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Another Look at Dollar Cost Averaging

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Abstract

Dollar cost averaging—spreading an investor's stock purchases evenly over time—is widely

touted in the popular press because of the mathematical fact that the average cost per share is less

than the average price. The academic press has generally been skeptical, and attributes dollar

cost averaging's popularity to investor naiveté and cognitive errors. Yet, dollar cost averaging

continues to be recommended by knowledgeable investors as a sensible way to avoid ill-timed

purchases. We argue that dollar cost averaging is, in fact, an imperfect, but helpful strategy for

diversifying investment decisions across time.

Another Look at Dollar Cost Averaging

An investor following a dollar cost averaging (DCA) strategy periodically invests a constant

dollar amount in stocks, adjusting the number of shares purchased as stock prices fluctuate.

When stock prices are high, fewer shares are bought; when prices are low, more shares are

purchased. Because the average cost weights the purchase prices by the number of shares

acquired at each price, the average cost is always less than the average price.

This mathematical fact has led many to recommend dollar cost averaging. In several *Barron's*

columns and multiple editions of *Successful Investing Formulas*, first published in 1947, Lucile

Tomlinson persuasively extolled the virtues of cost-averaging plans. In the 2012 edition, with a

preface written by Benjamin Graham, Tomlinson wrote that dollar cost averaging is "the

unbeatable formula," because, "No one has yet discovered any other formula for investing which

can be used with so much confidence of ultimate success, regardless of what may happen to

security prices, as Dollar Cost Averaging."

The academic literature has generally been scornful, concluding that DCA is an inferior

strategy. Constantinides (1979) finds it ironic that DCA's fixed investment rule is considered a

benefit. He argues that DCA's inflexibility is a fatal flaw because making portfolio decisions

based on the additional information accumulated as time passes must be superior to committing

oneself to a strategy based only on current information. One counterargument is that in an

efficient market, it may be difficult to profit from future information.

Knight and Mandell (1992/1993) argue that DCA is suboptimal because the DCA portfolio's

riskiness increases as stock purchases are made at regular intervals, but an investor with constant

relative risk aversion would not increase a portfolio's riskiness over time.

Thorley (1994) and Milevsky and Posner (2003) criticize a comparison of average price with average cost, since investors cannot sell the stocks they accumulate at the historical average price; however, this does not address the issue of whether DCA allows investors to acquire stocks at a relatively low average cost compared to an initial lump-sum (LS) investment. The answer obviously depends on whether stock prices subsequently increase or decrease. Thus, Grable and Chatterjee (2015) observe that, while LS does well in the early stages of a bull market, DCA may be more suitable for risk-averse investors who are fearful of a bear market. Luskin (2017) reports that DCA does better when initiated during historical periods when Shiller's cyclically adjusted price-earnings (CAPE) ratio is abnormally high.

Bierman and Hass (2004) argue that if an investor is considering an investment that has an expected return that is higher than from not being invested, then delaying purchases must reduce the investor's expected return. However, Cho and Kuvvet (2015) use a two-period mean-variance framework to argue that if the risk-return opportunity locus is concave, some investors may prefer DCA to LS.

Milevsky and Posner (2003) show that DCA can be financially attractive if investors have a fixed target portfolio value. In particular, DCA has a positive expected profit if the price of the stock being purchased is the same at the end of acquisition period as at the beginning, and the size of this profit increases with the stock's volatility. More generally, if the terminal stock price is known in advance, the higher the volatility, the higher the expected value of the return from DCA, so that there is some sufficiently high level of volatility such that DCA has a higher expected return than LS. They argue that investors do, in fact, have terminal target prices, and

conclude (presumably tongue in cheek) that

> perhaps the investing masses are more sophisticated than we are willing to admit. Indeed, they may be familiar with the Ito-Markovian structure of asset prices, yet they insist on dollar-cost averaging because they believe market prices are driven by Brownian *bridges* as opposed to Brownian *motions*. (Milevsky and Posner 2003, p. 191)

Some academics accept the argument that DCA is inferior to LS, and ask why so many investors continue to use and recommend it. They identify the main benefits as encouraging regular savings, protecting investors from trend chasing and portfolio churning, and avoiding a single ill-timed purchase that might scare them away from future stock purchases (Haley 2010). Others (Statman 1995; Dichtl and Drobetz 2011) argue that dollar cost averaging can be justified by Kahneman and Tversky's (1992) prospect theory; however, Leggio and Lien (2001) present evidence to the contrary and, in addition, argue that DCA is less successful with more volatile stocks—contradicting Milevsky and Posner.

