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Learning About ODEs Using Interactive Online Modules

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Abstract: This paper discusses the development and implementation of a set of online teaching and learning modules for the modeling and solution of simple linear Ordinary Differential Equations (ODEs). The paper describes how *constructivist* principles are used in the development of these modules and how in-built assessment has been used to enhance student learning. A discussion of the module structure is provided with particular emphasis on the concept of the module walk-through. Modules developed for the classical ODE problems of the simple pendulum and the mass on a spring are used to illustrate these module constructs. The paper concludes with a discussion of the effectiveness of these modules using data sourced from student questionnaires, focus groups and lab session evaluations.

1 Background

In 2003 one of the authors (Kuva Jacobs) was completing her undergraduate degree program at the University of South Australia (UniSA). The final year included a major project on developing interactive modules for the teaching and learning of a selection of mathematical topics. These topics covered a diversity of areas including Fourier Series and the Traveling Salesman Problem. At the end of 2003, Kuva decided that she would like to extend this work to the creation of *online* interactive modules for the learning of mathematics as a PhD dissertation under the supervision of Stephen Lucas and David Panton. Coincidentally and serendipitously our school hosted Professor Bob Borrelli as a visiting fellow in the first half of 2004 when Kuva commenced her thesis work and as a consequence of many discussions about how the research should proceed, a plan for creating some innovative online modules for the teaching of ODEs evolved. Kuva's work also involved developing a similar set of modules for the teaching of Discrete Mathematics and it was planned that both sets of modules would be trialled within undergraduate programs at UniSA in the following years, including an evaluation of their impact through questionnaires and focus groups.

Importantly this work was motivated not simply by a desire to implement some innovative teaching tools but also by a strong belief that the modules should be student

centered in the sense that the students should be in control of their learning processes. This *constructivist approach* to learning was the centerpiece of Kuva's thesis [10], and while we will not discuss these principles in depth here it will become apparent from a discussion of the online modules how these principles were implemented to good effect.

2 Rationale For Module Development

The traditional lecture approach to teaching places a strong emphasis on algebra, analysis and textual information. Recent research shows that a more visual, constructivist approach, which demonstrates the geometric representation of concepts as well as their technical significance, enables students to gain more insight into the structure and meaning of these concepts. The evolution of internet and online learning now gives us the opportunity to create course-ware that is both aesthetically engaging and insightful. A geometric overview of mathematics can be presented that is aesthetically superior to any hand drawn graph, especially with the additional features of interactivity and animation. Emerging software is particularly useful as revolutionary graphics programs demonstrate the behavior and significance of different models and animations can simulate realistic scenarios.

Despite the abundance of literature which explores the possibility for technology to revolutionize mathematics education, researchers such as Dalgarno [6], criticize the quality of the pedagogical principles which are underlying many current online resources, maintaining that often web based learning resources are "essentially printed materials, converted directly to an electronic format." Haughey and Muirhead suggest that the value of instructional media is dependent on how well their underlying educational principles are realized [9]. This leads to the important question of how we can create technological tools that are effective learning instruments.

A number of different criteria need to be addressed when designing instructional media. For example Dalgarno [6] and Karakirik and Durmus [11] discuss the benefits of using constructivism as a learning framework for instructional technology, highlighting the various aspects of computer media that support constructivist principles. The flexible presentation of information and navigational systems allow students to browse materials using a much more student centered approach. Cairncross and Mannion suggest that interactivity in online instruction promotes engagement, leading to a deeper level of learning [4]. This is a good realization of the notion that constructivism is centered around students constructing their own conceptual understanding of topics through active exploration. The inclusion of a recommended sequential learning path is indicated by Dalgarno to be beneficial, as it supports the principle of scaffolding, by guiding the learner through materials [6]. This balance of guidance and discovery gives the learner control while still providing them with a level of direction.

Appropriate technology allows topics to be presented with both visual and verbal modes of communication. Visualization can aid learning by not only providing multiple perspectives and conceptual insights which align with the constructivist viewpoint, but also the modeling of real world examples to teach mathematical concepts. In [12], Nam Hee explains that graphs enable students to understand various mathematical expressions

and see a resulting change by modifying a particular parameter. The exploration of graphs can be used to demonstrate complex and often unintuitive concepts to give students insight into the reasoning behind that particular concept, which can then lead to a much deeper level of understanding. When analyzing ODEs, computer graphing is particularly useful, as learning tools such as the use of phase planes provide a basis for meaningful qualitative analysis of a system of ODEs without requiring the use of algebraic techniques.

Interactivity is also a key theme of constructivist learning which can be incorporated into the design of instructional technology environments by focussing not so much on human interactions, but rather on human-computer interactions. Technology can support the learner by providing the opportunity to explore, manipulate objects and receive feedback in response to questions, providing a much more experiential learning environment. This enables students to grasp essential ideas and to build a cognitive base.

Dalgarno explains that putting newly learnt concepts into practice and receiving feedback is an important part of knowledge construction that can be facilitated in an online environment [6]. Assessment tasks can be structured to provide step-by-step guidance, with the software tailoring questions to the students' level. These assessments can be made authentic through modeling and the use of computer-based manipulatives to promote the constructivist ideals of providing a real world approach to teaching and learning.

3 Module Construction

A set of online modules to assist in the teaching of the modeling and solution of ODEs was developed in association with an undergraduate course in Differential Equations at UniSA. While the course covered a broad selection of topics in DEs, the modules were limited to dealing with only autonomous planar linear systems with constant coefficients. For the most part, modeling was discussed in lectures, while the modules presented in lab sessions following this discussion were designed to reinforce several of these techniques.

Some important features of the modules include:

- They are a supplement to traditional lectures, tutorials and a text book;
- Their online content covers necessary information, with high quality presentation to enhance their aesthetic appeal;
- They contain computer-based testing of knowledge including instantaneous, automatic feedback using various methods such as online quizzes;
- Guides to practical exercises step through software package(s) and offer checks to make sure students are working through them correctly;
- A balance between modeling exercises provides a context (why) and algebraic exercises that teach students necessary skills (how);
- An engaging, high quality user interface is presented in order to maintain attention more easily by increasing visual appeal;
- Text is minimized, emphasizing an overview of concepts rather than providing students with lengthy descriptions of procedures or theories which can be found in textbooks or lecture notes;
- Modules are easily accessible online, with small file sizes;

- Navigation systems are intuitive and user friendly.

