

Research Proposal: Minimal Circuits for Very Incompletely Specified Boolean Functions

Richard Strong Bowen

Faculty Advisor: Professor Nick Pippenger

1 Introduction

One topic in circuit complexity theory is the minimum circuit (made up of some set of gates, such as AND, NOT, and OR gates) which computes a particular boolean function (i.e., a function from $\{0, 1\}^n$ to $\{0, 1\}$).

Livnat and Pippenger([1]) discusses some applications of circuit complexity theory to theoretical biology. Specifically, they consider theoretical model organisms with some *computational limitation*, which they model as some limit on the number of gates in a boolean circuit describing the organism's binary choices. They are interested in systematic mistakes - mistakes due to limitations in the circuit. They show that, for most functions, the best circuits (i.e., the smallest ones which are correct on a certain fixed fraction ϵ of input bitstrings) of a certain size depend on all their inputs, and conclude from this that most such circuits make systematic mistakes.

Sholomov ([3]) considers incompletely-defined boolean functions and bounds on their circuit sizes. A boolean function is said to be incompletely defined if it is only defined on a subset of $\{0, 1\}^n$, i.e., there are rows of its truth table which are marked by a "don't care" symbol. The same paper also shows that, as long as the number of specified rows (N_n) of the truth table is at least

$$n \log_2^{1+\delta} n,$$

for some positive δ , then as n grows the minimum number L of gates in a circuit for a function, for most circuits, goes to

$$\rho \frac{N_n}{\log_2 N_n},$$

where ρ is a constant depending on the basis.

Sholomov ([3]) does not discuss the effects of allowable errors (the fraction ϵ in the paper of Livnat and Pippenger([1])) on the complexity of circuit sizes. This is considered by Pippenger([2]), where the size of a circuit

realizing a function having some fixed fraction of its inputs specified and allowed to make some fixed fraction of errors is asymptotically given. Unlike Sholomov([3]), Pippenger([2]) considers only circuits with a fixed fraction (i.e., growing exponentially) of specified rows in the truth table.

2 Proposed Research

I will try to find bounds on minimum circuits to realize a function which is specified on something smaller than a fixed fraction of its possible input strings, for example, one which is specified on polynomially many of them, and which are allowed a fixed fraction of errors. This is a sort of combination of the papers of Pippenger([2]) and Sholomov([3]), the first of which considers fixed fractions of allowed errors and the second which considers almost any polynomial number of input strings.

References

- [1] A. Livnat and N. Pippenger. Systematic mistakes are likely in bounded optimal decision-making systems. *Journal of Theoretical Biology*, 250(3):410–423, 2008.
- [2] N. Pippenger. Information theory and the complexity of Boolean functions. *Mathematical System Theory*, 10(1):129–167, 1976.
- [3] L. A. Sholomov. On the Realization of Incompletely-Defined Boolean Functions by Circuits of Functional Elements. *Problemy Kibernetiki*, 10:215–226, 1969. Trans: System Theory Research, 21 (1969) 211-223.