On the empirical front, Williams and Bacon (1993) look at DCA and LS investments in the S&P 500 and find that LS generally has a higher return, which is unsurprising since, historically, being in the stock market has, on average, been more profitable than being out of the market. Leggio and Lien (2003a, 2003b) consider the average return and various risk measures and conclude that DCA is inferior to LS. Thorley (1994) reports that LS has a somewhat higher average return and lower standard deviation than DCA. Similarly, Rozeff (1994) concludes that, relative to DCA, the LS strategy tends to have a higher return because it is more fully invested in stocks and the LS strategy tends to have a lower variance because it is more uniformly invested in stocks, as opposed to being lightly invested early on and heavily invested later. Using

simulations with historically based parameters, Abeysekera and Rosenbloom (2000) conclude that DCA generally has a lower return and less risk. Using bootstrapped CRSP index returns, Lei and Li (2007) report that the results from DCA and LS are statistically indistinguishable over short and long horizons, but that DCA has a higher probability of a negative return over intermediate-term horizons.

There is some empirical evidence in support of DCA. Balvers and Mitchell (2000) and Brennan, Li, and Torous (2005) argue that DCA may be advantageous if stock returns are negatively autocorrelated, as in the mean-reversion literature (Poterba and Summers 1988; Fama and French 1988). To exploit this mean reversion, Dunham and Friesen (2012) recommend an "enhanced DCA" strategy in which stock purchases are increased after a market decline, and reduced after a market advance. Dubil (2005a) and Trainor (2005) argue that DCA reduces shortfall risk, the chances of wealth dropping below a specified floor during the investor's lifetime.

Israelson (1999) uses mutual fund data to argue that DCA is a superior strategy, especially for low-volatilty mutual funds—which contradicts the more widespread argument that DCA is best suited for volatile investments. Atra and Mann (2001) argue that the January effect and other seasonal patterns in stock returns affect comparisons of DCA and LS. Paglia and Jiang (2006) report that the day of the month matters, too. These claims are examples of the more general point made by Dubil (2005b) that a comparison of DCA with LS depends on the specific stock price patterns that happen to occur; for example, LS will have an advantage if stock prices head up after the plan is started, while DCA has the advantage if prices drop. Comparing strategies that are always launched on January 1 may distort the evidence.

Overall, the academic verdict is that DCA is an inferior strategy both in theory and practice, but investors use it anyway for not particularly persuasive reasons. The unsettling thing about this academic conclusion is that many of those who endorse DCA are, in other respects, very sensible and experienced. Perhaps the academic verdict is too harsh and underestimates the wisdom of experienced practitioners?

Despite the generally negative conclusions in peer-reviewed academic research, dollar cost averaging is widely endorsed in the popular press. Damato (1994) wrote in *The Wall Street Journal* that,

> People who invest in stocks regularly get the benefit of "dollar cost averaging." Because a fixed sum is invested every month or quarter buys more shares when prices are down, the investor's average cost per share ends up being lower than the average price in the market over the same period.

Sylvia Porter (1979) describes dollar cost averaging as "how to beat the stock market." The Investment Company Institute (1984) says that, "It takes the ups and downs of the market and turns them to advantage." The *New York Times Financial Planning Guide* (1985) calls dollar cost averaging a "time-tested investing method" with a "seemingly magic result."

