Journal of Humanistic Mathematics

Volume 1 | Issue 2 July 2011

Pedagogy on the Ethnomathematics--Epistemology Nexus: A Manifesto

Ilhan M. Izmirli
George Mason University

Recommended Citation

Ilhan M. Izmirli, "Pedagogy on the Ethnomathematics--Epistemology Nexus: A Manifesto," *Journal of Humanistic Mathematics*, Volume 1 Issue 2 (July 2011), pages 27-50. DOI: 10.5642/jhummath.201102.04. Available at: https://scholarship.claremont.edu/jhm/vol1/iss2/4

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | http://scholarship.claremont.edu/jhm/

Pedagogy on the Ethnomathematics-Epistemology Nexus: A Manifesto

Ilhan M. Izmirli

George Mason University, Fairfax, VA
iizmirl2@gmu.edu

Synopsis

In this paper, we will elaborate on a pronouncement that should be at the onset of any study in epistemology and ethnomathematics, namely, we will argue that learners do think mathematically and it is our responsibility as educators to recognize and appreciate their modes of mathematical reasoning.

We will conduct our study in five parts. Following a brief introduction, in the second part, we will briefly discuss some of the critical tenets of epistemology especially as it applies to mathematics. The third part will be devoted to elucidating the basic nomenclature and hypotheses associated with ethnomathematics. In the fourth part we will expound on the organic and intrinsic relationship between these two fields. Lastly, we will propose some changes in the way academic mathematicians regard philosophy and pedagogy of mathematics that, in our opinion, will facilitate students' understanding of the cultural aspects of mathematics.

1. Introduction

One of the most crucial objectives of mathematics education is the incorporation of the learners' modes of mathematical reasoning into classroom discourse.[1] Consequently, academic mathematicians must constantly reassess and re-delineate the conventional notions of mathematical knowledge. They

[1]In the field of mathematics education this approach is referred to as the *genetic approach*. It requires that the method of teaching should be based, as far as possible, on natural ways and methods of knowledge inherent in the science. The teaching should follow ways of the development of knowledge. The term was first used as *genetic teaching* by the prominent German educator F.A.W. Diesterweg (1790-1866) in his 1835 book *Wegweiser zur Bildung fuer deutsche Lehrer und andere didactische Schriften*:

must also cultivate an understanding of the learners' cultural practices, and reclaim the histories of the contributions of all civilizations to mathematics, which, in the case of non-Western ones are rather obscure, even distorted [73]. Indeed, what is necessary to facilitate the learners' realization that they already think mathematically and, that therefore, they can excel in theoretical mathematics, is the deconstruction of this false history of mathematics followed by a meticulous revision of the theories on the epistemology of mathematics.

This deconstruction is at the crux of our refuting the inherently erroneous interpretation of the history of mathematics merely as a chronological narrative of the isolated successes of a few select societies, thus eradicating the arbitrary distinctions between Western and non-Western knowledge [22]. The proposed epistemological revision would ameliorate our synthesis of the seemingly dichotomous categories such as teaching and learning, theory and application, logic and intuition [24] by demonstrating the unity of practical (concrete) mathematical knowledge and its theoretical (abstract) counterpart.

2. Epistemology

Epistemology, from Greek επι′στημη (knowledge) and λο′γο′ς (science), is the branch of philosophy concerned with the nature and limitations of knowledge, and how these relate to notions such as truth, belief, and justification. Typical issues epistemologists deal with are: establishment of the necessary and sufficient conditions of knowledge, exploration of its sources, portrayal of its structure, a discernment of its limitations, as well as elucidation of the concept of justification, and instituting rules for determining what makes justified beliefs justified. Interpreted from a slightly broader perspective, epistemology is the study of issues related to the creation and dissemination of knowledge in particular areas of inquiry, in our case mathematics.

The various attempts to answer these questions are too numerous to list

---

> … The formal purpose requires genetic teaching of all subjects that admit such teaching because that is the way they have arisen or have entered the consciousness of the human … it is necessary to lead him/her [the student] to the target not sightless but sharp-eyed: he/she must perceive truth not as a ready result but discover it … [21].

in such a short paper. The honor of providing the first scholarly elucidation goes to Socrates, who defined knowledge as true belief followed by its justification. For obvious reasons, from the point of view of modern erudition, the Socratic definition is insufficient at best [39]. First of all, since belief implicitly implies absolute finality, this definition would lead to a rather sharp categorization of "knowledge" as either "complete knowledge" or "complete ignorance", whereas in reality, our knowledge of any given object or abstract concept is somewhere in between, and is in a continuous state of flux and transition. A good example would be laws of motion. These laws have undergone tremendous changes from Aristotle's *primus motor* (πρῶτον κινοῦν ἀκινητον)[2] or to Newtonian laws, to the laws of general relativity. Secondly, the criterion "truth" is too ambiguous. What constitutes the standards against which we can label beliefs as true or false?

We also completely reject the more recent perspective that knowledge is neutral, value-free, objective, and entirely detached from how people use it[3]. In our opinion, this view demotes the process of learning to unintelligible sequences of contrived discoveries of some static facts and their subsequent descriptions and classifications, and unreservedly disregards the social nature of knowledge [8].

Instead, we adhere to the *critical pedagogist*[4] standpoint, and emphasize

---

[2]That is, prime mover or unmoved mover.