The authoring tools used in the module construction were Macromedia Flash MX and Java Applets. Macromedia Flash MX is a powerful multimedia tool that has an in-built scripting language, unleashing the ability to merge mathematics with animation and graphics. This software allows for the easy construction of visually attractive animations and imagery and the scripting language provides an opportunity for adding complex interactivity. Key-framing also enables walk-throughs to be readily created that present information step-by-step, with the flexibility to modify the graphics on each screen. Flash programs can be embedded within web pages, allowing online accessibility. Java Applets provide a programmable tool that can be used on any platform, online. Java is a powerful and widely used programming language that gives developers the capability to create sophisticated programs which run quickly, although requiring more time to build than the Flash programs. These can be embedded into web pages, and accessed using a Java Virtual Machine, which is installed on most computers [3].

4 Module Online Walk-throughs

An important component of each module is the *online walk-through.* Online walk-throughs aim to provide key information on topics in an engaging and interactive format. Some of the key features of the walk-throughs include a navigational structure to guide students through information as well as interactive graphs, animations that use visualization to demonstrate concepts and the use of real world problems.

4.1 Navigational Structure

The ease with which modules may be navigated by students is a key factor in their appeal. The major module navigational structure is governed by a vertical linear sequence of main headings on the left side of each page (see Figure 1). This structure is provided as a guide only, giving the student full control to skip forwards or backwards between pages. New concepts, variables or definitions are introduced on each page using the walk-throughs, examining the simplest case and building with each page to more complex cases. All information for one page is visible on the screen, and scroll panes are deliberately not used, to avoid lengthy explanations.

4.2 Interactive Graphs

Interactive graphs form an important element of the online modules. Students are given control of sliders to change the value of certain parameters (see Figure 2). They can immediately discover the effect of this modification as the solution to the new equation is plotted, encouraging them to explore and investigate. These sliders enable the formation of very important conceptual understanding of initial conditions, such as the change of one parameter to form a solution with real, repeated or imaginary roots that is not easily achieved with a static graph. An analytic solution as well as a graph is provided

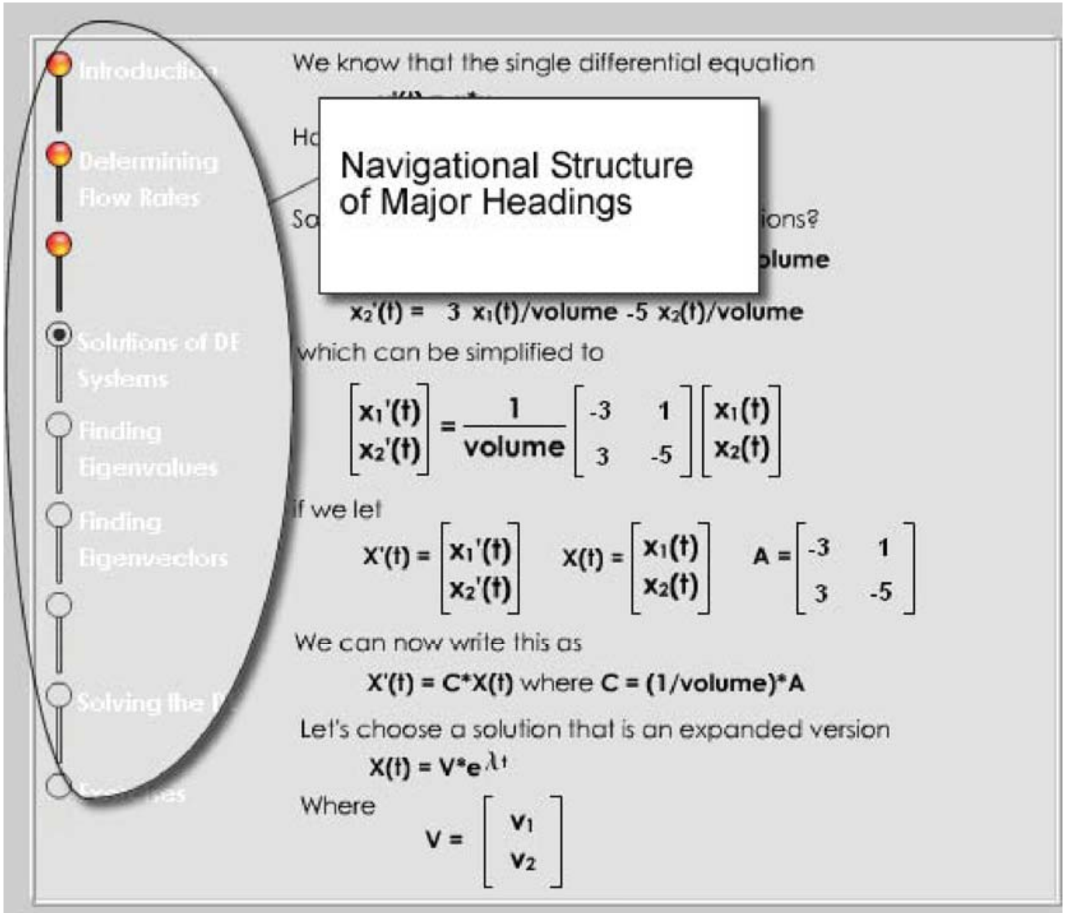

Figure 1: Navigational Structure

for solvable ODEs, which enables students to understand how this solution affects the graphs, such as the presence of trigonometric, or exponential solutions.

The various graphical depictions of solutions of ODEs give students a geometric appreciation of their behavior. Branton and Hale [2], point out that visual insight can be obtained by observing multiple solutions, an important aspect of ODE models that is often difficult to comprehend and that graphics such as slope and direction fields provide the capability to analyze these solutions before a more technical phase is reached.

Slope fields for first-order ODEs suggest a range of all possible solutions for an initial value problem, allowing students to visualize how the model behaves; see for example Borrelli and Coleman [1]. Direction fields enable students to observe equilibrium points, which are important attributes of autonomous differential equation models (see Figure 3).

4.3 Animations

Animations such as a shiny mass on a spring, or water tanks that fill and empty with yellow liquid (see Figure 4), are not only included as a stimulating way to capture students' attention, they also provide students with real world simulations that provide a conceptual bridge between reality and the graphs and equations of a standard mathematics course.

