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Review: On complex symmetric Toeplitz operators

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On complex symmetric Toeplitz operators. (English summary)

Let $\mathcal{H}$ denote a separable, complex Hilbert space. A conjugation on $\mathcal{H}$ is a conjugate-linear, isometric involution $C: \mathcal{H} \to \mathcal{H}$. A bounded linear operator $T: \mathcal{H} \to \mathcal{H}$ is $C$-symmetric if $T = CT^*C$. If there exists a $C$ with respect to which $T$ is $C$-symmetric, then $T$ is called a complex symmetric operator (CSO). It is known that $T$ is a CSO if and only if it is unitarily equivalent to a symmetric matrix acting on an $\ell^2$ space of the appropriate dimension.

Every normal operator is a complex symmetric operator, as are all truncated Toeplitz operators. It is suspected that “most” Toeplitz operators are not CSOs, and index considerations provide many Toeplitz operators that are not CSOs. The unilateral shift $T_z$ is such an example. As another example, if $T_\phi$ is analytic or coanalytic and a CSO, then $\phi$ is a constant function (Theorem 2.1 of the paper under review).

If $\phi \in L^\infty$ is real, then $T_\phi$ is self-adjoint, and hence a CSO. More generally, A. Brown and P. R. Halmos proved that a Toeplitz operator is normal if and only if its symbol is of the form $a\phi + b$, in which $a, b \in \mathbb{C}$ and $\phi$ is a real-valued function in $L^\infty$ [J. Reine Angew. Math. 213 (1963/1964), 89–102; MR0160136]. For several natural conjugations $C$ on the Hardy space $H^2$, the authors provide a characterization of the corresponding $C$-symmetric Toeplitz operators. These characterizations are given in terms of the Fourier coefficients of the symbol. As a consequence, they are able to construct non-normal, complex symmetric Toeplitz operators.

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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