[3]Namely, the positivist perspective. The defenders of positivism invoke the early Wittgensteinian point of view that all knowledge should be codified in a single standard language of science, and ordinary-language concepts should gradually be replaced by more precise equivalents in this standard language. According to Stephen Hawking, for a positivist

> ... a scientific theory is a mathematical model that describes and codifies the observations we make. A good theory will describe a large range of phenomena on the basis of a few simple postulates and will make definite predictions that can be tested. ([46, page 31])

[4]*Critical pedagogy* is an approach to education that encourages learners to question and challenge the imperious, mainstream, prevailing beliefs and practices in order to help them achieve critical consciousness. According to Shor, it should invoke in the learners

> Habits of thought, reading, writing, and speaking which go beneath surface meaning, first impressions, dominant myths, official pronouncements, traditional clichés, received wisdom, and mere opinions, to understand the deep meaning, root causes, social context, ideology, and personal consequences

the dialectical view that knowledge is a process which is incessantly created and re-created as people act and reflect on their surroundings and that gaining existing knowledge and producing new knowledge are coexisting occurrences. Implicit in this view is our realization that knowledge requires both objects and subjects, and that therefore, it is a negotiated outcome of the interaction of human consciousness and reality.

> Knowledge ... necessitates the curious presence of subjects confronted with the world. It requires their transforming action on reality. ([31, page 101])

Let us also affirm the impact of the subject and the object as well as their interdependence in epistemology. To reject the importance of the subject in the process of transforming the world and history is to assume a world without people. To reject the importance of the object in analysis or action is to assume people without a world [30, page 35].

Because of this incessant interaction between the subject and the object, one cannot completely know particular aspects of the world – no knowledge is finished or infallible, in other words, "... knowledge is never fully realized, but is continually struggled for" [63, page 117]. At particular moments in history, communities of people debate, revise, adopt, challenge, and reject concepts and theories.

The constant modifiability of knowledge is best expressed by the epistemological hypothesis known as fallibilism, which can be traced to Karl Popper.[5] In his *Logik Der Forschung* (The Logic of Investigation 1934),

---

> of any action, event, object, process, organization, experience, text, subject matter, policy, mass media, or discourse. ([77, page 129])

It regards education as a continuous process of unlearning, learning, relearning, reflection, and evaluation. It was heavily influenced by the works of the Brazilian educator Paolo Freier (1921-1997).

[5]Sir Karl Raimund Popper (1902-1994) was an Austrian born British philosopher who taught at the London School of Economics. He is generally regarded as one of the greatest philosophers of science of the 20$^{\text{th}}$ century. He is best known for his attempt to interchange the classical observationalist / inductivist form of scientific method by empirical falsification, and for his opposition to the classical justificationist account of knowledge. The philosophical approach he referred to as critical rationalism was the first non-justificational philosophy in the history of philosophy. Popper was a committed advocate and staunch defender of the "Open Society", and an implacable critic of totalitarianism in all of its forms.

Popper introduced the concept of critical rationalism into the philosophy of science and epistemology at large with the central tenet being the rejection of the idea that knowledge can ever be justified in the strong form that is sought by most schools of thought – in other words, "knowing" does not entail certainty.

Application of the fallibilist model to mathematical knowledge is known as quasi-empiricism. Quasi-empiricism was developed by Imre Lakatos,[6] inspired by the philosophy of science of Karl Popper. Quasi-empiricism in mathematics focuses on mathematical practice, rather than solely on issues in the foundations of mathematics. It espouses the idea that mathematics had accepted informal proofs and proof by authority, and had made and corrected errors all through its history. Hence mathematics is a "quasi" empirical branch of science subject to revision and correction as any other branch of science. A good example is given in Lakatos's *Proofs and Refutations*, a book published posthumously in 1976 based on his 1961 dissertation. Here, a professor and his students discuss how the idea of Euler characteristic evolved over time.

As stated in Lerman (1989)

> Certainty has a tendency to lead one to say "That's it, no more discussion, we have the answer." Fallibilism, a view which accepts the potential refutation of all theories, and counter-examples to all concepts, allows one to ask how does one know that this answer is better than that one, what might constitute a notion of 'better,' might they not both be possible, as with Euclidean and non-Euclidean geometries, or arithmetics with or without the Continuum Hypothesis. ([59, page 217])

Quasi-empiricism argues that in doing their research, mathematicians

[6] Imre Lakatos (1922-1974) was a Hungarian philosopher of mathematics and science, known for his thesis of the fallibility of mathematics. Lakatos viewed mathematical methodology as an incessant process of proofs and refutations. Born Imre (Avrum) Lipsitz to a Jewish family in Debrecen, Hungary, to avoid Nazi persecution, he changed his last name to Lakatos (locksmith). After the war, he continued to conduct mathematical research and translated George Polya's famous book *How to Solve It* to Hungarian. In the aftermath of the 1956 Soviet invasion, Lakatos fled to England. In 1960 he was appointed to a position in the London School of Economics and was introduced to Popper. He received a doctorate in philosophy in 1961 from the University of Cambridge. The book *Proofs and Refutations*, published after his death, is based on his doctoral dissertation.

test hypotheses as well as proving theorems. A mathematical argument can transmit falsity from the conclusion to the premises just as well as it can transmit truth from the premises to the conclusion. The quasi-empirical point of view of mathematics, by virtue of taking mathematics within a cultural context, is essential for the pedagogical model that will be developed in subsequent sections,

3. Ethnomathematics

The term ethnomathematics was coined by the Brazilian educator and mathematician Ubiratan D'Ambrosio (1932 -) in 1977 as, in broad terms, the study of the relationship between mathematics and culture. Its theoretical foundation and its intellectual stances and perspectives were established by D'Ambrosio [14, 15, 16, 17, 18, 19], Frankenstein [28], Gerdes [35, 37], Knijnik [51, 52, 53, 54, 55], Powell & Frankenstein [69] and Powell [68]. Although at the onset, most of these stances and perspectives seem to focus mainly on the non-Western cultures, the goal of ethnomathematics is not to diminish the role of Western contributions to mathematics, but to present the role of other cultures' inputs to mathematics in an unbiased, unprejudiced, and objective manner.