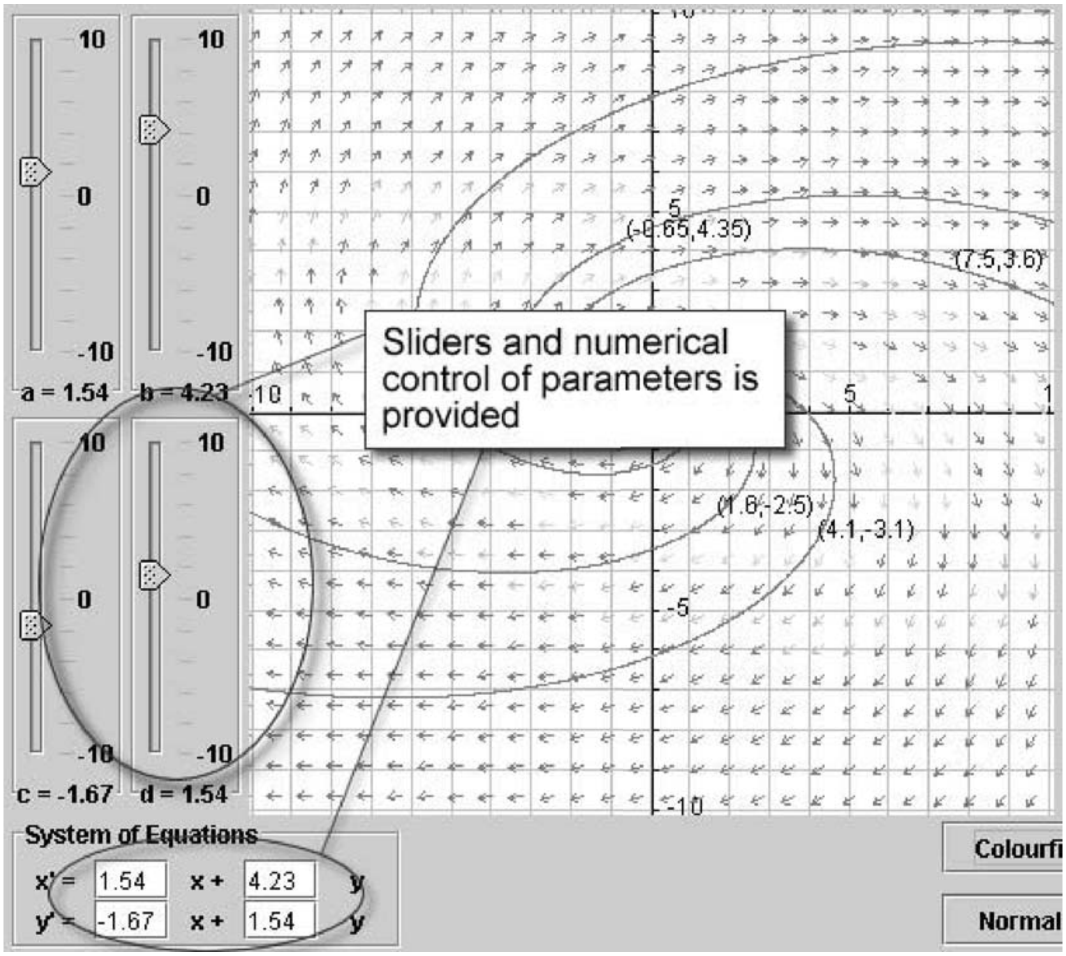

Figure 2: Graphical Control of Parameters

Students can see that a pendulum with initial angle π and initial velocity 0 is balanced upside down and will not move unless given a slight push.

4.4 Mathematical modeling of Real-World Problems

The use of models based on real-world problems is a vital aid in showing the relevance and application of mathematical knowledge; see for example Cooper and LoFaro [5]. Differential equations are widely reflected in the real world, with countless modeling examples such as the observation of population growth and decay, oscillations of a vibrating spring, movement of water waves and many more. Data is readily available, and software can be used to examine the effectiveness of equations that model solutions, visualize graphs depicting these solutions, and show graphical animations, such as a spring vibrating.

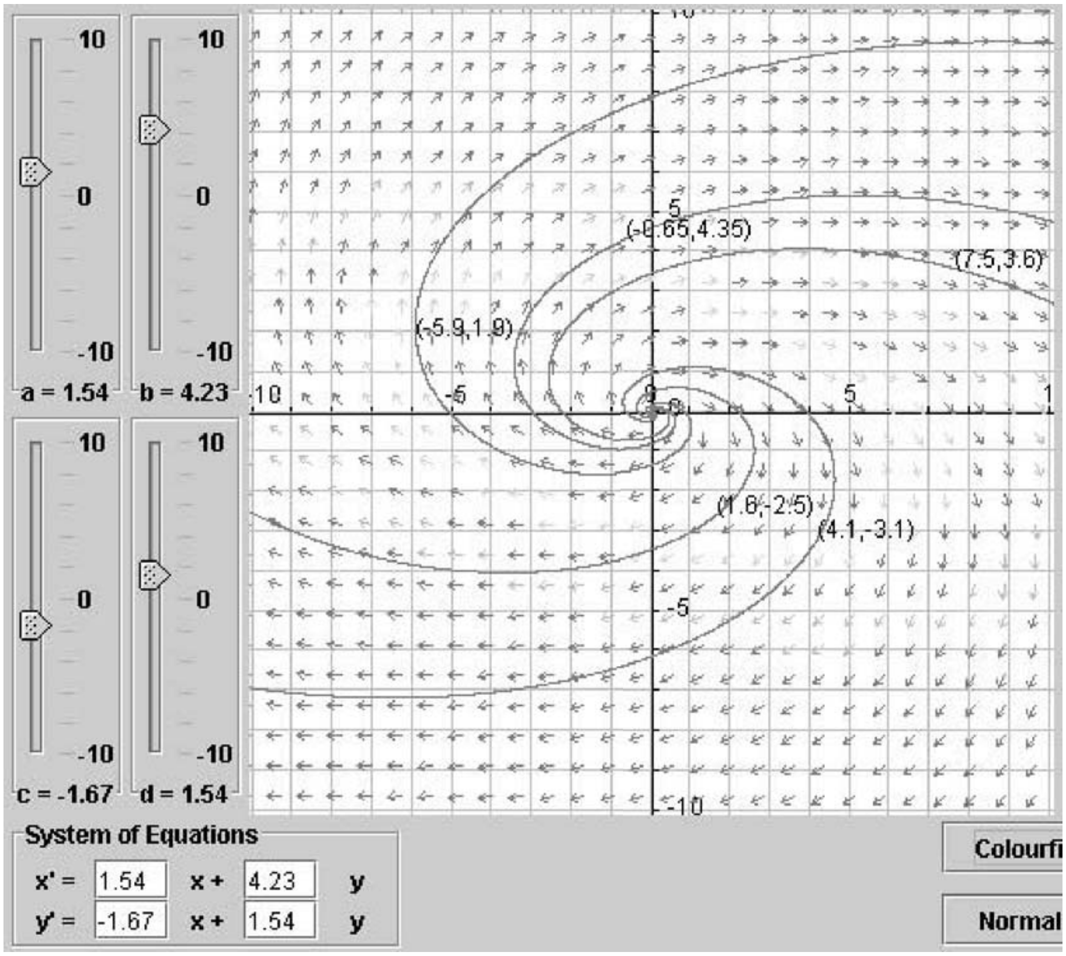

Figure 3: Direction Field

5 Online Assessment

Assessment is recognized as an important aspect of learning in the development of new knowledge and skills within a relevant problem domain. To assist in this development online assessments have been designed to test student understanding, by transferring and applying knowledge learnt in walk-throughs to suitable problems. Some of the key differences between traditional and standard styles of assessment and assessment used in these modules have been highlighted in Table 1.

