:

The purpose of algebra is to solve real problems. Since such problems are stated in words, not mathematical symbols, the first step in solving them is to translate the problem into one of more mathematical statements. This is the hardest step for most people. George Polya (1888-1985), a native of Budapest, Hungary, wrote the modern classic *How to Solve It*. In this book he proposed a four-step process for solving a problem:

1. Understand the problem
2. Devise a plan.
3. Carry out the plan.
4. Look back and check.<sup>21</sup>

Algebra solves real world problems. Even when taught specifically for other subjects, it has more worth than just the course material.

The next “connection” is the first applied problem and is a great follow-up to George Polya’s four-step process:

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<sup>21</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 103.

In order to solve some applied problems, we can use a ready-made equation, called a formula. For example, the U.S. Postal Service requires that any box sent through the mail have length plus girth (distance around) totaling no more than 108 inches. The maximum volume is obtained if the box has dimensions 18 inches by 18 inches by 72 inches. What is the maximum volume? The volume of a box (a rectangular solid) is  $V = LWH$ , where  $L$  is its length,  $W$  is its width, and  $H$  is its height.<sup>22</sup>

Let us use Polya's four-step process to solve this problem.

We first need to understand the problem. It is clearly stated in the second to last line: "What is the maximum volume [of a U.S. Postal Service box]."<sup>23</sup> Now we must devise a plan to solve this problem. The first line of the "connection" enlightens us that there are ready-made equations we can use. In this instance, the equation is given in the last line. We know that the volume of a box is the product of its length, width, and height. So what are the length, width, and height? Again, this information is given to us at 18, 18, and 72. Now we execute our plan by substituting out our variables for the given values.  $V = LWH = 18 \cdot 18 \cdot 72 = 23328$ .

Finally, we must look back and check. This is easier said than done. We must first decipher what "length plus girth (distance around)" is.<sup>24</sup> What is meant by length plus girth is misstated in the problem and is in fact the sum of the dimensions. Now, we look back and decide whether we trust that the maximum volume is achieved by the dimensions of 18 inches by 18 inches by 72 inches. We assume that the student has no

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<sup>22</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 115.

<sup>23</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 115.

<sup>24</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 115.

knowledge of calculus or maximization, so that leaves us with the guess and check method.  $LWH$  using the given values is equal to 23328. What should we try first to replace these values? There are infinitely many combinations of  $H, W, L$ , but a banal one, which lends itself a comfortable place to start is when  $H = W = L$ . We find the value with our girth equation:  $H + W + L = 108$ , but since all are equal, we can rewrite our equation as  $3H = 108$ , and find that  $H = W = L = 36$ . The volume of this box is  $HWL = 36^3 = 46656$ , twice the value of the given dimensions.

There are a host of concerns with this problem from an editorial stand point, but let's assume that the "connection" were true. Is this a good connection to real life? I say no. I do not believe that someone would know the volume of their package and not the dimensions. Thus a person would not go through this process to figure out whether or not they could deliver their package.

The next "connection" falls under the same criticism:

When you look a long way down a straight road or railroad track, it seems to narrow as it vanishes in the distance. The point where the sides seem to touch is called the vanishing point. The same thing occurs in the lens of a camera, as shown in the figure. Suppose  $I$  represents the length of the image,  $O$  the length of the object,  $d$  the distance from the lens to the film, and  $D$  the distance from the lens to the object. Then Image Length/Object Length = Image Distance/Object Distance or  $I/O = d/D$ . Given the length of the image on the film and its distance from the lens, then the length of the object determines how far away the lens must be from the object to fit on the film.<sup>25</sup>

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<sup>25</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. Beginning Algebra (New York, NY: HarperCollins College, 1996), 124.

The math behind vanishing points is interesting, but again lacks a connection to real life. Photographers do not measure out distances based on the ratios of image and object length and distance. They look through the lens and either the object fits or it does not.

The former two “connections” confuse the order of reasoning and sense. Both of these examples do not need to be reasoned out to find a solution. The solution is either it fits, or it does not. There is reasoning behind common sense, but, as the name common sense implies, the reasoning is not necessary in order to make the proper decision. The rules of perspective were important to the development of art, which will be discussed in a later section, but it still does not have relevance on a daily basis. It often occurs that math supports common sense, but does not create a new thought. Pythagorean Theorem proves that walking on a direct path is the fastest way to reach a destination, but the skill, if it could even be call it that, of walking straight to a destination is not reserved to those who understand geometry. For these examples, math answers the question: why is this true, not the question: is this true? When students are trying to apply math to their own lives, it fails if they can dismiss it due to common sense.

A quality “connection” comes nearly twice as far into the book as the first “connection.” It promotes the section on distance, rate, and time:

The winner of the first Indianapolis 500 race (in 1911) was Ray Harroun driving a Marmon Wasp at an average speed of 74.59 miles per hour. To find this time we need the formula giving the relationship between distance, rate, and time. This formula is used frequently in everyday life. In this section we look at some applications of the distance, rate, and time relationship.<sup>26</sup>

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<sup>26</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. *Beginning Algebra* (New York, NY: HarperCollins College, 1996), 140.

Initially, this seems like another wasted “connection.” I read it and initially thought back to the train problem, and that if I ever wanted to know a result of a race, I would just look up the official results, and moreover, I would never even know his average speed even if I wanted to go through the prolonged process of finding out his time in the way presented in the “connection.” But then I reread the conclusion and that the focus is on the formula, not the example. It is a great connection because it is specific enough to easily talk about, but general enough to apply to life.

To expand on what I took from this “connection,” I would never use the average speed of a racer to figure out his time in a race, but I need this equation often in life. If I am driving on the highway and I see a sign for my exit in however many miles, I would use this equation to get a sense of how long I would be driving until I reach my exit. This is a very useful driving tool. A teacher can have this conversation with their students so that a student will have some personal connection to the material.

The final “connection” we will discuss comes from the section on inequalities:

Many mathematical models involve inequalities rather than equation. This is often the case in economics. For example, a company that produces videocassettes has found that revenue from the sales of the cassettes is \$5 per cassette less sales costs of \$100. Production costs are \$125 plus \$4 per cassette. Profit ( $P$ ) is given by revenue ( $R$ ) less cost ( $C$ ), so the company must find the production level  $x$  that makes  $P = R - C > 0$ .<sup>27</sup>

It is perfect for an algebra class that has a professional focus. It shows how algebra is used in a subject that might be used for a career. It is also a good use of algebraic thought

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<sup>27</sup> Margaret L. Lial, E. John. Hornsby, and Charles David Miller. Beginning Algebra (New York, NY: HarperCollins College, 1996), 149.

because it translates the problem and forces the student to distinguish information.