It is hard to give a single comprehensive, lucid, and universally accepted depiction of what should constitute the subject matter of the topic and/or what is be placed in the periphery. D'Ambrosio himself gave slightly differing definitions. In his 1985 paper [14], ethnomathematics was defined as

> The mathematics which is practiced among identifiable cultural groups such as national-tribe societies, labor groups, children of certain age brackets and professional classes.

whereas in his 1987 paper [15], it was defined as

> The codification which allows a cultural group to describe, manage and understand reality.

He elucidated further in 1999 [18, page 146]:

> I have been using the word ethnomathematics as modes, styles, and techniques (tics) of explanation, of understanding, and of coping with the natural and cultural environment (mathema) in distinct cultural systems (ethnos).

Ascher gave two definitions, one in 1986 [4] as

> The study of mathematical ideas of a non-literate culture.

and a similar one in 1991 [3] as

> The study and presentation of mathematical ideas of traditional peoples.

There is an all-encompassing definition given by Bishop [7]

> Mathematics ... is conceived as a cultural product which has developed as a result of various activities.

and another one by Pompeu [67]

> Any form of cultural knowledge or social activity characteristic of a social group and/or cultural group that can be recognized by other groups such as Western anthropologists, but not necessarily by the group of origin, as mathematical knowledge or mathematical activity.

Knijnik's definition emphasizes the political aspect [53]:

> The investigation of the traditions, practices and mathematical concepts of a subordinated social group.

If we gather the commonalities in all these different definitions we can characterize ethnomathematics as an approach that accentuates the following:

- Mathematics is a cultural product. As such, non-literate, traditional cultures and social groups also have a mathematics.
- Mathematicians have to establish a dialogue between the mathematics of different cultures, especially between those that have been systematically excluded from the mainstream history of mathematics, and formal, academic mathematics, and thus restore cultural dignity to groups that have been traditionally marginalized and excluded [19, page 42]. Here, we are implicitly claiming that since mathematicians are members of a populace, they have some societal responsibilities that exceed far beyond "proving interesting theorems".

- All quantitative and qualitative practices, such as counting, weighing and measuring, comparing, sorting and classifying, which have been accumulated through generations in diverse cultures, should be encompassed as legitimate ways of doing mathematics.
- People produce mathematical knowledge to humanize themselves. Mathematics, along with music, arts, literature, and sciences is a distinct product of human societies and as any other such cultural phenomenon, is vital to our being human.
- The history and the philosophy of mathematics constitute essential components of ethnomathematics.

The interpretation of these characteristics specific to pedagogy will be explored in Section 5.

3.1. Ethnomathematics and History of Mathematics

An interdisciplinary and transcultural approach to the history of mathematics is important to address Eurocentrism, the oversimplified and yet the prevalent view that most worthwhile mathematics known and used today was developed in the Western world[7], and to, consequently, substantiate the fact that it took the contributions of numerous diverse civilizations throughout human history to compile mathematical ideas into a coherent whole.

Even the images of mathematicians presented in textbooks espouse and permeate the Eurocentric myth. For example, Euclid, who lived and studied in Alexandria, is not only referred to as Greek, but is depicted as a fair-skinned Westerner, even though there are no actual pictures of Euclid and no evidence to suggest that he was not a black Egyptian [60, page 104]. This discriminatory portrayal becomes even more palpable when some prominent scholars, such as Archimedes, Apollonius, Diophantus, Ptolemy, Heron, Theon, and his daughter Hypatia, whose mathematical backgrounds were the products of the scholarly environment created by the Egyptian/Alexandrian society, are referred to as Greek, whereas Ptolemy, whose work is depicted as more practical and applied, is often described as Egyptian [60].

---

[7] And to some extent in the Near East, China and India. In a huge majority of books that deal with the history of mathematics, mathematics of Africa, South America, and the first nations of North America is hardly ever mentioned.

Diop [22] discusses a number of cases in which European scholars used this practical-theoretical hierarchy to deny the sophisticated mathematical knowledge of the ancient Egyptians. In the case of the Egyptian formula for the surface of a sphere $s = 4\pi r^2$ demonstrated in problem 10 of the Papyrus of Moscow, Diop shows how Peet [66] "lets his imagination run its course" in a "particularly whimsical effort" to avoid attribution of this mathematical feat to the Egyptians. Instead, Peet tries to demonstrate that problem 10 represents the formula for the surface of a half-cylinder, knowledge which is consistent with the less sophisticated mathematics he believed the Egyptians understood:

> The conception of the area of a curved surface does not necessarily argue a very high level of mathematical thought so long as that area is one which, like that of the cylinder, can be directly translated into a plane by rolling the object along the ground (quoted in [40, page 198]).

Further, Diop points out that even Gillings, who argued forcefully for the sophisticated mathematical knowledge of the ancient Egyptians, gets caught up in the practical-theoretical dichotomy. After accepting the interpretation of problem 10 as the formula for the curved surface of a hemisphere, 1500 years ahead of Archimedes, Gillings speculates that [40, pages 200-201]:

> Whether the scribe stumbled upon a lucky close approximation or whether their methods were the results of considered estimations over centuries of practical applications, we cannot of course tell.

It is certainly absurd to think the scribe "stumbled upon a lucky approximation", without any theoretical reasoning, to such complex mathematical knowledge. Diop [22] remarks on how curious it is that

> if the ancient Egyptians were merely vulgar empiricists who were establishing the properties of figures only through measuring, if the Greeks were the founders of rigorous mathematical demonstration, from Thales onwards, by the systemization of "empirical formulas' from the Egyptians they would not have failed to boast about such an accomplishment. [40, page 255]

Indeed, racism has impacted research on history of mathematics in such profound ways that some European scholars, irrationally, changed the date of the origination of the Egyptian calendar from 4241 to 2773 B.C.E., because

> such precise mathematical and astronomical work cannot be seriously ascribed to a people slowly emerging from neolithic conditions. [78, page 24]

Recently, there have been initiatives to remedy this historic transgression. Books and papers detailing the history of the mathematical developments of non-European civilizations, such as the Japanese [64], Iraqi [75], Egyptian [74], Islamic [76], Hebrew [57], and Incan [41], have proliferated, and the intuitive mathematical thinking of some other indigenous cultures have been thoroughly investigated [3, 9, 34, 37, 45, 49, 56, 81].