The financial press is one thing, but it is striking that DCA is recommended by several extremely knowledgeable observers, including Levy and Sarnat (1972), Sharpe (1978), Dreman (1982), Loeb (1996), Bogle (2015), and Tobias (2016). In the 2015 edition of his classic book, *A Random Walk Down Wall Street*, Malkiel writes that,

> Dollar cost averaging can reduce the risks of investing in stocks and bonds….Periodic investments of equal dollar amounts in common stocks can reduce (but not avoid) the

risks of equity investment by ensuring that the entire portfolio of stocks will not be

purchased at temporarily inflated prices. (p. 355)

Many individuals and institutions practice what these advisors preach, ranging from Fidelity's

Automatic Account Builder, which transfers money each month from a participant's bank

account to a stock fund, to the MacArthur Foundation, which used dollar cost averaging in

1985-1986 (at the rate of $100 million a month) to invest $1.4 billion realized from the

liquidation of real estate and other tangible assets.

Neither unequivocal praise or scorn is warranted. The core argument in favor of dollar cost

averaging gauges performance by cost, rather than rate of return, and, when this is taken into

account, the alleged simple virtues of cost averaging vanish. On the other hand, many criticisms

of dollar cost averaging neglect the return on funds not invested in stock or don't compare

equivalent DCA and LS strategies (for example, equally risky strategies). When these factors are

taken into account, dollar cost averaging may be a reasonable approach to investing in volatile

stocks. We first demonstrate that the claimed low-cost virtue is an illusion and then use a novel

approach to analyze how dollar cost averaging can effectively diversify investment risk across

time.

The Illusion

The presumed advantages of dollar cost averaging are invariably based on calculations similar to

those shown in Table 1. A stock currently sells for $60 a share and its price will either rise or fall

by 50 percent. If $900 is invested each period, the average cost is less than the average price, no

matter whether the stock's price goes up or down. The intriguing implication is that, although the

expected value of the price change is zero, the expected value of the average cost is less than the

expected value of the price in the second period. In Table 1, the expected value of the average cost is 0.5($72) + 0.5($40) = $56, and the expected value of the price is 0.5($90) + 0.5($30) = $60. DCA appears to have a positive expected profit even if changes in stock prices are independent with a mean of zero.

Table 1 Dollar Cost Averaging Reduces Costs

Time period	Price Rises to $90			Price Falls to $30		
	Price	Shares	Cost	Price	Shares	Cost
1	$60	15	$900	$60	15	$900
2	$90	10	$900	$30	30	$900
Average	$75		$72	$45		$40

This conclusion is misleading. The mathematical fact that the expected value of the average cost is less than the expected value of the price does not imply a positive expected rate of return, because it neglects the varying amounts invested in the each scenario. The relevant data are shown in Table 2. DCA gains $18 a share if the price rises and loses $10 a share if it falls. The average of these two numbers is indeed positive. But it is also irrelevant since the number of shares is not the same. DCA gains $18 a share on 25 shares or loses $10 a share on 45 shares—a $450 profit on an $1,800 investment or a $450 loss on an $1,800 investment. Even though the average cost is less than the average price, if a stock's expected return is zero, DCA's expected profit is also zero.

This is reminiscent of the Martingale betting system (Mitzenmacher and Upfal, 2005) in which bets are doubled after every loss. If the outcomes are independent and each wager has a zero expected return, the expected return from the strategy must be zero—no matter how one

adjusts the size of the wagers as time passes. This is true of a Martingale betting strategy and it is true of a DCA investing strategy. What these strategies do change is the shape of the payoff distribution, including the variance.

Table 2 Dollar Cost Averaging Does Not Increase Expected Return

	Price Rises to $90	Price Falls to $30
Average cost	$72	$40
Number of shares	25	45
Total investment	$1,800	$1,800
Value of portfolio	$2,250	$1,350
Dollar gain	$450	- $450
Rate of return	25%	- 25%

A Mean-Variance Analysis

Malkiel and other supporters of cost averaging emphasize the risk of making a lump sum investment at an unfortunate time; i.e., shortly before a big drop in prices. Cost averaging essentially diversifies one's investments, not across different stocks, but across different purchase dates. We use mean-variance analysis with Tobin's separation theorem to demonstrate that time-diversification can be an important virtue.