Although the prompt asks to find  $x$ , the equation,  $P = R - C > 0$ , does not contain  $x$ . The student then needs to figure out where in the information  $x$  can be found, and will do so when changing the revenue and cost definitions into variable equations.

Real life application is not synonymous with word problems. They can be used together, but application can be discovered in other ways and word problems can serve other purposes. For the teacher, it is vital to understand this and communicate it to the students. If a student believes the train problem is a true real life application, they will be unsatisfied; but if the student does not think there is a point to the prompt, they will be equally unsatisfied. Word problems are a delicate tool, but an effective one in maximizing math education. But while word problems are not necessary, application to the real world cannot be neglected in a proper math education.

**“[A mathematician is a] scientist who can figure out anything except such simple things as squaring the circle and trisecting an angle.”**  
-Evan Esar, *Esar's Comic Dictionary*

Even if math were not necessary for career development, it is still an effective medium for expressing the ideas of algebraic thought. One reason is because of the accessibility of the language of math. It is universal and highly symbolic, which allows it to be easily understood. Other languages can be more complex and thus more confusing:

The word “equal,” for example, can refer to equality in size, shape, political rights, intellectual abilities, or other qualities. Hence the assertion that all men are born equal is vague. As used in an expression such as  $d = 16t^2$ , the equals sign stands for numerical equality. The comprehensibility gained through symbolism derives largely from the fact that the mind easily carries and works with symbolic expressions, but has considerable difficulty even in carrying the equivalent verbal statement.<sup>28</sup>

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<sup>28</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 13.

Moreover, there are multiple ways to say the same thing. The equal sign can be spoken as “equals”, “equal to”, “is”, and “the same as.” It is the same for the operation signs as well. An interesting example of this is the word “by.” A grid that is three by three has a total area of nine. In this case, “by” means multiplication. Four divided by two, which can be said informally as four by two, is a division problem using “by” to denote the division sign. In the English language, a same word can denote the opposite operations, however “/” will never be used to symbolize the same thing as “:” in an equation.

One must also consider the effect of translation in understanding math equations. In many countries, the operation symbols are the same, but the languages of those countries are not. Thus, using the language of math is a universal equalizer to understanding problems. This is especially necessary for languages with different alphabets or language cases. For example, in Russia, the grammatical case changes with the operation. Using the instrumental case means that a multiplication is taking place, while the accusative case implies that addition is occurring.

Although numbers and symbols offer a clear platform, there are many arguments against using the material in math as a medium for teaching algebraic thought. First is the idea that because it can be separate, it should be. If the students are given too many goals in a single subject, some may be overlooked. If a student is most successful in math by memorizing formulas and their usage without a deeper understanding of the logic and problem solving improves, would that student not be considered a successful math student?

I believe that mathematics is the combination of material and thought. To truly master math is more than to master just the material or the reasoning style. It is being able to use them together. And even though the reasoning is not restricted to mathematics, the material is.

Another criticism is that it is unfair for students who struggle with numbers or are not as quantity-minded as other students. For a student who struggles with math, the idea that a greater portion of their academic success hinges on their mathematic success can be overwhelming. This idea leads into a third criticism: there are other subjects that problem solving and logic can be taught in with similar effectiveness. There is logic in grammar, problem solving in history, and pattern recognition in music.

I believe that this is backwards logic. Algebraic thought is most important within the context of a subject. It is in the subject of algebra that this style of thought comes into its own because it is the most direct case of algebraic thought. Yet even if the material in algebra is not necessary for algebraic thought, algebraic thought is necessary for the subject. I cannot fairly estimate the need for algebraic thought in other subjects, but it is inseparable in a good math education. Algebra, as defined by Merriam-Webster, is “a generalization of arithmetic in which letters representing numbers are combined according to the rules of arithmetic.”<sup>29</sup> This, however, only describes the material of the subject. Algebra needs to be redefined so it is not taught as material alone. That does not serve the students. Knowing the material is not a life skill; the true skill is in knowing

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<sup>29</sup> "Algebra - Definition and More from the Free Merriam-Webster Dictionary." (Dictionary and Thesaurus - Merriam-Webster Online), <<http://www.merriam-webster.com/dictionary/algebra>>. 22 Apr. 2011.

how to use it. Math is reduced in the high school curriculum, so that it primarily “consists of the relations of number, the relations of quantity, and the relations of space. This is not a general definition of mathematics, which, in my opinion, is a much more general science.”<sup>30</sup> The narrow view of math in education offers the best format to utilize concepts without being overwhelmed by material. The formulas and definitions are worthwhile, but largely by their ability to be utilized by students.

One must be careful, however, when talking about math and its utility to students. Most high school math can fit into one of these four types in regards to usage. There is direct usage. Mathematics with direct usage is the formulas and material that are needed in life. In the previous section on “connections,” I look at the time, distance, speed equation and its application to driving on the highway.

Second, there is indirect usage. There is math, such as the train problem, that can be used, but is not efficient or practical to actually use. This can also be expanded to math that is necessary in the process of learning. For example, calculus classes spend time with Riemann sums for its progression to integrals. Riemann sums, in the eyes of calculus students, are just less exact integrals. They spend time learning something that they will never think about again once they reach integrals. It is important for students to recognize and teachers to convey when seemingly unnecessary math is not. Adler explains:

For example, one of the first results of the theory of integrals is a formula for powers of  $x$ . The formula, which is simple, is valid for all powers except the power  $-1$ ...Is there some mathematical object whose importance has never been fully appreciated and which is suddenly signaling its hidden meanings? The answer is known: the formula for the

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<sup>30</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 121.

case -1 requires the introduction of the logarithm – the logarithm of high-school algebra, which in high school is usually relegated to a computational role.<sup>31</sup>

For something taught without its true use realized until multiple years later, it is important to not let it be dismissed as useless. Moreover, logarithms are used outside of pure math in statistics and economics. This information can aid in the learning process to connect it with the futures of the students.

There is proof-based math, which does not have obvious application. Rather, it supports what is already known and useable. Most students will learn geometry as proof-based geometry. Students have done algebraic thought, but now learn to support their thoughts with truth in the form of proofs. It is obvious that the most direct path is a straight line, but now students know why. They learn proofs for similar triangles as well, which confirms their pattern recognition abilities.

Finally, there is abstract math. This math is not referenced in everyday life, nor does it support common sense. It does not directly apply to anything most people encounter. It is relevant for a small minority in engineering and sciences. These topics includes formulas about the relationship of  $a^3$  and  $b^3$ . Even the quadratic formula is rarely brought up on a day-to-day scale. It is hard for students who are searching for a connection between educational material and their lives to find it in abstract math.

