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Review: An Intertwining Property for Positive Toeplitz Operators

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An intertwining property for positive Toeplitz operators. (English summary)

This article concerns the question of whether an operator $V$ which satisfies the intertwining relationship $VT_{T_{\Gamma}} = T_{T_{\Gamma}}S$ exists, where $T_{T_{\Gamma}}$ denotes a given positive Toeplitz operator with operator-valued symbol $\Gamma$ and $S$ denotes the unilateral shift acting on a Hardy space $H^2(U)$.

In part, this paper is motivated by the authors’ previous results in the case where the symbol $\Gamma$ is scalar-valued. Specifically, they proved that if $a$ is a scalar-valued positive function on the unit circle, then there exists an operator $V$ on $H^2$ which satisfies $VT_a = T_aS$ if and only if $a^{-1}$ belongs to $L^1$ [A. Biswas, C. Foias and A. E. Frazho, Acta Sci. Math. (Szeged) 65 (1999), no. 3-4, 657–686; MR1737279 (2001b:47027)].

In this paper, the authors consider the more difficult case where $\Gamma$ is operator-valued. They obtain a necessary and sufficient condition for the existence of $V$ which involves the maximal outer factor corresponding to $T_{T_{\Gamma}}$ and a certain auxiliary function $\Phi$ (Theorem 3.2).

Theorem 4.1 provides a sufficient condition for the existence of $V$ which closely resembles the condition from the scalar-valued case. As the authors show, their condition is necessary in the case when the fiber space $U$ is finite-dimensional (Theorem 5.1 and Theorem 5.2) and therefore a complete analogue of the scalar-valued case holds when the fiber space $U$ is finite-dimensional.

On the other hand, the authors provide an example (Example 5.3) which demonstrates that the sufficient condition from Theorem 4.1 is not necessary when the fiber space is permitted to be infinite-dimensional.

The authors close with several open questions and some illuminating examples.

Reviewed by Stephan R. Garcia

References

1. H. Bercovici, private communication.


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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