More subtly and perhaps more perilously ubiquitous in the conventional presentation of the history of mathematics has been the characterization of mathematics as a male discipline that is to be practiced only by divinely anointed minds. This discriminatory approach, tacit and in most cases inadvertent, has hindered many, disproportionately people of color and women, from engaging in mathematics, and as such, as any other condition that prevents others from engaging in the process of inquiry, can be viewed as an act of violence [44, page 251].

History of mathematics is inundated by examples of sexism [45]. In 1732, the Italian philosopher Francesco Algarotti (1712–1764) wrote a book titled *Neutonianismo per le dame* (Newton for the Ladies).[8] Algarotti believed that women were only interested in romance[9], and thus attempted to explain Newton's discoveries through a flirtatious dialogue between a Marquise (la marchesa di E***) and her male companion (cavaliere).

When Sophie Germaine wanted to attend the Ecole Polytechnique, an institution reserved only for men, she had to assume the identity of a former male student at the academy, a certain Antoine-August Le Blanc. She went on to become a brilliant mathematician and physicist contributing to such diverse fields as number theory and elasticity. In fact, her exceptionally astute paper *Memoir on the Vibrations of Elastic Plates* established the foundations of the modern theory of elasticity. However, as H. J. Mozans, a

---

[8]The full name of the text, which was published in Napoli in 1737, was *Il newtonianismo per le dame ovvero dialoghi sopra la luce e i colori.*

[9]See Mazzotti, Massimo. Bologna Science Classics on-line: Newton For Ladies. Retrieved July 2011 from `http://www.cis.unibo.it/cis13b/bsco3/algarotti/introbyed/algintrobyed.pdf`. The original book is kept in the Biblioteca Comunale dell'Archiginnasio, Bologna.

historian and author of *Women in Science*, wrote in 1913[10]

> All things considered, she was probably the most profoundly intellectual woman that France has ever produced. And yet, strange as it may seem, when the state official came to make out her death certificate, he designated her as a "rentière-annuitant" (a single woman with no profession) – not as a "mathématicienne." Nor is this all. When the Eiffel Tower was erected, there was inscribed on this lofty structure the names of seventy-two savants. But one will not find in this list the name of that daughter of genius, whose researches contributed so much toward establishing the theory of the elasticity of metals – Sophie Germain. Was she excluded from this list for the same reason she was ineligible for membership in the French Academy – because she was a woman? If such, indeed, was the case, more is the shame for those who were responsible for such ingratitude toward one who had deserved so well of science, and who by her achievements had won an enviable place in the hall of fame. (Mozans, quoted in [65, page 42]).

Racism, sexism, and intellectual elitism in mathematics are, in part, instigated by the different values attributed to practical and to theoretical work, which in turn stem from our inability to see that the creation and development of mathematics and the concrete needs of societies have been inextricably interconnected. For example, in ancient agricultural societies, the need for recording information as to when to plant certain crops gave rise to the development of calendars, and as African women, for the most part, were the first farmers, they were most probably the first people involved in the struggle to observe and understand nature, and therefore, to contribute to the development of mathematics [2]. These points are tenaciously endorsed by D'Ambrosio [13, 14, 20] and Frankenstein and Powell [29].

3.2. Ethnomathematics and Philosophy of Mathematics

A lucid and intelligible acquaintance with the philosophy of mathematics is paramount to one's gaining full consciousness of the cultural nature of mathematics – any debate about the cultural nature of mathematics will

[10] Mozans, H.J. 1974. *Woman in Science*. Cambridge: MIT Press.

ultimately lead to an examination of the nature of mathematics itself. One of the most controversial topics in this area is whether mathematics is culture-free (the Realist-Platonist school) or a culture-laden (Social Constructivists), that is, whether ethnomathematics is part of mathematics or not.[11] Barton, who has offered the core of research about ethnomathematics and philosophy, asks whether "ethnomathematics is a precursor, parallel body of knowledge or precolonized body of knowledge" to mathematics and if it is even possible for us to identify all types of mathematics based on a Western-epistemological foundation [6].

Indeed, most of us are limited by our own mathematical and cultural frameworks, and this, for the most part, compels us to reformulate the mathematical ideas of other cultures within our own structural restrictions, and raises a "demarcation" question of determining what distinguishes mathematical from non-mathematical ideas. It is also quite possible for many ethnomathematical ideas to evade notice simply because they lack similar Western mathematical counterparts, for example, Russian peasant multiplication and Egyptian division. There are also many interesting games of mathematical nature. A mathematical game found in West Africa is to draw a certain figure by a line that never ends until it closes the figure by reaching the starting point (that is one is constructing an Eulerian path on a graph. Children usually use sticks to draw these in the dirt or sand. The African board game called *mancala* is comparable to chess and requires high levels of mathematical reasoning. Another example is the way mathematics appears through symmetries. Woven designs in cloth may exhibit one of the seventeen kinds of plane symmetry groups [12].

