Socrates and the Nonslave-boy

W. M. Priestley

University of the South
Compulsive teaching can be risky. I once began a conversation outside of class with a student by saying that calculus literally means “pebble” and that playing with pebbles, or calculating, was a primitive form of arithmetic. To illustrate, I showed how pebbles arranged in the shape of a right triangle could be regarded as making up half a rectangle. This, I explained, was how the ancient Greeks saw that the sum of the first four positive integers was half of four times five, that the sum of the first five integers was half of five times six, and so on. After a few more examples I asked him about the sum of the first 1000 positive integers and quickly elicited the desired reply that it was half of 1000 times 1001.

Waxing with pleasure at the success of my Socratic method, I remembered Plato’s tale relating how Socrates himself deftly led an ignorant slave-boy to the discovery of truth. It occurred to me that there must have been unrecorded instances of Socrates giving lessons to students playing with pebbles, just as I was doing now. What an intriguing idea! I confidently began a Socratic dialogue, featuring myself in the principal role.

[SOCRATES] What is the sum of the first million whole numbers?

[BOY] It’s half of a million times a million and one.

[SOCRATES] Great! Now, what is the sum of the first N whole numbers?

[BOY] (After a pause) What’s this N? N is not a number.

This was not going as smoothly as Socrates and the slave-boy.

[SOCRATES] Yes, but suppose N stands for a number. You just did the case when N was a million and you said the answer was half of a million times a million and one... Now, WHAT IS THE SUM OF THE FIRST N NUMBERS?

He seemed to need time to absorb this. I backtracked some more.

[SOCRATES] You said the sum of the first four was half of four times five; the sum of the first five was half of five times six; the sum of the first million was half of a million times a million and one. What is the sum of the first N?
(BOY) (Exclaims) OH! The sum of the first N is half of N times...

There followed a very long pause, during which I bit my tongue, determined to say nothing more. Socrates would allow the boy to discover the truth for himself...

(BOY) (Exclaims again) OH!

At last, success is imminent! Is there anything sweeter?

Yet only silence followed as I awaited the answer I expected. Inexplicably, the student lowered his eyes. He would say nothing more.

Clearly, he was waiting for me to speak.

What would Socrates do now?

Socrates would show infinite patience, of course. I took him slowly through the earlier drill again and finally got him to say that the sum of the first billion integers was half of a billion times a billion and one. At length we arrived back at the same point. This time I was sure to get the answer I anticipated!

(SOCRATES) So, now! What is the sum of the first N integers?

(BOY) It's half of N times...

(SOCRATES) Yes, times WHAT?

(BOY) (Growing agitated) TIMES... THE NEXT ONE!

This had gone on too long. I was losing control. Why couldn't the student be like the slave-boy? Socrates was no match for such an awful and obstinate student. I reverted to my normal self.

(ME) (Wild-eyed) Of course it is! WHAT'S THE NEXT ONE AFTER N?

I realized that shouting was a mistake as soon as I had done it. The student was eyeing me nervously, as if he knew that at least one of us had lost his mind. For three seconds the silence was electric as we glared at each other, eyeball to eyeball. Then...

(BOY) (Softly and tentatively) It's 'O',... isn't it?

He might as well have hit me in the stomach, so thoroughly had he knocked the hubris out of me. "Of course it is!" I said, "Of course it is. How stupid of me!" I hugged him as tightly as I could. "I love you, Thomas," I said to my bewildered, eleven-year-old son.