Mathematical Arguments in Favor of Risk in Andy Weir's The Martian

Sarah C. Cobb
Midwestern State University

Jeff B. Hood
Midwestern State University

Mathematical Arguments in Favor of Risk in Andy Weir's The Martian

Mathematical Arguments in Favor of Risk in Andy Weir's *The Martian*

Sarah C. Cobb

Department of Mathematics, Midwestern State University, Wichita Falls Texas, USA
`sarah.cobb@mwsu.edu`

Jeff B. Hood

Department of Mathematics, Midwestern State University, Wichita Falls Texas, USA
`jeffrey.hood@mwsu.edu`

Abstract

In Andy Weir's novel *The Martian*, the characters encounter high-stakes, life-or-death situations, in which they must make choices based on their assessment of risk and likely outcomes. They have different reactions to risky situations, based on their approaches to assessing risk and their perspectives on the stakes involved. In this paper, we examine the ways that characters in *The Martian* intuitively assess risk and compare them to mathematical analysis of the situations in the book.

1. Introduction

Andy Weir's novel *The Martian* [2] opens with astronaut Mark Watney realizing that he has inadvertently been stranded on Mars. (Note that this paper will contain significant spoilers for the novel.) He has supplies intended to last six astronauts for a thirty-day mission, no way to leave the planet, and no way to communicate with any other human. Over the next year and more, Mark, his crewmates, and NASA work to find a way for him to survive on Mars and ultimately return to Earth.

The decisions the characters make in this situation are high-stakes. A significant mistake could result in Mark's death, but the time and resources available are too limited for extensive testing or tried-and-true strategies.

The characters respond to this pressure in a variety of ways. Teddy, NASA's administrator, tends to make the most cautious decisions he can and is reluctant to try anything that might make the situation worse. Mark himself becomes willing to improvise, risking disasters in order to increase his chances of ultimately surviving. Other astronauts in the book repeatedly express their willingness to risk their lives if there is a chance to save Mark.

These differences stem from different approaches to assessing risk. Because Mark's situation presents so many new problems with so little time to investigate possible solutions, the outcomes of situations are highly unpredictable. Mathematics provides tools to deal with uncertainty in careful, logical ways.

While assessment of risk in *The Martian* is driven by high-stakes, life-or-death decisions, the practice of making decisions that weigh risks against potential gains is universal. Whether making decisions about investing money, considering whether to start a conversation with a stranger, or choosing between health care options, assessment of risk against reward is unavoidable. As we make decisions, we develop intuition for dealing with risk. Mathematical assessment can help check and correct our intuitive approaches to risk assessment.

In this paper, we examine the ways that characters in *The Martian* intuitively assess risk and compare them to mathematical analysis of the situations in the book.

2. Expected Value

2.1. Intuition

Throughout *The Martian*, characters make choices about risky strategies. Mark's limited options lead him to jump into potentially risky decisions relatively readily: his desperate need for water leads him to attempt to synthesize water in the Hab, a decision that nearly kills him, but ultimately provides him with enough water to survive. For the NASA team back on Earth, decisions are made in meetings and consultations after weighing a much larger range of options.

One memorable example is Teddy's decision between two proposed uses of the one large rocket NASA has available. One option is the Iris II supply probe, designed to supply Mark with enough food to last until the next scheduled Mars mission. According to NASA's estimates, the Iris II mission has a 30% chance of success. The other possibility is the Rich Purnell maneuver, which would send Mark's five crewmates back to Mars to retrieve him instead of returning directly to Earth. This maneuver has a higher probability of success, but risks six lives instead of one. Teddy justifies his choice of Iris II with the statement that he does not think Rich Purnell is "six times more likely to work" (page 206).

Teddy's math in this case is profoundly flawed: in order to be six times as likely to work, the Rich Purnell maneuver would have to have a 180% chance of success. So even if the maneuver had a 100% chance of success and was therefore unambiguously a better choice, it would fail to meet Teddy's casual requirement.

