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Teaching an Online Sophomore-Level Differential Equations Course with Mathematica Supplements

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Abstract: I have had the experience of developing and teaching a number of sections of online sophomore-level differential equations courses for the past eight years. This article is an attempt to recall my methods, the ideas and philosophies that guided me, give an informal summary of student achievement and course evaluations, and describe my creation and use of interactive *Mathematica* supplements in the course. Four of the supplements that I created with *Mathematica* are available in the online appendix to this article.

1 Introduction

I am a person who has consistently been on the lookout for new things to do in my career as a mathematician, instructor, and mentor, while at the same time desiring to have a balanced life and not let my professional work squeeze out everything else. The thought of developing and teaching an online sophomore-level differential equations course for our mathematics, physics, and engineering students at Bethel University was very exciting to me, though I was concerned about the amount of time it would take me. The administration encouraged me to teach the course in the spring semester of 2008, and I started preparing for it during the summer of 2007. This turned out to be enough preparation time, though some of it was at the last minute.

Since I wanted to make sure the students heard my “voice” as an instructor, I didn’t want to hand the course off, so to speak, to some course management system with content delivered by others. I knew from the start that I wanted to deliver content through video lectures that I produced myself. Moreover, because of my positive experiences using the computer-algebra system *Mathematica* in both research [3, 4, 6] and teaching [5], I also wanted to deliver extra written content by creating interactive *Mathematica* supplements for the textbook by Blanchard, Devaney, and Hall [1] (the online appendix for this article has the current version of four of these supplements). The chapters covered (some only partially) from this textbook with corresponding topics and approximate number of weeks spent is given in Figure 1.

My hope was that the *Mathematica* supplements would deepen student skills and understanding, while also giving them tools to create their own interactive dynamic

Chapter	Topics	Time
1	1st-order Eqs & Models: Symbolic, Qualitative, and Numerical Techniques	3 weeks
2	2nd-order Eqs & Models: Symbolic, Qualitative, and Numerical Techniques	2 weeks
3	Linear Systems: Straight-Line Solutions, Eigenvalues & Eigenvectors, TD-plane	3 weeks
4	Forced Harmonic Oscillators: Complexification & Resonance	2 weeks
5	Nonlinear Systems: Linearization, Nullclines, Hamiltonian & Dissipative Systems	3 weeks
6	Laplace Transforms: The Basics for 1st-order Equations	1 week

Figure 1: Chapter Numbers, Topics, and Approximate Time in Weeks for each.

content (“demonstrations”, for short), which is one of the main capabilities of *Mathematica*. The students then put these skills and tools to use as part of their two project reports for the course. Doing these things took a lot of time, but I was able to finish everything when needed.

It was also important to me to use these *Mathematica* supplements to help encourage the students to display an *exploratory mindset*: to experiment, conjecture, model, discover and describe new behaviors, find creative solutions to problems, and understand topics more deeply. I have some evidence that I was successful in doing this for some of the students who took the online course over the years, which is perhaps the best one can say without doing a well-designed experiment.

2 Content Delivery: Videos and Mathematica Supplements

Besides delivering content through the video lectures, it was initially a priority of mine to make sure there was *less* lecture time overall than in a traditional course, so that more time could be spent with the *Mathematica* supplements. The traditional course had 37 lectures at 50 minutes each, or about 31 hours of lecture time. I was hoping to deliver the content in 16-18 hours for the online course.

My mode of working, in general, has been to do small and easy tasks first so I can feel a sense of accomplishment before I attempt larger and harder tasks. Therefore, I started by making lectures with a computational “how-to” focus, with topics such as: separation of variables, integrating factors, guess-and-test for scalar linear equations, and Euler’s method. Next, I made videos on qualitative techniques that were relatively easy to describe: slope fields, phase lines, one-dimensional linearization near equilibrium points, and classification of equilibrium points for autonomous scalar differential equations. I then made four videos about more difficult topics: bifurcations of one-parameter families of autonomous scalar equations and existence and uniqueness theorems. After that, I went back and made a few introductory videos: introducing the course, introducing *Mathematica*, and giving an overview of chapter 1 of the textbook. Finally, by the end of the summer of 2007, I made five videos that focused on the models in chapter 2: interacting species and basic harmonic oscillator models. At that point, I had twenty-three videos and about 10.5 hours of content on chapters 1 and 2. It seemed that my videos were not saving much lecture time. Partially this was because I told jokes at the end of many videos to give them a more personal touch and partially this was because of the introductory and

1.1.3: The Logistic Growth Model (Major *Mathematica* functions used: [Table](#), [Length](#), [Transpose](#), [Insert](#), [Fit](#), [ListPlot](#), [Show](#), [Solve](#), [N](#), [%](#), [FindRoot](#), [Plot](#), [Manipulate](#))

There are a couple different ways to derive the [logistic population growth model](#) and confirm its validity for a particular data set. One possible derivation is theoretical and is described on [pages 9 and 10](#) of the [Differential Equations](#) text. The basic idea there is to write the differential equation in the form $\frac{dP}{dt} = f(P)$ and choose the function f to have the simplest form possible where $f(0) = 0$, $f(P) > 0$ for $0 < P < N$ for some $N > 0$, $f(N) = 0$, and $f(P) < 0$ for $P > N$ (see below).