3.3. Ethnomathematics and Pedagogy of Mathematics

Since in our opinion education is not about conservation and duplication, but about critique and transformation, each aspect of the educational process (teaching, learning, and curriculum) heavily impinges on cultural values, and

[11] This needs much deeper discussion and will be analyzed more fully in another paper. For some, ethnomathematics is an ideological stance rather than an independent field of inquiry. For those who interpret it as the "mathematics of the other", it can be viewed as an interdisciplinary field which combines anthropology and mathematics. However, the main question is indeed a philosophical one of defining mathematics: Is mathematics just a collection of abstract theories and constructs or is mathematics a social and cultural construct?

consequently, on the political and social dynamics of a culture, and vice versa.[12]

If we agree that individuals and cultures are active participants in the act of learning and creating mathematics, we must then, naturally, reject the prevailing methods of teaching which treat mathematics as a deductively discovered, pre-existing body of knowledge. This, in turn, necessitates a multicultural approach to mathematics, which can best be defined as exposing students to the mathematics of different cultures so as to increase their social awareness, to reinforce cultural self-respect, and to offer a cohesive view of cultures.

The ethnomathematical perspective is cognizant and receptive of the impact of various cultural conventions and inclinations, including linguistic practices, social and ideological surroundings, for doing and learning mathematics. It regards mathematics education as a potent structure that helps learners and teachers to critique and transform personal, social, economic, and political constructs and other cultural patterns.

4. The Ethonomathematics–Epistemology Connection

It is customary for a mathematician to accentuate the difference between practical and theoretical mathematics, a schism that can tenuously be traced to the historically persistent existence of intellectual elitism that has regarded mathematical discovery as a rigorous application of deductive logic and most prodigious research as belonging to the domain of theoretical mathematics. For example, we tend to dismiss mathematical discoveries of Mesopotamia as mere simple applications and venerate Greek mathematics for its theoretical rigor.[13] This elitism, combined with racism, considers non-intuitive, non-empirical logic a unique product of European and Greek mathematics, and

[12] This view is shared by several educators of mathematics. See for example, [5, 36, 38, 61, 25].

[13] For example, great English mathematician G.H. Hardy writes in his famous *A Mathematician's Apology*

> The Greeks were the first mathematicians who are still "real" to us to-day. Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing. The Greeks first spoke a language which modern mathematicians can understand: as Littlewood said to me once, they are not clever schoolboys or "scholarship candidates", but "Fellows of another college".

undermines mathematics of other cultures merely as simple applications of certain procedures as opposed to generalizations and proofs. See [48] for an elaboration of this argument.

The role of epistemology is to remind us that "proof" has different meanings, depending on the context and the state of development of the subject. See [47] for an analysis of this assertion. To suggest that because existing documentary evidence does not exhibit the deductive axiomatic logical inference characteristic of much of modern mathematics, some cultures did not have a concept of proof, would be misleading. Indeed, inaccurate and impetuous assumptions on the use of mathematics and deductive reasoning by other groups can lead to flawed inferences. For instance, if a person agrees to exchange an object A for an object B, but does not agree to exchange two A's for two B's, this is not to be immediately interpreted as this person cannot perform simple multiplication as a narrow application of rational thought would imply. There might many other reasons – arguably, all equally logical. For example, it might very well be the case that in that person's opinion, the objects were not uniform: the second B was not as valuable as the first one, or the person needed the second A and wanted to keep it, and so on.

4.1. Considering interactions between culture and mathematical knowledge

Since knowledge is not static, and since objectivity and subjectivity, and reflection and action are dialectically connected, educators have to consider culture and context – daily customs, language, and ideology – as inseparable from the practice of learning mathematics. All people are cultural actors, and thus, mathematics, a cultural product, is created by humans in the midst of culture; it involves counting, locating, measuring, designing, playing, devising, and explaining [7], and is not to be restricted to academic mathematics. There is substantial research to ascertain that unschooled individuals, in their daily practice, develop accurate strategies for performing mental arithmetic. For example, Ginsburg, Posner, and Russell [43] have shown that unschooled children of Dioula, an Islamic people of Ivory Coast, develop similar competence in mental addition as those who attend school. Research on Brazilian children who worked in their parents' markets, showed that

> performance on mathematical problems embedded in real life contexts was superior to that on school-type word problems and

> context-free computational problems involving the same numbers and operations. [10, page 21]

Walkerdine [79] goes on to suggest that to learn school mathematics, children must learn to treat all applications, all practices as undifferentiated aspects of a value-free, neutral, and rational experience. Because of this suppression of reference, the discourse of mathematics in schools supports an ideology of rational control, of reason over emotion, and of scientific over everyday knowledge. But even these "reference-free" mathematical concepts are shaped by specific philosophical and ideological orientations. For example, Martin [62, page 210] analyzes how the intense antagonism to "rationality" which existed in the German Weimar Republic after World War I pressured the quantum physicists to search for the Copenhagen interpretation and its associated mathematical framework.

The salient point of our discussion is that ideas do not exist independent of social context. The social and intellectual relations of individuals to nature, to the world, and to political systems, influence mathematical ideas. The seemingly non-ideological character of mathematics is reinforced by a history which has labeled alternative conceptions as non-mathematical [62].

5. A Modest Proposal

The preceding arguments should not be interpreted as a concerted condemnation of European mathematics - undoubtedly, the West has played an enormous role in the advancement of mathematical sciences; indeed, classical mechanics was developed solely by European mathematicians - but rather as an understanding that all cultures have contributed at different degrees, at different times, in different styles, to the massive compilation of ideas and procedures that we call mathematics. We have to understand and respect these differences and stop assuming that the same criteria can be applied to distinguish what counts as knowledge in different societies. For example, in some societies

> There are no distinct separations between science and religion, philosophy and psychology, history and mythology. All of these are viewed as one reality and are closely interwoven into the fabric of daily life. [1, page 43]

More research needs to be done to uncover how the logic of all peoples can interact with each other to help us all understand our natural and social environment and act more effectively in the world. One place for mathematics teachers to start this research is with students' ethnomathematics. Classrooms are rich samples of the diversities in societies, and these cultural diversities can be employed to introduce some basic mathematical concepts in ways meaningful to these students. See [71, 72] for more information. For a specific example, consider the book *African Fractals* [23] by Ron Eglash, which was based on interviews with African designers, artists, and scientists. Eglash investigated and found many examples of fractals in African architecture, traditional hairstyling, textiles, sculpture, painting, carving, metalwork, religion, games, quantitative techniques, and symbolic systems.