There are a number of mathematical models and tools for more rigorously assessing decisions containing an element of risk. One of these methods is by computing expected value. We will examine Teddy's decision using this framework.

2.2. Computing Expected Value

Expected value is a useful tool for predicting average outcomes when the results of any process are unpredictable. The concept developed in the 17th century and was initially applied to gambling problems, though it did not take its modern form until later.

Any process whose results cannot be reliably predicted is called an *experiment*. An experiment has a space of possible outcomes, each having its own probability. Each probability is expressed as a number between 0 and 1, with a probability of 0 representing an impossible outcome and a probability of 1 representing an outcome that will certainly occur. In general, the closer the probability of an outcome is to 1, the more likely the event is.

In any experiment, it is possible to compute the *expected value* of the outcome, representing the average value of the outcome if the experiment is repeated a large number of times. If the random variable can take on values $x_1, x_2, \ldots x_n$ with probabilities $p_1, p_2, \ldots p_n$ respectively, the expected value of the variable is given by

$$E = p_1 \cdot x_1 + p_2 \cdot x_2 + \cdots + p_n \cdot x_n.$$

As an illustrative example, suppose a box contains three \$1 bills, two \$5 bills, and one \$20 bill. If one bill is drawn from the box uniformly at random, what is the expected value of the bill?

This experiment has three possible outcomes: draw a \$1 bill (probability $\frac{3}{6}$ or $\frac{1}{2}$), draw a \$5 bill (probability $\frac{2}{6}$ or $\frac{1}{3}$), and draw a \$20 bill (probability $\frac{1}{6}$). Thus the expected value is

$$E = \frac{1}{2} \cdot 1 + \frac{1}{3} \cdot 5 + \frac{1}{6} \cdot 20,$$

or 5.50.

It is noteworthy that the expected value \$5.50 is not one of the possible outcomes of the experiment. The expected value predicts the average value of the variable if the experiment is repeated a large number of times, not the likely outcome of any particular experiment. If 1,000 people draw one bill each out of 1,000 identical boxes, each of them will get \$1, \$5, or \$20, but the average value of all their bills will be close to \$5.50.

The nature of randomness makes it impossible to reliably predict the outcome of any particular experiment. But using expected value, it is possible to predict the average outcome. Therefore, using expected value as a guide for decision-making does not guarantee the best decision in any particular case, but it does guarantee the best average outcome in the long term.

The overarching goal of most characters in *The Martian* is to save Mark Watney's life, but without costing lives of other astronauts. This invites the use of expected value computations where the value is in lives lost, with the goal of making the decision that has the lowest expected loss of life.

2.3. Expected Value Computations on Mars

Many of the decisions Mark makes during his time on Mars are easy to understand when framed as problems of expected value. No matter how dangerous a particular situation is, it is preferable to certain death—the fate Mark faces if he refuses to engage in risky activities. Two parallel examples of this dynamic are Mark's decision to dig up the RTG for use as a heat source and his decision to synthesize water in the Hab.

In each of these two cases, Mark has two possible choices: risk immediate death (by radiation or explosion) in order to improve his chances of getting back to Earth; or decline the risk and not implement his plan. Let p represent the probability that Mark's risky plan will work, resulting in 0 lives lost. Then there is a probability of $1 - p$ that the plan will fail, killing him and resulting in 1 life lost. Thus the expected number of lives lost is

$$p \cdot 0 + (1 - p) \cdot 1 = 1 - p.$$

Since p is a number between 0 and 1, $1 - p$ is also between 0 and 1. If the probability of success is small, the expected loss of life is near 1—Mark will probably die.

It is even simpler to compute the expected value of declining to take the risk. If Mark does not find a way to water his crops and to heat his rover during his journey to Ares IV, he will certainly die. Therefore the expected number of lives lost is 1, which is larger than the $1 - p$ expected value for taking the risk.