Gibbs and Simpson [7] outline the need for timely feedback, and this automated process provides feedback either immediately or upon completion of a stage of the assessment, to enable students to recognize and correct errors as well as building confidence when they give a valid response. The two types of assessment developed are randomized multiple choice quizzes and randomized exercises. They are both designed to be completed in conjunction with the online walk-throughs and all questions directly relate to concepts and materials covered in the walk-throughs. Randomized parameters allow users to repeat multiple question 'types' such as the following example.

Does the equation $y'' + \omega y = 20\sin(kt)$ have a solution with...

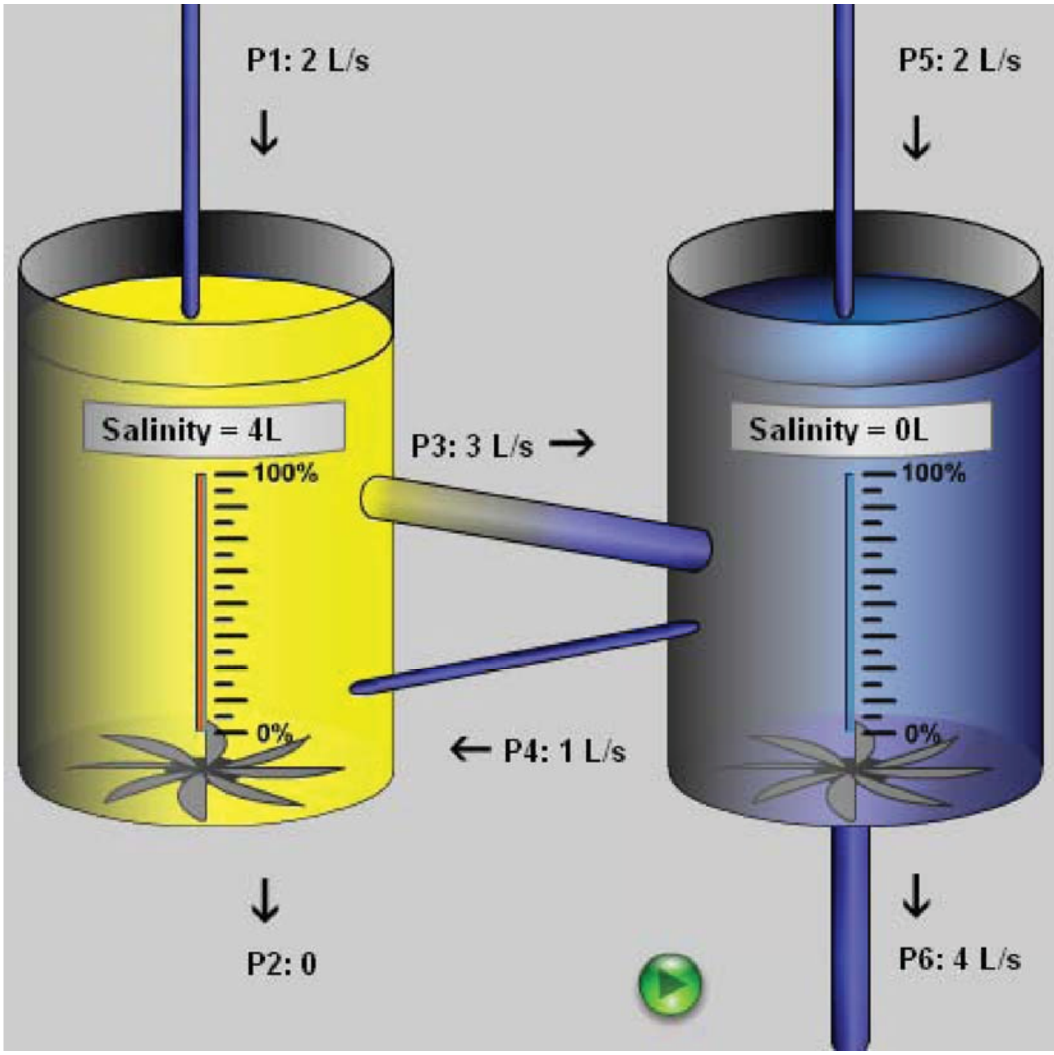

Figure 4: Water Tanks Simulation

1. Distinct beats?
2. Resonance?
3. Random oscillatory pattern?

This is presented with ω and k both defined randomly (as real numbers). The correct answer varies depending on whether ω and k are chosen to be equal, approximately equal or far apart. This question is followed by one that introduces a y' term, asking students to determine which of three graphs correctly describes the solution, further linking their understanding of graphs with equations (see Figure 5).

Prompting students to repeat question types until they are able to determine the right answer has two positive effects. Students are encouraged to answer correctly the first time so they do not need to do extra work and also to keep trying if they do not understand immediately. Practice and repetition encourages students to find the right solution and reflect upon incorrect choices.

Randomization has further advantages in terms of avoiding plagiarism. Students are

Traditional Assessment Structure	Online Assessment Structure
All students given identical assignments	Students given slightly different questions
Must be completed before deadline	Must be completed before deadline
Limited number of questions	Potentially infinite number of questions
Encourages plagiarism as all questions are identical	Allows students to explain problem solving techniques to peers who will not be asked to solve that particular problem
Incorrect answers result in poor grades for students and lower perception of their ability without any encouragement to reattempt topics that have been misunderstood	Incorrect answers result in re-testing a misunderstood topic until a correct answer is determined - emphasis on re-thinking and repetition
Can ask for help if they are stuck or lost, but usually just pointed in right direction	If they become stuck, can ask for help and reattempt question if assistance is not useful

Table 1: Differences Between Traditional and Online Assessments

able to compare and discuss the different question 'types' and their consequent problem solving techniques for each question without giving an exact answer. Each assessment requires them to initially enter their student ID which is displayed upon completion. Additional measures taken to deter plagiarism include the use of gradient backgrounds that prevent students from easily copying and manipulating digital images.

5.1 Online Multiple-Choice Quizzes

Randomized quizzes are a form of assessment that is becoming more widely practised. The most common form of randomization involves creating a large set of questions and selecting a random subset each time the quiz is performed by a student.

Some mathematics quiz systems have developed sophisticated techniques for creating questions with randomized parameters such as MacQTEX which was recently created by Griffin and Moore at Macquarie University in Australia [8]. Their package uses programs and server interactivity to generate PDF quizzes, enabling automatic grading and retrieval of student scores and information via a server. Questions are already written, and the lecturer can only choose from the set provided. Griffin and Moore highlight their MacQTEX quiz system as having several advantages over other similar quiz systems including the use of PDF output as opposed to HTML. Two advantages are the requirement of server interaction only upon initialization and completion of a quiz as opposed to continuous communication with a server, and a stand alone system that does not require algebra packages or databases. The disadvantages of this quiz system are large initial downloads, lack of connections between questions and an inability to include dynamic graphics.