But consider the latter three types. Whitehead urges that subject matter not utilized is harmful, which begs the question, what can be drawn these types of math. The

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<sup>31</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 9.

first of those three, applied but inefficient math, has been discussed in the section on word problems. Next is the idea of proof-based math. If students are exposed to material by a trusted teacher, it should suffice as an assurance of its truth so it does not need to be formally proved in order to master it. But students need to explore ideas within the restriction of truth. It is a different way of thinking:

Though mathematical proof is necessarily deductive, the creative process practically never is. To foresee what to prove or what chain of deductive arguments will establish a possible result, the mathematician uses observation, measurement, intuition, imagination, induction, or even sheer trial and error. The process of discovery in mathematics is not confined to one pattern or method. Indeed, it is in part as inexplicable as the creative act in any art or science.<sup>32</sup>

The process of deductive reasoning in math can be part of the creative act in art and science. Deductive laws can reveal new truths and also teaches students how to think and be expressive within a confined world, which can lead to creativity.

The use of deductive reasoning to reveal new truths can be found in art when examining the two portraits *The Arnolfini Wedding* (See Appendix A) and *The Giving of the Keys to Saint Peter* (See Appendix B) by Jan van Eyck and Pietro Perugino, respectively. Both painters try to accurately capture the natural world. It is evident in the human figures, the different textures, and the shadows. From this, it would seem that both artists also tried to capture true perspective, that is to say diminishing perspective, including a horizon line and vanishing point. Van Eyck, the earlier of the artists only used his eye to try and capture this perspective, while Perugino used lines and rulers, coupled

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<sup>32</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 12.

with a geometric understanding of perspective. The human “senses are limited and inaccurate. Moreover, even if the facts gathered for the purposes of induction and analogy are sound, these methods do not yield unquestionable conclusions.”<sup>33</sup> Needless to say, Van Eyck does not capture true diminishing perspective in his painting. This is most evident when examining the floor. It appears as though the floor is slanted upward so that if a ball were placed at the back wall, it would roll down to the couple in the front. Upon deductive inspection, it can be proved that van Eyck does not capture true perspective. There are many lines parallel to the ground present in the painting. The rules of true perspective say that all of these lines converge at the vanishing point. In the portrait, they do not. The center crack in the floor is vertical on the canvas, which implies that the vanishing point is in the center of the painting; however, the lines of the windowsill and bedpost do not intersect in the center.

Perugino follows the rules of perspective in his painting. There is a clear horizon line, which is marked by the end of the terrace. There are many lines parallel to the ground that all converge at a single point on the horizon line. Adding to the perspective are the human figures present at multiple depths in the portrait, which aid to the size consistency of each square in the gridded floor pattern. By using deductive reasoning, in this case calculating with respect to the truth generated by calculations instead of the truth generated by the senses, Perugino is able to advance art and more accurately capture the

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<sup>33</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 12.

sensual experience of diminishing perspective, which until that time was impossible to completely capture, even by the great artists such as van Eyck.

An example of creativity through expression within a confined structure is the music of Thelonius Monk. He is known for his dissonant harmonies and harsh breaks from the melody in his compositions. Although not a deductive science, by fully understanding the rules of chord and melody structures, he was able to manipulate them in order to create unique and brilliant music. It is not true deductive reasoning because the mastery of harmony and melody did not generate his art, but his creativity stemmed from working within the patterns developed by harmony and melody. While similar music could have been made with a disregard for structure (we hear something like this in atonal jazz), the mastery of structure was what made Monk's music possible.

Geometry is a very effective gateway to this way of thinking. Whitehead notes, "We must remember that owing to the aid rendered by the visual presence of a figure, Geometry is a field of unequalled excellence for the exercise of the deductive faculties of reasoning."<sup>34</sup> That is, common sense can be easily found in aspects of geometry so that the student tends to support their intuition instead of testing it. Intuition, as Morris Kline puts it, is inductive reasoning and reasoning by analogy. In geometry, most often, intuition will be correct. Kline offers this example of inductive reasoning: "By measuring the angles of a dozen or so triangles of various shapes and sizes a person would find that the sum in any one triangle is 180 degrees. He could then conclude by inductive

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<sup>34</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 16.

reasoning that the sum of the angles in every triangle is 180 degrees.”<sup>35</sup> It is simple to prove formally, but this style of guessing and checking gives the student enough confidence to trust it. Moreover, intuition is correct in its assertion. Similar is reasoning with analogy, which Kline also offers an example of: “The circle plays about the same role among curves that the sphere does among surfaces. Since the circle bounds more area than any other curve with the same perimeter, a person might conclude that the sphere bounds more volume than any other surface with the same area.”<sup>36</sup> Again, intuition serves the student well as this is a correct assumption. But it is not absolute truth until it can be proven. Introducing deductive reasoning in geometry lets students affirm their intuition. Proving something that is already thought to be true is a more comfortable transition to formal proofs than using material that goes against intuition or is too abstract for the student to possess any intuition of.

As mentioned in the section on word problems, deductive reasoning is not meant to replace intuition. These modes of thought are to be used separately, and it is important to possess both. It is wrong to think that one would try to verify every intuition with a formal proof before acting upon it. Similarly, it is wrong to think that one should try to replace their intuitive or deductive reasoning with the other. Using intuitive reasoning as a gateway into deductive reasoning is only effective if those two points are made clear.

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<sup>35</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 11.

<sup>36</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 11.

The last of the four types of math is abstract math. This is the math, whose material cannot be applied to everyday life. I remember in high school reciting the quadratic formula to the tune of *Pop Goes the Weasel*. It is perhaps the most memorable part of algebra. Its memorability, coupled with the amount of time spent learning it, convinced me of its importance. But it does not seem to have much value. First, finding the root of a function is not that helpful outside of itself. And the next year in math, I was required to buy a graphing calculator, which solved roots with a few button presses. Was there a point to me learning the quadratic formula if I could have just bought the calculator a year earlier? Although students cannot use this math, it is not useless. Beyond the material and algebraic thought, math teaches students how to handle abstract ideas. Whitehead remarks, “The main ideas which lie at the base of mathematics are not at all recondite. They are abstract. But one of the main objects of the inclusion of mathematics in a liberal education is to train the pupils to handle abstract ideas.”<sup>37</sup> The ability to handle abstract ideas is vital for a functioning member of society. Morals, ethics, planning for the future; these are all abstract ideas.