An evaluation of dollar cost averaging should take into account the rate of return on funds not invested in stocks. We assume two assets, safe Treasury bills and a risky stock. We assume that the return R on Treasury bills is constant and that the stochastic stock return S_t in period t is independent of the stock's return in other periods and has a constant mean μ and standard deviation σ, with the stock's expected return larger than the return on T-bills. DCA does not

benefit from mean reversion in our model, and LS has the built-in advantage of a stock expected return that is higher than the return on Treasury bills. Nonetheless, DCA has some appealing features.

The LS strategy involves an immediate investment of all wealth in stocks. The DCA strategy is to make regular, periodic constant stock purchases over an investment horizon of T periods. Money not initially invested in stocks is parked in Treasury bills.

Mean-variance analysis is usually applied to a decision on how to allocate wealth among a portfolio of different stocks or asset classes. It can be applied to dollar cost averaging by considering each periodic investment in stocks as an initial investment in Treasury bills that is converted into stocks at the appropriate time.

Specifically, one way to frame the timing question is to identify the delayed purchase of stock when there are j periods left in the investment horizon as an asset that yields $(1 + R)$ for each of the first $T - j$ periods and $(1 + S_t)$ for the remaining j periods. The gross return over the full T-period horizon is

$$Z_j = (1 + R)^{T-j} \prod_{t=T+1-j}^{T} (1 + S_t) \qquad j = 0, \ldots, T \qquad (1)$$

Z_0 is the gross return on an investment in Treasury bills for all T periods; Z_1 is the return on an investment in Treasury bills for $T - 1$ periods, followed by a stock investment in the last period; and so on. A decision to spread stock purchases over a T-period horizon is a portfolio allocation of wealth among the T assets j = 1, 2, … T. The overall return is

$$Z = \sum_{j=1}^{T} \lambda_j Z_j \qquad (2)$$

where a fraction λ_j of wealth is invested in asset j. For each value of $1 \leq j \leq T$, λ_j is invested now

in the safe asset and, after $T - j$ periods, $\lambda_j(1 + R)^{T-j}$ is invested in stocks. Dollar cost averaging corresponds to

$$\lambda_j = \frac{C}{(1+R)^{T-j}} \qquad\qquad j = 1, \ldots, T \qquad (3)$$

where C is set so that the sum of the λ_j is 1. For the lump-sum strategy, $\lambda_T = 1$, and $\lambda_j = 0$ for $1 \leq j \leq T$.

If the stock returns are independent across time, then a standard mean-variance portfolio analysis can be conducted using

$$E\left[Z_j\right] = (1+R)^{T-j}(1+\mu)^j$$
$$\text{Var}\left[Z_j\right] = (1+R)^{2(T-j)}\left((1+\mu)^2+\sigma^2\right)^j - (1+\mu)^{2j}\right) \qquad 1 \leq j \leq T \qquad (4)$$
$$\text{Cov}\left[Z_iZ_j\right] = (1+R)^{T-i+T-j}(1+\mu)^{j-i}\left((1+\mu)^2+\sigma^2\right)^i - (1+\mu)^{2i}\right)$$

Since asset 0 is a safe asset with return $(1 + R)^T$, Tobin's (1957) separation theorem applies, and we can identify the optimal ratios λ_j/λ_T for $j \geq 0$, regardless of risk preferences. These can be rescaled to give the optimal fractions γ_j of the risky portfolio:

$$\gamma_j = \frac{\lambda_j / \lambda_T}{\sum_{j=1}^{T}\left(\lambda_j / \lambda_T\right)} \qquad\qquad 1 \leq j \leq T$$

The optimal proportions depend on the volatility of stock prices—the more volatility, the stronger is the case for time diversification. The time-diversification argument is consequently more persuasive for individual stocks than for stock indexes. We assume a constant 3 percent annual return on Treasury bills and a 7 percent expected return on stocks, and use different values for the standard deviation.

The average annual return on Treasury bills since 1926 has been approximately 3 percent and the average annual return on large-cap stocks has been 10 percent (Ibbotson, Grabowski, Harrington, and Nunes 2016). However, a study of the twentieth-century performance of the stock markets in 39 different countries found that the U. S. stock market beat all the rest (Jorian and Goetzmann 1999). It seems unlikely that investors worldwide knew that the twentieth century would turn out to be America's century and the U. S. stock market would turn out to be the winner. It is more likely that the remarkable performance of the U. S. stock market was a pleasant surprise to investors who owned U. S. stocks—and we can hardly count on pleasant surprises indefinitely.