There is a need to empower all actors in various settings of mathematics education. Not only should students and instructors evaluate other students' work, but students should critique instructors' pedagogical approaches [70]. To give authority to the students and to incorporate their perspectives in transforming mathematics pedagogy, instructors must begin by listening to them[14] and finding in-depth ways to incorporate their perspectives into educational research. In such a revolutionary pedagogy, as we listen to students' themes, we organize them using our critical and theoretical frameworks, and we re-present them as problems challenging students' previous perceptions. We also suggest themes that may not occur to our students, themes we judge are important to shattering the commonly held myths about the structure of society and knowledge that interfere with the development of critical consciousness.

To achieve these goals, we must implement some changes in our philosophy and pedagogy of mathematics.

We must cultivate a philosophy of mathematics that realizes that mathematics is

(i) One (but not the only) way of understanding and learning about the world;
(ii) Not a static, neutral, and determined body of knowledge; instead, it is knowledge that is culture-laden;
(iii) A human enterprise in which understanding results from actions; in which process and product, theory and practice, description and analysis, and practical and abstract knowledge are impeccably interconnected [58];

[14] For instance by non-departmental polls and surveys.

(iv) In constant interaction with other disciplines, and its knowledge has social, economic, political, and cultural effects.

We must have a pedagogy of mathematics that is based on

(i) Giving due status to hitherto excluded, marginalized, trivialized, and distorted contributions from all cultures;

(ii) Recognizing and respecting the intellectual activity of students, to counter the present status quo where "the intellectual activity of those without power is always characterized as nonintellectual" [33];

(iii) Insisting that students take their own intellectual work seriously, and that they participate actively in the learning process [70]. Learners are not merely "accidental presences" [32] in the classroom, but are active participants in the educational dialogue, capable of advancing the theoretical understanding of others as well as themselves.

(iv) Realizing that learners are of various genders, of diverse ethnicities, of different sexual orientations, and that they come from a variety of ethnic, cultural, and economic background, have made different life choices and that people teach and learn from these corresponding perspectives;

(v) Believing that most cases of learning problems or low achievements in schools can be explained primarily in social, economic, political, and cultural contexts [42];

(vi) Rejecting racism, sexism, ageism, heterosexism, and other alienating institutional structures and attitudes;

(vii) Understanding that no definition is static or complete, and that all definitions are unfinished. Mathematics, as well as language, grows and changes as the conditions of our social, economic, political, and cultural reality change.

We believe these proposed changes in the way academic mathematicians regard philosophy and pedagogy of mathematics will facilitate students' understanding of the cultural aspects, and consequently the major themes of mathematics.

References

[1] Adams III, H. H. 1983. "African observers of the universe: The Sirius question". In I. V. Sertima (Ed.), *Blacks in science: Ancient and modern* (pages 27–46). New Brunswick, NJ: Transaction.

[2] Anderson, S. E. 1990. "Worldmath curriculum: Fighting eurocentrism in mathematics". *Journal of Negro Education*, **59**(3), 348-359.

[3] Ascher, Marcia. 1991. *Ethnomathematics: A Multicultural View of Mathematical Ideas*. Pacific Grove, Calif.: Brooks/Cole.

[4] Ascher, M. and R. Ascher. 1986. "Ethnomathematics". *History of Science*, **24**, 125-144.

[5] (Nelson-) Barber, S. and E. Estrin. 1995. "Bringing Native American Perspectives to Mathematics and Science Teaching". *Theory into Practice*, textbf34(3), 174–185.

[6] Barton, B. 1995. "Making sense of ethnomathematics: Ethnomathematics is making sense". *Educational Studies in Mathematics*, **31**(1-2), 201-233.

[7] Bishop, A.J. 1988. "Mathematics education in its cultural context", *Educational Studies in Mathematics*, **19** 2:179–191.

[8] Bredo, E. and W. Feinberg. (Ed.). 1982. *Knowledge and values in social and educational research*. Philadelphia, PA: Temple.

[9] Carraher D. W. 1991. "Mathematics in and out of school: A selective review of studies from Brazil". In M. Harris (Ed.). *Schools, mathematics, and work*. London: Falmer, pages 169–201.

[10] Carraher, T. N., Carraher, D. W. and A. D. Schliemann. 1985. "Mathematics in the streets and in schools". *British Journal of Developmental Psychology*, **3**, 21-29.

[11] Carraher, T. N., and A, D. Schliemann. 1985. "Computation routines prescribed by schools: Help or hindrance?", *Journal for Research in Mathematics Education*, **16**(1), 37-44.

[12] Crowe, Donald W. 1973. "Geometric symmetries in African art". Section 5, Part II, in Zaslavsky (1973).

[13] D'Ambrosio, U. 1980. "Mathematics and society: Some historical considerations and pedagogical implications". In *International Journal of Behavioral Development*, **11**, 479–488.

[14] D'Ambrosio, U. 1985. "Ethnomathematics and its place in the history and pedagogy of mathematics". *For the Learning of Mathematics*, **5**(1), 44-48.

[15] D'Ambrosio, U. 1987. "Reflections on ethnomathematics". *International Study Group on Ethnomathematics Newsletter*, **3**(1), 3-5.

[16] D'Ambrosio, U. 1988. "Ethnomathematics: A research program in the history of ideas and in cognition". *International Study Group on Ethnomathematics Newsletter*, **4**(1), 5-8.