The set of randomized online multiple choice quizzes for the online modules used at UniSA are tailored to complement the online walk-throughs, featuring small file sizes (32KB) and random generation of parameters on the fly, using embedded programming. Graphics are included, allowing a focus on questions which test conceptual understanding

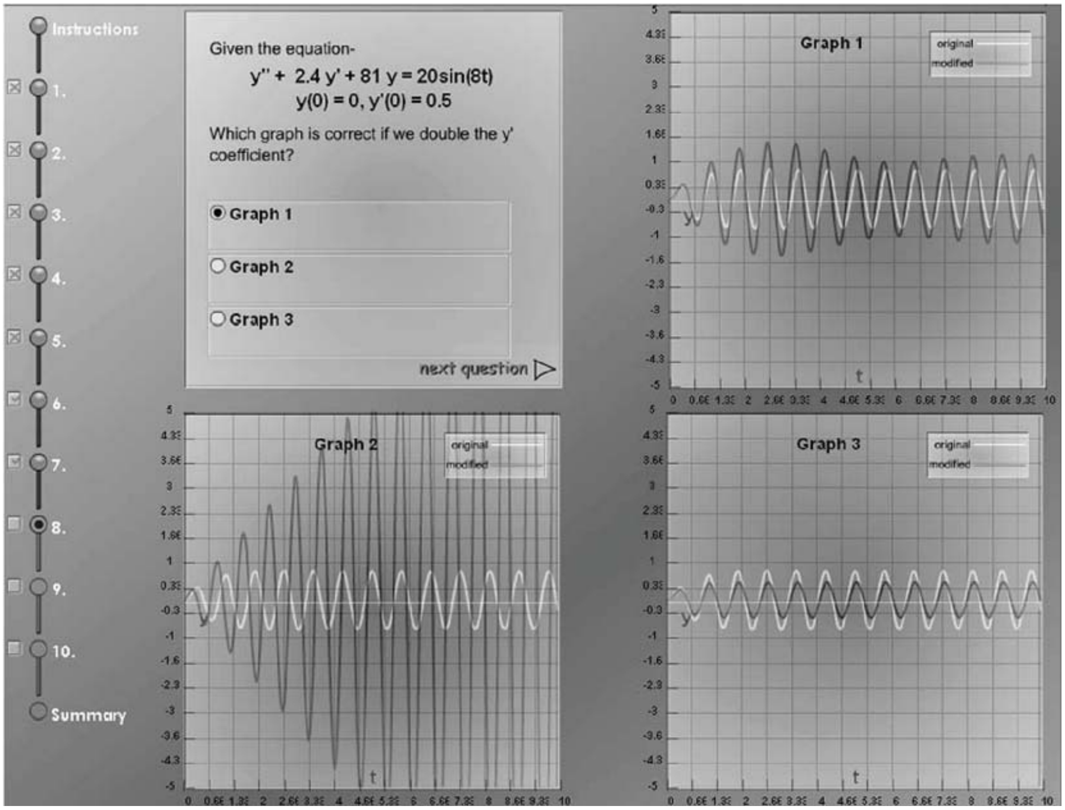

Figure 5: Mass on a Spring Question

of graphs and relationships between parameters as opposed to lengthy algebra questions that are easier to solve using pen and paper.

The quizzes continue testing students until the correct answer is eventually achieved for all question types, with two different types of sequential systems that have been implemented. For one system, the quiz steps through each question type, prompting the correct selection of three possible answers. Upon completion of all question types, a student will return to those previously answered incorrectly, which are re-tested with a different set of numbers. This process continues until completion, when all questions have been answered correctly. The second system requires a user to complete an entire question correctly, which often consists of multiple sections. If an incorrect answer is given for the question or section, the user must reattempt the same question starting at the first section. This system enables exercises to be completed that require a building of knowledge. Both navigation systems are finalized with a summary screen which presents the number of attempts for each question to be printed and handed up. A perfect score is awarded upon completion.

One weakness of this form of assessment is the possibility for students to simply guess solutions. Indeed, a poor student with little concern for understanding their subject may simply guess, and this random selection will quickly result in completion (by continuously pressing the 'next' button the quiz can easily be completed within five minutes). Students who actually want to understand their course will perceive any assessment to have

educational value. Furthermore if they are conscientious, they will want to answer any question correctly and consider it carefully before answering. This is reinforced by presenting the number of attempts on the summary screen, making multiple tries more transparent.

5.2 Online Exercises

The online exercises may include multiple answer questions that are sequentially structured to form the entire working of a lengthy problem. In some cases students will be given a new set of randomized parameters and must fill in the blanks to set up a model and solve it. Different types of automatic grading may include ticks that appear as the student inputs a section of a page correctly or a grading scheme that displays scores for each section only upon completion of all sections. Either scheme allows students to revise their answers in order to achieve a perfect score.

In contrast to the quizzes, the open style questions of these exercises are not easily guessed. This scaffolded scheme gives a clear framework for solving mathematical problems, focusing on both modeling concepts and the associated analytic problem solving procedures.

6 Ordinary Differential Equation Modules

Four modules demonstrating the modeling and solution of simple ODEs or systems of ODEs have been created. These involve the *mass on a spring*, the *non-linear systems* (using the motion of a simple pendulum), *movement of water between tanks* and the solution of a simple linear system of ODEs using *eigen values*.

While ODEToolkit is not used directly as a 'solver' in the execution of the modules, students were encouraged to utilize this software package to enhance their understanding of the subject material in the course. To facilitate this, an ODEToolkit module was also created to lead the students through the use of this package. Following a brief discussion of this module we will consider two of the other four modules created, namely the classical problems of a mass on a spring and non-linear systems.

6.1 ODEToolkit walk-through

The ODEToolkit [13], a software package, is introduced to students in the first lab session of the course. This state-of-the-art software is specifically designed for easily setting up and automatically solving differential equations numerically, visualizing the results, and experimenting with the effects of changing system parameters or initial conditions.

An important feature of the module development was the utilization of the interactive functionality within ODEToolkit to enable students to better understand the solution processes.