This kind of computation drives much of Mark's fearless risk-taking throughout his time on Mars. Any action that makes it possible for him to get back to Earth is a gain compared with the certain death of taking no action.

2.4. Expected Value Computations on Earth

Teddy's situation is somewhat different from Mark's in that he has a larger range of possible actions with a corresponding variety of outcomes. In particular, his decision about whether to send the Iris II probe to resupply Mark

or to use the Rich Purnell maneuver to send the Hermes back to pick him up is mathematically more complicated than the decisions Mark makes.

Iris II has two possible outcomes: success, in which case the total loss of life is zero, with a probability of 30%; and failure, in which case the total loss of life is one, with a probability of 70%. This means that the expected loss of life from Iris II is

$$(.3)(0) + (.7)(1) = .7.$$

The Rich Purnell maneuver has an unspecified probability of success. Let p represent this probability. The two possible outcomes, then, are success, in which case the total loss of life is again zero, with probability p; and failure, in which case the total loss of life is six, with a probability of $1 - p$. The expected loss of life, then is

$$(p)(0) + (1 - p)(6) = 6 - 6p.$$

Using these expected value numbers, Teddy should choose whichever of the two plans has the lower expected loss of life, which will depend on the value of p. Specifically, if $p > .883$, the Rich Purnell maneuver has the lower expected loss of life. If $p < .883$, Iris II has lower expected loss of life.

While the NASA scientists and administrators are readily able to give probabilities of success on many occasions, this is not one of them. Because the Rich Purnell maneuver is a closely-held secret, the scientists who work to determine those probabilities cannot be asked for a detailed analysis. The decision is not necessarily easier in light of this computation, but the threshold is clearer: if the Rich Purnell manuever has at least an 88.3% chance of working, Teddy should make that choice.

Note that both computations are somewhat simplified, since there are more than two possible outcomes in each case. For example, even if Iris II is a success, Mark could still die during his journey to the Ares IV site, or the Hermes crew could execute the Rich Purnell maneuver perfectly and still fail to rendezvous with Mark—in fact, this nearly happens. Still, this model captures the essential features that Teddy is using in his computation and frames them in a more mathematically rigorous way.

The mathematical analysis in each of these situations favors risky decision-making: the expected outcome is improved by making decisions with high levels of uncertainty and bad worst-case scenarios, but high probability of

success. Mark's intuitive assessment of risk leads him to adopt this risk-friendly attitude easily, whereas Teddy's more cautious nature leads him to avoid as much risk as possible. *The Martian* presents a world in which risk is rewarded and caution is the least safe option.

3. Markov Chains and Probability

3.1. Intuition

Throughout *The Martian*, the measures Mark takes to survive involve greater and greater risk, seemingly putting him at such an extreme level of peril that his salvation would seem unlikely in the extreme. But is it really, or is it just our intuition that is tricking us?

Logic would dictate, for example, even in a rudimentary thought experiment, that the probability of his surviving any one of the extreme risks he takes in his endeavor to save his own life might have been low. But for the sake of argument, let us assume that the probability of his surviving any one risk is 50%. Mathematically speaking, we would say that is $\frac{1}{2}$, or one chance of every two attempts. If he takes two such risks, standard mathematics would dictate that the probability of surviving *both* such risks is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. One's intuition would then suggest that with greater number of such risks (Mark's risks might be reckoned at the order of about 30), the probability of survival is an ever decreasing value, rapidly approaching 0; $(\frac{1}{2})^{30}$ represents only a 1 in 1,073,741,824 chance of survival.

It is worth mentioning that even if we consider Mark's probability of surviving one risk to be much higher in this rationale, say 90%, after 30 risks we would conclude that he has approximately a 4% chance of survival. This suggests that the fallacy is not with the number we assign to the probability of surviving one risk, but rather with the selection of the model we are using. We therefore must find a better model to adequately measure the long-term risk that Mark assumes.