There is a large factor of trust in this method of teaching, however. If students understand that they are learning inapplicable material in order to learn how to deal with abstractions, there will be a disconnect for them in the material. Similarly, if a student is led to believe that the idea is important for the applied world, they may move past handling the abstraction in a pursuit to apply it. Whitehead offers further insight “that the very reasons which make this science a delight to its students are reasons which obstruct

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<sup>37</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 121.

its use as an educational instrument – namely, the boundless wealth of deductions from the interplay of general theorems, their complication, their apparent remoteness from the ideas from which the argument started, the variety of methods, and their purely abstract character which brings, as its gift, eternal truth.”<sup>38</sup> This is when the student’s motivation needs to come from excitement about learning and discovery, not from a need to know why it is necessary for the future.

**“You never know what is enough unless you know more than enough.”  
-William Blake**

There is a cliché about being a math person. Some are, some aren’t, and that is just the way it is. Normally these people realize it at an early age. Friends, who at the college level still excel in math, say that they knew they were good at math in middle school and prior. One even told me she knew it when she was four. However, what is misstated as math is in fact numbers. Those who are in the top-percentile of dealing with numbers generally realize it early. However, because math tends to focus on the material

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<sup>38</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 118.

more than the reasoning, it is not unfair suggest that there is a correlation between numbers and mathematics in its current academic sense, so at the high school level, there will be a larger and more defined range of math students. And the children know who they are. But in school, not all successful math students are going to go into math-based professions. Not even all of the students in that top-percentile will. The notion that students are at a wide range of skill regarding math, but are required to take it to complete their education seems to be inconsistent with a high school math program that is focusing on preparing all the students for careers, and from this logic, math-based professions.

The first step to managing this inconsistency is educational awareness. Educators need to be aware that even if all children have equal opportunity, they will not all be equally successful. While the education system should be able to offer the same education to every child, there is a range of inherent intelligence that plays a role in absolute potential. There are three outcomes that can transpire in the case of two students with unequal intelligence in the same education system. First, the school paces itself to the level of the better math student and the other student is left behind. Second, the school paces itself to the level of the weaker math student and the other does not capitalize on his potential. Third, the school paces itself in a way that allows both students to maximize their potential without hurting the other. Obviously, the third outcome is the favorable one, but its success hinges on the whether people are willing to admit that some students are better than others. The problem is that admitting this is not American. It seemingly goes against the idea that all men are created equal. This notion of equality, however, is in regards to the treatment of men, not their genetic predisposition. Regardless,

politicians do not want to distinguish children as different intelligence levels. Intelligence, as it pertains to education, is relative – by identifying more intelligent students, politicians are also recognizing less intelligent students. This is hard to do eloquently and without offending the voters who are the parents of “dumb” children. Even if a politician were to openly admit this truth, it would be difficult to use it to aid the education system. By merely acknowledging that there is a difference in intelligence levels, a politician is not believable when he says he wants to change the education system in a way that would help everyone. Now think of the loss of credibility if he openly admitted only the smartest of the smart would make a life changing impact on society.

Similarly, most parents will not admit that their child is not as smart as the next one. America, as a youth development nation, has delayed specialization compared to other countries. Even if children are at the age when specialization could occur, their development is quite generalized. They participate in many sports, in many arts, and are active in many subjects in school. They do not specialize in one or two areas where they show early indicators of success. This is because the mindset in America is that people can achieve anything they want if they work hard enough at it, which is simply not true, but it is also a hard fact to admit. However, even if America were more concerned with specialization, it would not necessarily be a good idea. Without being overstated, the students that would give up math for specialization in trades or other subjects would miss out on a very important part of their education. The result, however, is that math education, in its goal to prepare children for math-based careers, cannot focus only on the

highest-level professions, but must give a larger number of students the opportunity to pursue a larger quantity of math-based careers. But the students who choose to specialize in math would also miss out on a very important part of their education. I think it is harmful for students to focus their mental development in one medium, such as numbers, as well as it being harmful to close off opportunities. For career development, the value is not just in preparation, it is also in ensuring that the options remain open for when a child does decide. If there were two math programs, one for career placement that focused on the necessary knowledge of material to continue study of math and the other that only explored algebraic thought without concern for advancing in the material, then the students in the latter class would miss out on the opportunity to pursue a career in mathematics. Similarly, those in the former class would miss out on the merits of math beyond career development.

In reality, it is a very small group that will actually be successful in significantly moving society forward. But this small group has more value to society than nearly any other group. Therefore, it benefits society to give these students the necessary pace and breadth of education to maximize their potential. The appropriate pace and breadth is beyond that of the majority of students, so, as previously mentioned, a school must be able to manage the range of intelligence levels. A change in curriculum, either shortened or extended, would not satisfy both groups. Until the education system embraces algebraic thought as an equal part of a sufficient math education, the need to satisfy both groups' material needs halts education policy changes because it is impossible to weigh the expected value of the different groups of students.

The government cannot dissuade children from pursuing math in order to utilize its resources for the elite math students. On the contrary, the government needs to require a full math education from its students. A student should also not be allowed to choose whether or not he wants to continue math in high school. A student is not old enough or experienced enough to make a decision of that magnitude. Students change majors junior year of college, there is no telling what kind of changes will go on in a child's life freshman year of high school and beyond that could guide his career choice. Regardless of whether or not the student could make the appropriate decision on when and what to specialize in, this type of specialization, as previously stated, is harmful to a student's success. Specialization is good, but needs to come later. It is good and "undoubtedly the chief reason is that the specialist study is normally a study of peculiar interest to the student. He is studying it because, for some reason, he wants to know it. This makes all the difference. The general culture is designed to foster an activity of mind; the specialist course utilises this activity."<sup>39</sup> Whitehead leaves to the reader what the reason is, but that decision need not be rushed and when it is made, it should be made for the right reasons. And, because of the nature of math in high school, such decisions can be irrecoverable to a student's educational pursuits.

The nature I am referring to is the linear quality of mathematic leading up to calculus. Only when a student learns his numbers is he able to start his arithmetic. Once that is completed, he is able to go to algebra, then geometry, trigonometry, pre-calculus, and calculus. It has to happen in this order because of the building nature of the subjects.

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<sup>39</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 18.

The subjects are continuations of one another in many ways. Typically, one does not even use the phrase “leading up to” to refer to subjects other than math, which supports the notion of linearity. If a child decides he does not want to pursue his math education, only to change his mind, he is now behind. Also, if a student does not understand a certain topic in math, or has a bad experience in a class, he is now behind.

In line with this causal nature, tribulations in the process will be projected onto math as a whole. That is, a bad experience with a teacher or a specific class will settle as a bad experience with math. Perhaps this is best understood from the opposite angle, as it is unlike many other subjects. Irish literature is not indicative of American literature. Thus a student who is uninspired by Irish literature will understand it as Irish literature, and not literature as a whole. A boring teacher in American history will have his shortcomings understood as shortcomings of the teacher, not of American history and not of history as a whole. The same is so for many other subjects. Why this is so will be discussed further in a later section. While courses in other subjects are not completely independent, each tends to be its own unique experience.