The Introduction to ODEToolkit module (see Figure 6) aims to introduce the software package in an interactive, step-by-step format. The module explores the significance of solutions and investigates qualitative properties of differential equations. Upon completion

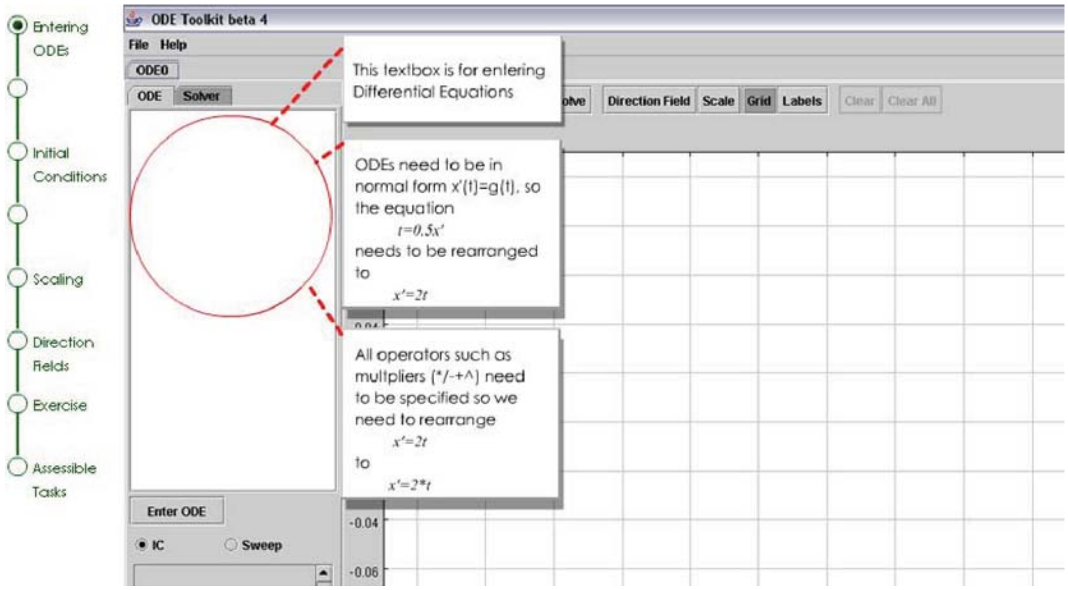

Figure 6: ODEToolkit Introduction Module

of the module, students are given a set of assessable exercises, testing their understanding of differential equations and the software package.

6.2 Mass on a Spring walk-through

The Mass on a Spring module (see Figure 7) introduces the differential equation describing the movement of a mass on a spring. Variables are introduced step-by-step, by first examining the simplest case and observing the effects of the initial position and velocity. Later, damping and finally a driving function are introduced, with a careful examination of the relationships between parameters and their effect on the solution. Each section appears on a single page, with a clearly labeled navigation system on the left hand side that implies a linear order while still allowing the user to jump forwards or backwards.

Solutions are plotted in three adjacent graphs which show position over time $y(t)$, velocity over time $y'(t)$ and orbital portraits of y' versus y. Animated dots allow the students to see any of the graphs as a function of time. Students are given control of the parameters at any stage via sliders so that they can test the effect of modifying their values. To further understand how the solution behaves, an animated spring is included, simulating a real world example of what the mathematical graphs and equations are describing.

Text is minimized, with further discussion separated from the main panel, allowing the rapid acquisition of key terms and ideas, with additional information for those who wish to learn more detail. The differential equation is shown in the main panel with all parameters being adjustable via sliders. The value of each parameter and the analytical solution are shown in the top right panel.

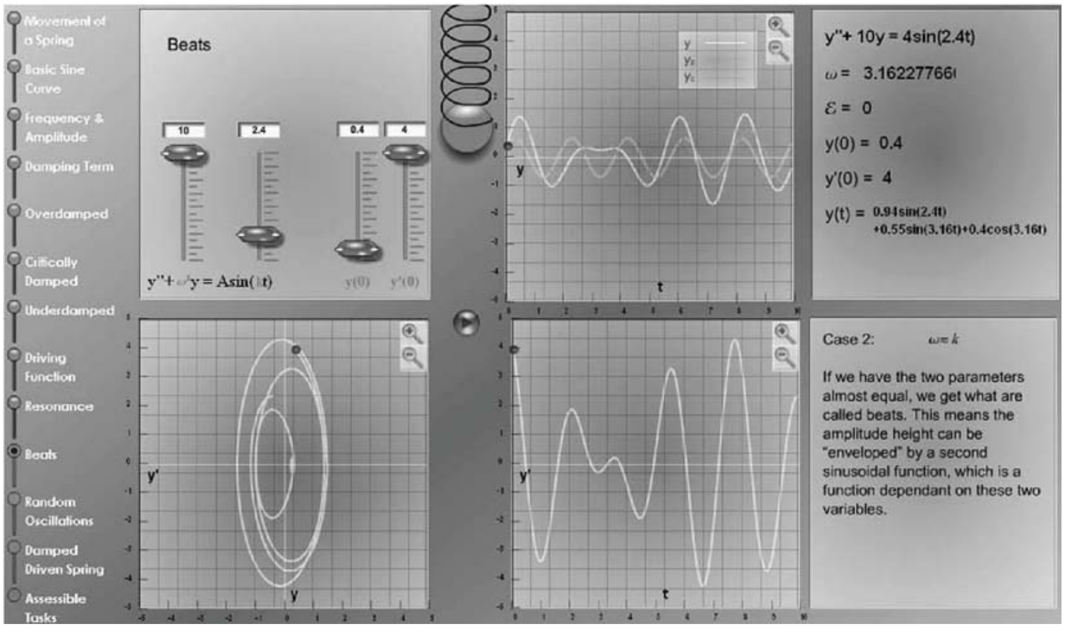

Figure 7: Mass on A Spring walk-through

6.3 Non-linear systems walk-through

The aim of the non-linear systems (which we will refer to as the *pendulum*) walk-through (see Figure 8) is to explore the accuracy of replacing non-linear systems of differential equations with analytically solvable linear systems. The pendulum model demonstrates a real-world example of this approximation, showing movement over time as well as graphs that plot numerical solutions of the non-linear system and the linear approximation. This approximation is found using a Taylor expansion (centered at the origin) of the non-linear term, when the pendulum is hanging downwards with no movement. Students are then asked to find an approximation around the point where the pendulum is balanced perfectly still but upside down. They are encouraged to explore this phenomenon, and to investigate what happens if the pendulum is slightly off balance, further demonstrating important concepts about the stability of systems.

Methods for finding linear approximations are explored through analysis of the equilibrium points of the non-linear systems and use of transformation functions. Graphs are used to compare the behavior of solutions of the non-linear system and its linear approximation around both stable and unstable equilibrium points, with the flexibility to change the initial conditions. A graphing tool is also provided that enables students to enter the coefficients of their own linear and non-linear systems, allowing them to compare their approximations. A set of online exercises is provided to ensure students understand connections between concepts and mathematical procedures.