We cannot assume that the outcomes of Mark's risk-taking are always divided into absolute success or utter failure (see Figure 1); this is where our intuition is flawed. There are varying degrees of success or failure, only a small fraction of which would result in his immediate demise, and a slightly larger fraction implying his ultimate demise on Mars (see Figure 2).
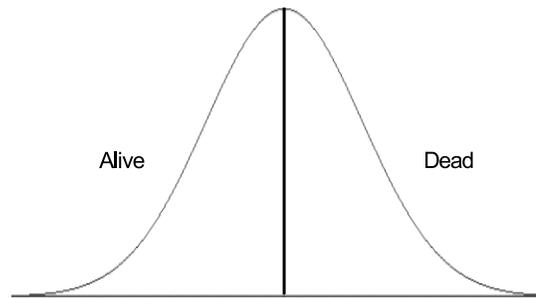
Figure 1: A typical intuitive interpretation of Mark's risk assessment.

For the vast majority of outcomes classified as 'failure', there are things that he can do to rectify the situation (albeit by taking further, usually more cautious, risk). This is evidenced multiple times in the story, such as in Chapter 5, *Sol 40*, when an explosion occurs in the Hab while Mark is attempting to synthesize water. Mark is soon able to continue the water synthesis by assessing the reason for the explosion and cautiously proceed without the problematic elements. Also, in Chapters 13 and 14, *Sol 119*, when the airlock blows off of the Hab and his faceplate is smashed, he is able to achieve a lower level of risk by first increasing his risk: he must limit his mobility by removing the arm of his suit and using it to create a seal on his broken faceplate. In both situations, Mark's risk-taking is vindicated when he salvages a perilous situation, demonstrating that the probability of a catastrophic outcome is much less than intuition might suggest.
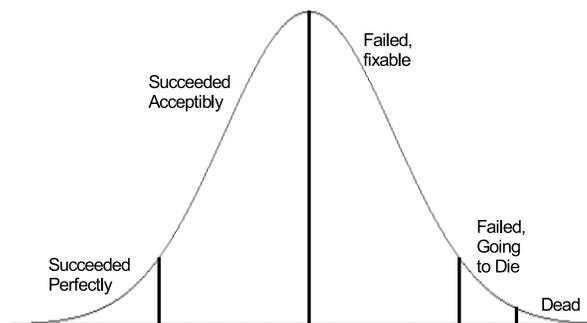

Figure 2: A more realistic interpretation of Mark's risk assessment.

The second argument in favor of Mark's continued risk-taking is that the idea that these events are compounding in the way described above is also flawed. It would be a more accurate measurement of his survivability to use state-transitions in a Markov Chain to simulate his likelihood of survival. This process is described in the following section.

3.2. Risk Assessment with Markov Chains

In order to understand how we will calculate Mark's likelihood of survival, we must first examine what a Markov Chain is. We create a Markov Chain by taking a state-transition matrix and multiplying it by itself a number of times to determine the outcome of several iterations of connected events. To explain:

A *state-transition matrix* is a square matrix of numbers that represent the probabilities of certain events, given certain starting conditions. A simple example would be: If it is raining today, there is a 50% (i.e., .5) chance it will rain tomorrow, and a 50% chance it will be sunny; if it is sunny today, there is a 75% chance it will be sunny tomorrow, and a 25% chance it will rain. We arrange these probabilities in such a way that in a row, the probabilities add to 1 (i.e., 100%).

$$A = \begin{bmatrix} \text{will rain} & \text{will sun} & \\ .50 & .50 & \text{rain} \\ .25 & .75 & \text{sun} \end{bmatrix}$$

or, for mathematical usage, simply

$$A = \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix}.$$

This matrix is usable in its pure form. For example, if we know whether or not it is raining, that corresponds to a vector of the form

$$\mathbf{v} = \begin{bmatrix} \text{rain} & \text{sun} \end{bmatrix},$$

and if it is indeed currently raining, we get

$$\mathbf{v} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

So, if we multiply the state-transition matrix by this vector, we see that

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix} = \begin{bmatrix} .50 & .50 \end{bmatrix},$$

which gives us the prediction as already dictated that there's a 50/50 chance of rain or sun.