There are two remedies that can help to advance the math system. The first change is to get more students excited about math. If more students migrate towards math, the pace will organically increase. Students will not just see math as textbook material that is required of them and, as suggested by Secretary Duncan, the real educational experience can take place once students think beyond the surface subject matter. While this is easily stated, it is much tougher to accomplish.

The other remedy is to be able to offer a variable pace to the educational experience. Because of the linearity, students cannot afford to miss out on material because of the pace, but rather, need the opportunity to spend the necessary time with the material before moving on. There are two conditions for such an environment. Rather, there are two conditions that cannot be present. The environment cannot be passive in its acceptance nor can it be able to slow down the faster paced students. It is unacceptable for a student to be deterred in his exploration of a subject. The pace can occur in the concept, not necessarily in the material itself. That is to say, the algebraic thought and reasoning that is learned in class can be paced independently of the material. Thus, an accelerated student does not need new material to advance his math education. He can learn to better identify and apply what he knows, as well as use algebraic thought in different modes to continually be stimulated and challenged.

An example of this in math is the Bongard Problems.<sup>40</sup> These problems were created by Mikhail Moiseevich Bongard in the middle of the 20<sup>th</sup> Century. The set up is two sets of six squares, each with a figure inside (See *Appendix C*). The problem is to identify the characteristic that separates the two sets. Students are exposed to pattern recognition and taught to apply their problem solving skills in a different way. It closely follows the four-step process for solving problems presented in “connections,” which gives it an algebraic flavor. Similarly, it deals with the idea of truth and translating problems into useable language. There are plenty of nuances that can be applied to the problems, but understanding the true variables is what creates success. The answer to the

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<sup>40</sup> M. M. Bongard. *Pattern Recognition* (New York: Spartan, 1970).

first Bongard problem is that the squares in the left set do not have figures. A true statement is that if the squares were divided into nine equally sized squares (3x3), the center squares in the right set would contain figures and the left set would not. That is not the answer, and to have students explore the problem to find the true question and true answer has value in algebra. Similarly, it teaches students to use the information presented. If the left set possesses some quality about triangles, but there are no triangles in the right set, it is unnecessary to the answer. This is algebraic thought, but instead of translating words into variable equations, Bongard Problems require translating pictures into words. In the book *Gödel, Escher, Bach: An Eternal Golden Braid*, the author, Douglas R. Hofstadter notes to be successful at the Bongard Problems, one must

devise explicit rules that say how to  
make tentative descriptions for each box;  
compare them with tentative descriptions for other boxes of either  
Class;  
restructure the descriptions, by  
    (i) adding information,  
    (ii) discarding information...  
    (iii) viewing the same information from another angle;  
iterate this process until finding out what makes the two Classes  
differ.<sup>41</sup>

Essentially this process is translating the problem and using the information and tools in the new form, solve the problem and reapply it back to the original form. There can be many things that define a single box, but the Class has general characteristics, so using generalized variables will expose the connection within a Class. That is algebraic thought. Working through these problems enriches the

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<sup>41</sup> Douglas R. Hofstadter. *Gödel, Escher, Bach: an Eternal Golden Braid* (New York: Vintage, 1980), 649.

learning experience and expands on concepts from the course material. It does this without introducing new course material so the slower students are not left behind.

Finally, that it is necessary for a system to not be passive is why, as mentioned previously, teachers must be proficient in math. A student needs to be paced based on potential. A student who does not care to reach his potential will go slower than if he were to be pushed and appropriately pacing students aids to the success of a math program. School is a wasted resource if the students are not pushed to their full potential. A student needs to experience through practice and credible role models that math is an indispensable part of their education.

**“It's psychosomatic. You need a lobotomy. I'll get a saw.”**  
**-Calvin and Hobbes**

Algebra is usually taught between the ages of 11 and 14. Psychologist Lawrence Kohlberg refers to this age group as the conventional level of development. Kohlberg developed six stages of morality that he coupled into three levels. The first level, the

preconventional level, contains stages one and two. The second level, the conventional level, contains stages three and four. The third level, the postconventional level, contains stages five and six.<sup>42</sup>

Kohlberg explains, “The term conventional means conforming to and upholding the rules and expectations and conventions of society or authority just because they are society’s rules, expectations, or conventions.”<sup>43</sup> The two stages in the conventional level (Stage 3 and Stage 4) differ “in terms of: (a) what is right, (b) the reason for upholding the right, and (c) the social perspective behind each stage.”<sup>44</sup>

Stage 3 is indicative of mutual interpersonal expectations, relationships, and interpersonal conformity. It defines right as “Living up to what is expected by people close to you or what people generally expect of people in your role as son, brother, friend, etc. ‘Being good’ is important and means having good motives, showing concern about others. It also means keeping mutual relationships, such as trust, loyalty, respect, and gratitude.”<sup>45</sup> It defines the reason for doing right as “The need to be a good person in your own eyes and those of others. Belief in the Golden Rule. Desire to maintain rules

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<sup>42</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987).

<sup>43</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 283.

<sup>44</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 283.

<sup>45</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 284.

and authority which support stereotypical good behavior.”<sup>46</sup> And it defines the social perspective of the stage as “*Perspective of the individual in relationships with other individuals*. Aware of shared feelings, agreements, and expectations which take primacy over individual interests. Relates points of view through the concrete Golden Rule, putting yourself in the other person’s shoes. Does not yet consider generalized system perspective.”<sup>47</sup>

Stage 4 is indicative of the social system and conscience. It defines right as “Fulfilling the actual duties to which you have agreed. Laws are to be upheld except in extreme cases where they conflict with other fixed social duties. Right is also contributing to society, the group, or institution”<sup>48</sup> It defines the reason for doing right as “To keep the institution going as a whole, to avoid the breakdown in the system ‘if everyone did it,’ or the imperative of conscience to meet one’s defined obligations.”<sup>49</sup> And it defines the social perspective of the stage as one who “*Differentiates societal point of view from interpersonal agreement or motives*. Takes the point of view of the system

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<sup>46</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 284.

<sup>47</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 284.

<sup>48</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 285.

<sup>49</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 285.

that defines roles and rules. Considers individuals relations in terms of place in the system.”<sup>50</sup>

For a point of clarity, as I have been discussing math problems, by the term *right*, I am referring to morally correct, not giving the right answer for questions. More specifically, right, in this context, is being a diligent student. According to the conventional level of morality, it is important to be a diligent student because parents or teachers say so. It is important because society expects it and because it is part of the process to having a successful career. It is important because everyone else is doing it. The student, who views the world with this morality, does not justify his efforts with an appreciation of the material or of the learning experience.