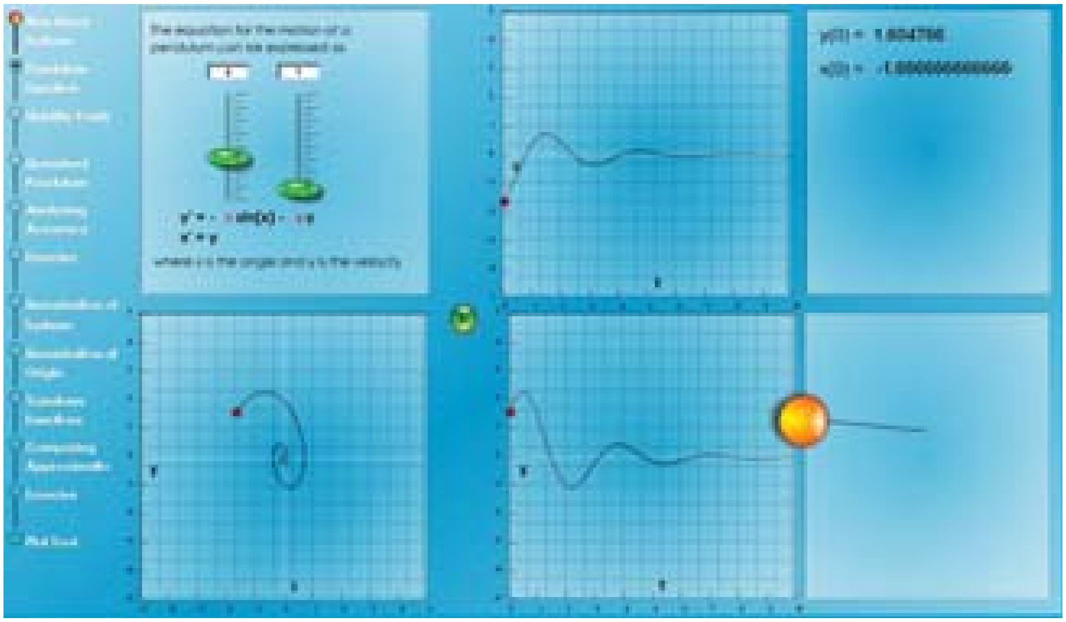

Figure 8: Pendulum walk-through

7 Evaluation of Module Effectiveness

Following the creation of the online modules, and throughout their implementation within the course, feedback from students was captured in three separate ways, namely via a questionnaire, a focus group and by observations and interactions within the lab sessions. Tables 2–5 display frequency response data from 25 students in the Differential Equations course at UniSA. The same questionnaire was given to students in a Discrete Mathematics course but for the most part this data and any comparison with the ODE data will not be discussed here. Questions have been categorized to capture information on the ease of use of the modules (questions 1–6), a comparison of online assessment with written assignments (questions 7–17) and as a benchmark, questions gauging students mathematical preferences (questions 18–22).

A detailed qualitative analysis of all data was carried out, with a quantitative evaluation of the questionnaire data using Principal Component Analysis (PCA). A brief summary of the most significant outcomes of an evaluation of the questionnaire data follows. A more detailed analysis of the outcomes of these evaluations and all other data can be found in Jacobs [10].

7.1 Using the Online Modules

The ease with which students were able to use the modules was determined by questions 2, 5 and 6. The majority of students thought that the modules were visually appealing and that the amount of text to read on the screen was not excessive. On the other hand it is clear from question 5 that most students felt more explanation of topics was required, evidentally relating to their comprehension of these topics. Students were encouraged to refer to their text book during the lab sessions, if necessary, to obviate this problem.

Using the Online Modules	SA[1]	A	N	D	SD
1. I liked the interactive features of the modules.	13	9	3	0	0
2. I found the modules visually appealing.	13	9	3	0	0
3. I could easily navigate through the topics within each module.	10	11	3	1	0
4. Analyzing graphs and diagrams helped me to understand the concepts.	7	14	4	0	0
5. Explanations were not detailed enough.	0	5	13	7	0
6. There was too much text to read.	0	2	4	17	2

Table 2: Frequency of Responses for Selected Mathematical Preferences Questions.

Comparing Online Assessments with Written Assignments	SA	A	N	D	SD
7. I felt that the online assessments helped me learn.	2	20	3	0	0
8. I guessed answers to the online quizzes.	0	4	7	13	1
9. I liked doing the online quizzes as a form of assessment.	8	13	3	1	0
10. I preferred the online assessments rather than the written assignments.	9	8	5	3	0
11. I felt less stressed by knowing my grades for the online assessments with automatic grading.	14	7	3	1	0
12. The assessments helped me to apply the knowledge learnt in the walk-throughs.	4	14	7	0	0

Table 3: Frequency of Responses for Selected Mathematical Preferences Questions.

7.2 Visualization

As discussed in Section 2, visualization is an important aid to students when learning mathematics and is one of the major advantages in using online modules compared with traditional modes of teaching. Results from question 4 clearly show that the analysis of graphs and the use of sliders helped to provide insight into concepts and the behavior of solutions. Students were frequently observed during the lab sessions playing with the interactive elements and adjusting sliders to see the effects of changing parameters. One student commented during a lab session that the presentation of three images showing the function graph, the slope and orbital portrait enabled the student to understand concepts which had previously been confusing during lectures. The focus group also provided additional evidence of students' appreciation of the visual elements.

7.3 Interactivity

Constructivist learning theories have emphasized the importance of creating an environment in which the student is actively participating in learning. The modules feature

[1] SA- Strongly Agree, A- Agree, N- Neutral, D-Disagree, SD- Strongly Disagree

Comparing Online Assessments with Written Assignments	SA	A	N	D	SD
13. I could use the online modules without much assistance from an instructor.	3	17	5	0	0
14. I could do the tutorials without needing any explanation from an instructor.	6	10	7	2	0
15. I enjoyed using the online modules more than doing tutorial exercises.	8	11	6	0	0
16. Using the online modules in addition to tutorials helped me understand concepts better.	1	10	4	10	0
17. I would prefer to have-					
online modules (without written tutorials)	0	4	9	5	3
written tutorials (without online modules)	0	2	10	6	3
both online modules and written tutorials	11	10	4	0	0

Table 4: Frequency of Responses for Selected Mathematical Preferences Questions

Mathematical Preferences	SA	A	N	D	SD
18. I enjoy mathematics.	10	12	3	0	0
19. I enjoyed taking this course.	3	6	13	3	0
20. I like doing maths on the computer.	2	9	8	4	2
21. I generally find mathematics easy.	0	5	11	7	2
22. I found this course easy.	0	2	12	9	2

Table 5: Frequency of Responses for Selected Mathematical Preferences Questions

interactive elements such as sliders to adjust parameters, assessments with automatic feedback and walk-throughs with dynamic navigation systems that encourage students to explore and investigate.