The equity premium seems to have declined over time and the *ex ante* premium is likely to be significantly smaller in the future than the *ex post* premium has been in the past. Fama and French (2002) estimate a risk premium of 2.55 to 4.32 percent for 1951 to 2000 relative to short-term risk-free bonds. Siegel (1999) suggests it might be as low as 1 percent to 2 percent going forward. We use 4 percent as our baseline premium (a 7 percent expected return on stocks compared to a 3 percent return on safe Treasury securities), but also consider 2 percent and 6 percent premiums.

Table 3 shows the means, standard deviations, and correlation coefficients for annual investments over a five-year horizon, in accordance with Equation 1, and the optimal allocation in the risky portfolio using Tobin's Separation theorem. The fifth asset is an initial investment in stocks and, because its return is the least correlated with the return on the first asset (four years of Treasury bills, followed by a stock investment), the first asset has the second largest optimal allocation. The spread of investments across all five assets is much more dispersed than is the LS

all-in strategy, which sets $\gamma_5 = 1$ and the other $\gamma_j = 0$.

Table 3 Mean-Variance Analysis with a 5-Year Horizon and $R = 3\%$, $\mu = 7\%$, and $\sigma = 50\%$

| Asset | Mean, % | SD, % | correlation coefficients | | | | | optimal γ_j |
			1	2	3	4	5	
1	20.43	56.27	1.00	.67	.52	.42	.36	.229
2	25.11	87.07		1.00	.77	.63	.54	.174
3	29.96	116.86			1.00	.87	.69	.132
4	35.01	148.11				1.00	.85	.101
5	40.26	182.04					1.00	.365

Figure 1 shows the percentage of the risky portfolio invested initially in stocks for different investment horizons and 30 percent and 50 percent annual standard deviations of the stock return. Time diversification is most attractive for less correlated alternatives, and the correlations among the alternatives drop for riskier stock and longer horizons. In Figure 1, as the investment horizon lengthens or the standard deviation increases, the initial investment in stocks approaches that recommended by dollar cost averaging.

