Harvey Mudd College
Mathematics

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Research Proposal: Discrete Signal Processing and Representations of Groups

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1 Introduction

The ability to process and analyze signals (values that vary with time or space) is crucial to a wide variety of uses, from engineering to music production. Beyond its applications in other fields, the mathematics at the heart of signal processing are interesting to study on their own. Markus Püschel and the SMART (Signal Models, Algebra Representations, and Transforms) project at Carnegie Mellon University have developed a theory for viewing signal processing as a collection of algebraic structures [Püschel and Moura(2006)].

2 Proposed Research

The classical discrete Fourier transform (DFT) is well known to be the representation-theoretic discrete Fourier transform of the cyclic group. In [Püschel and Moura(2006)], signals and filters are viewed as elements of polynomial algebras, with the signal processing Fourier transforms given by applications of the Chinese remainder theorem. I would like to explore connections between signal processing and group algebras, specifically searching for a similar general construction of the discrete sine and cosine transforms as representation-theoretic DFTs for group algebras.

3 Prior Research

Again, [Püschel and Moura(2006)] gives a general algebraic interpretation of signals and filters as elements of polynomial algebras, with corresponding Fourier transforms derived using the Chinese remainder theorem. Furthermore, [Püschel and Moura(2006)] indicate that their background research uncovered methods for deriving some, but not all, of the discrete sine and cosine transforms in terms of group representations. However, they did not

discover a general group-theoretic framework that could derive all of these transforms.

I took Representation Theory in the spring of my junior year, giving me a basic background in the representations of finite groups. In addition, I am familiar with the representation-theoretic idea of a Discrete Fourier Transform. However, I have relatively little background in signal processing and representations of polynomial algebras or non-finite groups; I will be learning more about these topics.

References

[Püschel and Moura(2006)] Püschel, Markus, and José M. F. Moura. 2006. Algebraic signal processing theory. *CoRR* abs/cs/0612077.