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Review: The Dirichlet Space: A Survey

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This paper is a thorough survey of many recent and historical function-theoretic results about the classical Dirichlet space $D$ on the open unit disk $D$. As the authors themselves point out, generalizations to other domains or to several variables and connections to operator theory are not discussed. Nevertheless, there is still a wealth of information covered in this survey.

The authors begin with the basic definitions, including several alternate characterizations of $D$ due to R. Rochberg and Z. J. Wu [Illinois J. Math. 37 (1993), no. 1, 101–122; MR1193132 (93j:30039)] and B. Böe [Proc. Amer. Math. Soc. 131 (2003), no. 1, 235–241; MR1929043 (2003g:46024)]. An in-depth study of Carleson measures for $D$ is undertaken in Section 3. In particular, the original characterization of Carleson measures for $D$ due to D. A. Stegenga [Illinois J. Math. 24 (1980), no. 1, 113–139; MR0550655 (81a:30027)] and several more recent approaches of E. Tchoundja [Ark. Mat. 46 (2008), no. 2, 377–406; MR2430733 (2009g:32012)] and the first three authors [Rev. Mat. Iberoamericana 18 (2002), no. 2, 443–510; MR1949836 (2003j:30080)] are treated. A detailed exposition of the tree model of the unit disk and its application to the Dirichlet space are explored in Section 4. In particular, the authors study Carleson measures, capacities, and testing conditions from this viewpoint. A brief discussion of the complete Nevanlinna-Pick property is conducted in Section 5 (a more complete treatment can be found in the book [J. Agler and J. E. McCarthy, Pick interpolation and Hilbert function spaces, Grad. Stud. Math., 44, Amer. Math. Soc., Providence, RI, 2002; MR1882259 (2003b:47001)]), and Section 6 studies the multiplier space $M(D)$ and other spaces which are intrinsic to $D$ theory. In particular, the weakly factored space $D \odot D$, the $\overline{\partial}$-equation in the Dirichlet space, and the corona theorem for $D$ are considered. After a detailed discussion on interpolating sequences for $D$ and its multiplier space $M(D)$, the paper concludes with several open problems.

This paper will no doubt become a standard reference on the subject and also the starting point for many graduate students.

Reviewed by Stephan R. Garcia

References


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33. Jones, Peter W. \( L^\infty \) estimates for the \( \overline{\partial} \) problem in a half-plane. *Acta Math.* 150 (1983), no. 1–2, 137–152. MR0697611 (84g:35135)


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*