HARVEY MUDD

Introduction

Surgically removable cataract is the leading cause of blindness worldwide. The VISION 2020 project of the World Health Organization (WHO) aims to raise cataract surgical rates (CSR) in Africa. To accomplish this goal, CSR targets should at least equal cataract incidence in each region (Lewallen et al., 2010).

Cataract incidence is difficult to measure, but Rapid Assessment of Avoidable Blindness (RAAB) surveys provide age-specific cataract prevalence data. They

- Sample from feasible districts of 1–2 million people;
- Distinguish several causes of blindness or near-blindness, including cataract; and
- Are standardized over the 7 districts surveyed so far.

Research Goal

From RAAB data, age-specific cataract prevalence (static percentage of the population with cataract) can be compared across districts. Our research challenge is to estimate cataract **incidence** (rate at which people develop cataract) from prevalence data.

Previous Research

Podgor and Leske (1986) show a way to estimate incidence from age-specific prevalence.



Figure 1: Single-Disease Incidence Estimation Model. They make assumptions reasonable for cataract:

- People over 50 years old in each district form a closed system (little immigration or emmigration);
- Prevalence and mortality are time-independent; and
- Transitions follow exponential distributions.

The model then uses the age-dependence of prevalence in place of time to estimate incidence rates.

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Extended Incidence Model

For cataract, our extended model distinguishes between cataract in one eye (one surgery needed) or both eyes (double surgery needed).



- *H* : Healthy individuals
- *U* : Unilateral cataract
- *B* : Bilateral cataract
- *D* : Deceased individuals

Figure 2: Incidence Model for Bilateral Diseases.

We interpolate transfer rates for each 5-year interval between prevalence data points. The number N_1^i of people in any state at the interval's end is based on the initial number N_0^i of people in its source states times the transition probability P_{ij} . This gives

> $N_1^H = N_0^H P_{HH}$ $N_1^U = N_0^H P_{HU} + N_0^U P_{UU}$ $N_1^B = N_0^H P_{HB} + N_0^U P_{UB} + N_0^B P_{BB}.$

Each *N* can be expressed as the prevalence (proportion) of that state times the total population. The transition probabilities P_{ii} can be expressed in terms of transition rates λ_{kl} . For example,

$$P_{HH} = e^{-(\lambda_{HD} + \lambda_{HU})}, \text{ and}$$

$$P_{HB} = \int_0^1 \int_{t_U}^1 \int_1^\infty \lambda_{HU} e^{-(\lambda_{HD} + \lambda_{HU})t_U} \lambda_{UB} e^{-(\lambda_{UD} + \lambda_{UB})(t_B - t_U)}$$

$$\cdot \lambda_{BD} e^{-\lambda_{BD}(t_D - t_B)} dt_D dt_B dt_U.$$

Our four-compartment model has new transition probabilities, notably P_{HB} (a two-state progression in one time step). Since mortality rates λ_{HD} , λ_{UD} , and λ_{BD} are known, we can solve for λ_{HU} and λ_{UB} .

We further extended our model to an arbitrary, *n*stage progressive disease by finding probabilities of transitions through arbitrarily many states. Our conservation equations also generalize in this way.

Results

District incidence results with 95% confidence intervals are shown in Figure 3.



Figure 3: Cataract Incidence Results.

RAAB surveys fell into two groups based on 95% confidence intervals for prevalence. Prevalence in Group 2 (Eritrea, Mali and The Gambia) is about 2.5 times greater than in Group 1 (Kenya, Tanzania, and Rwanda) (Lewallen et al., 2010). Except The Gambia, which has a smaller sample size, this trend is supported by incidence data.



Figure 4: Age Dependence of Cataract Incidence.

Group 2 districts (red) have 2–2.5 times the incidence of Group 1 districts (blue). Figure 4 shows these geographic differences are maintained as incidence increases with age.



Figure 5: Geographic Variation in Cataract Incidence.



Country groups correspond to rough geographic regions, although more districts are needed to understand geographic trends.

Conclusions

Our incidence estimates match the sharp trend in initial prevalence data, confirming that it important to consider regional variations when setting CSR targets. Future research goals include:

- Make our model accessible for continued work with new RAAB surveys.
- Improve data clustering methodology.
- Allow the mortality ratio $\lambda_{UD}/\lambda_{HD}$ to vary with age, modeling age-dependent mortality due to cataract.
- Apply the model to other progressive diseases.

References

Lewallen, S., T. Williams, A. Dray, B. Stock, W. Mathenge, J. Oye, J. Nkurikiye, K. Kimani, A. Muller, and P. Courtright. 2010. Estimating incidence of visionreducing cataract in Africa: a new model with implications for program targets. In press.

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For Further Information

This work was conducted as a senior thesis project at Harvey Mudd College. See our publication above, http://www.math.hmc.edu/~adray/thesis/, or contact Alyssa Dray at alyssadray@gmail.com.