Figure 2 shows the Markowitz frontier using the assumptions in Table 3: annual investments over a five-year horizon with a 50 percent annual standard deviation in the stock return. The DCA portfolio is very close to the optimal portfolio implied by Tobin's separation theorem.
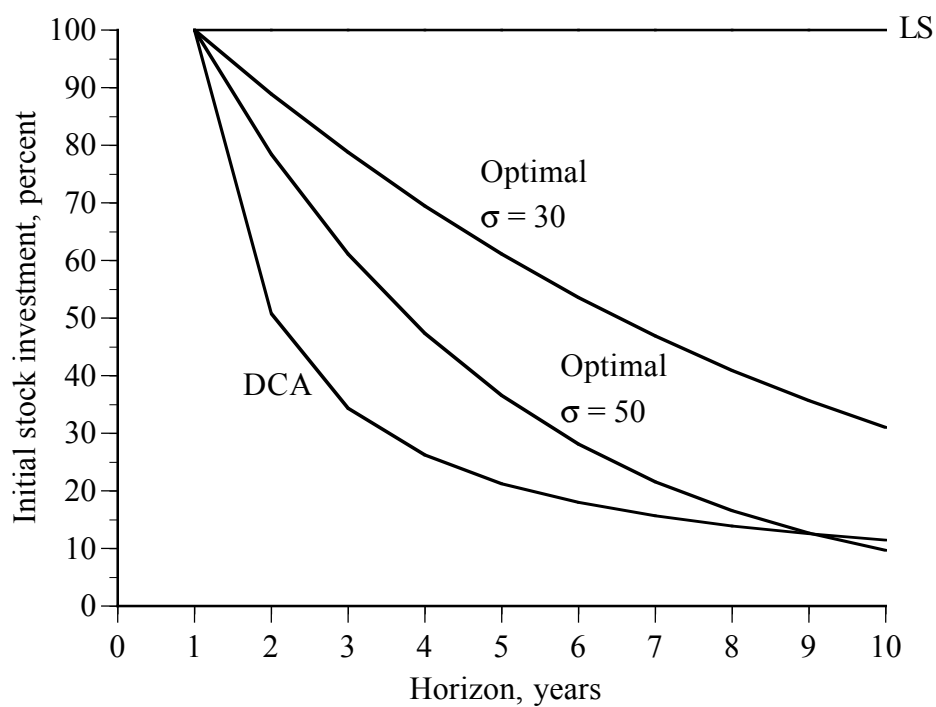
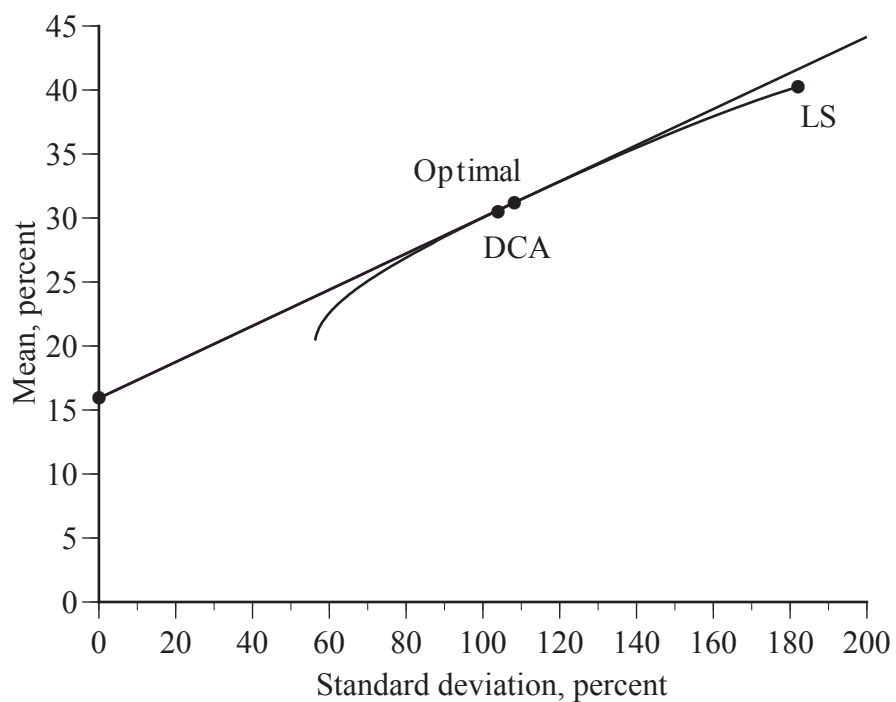Figure 1 Fraction Initially Invested in Stocks, annual investments

Figure 2 Markowitz Frontier, annual investments over a 5-year horizon with $\sigma = 50\%$

Figure 3 is the analogue of Figure 1, but now with monthly investments over horizons up to

60 months and with 20 percent and 30 percent standard deviations of stock returns. Again, a

longer horizon or standard deviation supports the DCA idea of diversifying stock purchases

across time. Figure 4 shows the corresponding Markowitz frontier for monthly investment over a

60-month horizon with a 20 percent standard deviation. Again, the DCA portfolio is close to the