[17] D'Ambrosio, U. 1990. *Etnomatemática: Arte ou técnica de explicar e conhecer*. São Paulo: Editora Ática.

[18] D'Ambrosio. 1999. "Literacy, Matheracy, and Technoracy: A Trivium for Today". *Mathematical Thinking and Learning* **1**(2), 131-153.

[19] D'Ambrosio, U. 2001. *Etnomatemática: Elo entre as tradições e a modernidade* [Ethnomathematics: Link between tradition and modernity]. Belo Horizonte, MG: Autntica.

[20] D'Ambrosio, U. 2001. "What is ethnomathematics and how can it help children in schools?" Retrieved from `http://etnomatematica.univalle.edu.co/articulos/Ambrosio1.pdf` accessed July 28, 2011.

[21] Diesterweg, F.A.W. 1962. *Wegweiser zur Bildung fuer deutsche Lehrer und andere didactische Schriften*. Berlin.

[22] Diop, C. A. 1991. *Civilization or barbarism: An authentic anthropology.* Brooklyn, NY: Lawrence Hill.

[23] Eglash, Ron. 1999, *African Fractals: Modern Computing and Indigenous Design*. New Brunswick: Rutgers University Press.

[24] Fasheh, M. 1988. "Mathematics in a social context: Math within education as praxis versus within education as hegemony". In C. Keitel, P. Damerow, A. Bishop, & P. Gerdes (Ed.), *Mathematics, education, and society* (pages 84-86). Paris: UNESCO.

[25] Flores, A. 1997. "Si Se Puede, 'It can be done': Quality Mathematics in More Than One Language". In *Multicultural and Gender Equality in*

the Mathematics Classroom: The Gift of Diversity. Yearbook, NCTM, Reston, VA, 181–191.

[26] Frankenstein, M. 1987. "Critical mathematics education: An application of Paulo Freire's epistemology". In I. Shor (ed.), *Freire for the classroom* (pages 180-210). Portsmouth, NH: Boynton/Cook.

[27] Frankenstein, M. 1989. *Relearning mathematics: A different third R-Radical maths*. London: Free Association.

[28] Frankenstein, M. 1997. "In Addition to the Mathematics: Including Equity Issues to the Curriculum". In J. Trentacosta (Ed.),*Multicultural and Gender Equity in the Mathematics Classroom: The Gift of Diversity* (1997 Yearbook). Reston, VA: National Council of Teachers of Mathematics.

[29] E. Frankenstein and Powell. 1994. "Toward liberatory mathematics: Paulo Freire's epistemology and ethnomathematics". In Mc Laren, P. L. and C. Lankshear (eds.) *Politics of liberation: Paths from Freire.* London: Routledge., 74–99.

[30] Freire, P. 1970. *Pedagogy of the oppressed.* New York: Seabury.

[31] Freire, P. 1973. *Education for critical consciousness.* New York: Seabury.

[32] F. Friere, P. 1982. *Education for consciousness.* New York: Continuum.

[33] Freire, P., Macedo, D. 1987. *Literacy: Reading the word and the world.* South Hadley, MA: Bergin & Garvey.

[34] Gay, John and Michael Cole. 1967. *The new mathematics and an old culture. A study of learning among the Kpelle of Liberia.* New York: Holt, Rinehart and Winston.

[35] Gerdes, P. 1985. "Conditions and strategies for emancipatory mathematics education in underdeveloped countries". *For the Learning of Mathematics*, **5**(1), 15-20.

[36] Gerdes, Paulus. 1988b. "On culture, geometrical thinking, and mathematics education". *Educational Studies in Mathematics*, **19** 3:137–162.

[37] Gerdes, P. 1999. *Geometry from Africa: Mathematical and educational explorations*. Washington, DC: The Mathematical Association of America.

[38] Gerdes, Paulus. 2001. "Exploring the game of Julirde", *Teaching Children Mathematics*, **2**, 321–327.

[39] Gettier, Edmund 1963. "Is Justified True Belief Knowledge?". *Analysis* **23**:121–23.

[40] Gillings, R. J. 1972. *Mathematics in the time of the pharaohs*. Cambridge, MA: MIT (Dover reprint, 1982).

[41] Gilsdorf, Thomas. 2001. "Inca Mathematics". In Helaine Selin (ed.), *Mathematics Across Cultures. The History of Non-Western Mathematics*, page 189. Dordrecht/Boston/London: Kluwer Academic Publishers.

[42] Ginsburg, H. P. 1986. "The myth of the deprived child: New thoughts on poor children". In U. Neisser (Ed.), *The school achievement of minority children: New perspectives* (pages 169–189). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

[43] Ginsburg, H. P., Posner, J. K., Russell, R. L. 1981. "The development of mental addition as a function of schooling and culture". *Journal of Cross-Cultural Psychology*, **12**(2), 163-178.

[44] Gordon, M. 1978. "Conflict and liberation: Personal aspects of the mathematics experience". *Curriculum Inquiry*, **8**(3), 251-271.

[45] Harris, M. 1987. "An example of traditional women's work as a mathematics resource". *For the Learning of Mathematics*, **7**(3), 26-28.

[46] Hawking, Stephen W. 2001. *The Universe in a Nutshell*. New York: Bantam Books

[47] Izmirli, Ilhan M. 2009. "Examples of Gonseth's Idonéisme in Pedagogy of Mathematics: Time Dependence of Mathematical Concepts and Their Proofs". *Epistemologia*, XXXII (2009), pages 209–218.

[48] Joseph, George Gheverghese. 1987. "Foundations of eurocentrism in mathematics". *Race & Class*, **28**(3), 13-28.

[49] Joseph, George Gheverghese. 2000. *The Crest of the Peacock: Non-European Roots of Mathematics*. 2nd. ed. London: Penguin Books.