Another possible derivation is [empirical](#) (data-based) and can be thought of as being valid when the data for the values of the [relative growth rate](#) $\frac{1}{P} \cdot \frac{dP}{dt} = \frac{\frac{dP}{dt}}{P}$ follow a decreasing linear trend, *when thought of as a function of P* . In other words, $\frac{1}{P} \cdot \frac{dP}{dt} = \frac{\frac{dP}{dt}}{P} = -aP + b$ for some constants $a, b > 0$ (if you studied the optional subsection at the end of the previous section 1.1.2, what we are about to do is quite similar to the derivation there, *but not exactly the same*).

Let's re-enter the census data from the previous section 1.1.2 to help us find this linear model (i.e., find m and b) and see how well it matches the data.

```
CensusTimes = Table[10 * i, {i, 0, 21}];
CensusPops = {3.9, 5.3, 7.2, 9.6, 12, 17, 23, 31, 38, 50, 62, 75, 91, 105, 122, 131, 151, 179, 203, 226, 249, 281};
AllCensusData = Transpose[{CensusTimes, CensusPops}];
```

Data to estimate the values of the relative growth rates $\frac{\frac{dP}{dt}}{P}$ can be computed in this situation by calculating the ratios $\frac{\frac{P(t+10) - P(t)}{10}}{P(t)} = \frac{P(t+10) - P(t)}{10 P(t)}$ for $t = 0, 10, 20, 30, \dots, 200$. [Table](#) can help us do this. Recall the use of the [Part](#) double-bracket operator `[[]]` and [Length](#) (think carefully about how this code is constructed and why these values give logical approximations to the relative growth rates).

```
RGRestimates = Table[ $\frac{\text{CensusPops}[[t + 1]] - \text{CensusPops}[[t]]}{10 \text{ CensusPops}[[t]]}$ , {t, 1, Length[CensusPops] - 1}]
```

Now we have to be careful. Note that `CensusPops` and `RGRestimates` have different lengths. Because of this, we would not be able to

Figure 2: Screenshot from a page of the *Mathematica* supplement for Section 1.1. Hyperlinks are in orange and blue.

overview videos I made. But it was also clear that I would need to be more efficient in my content delivery if I was going to have significantly less lecture time overall.

During the fall semester of 2007, I focused mostly on writing the interactive *Mathematica* supplements for many of the sections in (an earlier edition of) the textbook [1], making videos about chapter 3 content only occasionally. While the authors of the textbook took a modeling approach with a very conversational style, which I appreciated, I wanted my supplements to contain a fair amount of pure mathematics to solidify understanding and broaden perspectives. I also wanted the supplements to contain instruction on using *Mathematica* so that the students could ultimately create their own demonstrations to increase their understanding and to analyze models for projects. As already mentioned, these supplements were truly *interactive*. Students could experiment with and enter embedded code as they read to see what *Mathematica* was capable of. Hyperlinks to relevant topics were also included. Electronic exercises to gain skill in applying what they were learning were assigned within these supplements. A screenshot of one of the pages of a supplement for the first section of the textbook is shown in Figure 2. The online appendix for this article, which contains full copies of the supplements for the first four sections of the textbook, will show this approach more fully. Code and output for one demonstration for one of the projects from the textbook is given in the written appendix of this article.

As the spring semester started in February 2008, I still had more videos to create on chapters 3, 4, 5, and 6, and I still had many *Mathematica* supplements to make on the same chapters, but I had gotten a good start. I was able to finish everything in time for when it was needed in the course. In the end, I made forty-five videos with about twenty-one hours of lecture time, representing about 68% of the lecture time in a traditional class. I also made *Mathematica* supplements for every major section of chapters 1-4, as well as the first two sections of chapter 5.

Week	To Do
1, 2/4/08 – 2/8/08	Lots of videos, get going on homework
2, 2/11/08 – 2/15/08	Large amount of videos and of HW, both textbook & <i>Mathematica</i> (Mma)
3, 2/18/08 – 2/22/08	Small amount of HW & videos, Prepare for Exam 1
4, 2/25/08 – 2/29/08	Moderate amount of HW and videos
5, 3/3/08 – 3/7/08	Large amt of regular HW, moderate amt of videos and Mma HW
6, 3/10/08 – 3/14/08	Small amount of HW & videos, Prepare for Exam 2
7, 3/17/08 – 3/20/08	Large amt regular HW & intense work on Project 1 (before Spring Break)
SB, 3/21/08 – 3/30/08	Spring Break
8, 3/31/08 – 4/4/08	Finish Project 1, moderate amt reg HW, moderate amt videos
9, 4/7/08 – 4/11/08	Large amt reg HW, moderate amt Mma HW, moderate amt videos
10, 4/14/08 – 4/18/08	Moderate amt reg HW, Mma HW, & videos; Prepare for Exam 3
11, 4/21/08 – 4/25/08	Large amt reg HW & Mma, moderate amt videos
12, 4/28/08 – 5/2/08	Moderate amt reg HW & videos, small amt Mma HW
13, 5/5/08 – 5/9/08	Intense work on Project 2
14, 5/12/08 – 5/16/08	Finish Project 2 & prepare for Final Exam
FW, 5/19/08 – 5/23/08	Finals Week

