1-1-2008

Review: Shift-Type Invariant Subspaces of Contractions

Stephan Ramon Garcia

Pomona College

Recommended Citation
(Reviewer: Stephan R. Garcia)
Using the Sz.-Nagy-Foias theory of Hilbert space contractions, the author proves a factorization theorem for asymptotically non-vanishing, absolutely continuous contractions. To be more specific, recall that a contraction $T$ on the Hilbert space $\mathcal{H}$ is an absolutely continuous (a.c.) contraction if the spectral measure of the unitary component of $T$ is a.c. with respect to normalized Lebesgue measure $m$ on the unit circle $\mathbb{T}$.

Let $n \in \mathbb{N} \cup \{\aleph_0\}$ denote a cardinal number and let $\mathcal{G}_n$ denote an $n$-dimensional Hilbert space. The author shows that if the spectral-multiplicity function of the unitary asymptote of the contraction $T$ is at least $n$ on the Borel subset $\gamma$ of the unit circle $\mathbb{T}$, then the natural embedding of the Hardy space $H^2(\mathcal{G}_n)$ into $\chi_\gamma L^2(\mathcal{G}_n)$ defined by $Jf = \chi_\gamma f$ can be factored as $J = ZY$ where $Y$ intertwines the unilateral shift $S_n$ on $H^2(\mathcal{G}_n)$ with $T$, and $Z$ intertwines $T$ with the unitary operator $M_{n,\gamma}$ of multiplication by the independent variable on $\chi_\gamma L^2(\mathcal{G}_n)$. Furthermore, it is proved that for each $\varepsilon > 0$, $Y$ and $Z$ may be chosen so that $\|Y\| < 1 + \varepsilon$ and $\|Z\| < 1 + \varepsilon$. These norm estimates improve upon previous work of the author [Acta Sci. Math. (Szeged) 61 (1995), no. 1-4, 443–476; MR1377377 (97b:47007)] and they require substantial modifications of the original proof.

The paper concludes with several observations relevant to the study of invariant subspaces. In particular, it is proved that if the residual set of a contraction covers the whole unit circle, then those invariant subspaces where the restriction is similar to the unilateral shift with a similarity constant arbitrarily close to 1 span the whole space. Moreover, the hyperinvariant subspace problem for asymptotically non-vanishing contractions is reduced to this special case.

Reviewed by *Stephan R. Garcia*

**References**


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.