A Tale of Two Workshops: Two Workshops, Three Papers, New Ideas

Gizem Karaali

Pomona College

Recommended Citation
Opening Futures to Opportunity
Research Experiences for Undergraduate Faculty

Participating in a research experience in mathematics as an undergraduate can change a student’s life, inspiring him or her to pursue a career in the mathematical sciences. Approximately 60 percent of bachelor’s degrees in mathematics are awarded by primarily undergraduate institutions, or colleges and universities that do not have doctoral programs. Faculty at these institutions typically teach heavy course loads and have limited time to invest in their own research pursuits, which means that engaging in research with students can be challenging. AIM’s Research Experiences for Undergraduate Faculty (REUF) program encourages faculty to do research with undergraduates, while providing an avenue for research re-engagement for the faculty themselves and a network for professional support. Many of the faculty participants initiate research with undergraduates following the workshop; some have published papers with REUF collaborators.

The REUF program originated with a workshop proposal by three faculty members at historically black, primarily undergraduate institutions: Roselyn Williams of Florida A&M University, Yewande Olubummo of Spelman College, and Joe Omoyika of Southern University at New Orleans. Since the first REUF workshop in 2008, AIM has organized three additional workshops and follow-up activities, all supported by the National Science Foundation. Williams, Ulrica Wilson of Morehouse College, and AIM’s Associate Director for Diversity, Leslie Hogben, oversee the program, working closely with AIM’s Director of Special Projects, Brianna Donaldson. Recruitment efforts focus on faculty at minority-serving institutions and underrepresented minority faculty at undergraduate colleges.

Each REUF workshop involves approximately 20 fully funded participants and four mathematical leaders, senior mathematicians who have experience doing research with undergraduate students. Most of the time is spent doing mathematics, but there are also whole group discussions about topics such as best practices in undergraduate research. The workshop also includes instruction in using the free open-source mathematics software Sage.

In addition to the workshop, each annual cycle of the REUF program includes follow-up activities for participants to support continuation of research engagement sparked by the workshop. One research group per year receives funding to return to AIM for a week to continue their collaboration.

Beginning this year, the Institute for Computational and Experimental Research in Mathematics (ICERM), in Providence, Rhode Island, has partnered with AIM on the REUF program. ICERM hosted the REUF 4 workshop in June 2012 in their new facility and will do so again in summer 2013. AIM looks forward to continuing this partnership.  

– Brianna Donaldson and Leslie Hogben

A Tale of Two Workshops
Two Workshops, Three Papers, New Ideas

In July 2009, together with my colleague and spouse Stephan Ramon Garcia (and our five-month old baby, Reyhan), I attended an AIM REUF workshop. The main motive of the workshop appealed to me, since I had been teaching at a liberal arts college since right after my post-doc. Because most of my research focused on quantum algebra, a topic not easily accessible to even the most well-prepared undergraduates, I had been struggling to find a way to initiate and sustain successful collaborations with my students.

On the first day of the workshop, the organizers introduced us to four possible topics, all intriguing. But Stephan’s deep interest in number theory and my training in representation theory led us both to eventually select the project proposed by Phil Kutzko from the University of Iowa, a problem that combined classical character theory with number theory. When the workshop ended, most people went back to their regular routine, but Stephan and I, together with one of our team members, Patrick Fleming from the South Dakota School of Mines, kept at the problem. We were quite intrigued by how a simple finite group seemed to have embedded in its character theory some significant identities relating to a mathematical quantity called a Kloosterman sum. After additional focused work we ended up with a lengthy preprint, which, when posted on the Internet arXiv of mathematics papers, generated several interesting follow-up e-mails from around the world. Our paper, “Classical Kloosterman sums: Representation theory, magic squares, and Ramanujan multigraphs,” has already been published by the Journal of Number Theory.

We were pleased; none of us were experts in number theory, but this was a solid publication, and the conversations that opened up through the arXiv preprint itself were quite exciting. However, there was an element of redundancy in our work and we had a sneaking suspicion that there was something we were missing. We thought that somehow there had to be a way to clean things up so that we did not need to employ ad hoc methods to get rid of irrelevant material and reveal the desired information.

Meanwhile, back in December 2009, while we were still working on our Kloosterman sums paper, I received a most welcome invitation to attend another AIM workshop in May 2010 on supercharacters and combinatorial Hopf algebras. I had recently completed a project on combinatorial Hopf algebras and was looking to start a second one. I knew nothing about supercharacters at the time, but found the second topic of the workshop exciting. My main goal was to further connect with the active members of the combinatorial Hopf algebra research community and to possibly foster some new collaborations. However, the very first day of the workshop, when I actually understood what supercharacters were about, I had an epiphany. This was exactly what we needed!

The workshop itself brought forward some amazing results, which culminated in, with 28 authors, possibly the most co-authored paper in the history of pure mathematics. See http://www.mathinstitute.org/nuggets/secrete_identity.html for more details. Later, we focused on how supercharacter theory could help us get rid of the redundancy in our previous work. We could see that the previous Kloosterman identities could be obtained through a particular supercharacter theory for the associated finite group, which then led us to wonder whether the theory could be extended to other situations. This led to fertile land for investigation. In particular, we were able to prove recently, together with Pomona undergraduate Christopher Fowler, that certain supercharacter theories for cyclic groups could be used to churn out several (known and unknown) identities related to other important quantities called Ramanujan sums. As we got more into this investigation, many new possible avenues opened up in front of us, and several undergraduate researchers joined our quest. Stephan directed a team project this summer to push this thread further with the result of a second paper already posted on the arXiv.

All in all, this has been a fantastic ride so far! And we could not have done it if we weren’t for serendipity and, of course, the two distinct workshop opportunities, provided generously by the American Institute of Mathematics.

– Gizem Karaali