[50] Kline, M. 1953. *Mathematics in western culture*. New York: Oxford.

[51] Knijnik, G. 1996. *Exclusão e resistĕncia: Educação matemática e legitimidade cultural* [Exclusion and resistance: Mathematics education and cultural legitimacy]. Porto Alegre: Artes Médicas.

[52] Knijnik, G. 1997. "An ethnomathematical approach in mathematical education: A matter of political power". In A. B. Powell & M. Frankenstein (Eds.), *Ethnomathematics: Challenging eurocentrism in mathematics education*. Albany: State University of New York.

[53] Knijnik, G. 1998. "Ethnomathematics and political struggles". *Zentralblatt für Didaktik der Mathematik*, **30**(6), 188-194.

[54] Knijnik, G. 1999. "Ethnomathematics and the Brazilian landless people education". *Zentralblatt für Didaktik der Mathematik*, **31**(3), 96-99.

[55] Knijnik, G. 2002. "Two political facets of mathematics education in the production of social exclusion". In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (Vol. **2**, pages 357–363). Copenhagen: Centre for Research in Learning Mathematics.

[56] Kyselka, W. 1987. *An ocean in mind*. Honolulu, HI. University of Hawaii Press.

[57] Langermann, Tzvi T. and Shai Simonson. 2001. "The Hebrew Mathematical Tradition. Part One (Langermann, Tzvi T.) Numerical Speculation and Number Theory. Geometry. Part Two (Simonson, Shai) The Evolution of Algebra". In Helaine Selin (ed.), *Mathematics Across Cultures. The History of Non-Western Mathematics*, page 167. Dordrecht/Boston/London: Kluwer Academic Publishers.

[58] Lave, J. 1988. *Cognition in practice*. Cambridge, England: Cambridge.

[59] Lerman, S. 1989. "Constructivism, mathematics and mathematics education". *Educational Studies in Mathematics*, **20**, 211–223.

[60] Lumpkin, B. 1983. "Africa in the mainstream of mathematics history". In I. V. Sertima (Ed.), *Blacks in science: Ancient and modern* (pages 100–109). New Brunswick, NJ: Transaction.

[61] Malloy, C. E. 1997. *Including African-Americans in the Mathematics Community.* Yearbook, NCTM, Reston, VA, 23– 33.

[62] Martin, B. 1988. "Mathematics and social interests". *Search*, **19**(4), 209–214.

[63] McLaren, P. 1990. "Review of Freire for the Classroom: A Sourcebook for Liberating Teaching". *Journal of Urban and Cultural Studies*, **1**(1), 113–125.

[64] Ogawa Tsukane. 2001. "A Review of the History of Japanese Mathematics". *Revue d'histoire des mathematiques*, **7** (2001), pages 137–155.

[65] Osen, L. M. 1974. *Women in mathematics.* Cambridge, MA: MIT.

[66] Peet, E. T. 1931. "A problem in Egyptian geometry". *Journal of Egyptian Archeology*, **17**, 100–106.

[67] Pompeu, G. Jr. 1994. "Another definition of ethnomathematics?". *Newsletter of the International Study Group on Ethnomathematics*, **9** (2), 3.

[68] Powell, A. B. 2002. "Ethnomathematics and the challenges of racism in mathematics education". In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society Conference* (Vol. **1**, pages 15–29). Copenhagen: Centre for Research in Learning Mathematics.

[69] Powell, Arthur B., and Marilyn Frankenstein (eds.) 1997. *Ethnomathematics: Challenging Eurocentrism in Mathematics Education.* Albany, NY: State University of New York Press.

[70] Powell, A. B., Jeffries, D. A., and Selby, A. E. 1989. "An empowering, participatory research model for humanistic mathematics pedagogy". *Humanistic Mathematics Network Newsletter* #4, pages 29–38.

[71] Powell, A. B. and López, J. A. 1989. "Writing as a vehicle to learn mathematics: A case study". In P. Connolly & T. Vilardi (Ed.), *The Role of Writing in Learning Mathematics and Science* (pages 157–177). New York: Teachers College.

[72] Powell, A. B. and Ramnauth, M. 1992. "Beyond questions and answers: Prompting reflections and deepening understandings of mathematics using multiple-entry logs." *For the Learning of Mathematics*, **12** (2), 12–18.

[73] Powell, A. B., and Temple, O. L. 2001. "Seeding ethnomathematics with oware: Sankofa". *Teaching Children Mathematics*, **7**(6), 369–375.

[74] Ritter, James. 2000. "Egyptian Mathematics". In Helaine Selin (ed.), *Mathematics Across Cultures. The History of Non-Western Mathematics*, pages 115–136. Dordrecht/Boston/London: Kluwer Academic Publishers.

[75] Robson, E. 2008. *Mathematics in Ancient Iraq: A Social History*. Princeton: Princeton University Press.

[76] Sesiano, Jacques. 2001. "Islamic mathematics". In Helaine Selin (ed.), *Mathematics Across Cultures. The History of Non-Western Mathematics*, page 137. Dordrecht/Boston/London: Kluwer Academic Publishers.

[77] Shor, I. 1992. *Empowering education: Critical teaching for social change*. Portsmouth, NH: Heinemann.

[78] Struik, Dirk J. 1987. *A Concise History of Mathematics* (Fourth Edition). Mineola, N.Y.: Dover Publications.

[79] Walkerdine, V. 1988. *The mastery of reason*. London: Routledge.

[80] Youngman, F. 1986. *Adult education and socialist pedagogy*. London: Croom Helm.

[81] Zaslavsky, Claudia. 1973. *Africa Counts: Number and Pattern in African Culture*. Third revised ed., 1999. Chicago: Lawrence Hill Books.