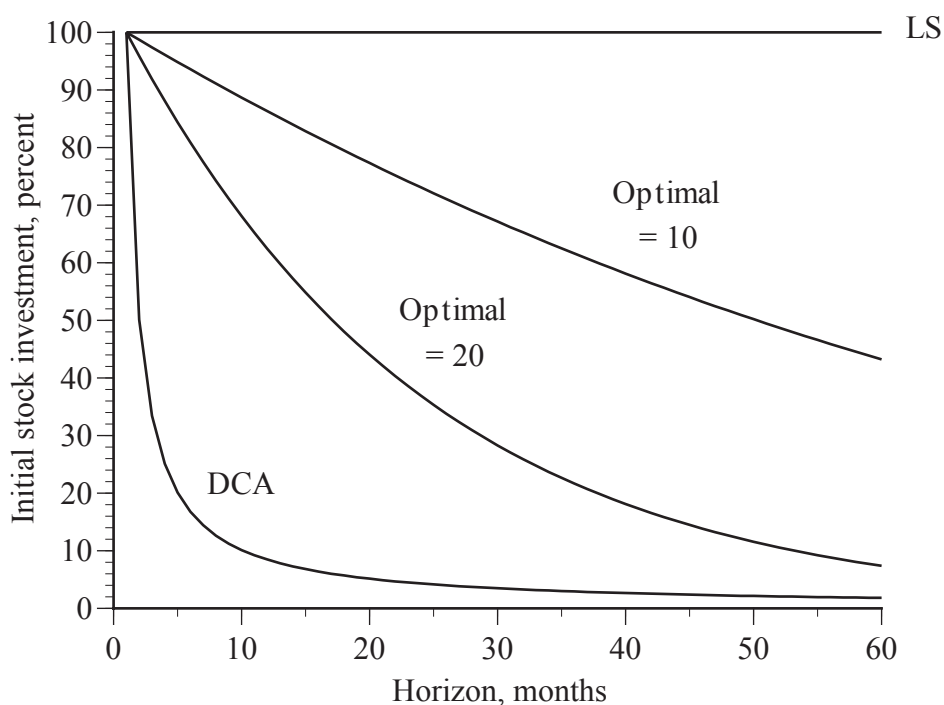optimal portfolio implied by Tobin's separation theorem.

Figure 3 Fraction Initially Invested in Stocks, monthly investments

One might think that a higher equity premium would pull the optimal portfolio towards a

larger initial investment in stocks, because it increases the opportunity cost of investing in

Treasury bills. However, the equity premium has little effect on the curvature of the Markowitz

frontier and, looking at Figures 2 and 4, an upward shift in the Markowitz frontier slides the

optimal portfolio away from LS. Table 4 confirms this. As the equity premium increases, the

optimal initial investment in stocks moves away from the 100 percent figure used by LS towards
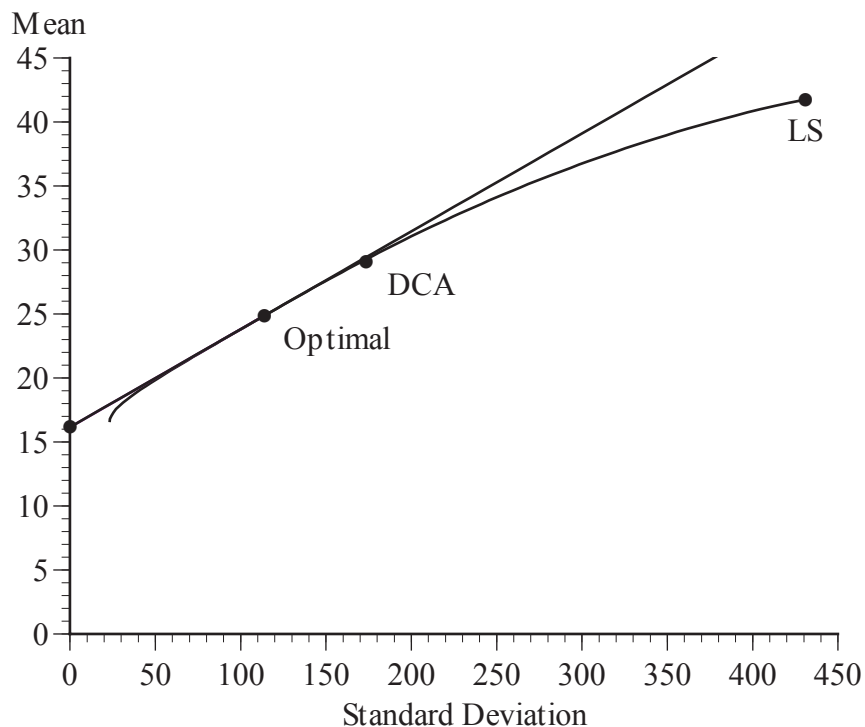
the 21.2 percent figure used by DCA.