Perhaps the idea is not terribly interesting just using it one time. What makes it interesting is when we chain the matrices together. Let us say we want to know, if it is raining today, what is the probability it will rain in three days? We can simply take the matrix and multiply it by itself three times, then by the vector:

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix} \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix} \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix} = \begin{bmatrix} .3438 & .6563 \end{bmatrix},$$

which gives us a 34.38% chance of rain in three days.

This notation can tend to be clunky with larger numbers of state transitions, so we can abbreviate it to

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix}^3 = \begin{bmatrix} .3438 & .6563 \end{bmatrix}.$$

What ends up happening though, is eventually (if you are predicting sufficiently far into the future), the probability of rain or shine is independent of whether it is currently raining, and the Markov chain reveals the trends inherent in the system we are looking at. In other words, we would look more deeply at the trend of multiplying the state-transition matrix by itself as the number of multiples gets very large. Very large, being subjective, can actually be as small as 30. Indeed, for our example,

$$\begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix}^{30} = \begin{bmatrix} .3333 & .6667 \\ .3333 & .6667 \end{bmatrix} \text{ and } \begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix}^{31} = \begin{bmatrix} .3333 & .6667 \\ .3333 & .6667 \end{bmatrix}.$$

At this point, the chain has stabilized and we may say

$$\begin{bmatrix} .50 & .50 \\ .25 & .75 \end{bmatrix}^n \xrightarrow{n \to \infty} \begin{bmatrix} .3333 & .6667 \\ .3333 & .6667 \end{bmatrix}.$$

Thus on any random day, for that location, there is a roughly 1-in-3 chance of rain, and 2-in-3 chance of sun.

To apply this to Mark's chances of survival, we will look at the risk not as a chance of survival or death, but as differing states of safety versus un-safety, and use Markov Chains to determine his long-term safety (survival). In this scenario, we consider four different possible changes of state: Safe-to-Safe (SS), Safe-to-Unsafe (SU), Unsafe-to-Safe (US), or Unsafe-to-Unsafe (UU). So, for example, in any of his endeavors, if Mark starts out safe, but ends unsafe, we would classify that as SU.

Assessing the number of these state transitions was a challenge in itself. We noticed that one person's assessment of Mark's safe/unsafe levels was liable to be biased one way or the other. So, we asked that a group of students also render their counts of the number of times that Mark's state changed. Then, the average probability of each state-transition was found and used in the appropriate position of the transition matrix:

$$M = \begin{bmatrix} SS & SU \\ US & UU \end{bmatrix} = \begin{bmatrix} .723477 & .276523 \\ .529157 & .470843 \end{bmatrix}.$$

When this matrix is subjected to repeated self-multiplication we find that

$$M^n = \begin{bmatrix} .723477 & .276523 \\ .529157 & .470843 \end{bmatrix}^n \xrightarrow{n \to \infty} \begin{bmatrix} .656783 & .343217 \\ .656783 & .343217 \end{bmatrix}.$$

This implies that while Mark's situation is indeed dangerous, with an appreciable probability of his eventual death on Mars, his long-term probability of surviving Mars is more along the lines of 65.68%. This is certainly a better chance of survival than the 1 in 1,073,741,824 chance supposed by intuition.

4. Mark the Invincible

Over the course of the story, Mark takes a more and more cavalier attitude toward the risks he is taking. Initially, he makes risky decisions carefully after considering the consequences; by the end of the book, he impulsively proposes punching a hole in the glove of his space suit and using the air pressure to fly like Iron Man.

Mark's embrace of necessary risk may have led him to believe in his own near-invincibility. In part, this may be because of what we called the intuitive approach to risk that suggests an astronomically low probability of survival.