Questionnaire responses generally showed that the students had no difficulties navigating through various topics (see results for questions 1 and 3 in particular). Comments during the focus group also suggested that the modules were easy to use, without navigational difficulties and that there was an adequate level of scaffolding.

The autonomy of modules was demonstrated by student response in question 13, namely that they did not need much assistance from an instructor with the modules. This was also observed during the lab sessions, where students asked few or no questions and were able to complete most or all parts of each session without assistance.

While the general consensus in both the questionnaire data and the focus group was that students would be capable of completing the lab sessions on their own, they also suggested in the focus group that it was useful to have an instructor nearby so that they could ask additional questions.

Results for question 14 were not as positive however. This question was aimed at benchmarking the lab sessions against tutorial sessions in which the online modules were not used. In the focus group students described the tutorials as being the "real hard nitty-gritty of the course", requiring them to "get in and do the math" whereas they suggested that the lab sessions were free from the algebraic complexity which they saw as being an important but also a difficult part of learning mathematics.

7.4 Online Randomized Assessment

Online assessments were created with the aim of assisting students to learn by providing assessment tasks that are engaging and motivating, testing student understanding in an interactive and innovative format as discussed in Section 2. Most students expressed a preference for the online assessments over written assessments, as indicated by the results of questions 7 and 10. In relation to question 10, students verbally expressed their opinions during class that the first written assignment was long and difficult. These issues were reflected in the high variation of grades for the written assignments. The difference in performance for these two types of assessment is a reflection of their nature, as the online assessments provide ongoing feedback and give students an opportunity to continue practising until a correct answer is determined as opposed to written assignments which give feedback only after grades had already been assigned. Results from the question 11 showed that students felt less stressed using the automatic marking as they could attempt questions until full marks were achieved.

The randomized nature of assessments encouraged students to work together without directly copying, which they often did by describing their own qualitative understanding or techniques for solving a problem rather than directly telling each other the answers. Initial observations in the lab sessions showed that students completed quizzes slowly and carefully, using the walk-through to help them explore different solutions as well as a pen and paper to calculate numerical solutions. Perhaps they may intuitively guess based on prior knowledge, but often when reattempting a question 'type' after an incorrect guess, the student would reconsider it carefully, further analyzing the walk-through before

answering again. This was reflected in their questionnaire response to question 8.

7.5 Understanding

In Section 2, the importance of providing a deeper approach to learning was discussed, linking this to the promotion of a richer and more meaningful learning experience for the student. Section 3 described how the modules aimed to provide students with a more comprehensive overview of concepts through graphing, modeling and the use of real world examples. Students indicated in their response to question 16 and their comments in the focus group, that the modules did help them to achieve a better conceptual understanding.

> Male 2: "Like yesterday's lectures I didn't have a clue what was going on, but now I feel like I actually understand it completely."
> Female 1: "Yeah, I think the online, like assessment just kind of helps with the basics of the written assessment."
> Male 2: "That I actually do understand things, and then look to take on sort of bigger things, or harder things, given that at least at that level, at the easier level, the fundamental level, I'm on track."

7.6 Engagement

In Section 2 we outlined the importance of the goal of creating a stimulating and enjoyable learning environment when developing the online modules. Positive affective factors such as enjoyment can lead to better learning conditions and improve motivation, engagement and confidence. The interactive features and aesthetic quality of the modules was designed to enhance engagement and enjoyment, towards which positive responses in questions 9 and 15 demonstrated the students' appreciation. This was also the primary Principal Component for this course, which showed positive responses towards the use of the online modules. Additionally, it was also indicated in the focus group, as many students expressed their enjoyment for using the online modules.

> Female 2: "User friendly as well, like they weren't boring, you know, it's all up there and it's like (inaudible) questions, but having like even the animations and stuff like that, the layout was pretty nice."
> Male 2: "I think that's a very important point, and I felt that way too that the labs, the online sessions, were enjoyable to do and so I really got into them and I got a lot out of them for that reason, whereas boring old maths all the time, yeah, you can switch off, you know, and get lazy with it, but it sort of rejuvenates you and I found it built my confidence, particularly when I got immediate feedback that 'Yes, what you're doing is correct', right, I feel good about that, and sort of spurs me on to, back to the textbook."

It is interesting to note, in student response to question 17 however, a more balanced attitude to the use of online modules, suggesting, as was the rationale for developing the online modules, that they should be used as a *supplement* to existing modes of teaching.

7.7 Mathematical Preferences

Attitudes towards mathematics are known to play an important part in influencing students' motivation and engagement when learning. Students were asked about their level of enjoyment towards mathematics in questions 18–22, as a benchmark for their attitudes towards the online modules. This was regarded as of particular importance when assessing the reaction of students in the *Discrete Mathematics* course, as the majority of students in this course were not taking a mathematics major.

Results from the Principal Component Analysis showed that mathematical preferences were a major part of the second Principal Component for both courses, with some variation in the structure of this construct across the two courses. For *Discrete Mathematics*, strong negative feelings towards ease and enjoyment of mathematics both in general and specifically for the course were very influential in constructing the second component and this group of students also preferred to have online modules only. This was also true for the *Differential Equations* students in regard to their enjoyment towards mathematics and the course, except that it was correlated with finding the course easy. In contrast, for *Differential Equations* students, the first PC also showed a correlation between students who enjoyed various aspects of the online modules and those who enjoyed mathematics.

8 Conclusions And Future Work

In summary, results showed that the online modules were indeed effective in fulfilling two goals. First they demonstrated the advantages of using technology in addition to traditional methods of teaching and second they satisfied their underlying educational principles, offering a comprehensive online learning environment that fostered engagement and enjoyment as well as helping students to better understand concepts. The online modules were developed with the aim of supplementing traditional modes of teaching by providing assistance through exploiting the power of technological features such as visualization and automatic grading. Students expressed appreciation for the visual approach of the online modules, emphasizing the usefulness of being able to see the effects of manipulations which enabled them to get a better understanding. Immediate feedback and multiple attempts to answer questions are just some of the aspects of the randomized assessments that students found useful as compared to traditional styles of assessment. Students appreciated the opportunity to learn with the modules, utilizing an active, hands on approach that allowed them to work at their own pace. Furthermore they suggested that information could be easily understood due to the step-by-step nature and the presentation of a conceptual overview.

While there are deeper educational issues that can be explored, such as how effective these modules really are in enhancing the learning process compared with the use of more traditional methods of teaching, there is clearly an opportunity to build on this approach to learning, not only in the teaching of ODEs but also in other areas of mathematics. Indeed there is compelling evidence from the outcomes of student questionnaires (particularly from the Discrete Mathematics class) that those students with a weaker background and with a relatively negative attitude to mathematics may have an enhanced learning

experience through using learning modules designed in this way.

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