Students will try to live up to social roles and expectations and are concerned with how their actions affect relationships. As a student, that means not challenging the teacher. The expectation to be a good student, as well as the concern about what the teacher thinks about the student generally drives diligence. Students will typically not ask why they need to learn algebra. They accept it because of the moral stage they are in when it is taught. Moreover, they do not inquire as to whether it is being taught properly or whether or not there is more that they are missing.

There is an episode of *Pete & Pete*, a children’s television show from the 1990s, where a student, Ellen, does just that. She raises her hand to answer a word problem and when called on she replies, “Why?” At first the teacher thinks she means, “y,” which is

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<sup>50</sup> Lawrence Kohlberg. *Child Psychology and Childhood Education: a Cognitive-developmental View* (New York: Longman, 1987), 285.

an incorrect answer, but eventually it is understood that Ellen is questioning the applicability of the word problem. The episode sets itself up by exploring many of the concerns of this thesis. However, the student is not trying to reach a higher state of awareness concerning her education. She is frustrated with algebra and rather than make an effort to understand the material or the specific problem, she decides to challenge the teacher, who cannot give her a sufficient answer. The teacher, whose life's passion and work was algebra, has a crisis of confidence and quits. Ellen continues to challenge the substitute teachers that come in, none of whom can provide her a sufficient answer. Ellen even challenges her own father when he comes in as a substitute teacher. Ellen is revolting against schoolwork and it is only a consequence that she engages herself in a search for meaning in algebra. The conclusion of the episode fails to find real meaning in algebra, but it does shed light on the Kohlberg stage that Ellen is in when learning algebra. The climax occurs when Ellen finds out her former teacher is leaving forever, which was never Ellen's intention in challenging her. The teacher, in one last passionate act towards math, writes out the train station she is leaving from in the form of an algebraic word problem. Ellen struggles with, but eventually solves the problem with help from her friends and father. And, in the nick of time, Ellen manages to find her teacher, restore her friendship, and find the usefulness of algebra. The final resolution of finding usefulness in algebra is not so much discovered, but merely stated in the closing monologue. The episode does not offer any insight into the meaning of algebra, but much of the motivation of an algebra-aged student. Ellen's quest for true knowledge was a mature notion, even if executed poorly. Her quest, however, is not put to rest when she

receives sufficient and deserving information, but rather, when she believes it has gone too far in ruining her relationships. The need to remedy her relationships with her teacher and father is based on her perception of morality at this stage in her life. Children do not question why they learn algebra. And if they do, or it is explained why preemptively, the answer usually involves its necessity in preparing students for higher-level math, which directs the focus to the material.

Although there is a high risk to teaching algebra at this age, the reward is even higher. It is perhaps the earliest a student can fully comprehend the material, both in mental capacity and prior knowledge. By learning it early, a student has more time to master the material and use it. It is important “In training a child to activity of thought, above all things we must beware of what I will call ‘inert ideas’ – that is to say, ideas that are merely received into the mind without being utilised, or tested, or thrown into fresh combinations.”<sup>51</sup> Until a student fully grasps the material, he is unable to make it his own, whether it comes in further math or other disciplines. If the material comes too late, it becomes inert because the student has moved on or does not have time to apply it. Whitehead continues that “the science students will have obtained both an invaluable literary education and also at the most impressionable age an early initiation into the habits of thinking for themselves in the region of science.”<sup>52</sup> But, if done correctly, these habits will be present in whatever discipline the student enters.

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<sup>51</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 1.

<sup>52</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 39.

As will be discussed later, math is reliant on the teacher to capture the full meaning beyond the material. Thus, when the student is most willing to pursue diligence for and is most susceptible to be influenced by the teacher is the best time to explore algebra. It is almost contradictory to tell students to think for themselves because if a student is really thinking for himself it will not come from a teacher's order. As students continue to move forward, they will question more of the world around them. They no longer trust their teacher simply because he is their teacher. Because of the seemingly wide gap between algebra material and application, this questioning can be detrimental to a student's development. It is important that "different subjects and modes of study should be undertaken by pupils at fitting times when they have reached the proper stage of mental development," and that algebra is taught at the correct age.<sup>53</sup>

Students will have started to act out against their parents in this stage. I believe, however, that there will be a delay in acting out against teachers. Kohlberg suggests that students in the conventional level are concerned with societal expectations. As children grow and are exposed to more of the world, the home becomes less a part of society as a whole. The same happens eventually with school, but because it is a larger part of their exposure to society, children will not rebel against teachers in the same way they do against their parents at that age.

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<sup>53</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 24.

**“If you don't have the *ganas*, I will give it to you because I'm an expert.”  
-Jaime Escalante, *Stand and Deliver***

More than in other subjects, poor math students, and even those with some mastery of the material, find it boring. Perhaps worse, they find the material inapplicable to their lives. For most early math, there is a lack of discovery. Students need the sensation of learning instead of feeling like they are being taught. Whitehead states of discovery:

Let the main ideas which are introduced into a child's education be few and important, and let them be thrown into every combination possible. The child should make them his own, and should understand their application here and now in the circumstances of his actual life. From the very beginning of his education, the child should experience the joy of discovery. The discovery which he has to make, is that general ideas give an understanding of that stream of events which pours through his life, which is his life.<sup>54</sup>

Consider almost every other subject in regards to discovery. In literature, the material is surrounded by stories and adventures. Similarly in history, what is taught has been deemed worth remembering, which makes it worth learning. In the sciences, it is a chance for students to really apply what they already know. Regardless of its actual truth, it feels true that the essence of the subject is the solving of problems, unlike most mathematics, where the essence of the subject seems to be the material, where problem solving is only a vehicle to the mastery of that material. In that likeness, testing knowledge of math is done primarily by giving the material to the student to memorize and seeing whether or not he has done so. It is paradoxical for a student to struggle with

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<sup>54</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 3.

variables and excel at algebraic word problems, while it is quite common that a student who has trouble with grammar and vocabulary can give meaningful criticisms to works in a literature class. Students take the material in literature and use it to learn about the human experience. Students take the material in history and learn lessons of the past. Students take the material in math class, and regurgitate it for a grade.

Teachers play a large role in the sense of wealth and discovery in the material they teach. There are two things that almost all good teachers have: a passion for their subject, and an ability to pass that passion onto their students. Math teachers are at an advantage: their ability to pass the passion onto their students is present in the mere fact that they are passionate. Because math needs to be passed to students in such a linear manner, successful teachers are the ones who can do more than just present it. It is in some ways its own language so the material does need to be presented and memorized. This is generally a boring endeavor. Alfred Adler, in his essay *Mathematics and Creativity*, remarks that “almost every good mathematician is also a good math teacher, while almost no mediocre mathematician can teach the subject adequately even at an elementary level.”<sup>55</sup> Because math can simply be presented as material, it is necessary for students to see an authority figure and hopefully a role model loving the material. Even if they are not yet able to make the material their own through its application when it is taught, they can make it their own by the excitement they feel towards it. Like diligence,

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<sup>55</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 6.

this excitement is a result of the stage of moral development. When a teacher is excited, students will become excited to maintain a positive relationship with the teacher.