Figure 4 Markowitz Frontier, monthly investments over a 5-year horizon with $\sigma = 20\%$

Table 4 Initial Stock Investments with a 5-Year Horizon and $R = 3\%$

	LS	DCA	Optimal Portfolio	
			$\sigma = 30\%$	$\sigma = 50\%$
$\mu = 5\%$	1.00	.212	.665	.396
$\mu = 7\%$	1.00	.212	.611	.365
$\mu = 9\%$	1.00	.212	.558	.335

The time-diversification value of DCA depends on the investor's assumptions, with a longer

horizon and larger standard deviation of stock returns moving the optional portfolio away from

LS and closer to DCA. Dollar cost averaging is obviously not always a reasonable approximation

to the optimal portfolio implied by Tobin's separation theorem. Our point is simply that Bogle,

Malkiel, Tobias, and other wise and experienced investors are neither naïve or foolish. When

contemplating an especially risky investment, there is merit in time diversification even if the

stock returns are independent with a higher expected return than Treasury bills.

Empirical Evidence

We used historical data to compare the DCA and LS strategies. Even though *ex post* returns may be an unreliable guide for *ex ante* decisions, it seems worthwhile to consider how these strategies would have fared in the past.

We looked at the 84 stocks that have been in the Dow Jones Industrial Average from October 1, 1928, when the Dow was expanded from 20 to 30 stocks, through December 31, 2016, a total of 23,219 trading days. These are all prominent companies, widely followed by investors, with reliable data, though the fluctuations in the daily returns are presumably smaller than those for many stocks that might be DCA candidates. The daily returns on Treasury bills and these 84 stocks were taken from the Center for Research in Security Prices (CRSP) data base.

To avoid biases that might arise because stocks added to the Dow generally did well before their inclusion, we only looked at purchases of Dow stocks that were made while the stocks were in the Dow. In order to avoid distortions that might be caused by seasonal or day-of-the-month patterns, our portfolio simulations used every Dow stock and every starting date; for example, IBM on April 18, 1978. When occasional gaps in the CRSP data base of returns arise, we assume that the portfolio temporarily earns the Treasury-bill return. All initial purchases were made on dates such that the investment horizon ended on or before December 31, 2016.

The LS strategy purchases the selected stock on the selected day and holds it until the end of the investment horizon. The DCA strategy uses the asset allocation described by Equation 3 to invest in the *T* assets that make up the DCA strategy. A specified fraction of the portfolio is invested in the stock initially, with the remainder parked in Treasury bills. Each successive

period, an additional investment (equal to the initial investment) is made in the stock. After looking at Figures 1 and 3, we also considered a 50-50 strategy of investing 50 percent of wealth immediately in stocks and spreading the remainder equally over the horizon.

We considered two horizons consistent with the earlier theoretical analysis—annual purchases for five years and monthly purchases for five years. There are approximately 250 trading days in a year, so we assumed that the annual purchases were made every 250 trading days after the initial purchase and that the monthly purchases consisted of purchases made every 20 trading days after the initial purchase.

Table 5 shows the results. As expected, the LS strategy had the highest average return and the highest standard deviation, and the DCA strategy had the lowest values, with the 50-50 strategy in between. The LS strategy had the lowest Sharpe ratios, while DCA and 50-50 were very similar. Historically, an investment in Treasury bills and either DCA or 50-50 would have dominated an equally risky investment in Treasury bills and LS.

Table 5 Simulated of Purchases of Dow Stocks Over a Five-Year Horizon, 1928-2016

	Average Annual Return, %	Standard Deviation, %	Sharpe Ratio
Annual Purchases			
LS	73.63	106.54	0.41
DCA	51.76	60.55	0.49
50-50	59.63	74.48	0.50
Monthly Purchases			
LS	69.73	101.86	0.40
DCA	44.32	50.36	0.47
50-50	56.55	71.76	0.49

Conclusion

Dollar cost averaging is not as foolish as it is sometimes portrayed. It is well known that its mechanical nature may encourage saving and reduce the emotional anxiety associated with making decisions. It is not well known that cost averaging can also be a valuable way of diversifying investment decisions across time, which is particularly appealing when investing in volatile stocks over a substantial horizon. Dollar cost averaging is not always optimal but, in some circumstances, it may be a reasonable approximation.

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