Review: Nevanlinna-Pick spaces with hyponormal multiplication operators

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The main result of this paper (Theorem 1.1) shows that the classical Hardy space $H^2$ is essentially the only complete Nevanlinna-Pick space whose multiplication operators are all hyponormal. To be more specific, if $\mathcal{H}$ is an irreducible complete Nevanlinna-Pick space on a set $X$ with kernel $K$ such that all multiplication operators on $\mathcal{H}$ are hyponormal, then either:

1. $X$ is a singleton and $\mathcal{H} = \mathbb{C}$.
2. There is a set of uniqueness $A \subset \mathbb{D}$ for $H^2$, a bijection $j: X \to A$, and a nowhere vanishing function $\delta: X \to \mathbb{C}$ such that

$$K(\lambda, \mu) = \delta(\lambda)\overline{\delta(\mu)} k(j(\lambda), j(\mu)).$$

Here $k(z, w)$ denotes the Szegö kernel on $H^2$.

In particular, the map $f \mapsto \delta(f \circ j)$ is a unitary operator. Continuity and analyticity are also respected: if $X$ is endowed with a topology such that $K$ is separately continuous on $X \times X$, then $j$ is also continuous. If $X \subset \mathbb{C}^n$ and $K$ is holomorphic in the first variable, then $j$ is holomorphic.

An interesting consequence of the main theorem is Corollary 1.3, which asserts that if $n \geq 3$ and $U \subset \mathbb{R}^n$ is an open set, then there is no irreducible complete Nevanlinna-Pick space on $U$ which consists of continuous functions and whose multiplication operators are all hyponormal. The proof appeals to Brouwer’s invariance of domain theorem.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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