But this seems useless if the essence of math is merely the material. But Adler believes there is more to math than the misconceived conventional idea that it is just boring material. He states: “Textbooks, course material – these do not approach in importance the communication of what mathematics is really about, of where it is going, and of where it currently stands with respect to the specific branch of it being taught.”<sup>56</sup> This conversation needs to be initiated by teachers, so that as students are learning the material they believe it has value instead of just being ink on a page. The way the sciences offer a sense of discovery for the world, math should be so for the mind. Adler continues, “What really matters is the communication of the spirit of mathematics. It is a spirit that is active rather than contemplative – a spirit of disciplined search for adventures of the intellect. Only an adventurer can really tell of adventures.”<sup>57</sup>

It has been established that math needs good teachers, and good math teachers are those that are good mathematicians. This is not in the sense of research, but in an appreciation for and a mastery of the material and thought needed to be successful at math. Again, Adler enlightens:

This phenomenon is easier to recognize than to explain. Students, even though in most cases they do not know what constitutes good mathematics or which are the best mathematicians they have encountered, will

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<sup>56</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 7.

<sup>57</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 7.

unfailingly pick out the best mathematicians when asked to identify their best mathematics teachers. Love of mathematics and active involvement in its development forge ties between the teacher and his students; the latter are rarely fooled by style or dramatic effect. The usual confusions are absent: confusions between content and presentation, between the subject and the man, between profound inspiration and trivial manipulation – in short, those confusions common in the classrooms of so many other subjects, and common, in fact, in so great a part of life. There is no such thing as a man who does not create mathematics and yet is a fine mathematics teacher.<sup>58</sup>

Similarly to the student, a literature teacher can offer worthy criticisms to a work while lacking mastery of a subject. And these words do not have to be, nor are they expected to be original thoughts. Classic works have been analyzed for hundreds of years in every depth and from every angle. For many, there is nothing original left to think about it. For the greatest of those, many of those thoughts have even been organized so that anyone can find them and present them in a neat and formal fashion. The same is so for history. The style and dramatic effect that Alder believes cannot veil a deficiency of passion of math can foster success in the classroom of almost all other subjects.

This is not to sell short teachers in other subjects. To truly be an excellent teacher, the teachers in those subjects need to possess both passion and ability, while math teachers only need passion. This is not to sell short math teachers, either. Developing a passion in math is difficult; the legitimacy of this thesis is dependent on that fact. Alder is vague when he talks of creating mathematics. Personally, creating math is not just in research, but also in making it more than material. Creating math is falling in love with it for its beauty and application. In addition to pursuing it directly, for some, this passion

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<sup>58</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 7.

and creation is built by a passion for education. If a teacher recognizes math as an indispensable part of education, then his passion for education can sprout a passion for mathematics in the classroom. Because the relationship of algebra teacher and student is cyclical – that is, students become teachers – it is necessary to address the lives of mathematicians in regards to developing a passion for mathematics.

There are several motivating factors that can lead one to study mathematics. There is ego and a quest for greatness in the field of math. There is an appreciation for mathematics as the foundation of the sciences and its necessity as a pragmatic tool. There is a notion that mathematics is the foundation of thought and there is a beauty in its relationship to logic and the mind. The obvious question to ask next is: how can these ideas be instilled in the minds of math students? The first is at the mercy of the individual. There is a unique, but small appeal to pursuing excellence in math versus a different intellectual field, or even other fields outside of the intellectual realm. The main appeal of math is the purity of it, which nullifies extraneous circumstances to explain the greatness. This idea is better understood with counterexamples.

There are no competitive interactions that affect performance in mathematics. Consider the scenario in a baseball game when a strong pitcher, who has a very effective fastball, faces a weak batter. The batter decides that he has no chance of hitting the fastball, so he waits for a change-up. The pitcher eventually throws a change-up and the batter gets a hit. In this one instance, the batter was successful, and if they never meet again, the batter will be on some level, better than the pitcher. However, if there were future encounters, the pitcher would acquire a positive record against the batter and be

considered better. Similarly, a different weaker pitcher could in a single meeting, strike out the same batter and on some level be considered the greater pitcher. A component of some athletic success is to deter the opposing athlete. Competitors in math do not directly affect each other's performance. There is no component to taking an exam, or trying to prove a theorem that involves deterring others. Comparative success in mathematics is judged on the result – if a mathematician is considered greater, it is because he is.

There is no luck in mathematics. Many discoveries in science and technology are accidental. Such is the case with penicillin:

After returning from a vacation trip in 1928, Alexander Fleming, a Scottish bacteriologist, noticed that mold had started to grow on some of the staphylococcus bacteria cultures he had left exposed. Oddly, though bacteria dotted the dish, none grew where the mold was. Fleming eventually figured out that this mold, called *Penicillium notatum*, was causing the bacteria to undergo lysis, or membrane rupture, and killing it.<sup>59</sup>

These lucky encounters do not happen in math.

Alder expands on how the purity of mathematics gives its victors a distinct greatness comparatively:

Mathematics, like chess, requires too direct and personal a confrontation to allow graceful defeat. There is no element of luck; there are no partners to share the blame for mistakes; the nature of the discipline places it precisely at the center of the intellectual being, where true cerebral power waits to be tested. A loser must admit that in some very important way he is the intellectual inferior of a winner. Both mathematics and chess spread before the participant a cast domain of confrontation of intellect with

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<sup>59</sup> "Accidental Inventions and Discoveries - Newsweek." (Newsweek - National News, World News, Business, Health, Technology, Entertainment, and More – Newsweek), <<http://www.newsweek.com/photo/2010/08/31/famous-accidental-discoveries.html>>. 22 Apr. 2011.

strong opposition, together with extreme purity, elegance of form, and an infinitude of possibilities.<sup>60</sup>

Alder believes in the greatness of mathematicians, but in math, perhaps more than any other discipline, the glory may not be worth the sacrifice. The greatness of a mathematician may be unparalleled, but the life of a great mathematician is not similar to other celebrities.