Since he has survived so long in a seemingly unsurvivable situation, he develops a sense of "luck" to explain it. What he needs to remember is that his situation is not as immediately dire as it would seem, but it is indeed dire throughout. As we see, even with our Markov Chain calculation, his chances of surviving being stranded on Mars are still considerably lower than the probability of surviving more than a year on Earth.

He is also trapped by the difficulty of computing expected value in dynamic situations. Dan Gilbert, in his book *Stumbling on Happiness* [1, pages 235-238], describes just why humans are particularly bad at assessing the expected outcome of a particular situation. Gilbert's arguement is that while the expected value formula is a useful tool for making decisions, it is often poorly used because people are bad at determining the two unknown quantities involved. In other words, people do not know what something is actually worth, and they do not know how to determine the probability of its occurrence.

Since this calculation is difficult, Mark develops his own shortcut: if he takes a risk, he might die; if he does not take risks, he will definitely die. This leads him to impulsively take risks that would give pause to anyone in a less grave situation.

Mark's decision-making process comes into conflict with NASA's when Mark's water reclaimer malfunctions. NASA would have had Mark wait to complete the project while they debated millions of miles away about how he should fix it, because it was not considered his field of expertise. Mark on the other hand, standing two feet from the equipment, could see the obvious problem, determine the method to fix it, and assess that the risk was minimal if any that it would go wrong. So, he relied on his own judgement, ignored his instructions from NASA, and fixed the water-reclaimer with no ill effects. In this instance, Mark's perception of the risk and NASA's were different. Mark takes a slightly riskier route, trusting his expertise and his "luck" and taking the action he deems necessary.

Once the rest of the crew re-enters the situation, Mark's computational shortcut leads him astray. More people and more resources mean more choices. It is not a matter of risk-or-death, but of choosing the best course among a wider (but still limited) range of options. Mark's "Iron Man" proposal relies on his own resources and risks only his own life—the sort of decision that becomes natural after a year spent in extreme isolation.

Commander Lewis counters Mark's "Iron Man" idea by blowing an airlock on the ship that will take the crew back to Earth, causing the ship to decelerate so that Mark can be retrieved. Commander Lewis has embraced Mark's willingness to take risks, but she has not relied on his computational shortcuts. In fact, she is able to run simulations on the ship's onboard computers to determine the overall outcome and probability of success. She is therefore able to propose a solution that has a high probability of success and uses the available resources efficiently.

Lewis' plan walks the boundary between Teddy's over-cautious approach and Mark's wild improvisation. Here, as throughout *The Martian*, a willingness to take risks is vindicated. Carefully assessing the risks of various options and choosing the best one allows characters to make the best possible decisions.

5. Concluding Remarks

Mark Watney's situation in *The Martian* provides a small, relatively uncomplicated setting in which to explore modeling techniques and the various heuristic approaches people take for making decisions in risky situations. These techniques provide a possible basis for risk assessment and decision-making in real situations.

Many of the decisions that humans are faced with every day involve measuring risk against potential reward. Two well-known proverbs speak to opposite approaches to making risky decisions: "nothing ventured, nothing gained" argues for Mark's bold, risk-embracing strategy, while "better safe than sorry" promotes Teddy's inclination to avoid risk. The contradictory nature of these two pieces of received wisdom illustrates the balance necessary in making these decisions. As in *The Martian*, it is possible to err on the side of too much caution or too much risk.

In our day-to-day life, using careful mathematical modeling is often impractical. We primarily make quick, low-stakes decisions, and pausing to do extensive calculation for these everyday decisions would paralyze us. At crucial moments, however, careful assessment is necessary, and mathematics provides useful tools to conduct that assessment.

References

[1] Daniel Gilbert, *Stumbling on Happiness*, Knopf, New York NY, 2006.

[2] Andy Weir, *The Martian*, Broadway Books, New York NY, 2014.