Mathematicians are trained to doubt. It is necessary to be successful at math to question and “mathematics, by its nature, forces skepticism on its students as a first requirement...Mathematics is a field in which much that appears obviously true is in fact false.”<sup>61</sup> The cynicism necessary is enough that it has the ability to consume the other aspects of one’s life. Alder also notes that mathematicians as a group have an abnormally high rate of divorce and depression, which is most likely a result of the isolating and incommunicable nature of research and the thought process required to be a great mathematician.<sup>62</sup>

There is no guaranteed success in the field of mathematics. The glory comes from the findings, but there is no assurance the magnitude of the finding will gain that glory, nor is it even assured that the pursuits of a mathematician will yield findings. Similarly, the careers of mathematicians are often short-lived. Many monumental findings are from

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<sup>60</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 3.

<sup>61</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 5.

<sup>62</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 9.

the young minds in the field. The idea of longevity is appealing in a career. Even in careers like investment banking, which require a lot of time and effort, are usually ephemeral, and are populated with young talent, there is still more levels to advance to afterwards in trading and hedge funds.

Finally, and probably most significant, great mathematicians are not popular or revered in society. Alder notes that the equivalent of “Tolstoy, Beethoven, Rembrandt, Darwin, [and] Freud” in math are unrecognizable names.<sup>63</sup> Children do not have posters of mathematicians like they do with sports heroes. Children do not talk about mathematicians in school or at home like they do with CEOs, politicians, or scientists. Children do not learn to appreciate mathematicians through interactions like they do doctors, lawyers, or teachers. The findings are not appreciated either. Even if a child does not personally know any architects, computer scientists, or engineers, he understands that those professions are the reason he has a house, a computer, and a car, and appreciates those careers. Children do not know Andrew Wiles, or Fermat’s Last Theorem, and even if they did, they would quickly understand it does not impact their lives. This again might trace back to the incommunicable nature of math research. There is nothing that could be talked about even if students wanted to.

These are the reasons why math is so unpopular. The other reasons for pursuing math fall on the teacher. The material needs to be introduced like history or literature. These concepts in math are being taught because they are for some reason worth

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<sup>63</sup> Douglas M. Campbell and John C. Higgins, eds. *Mathematics: People, Problems, Results* (Vol. II. Belmont (Calif.): Wadsworth, 1984), 7.

remembering. Their importance to society in practical applications can be immediately apparent, or accepted trustingly when given from teacher to student. The stories and adventures of math lie in the exploration of thought. Empowering the mind with tools and resources from the material of math opens new doors and challenges for the intellect to conquer. This portrayal of math will not be realized by the student unless they are introduced and made real by the teacher.

I realize now that in fourth grade, I was not good at math, but rather I was good at numbers. The multiplication tables tested my memorization of numbers, not my ability to calculate numbers. Although it was strictly memorization, I understand that it was necessary to learn my multiplication tables in that manner because it equipped me with tools and resources I could use later on when calculations and algebraic thought came into play.

I realize now that the algebra problem Teddy Meeks put up on the board was my first exposure to math and not just numbers. Even though I was good with numbers, I had never had to think in the way required to solve variable equations. The children who are good with numbers still need a proper introduction to the idea of abstraction in math.

I realize now that math journals in eighth grade were not a stupid waste of time. It had two valuable uses for me. First, it forced me to translate my math into words, which is an important concept of algebraic thought. Also, it forced me to think about math besides just trying to find a numerical answer. When math is just material, “nothing can be worse than the aimless accretion of theorems in our textbooks, which acquire their position merely because the children can be made to learn them and examiners can set neat question on them.”<sup>64</sup> My teacher could have just resorted to only testing our

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<sup>64</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 120.

memorization and reduced the math to only numbers. Perhaps the reason I disliked math journals was because I knew the alternative was to test our comfort with numbers, which I greatly possessed.

I realize now from the amount of tutoring I did in high school how much of an impact a teacher can have on his students. I know the people I tutored were only concerned with their grade in class, but I like to think my passion of math was effectively communicated to them. Even if they are not passionate about math, they understand its importance, and my excitement toward it hopefully gave them an easier learning experience.

All these anecdotes come from formidable times in my math career. Times when I was developing a passion for math and undergoing a transformation as a mathematician. This leads me to a final memory. I remember taking a Calculus II class during my third semester of college. I had already taken multivariable calculus and two other upper-level classes, but I decided to take it because although I did not need it graduate, I thought it would be worthwhile material for when I took math analysis the next semester. I remember Professor Sam Nelson took the first part of the lesson to introduce the class and make an analogy that compared math to lifting weights. He said that math was like weight training because you are not able to just sit around and then expect to lift huge weights, you need to be constantly lifting, the same way that you need to be constantly doing problems; you cannot just walk into the exam and expect to be able to do everything. As a weight lifter, the analogy made an impression on me and the more I thought about it, the more sense it made.

The analogy can be expanded. Lifting weights is primarily done to improve performance in sports, but can also be done for leisure or for lifting weights as its own pursuit. The same is true of math, if the complete education is the sport. The first priority of a weight lifting program is injury prevention, just like math cannot be effective in improving education if students are dropping out (getting injured). But simultaneously with injury prevention, a lifting program can improve strength, speed, balance, or flexibility. In an environment where students are not dropping out, they can also learn new material, improve their logical reasoning, pattern recognition, and problem solving. There are aspects of sports that are directly related to lifting and others that require some trust. But ultimately, the purpose of lifting weights is to improve performance in the sport, and the purpose of math is to advance a student's intellectual pursuits.

A successful math education program can encompass many aspects of education. It can be conducted in an environment that supports students of different levels and does not overwhelm the bottom or slow down the top. It can introduce and produce a mastery of material. It can teach concepts like logical reasoning, pattern recognition, and problem solving. Those skills can be brought to other fields in the educational pursuit, as well as applied to the mathematical material. Math can blend a sense of exploration and discovery with learning necessary for educational fulfillment. And these benefits serve the ultimate goal:

Here we are brought back to the position from which we started, the utility of education. Style, in its finest sense, is the last acquirement of the educated mind; it is also the most useful. It pervades the whole being. The administrator with a sense for style hates waste; the engineer with a sense

for style economises his material; the artisan with a sense for style prefers good work. Style is the ultimate morality of mind.<sup>65</sup>

The mutual relationship of material and critical reasoning allows for a deep and blended math education program that can effectively and appropriately advance students in their intellectual pursuits.

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<sup>65</sup> Alfred Whitehead. *The Aims of Education* (New York: MacMillan, 1929), 19.

## **Appendix**

*Appendix A: The Arnolfini Wedding, by Jan van Eyck, 1434.*

QuickTime™ and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture.

*Appendix B: The Giving of The Keys to St. Peter, Pietro Perugino, 1482*

QuickTime™ and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture.

*Appendix C: Bongard Problem #1*

QuickTime™ and a  
TIFF (Uncompressed) decompressor  
are needed to see this picture.

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