Figure 3: Weekly Schedule for Spring 2008

3 Course Execution: Scheduling, Exams, and Video Projects

In reality, the course I’ve been describing was *not fully* online; it was not “distance-ed”. Some people might call it a “hybrid course”. In fact, I started out the spring semester of 2008 with a class of twenty students by personally meeting with the students in groups of about five at a time to talk about the syllabus, goals, expectations, and habits that would help them be successful. I also hoped at this initial meeting to encourage them to make use of office hours, which I decided to hold in a computer lab about twenty times bigger than my office. I didn’t know what kind of crowd to expect at office hours, but in the end, only three or four of the students attended consistently.

I decided to structure the course to have certain emphases within each week. My thought was that this would provide some variety and “flow” to the course that might keep students more engaged and less dulled by the monotony that might occur by just focusing on reading the book, watching the videos, and doing the assignments. This “flow” might be described, in brief, as: 1) lots of videos and homework for two weeks, then test; 2) lots of videos and homework for two weeks, then test; 3) project 1 and a moderate amount of videos and homework for two weeks; 4) lots of videos and homework for two weeks, then test; 5) lots of videos and homework for two weeks, then project 2; 6) review, then test.

The table in Figure 3 displays these emphases in a bit more detail in terms of amounts of videos, regular homework, *Mathematica* supplement homeworks, projects, and preparation for exams. Note also that I decided to have my first two exams quite early in the semester. My idea was to give them feedback on their learning progress as soon as possible. Exam 1 (covering sections 1.1 – 1.6) was on Thursday, February 21; Exam 2 (covering sections 1.7 – 2.4) was on Thursday, March 13; Exam 3 (covering sections 2.5 – 4.1) was on Tuesday, April 22; and the cumulative Final Exam was on Wednesday, May 21. Project 1 was due Thursday, April 3 and Project 2 was due Thursday, May 15.

This information might be easier to digest in a graph. Figure 4 shows approximate

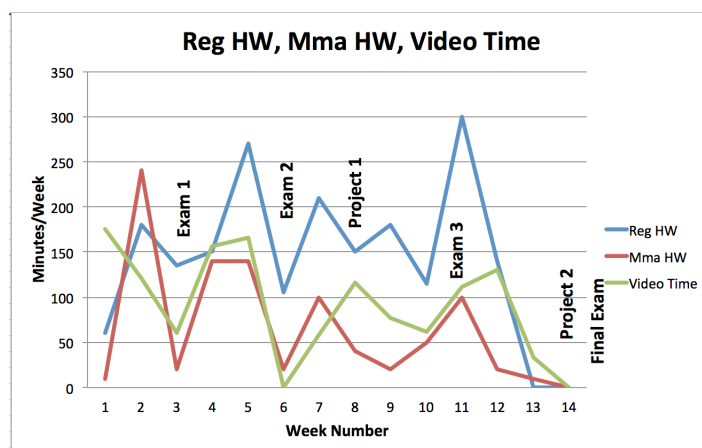


Figure 4: Regular HW vs. Mma HW vs. Exam & Project Weeks

exam and project weeks with very rough approximations to the number of minutes per week students spent on regular homework, *Mathematica* homework (the supplements), and watching video lectures. Note that Exam 3 was early in Week 11 so that the majority of the regular homework they did that week was after that exam. The students worked on the projects for about 1.5 weeks before the project due dates.

For the two group projects, I decided to try something new. I had the students produce video reports rather than written reports. We then watched these video reports together on the project due dates. For the first project the students studied content from one of the “labs” at the end of chapter 1 or 2, which typically involved real-life models, answered the questions asked in the text, created dynamic *Mathematica* demonstrations to illustrate solutions and applications, and then presented the report in video format as if they were teaching their peers. For the second project they made a similar video report with dynamic *Mathematica* demonstrations, but this time they focused on explaining regular content from a section or two of the chapters of the textbook that we didn’t have time to get to during the main part of the course. The topic options were: 1) Hamiltonian and/or dissipative systems (we didn’t fully get to this topic for the online course, though it is listed in the content of Chapter 5 in Figure 1); 2) Laplace transforms for basic 2nd-order equations; 3) more advanced numerical methods and error analysis; and 4) basic discrete dynamical systems.

Individual accountability for project content was handled as follows: 1) each student kept detailed notes on their personal tasks and time on the project, with a goal that the students in each group should be putting in roughly the same amount of time and effort, 2) each student rated their own contributions as well as those of their peers (each person received a personal accountability score based on the average of their assessment of their own contributions as well as their peers’ assessments of their contributions), 3) each student had to have roughly the same amount of screen time in the videos, 4) I asked each student at least one question while we watched (and paused) their videos together, and 5) one or two questions on the final exam were tailored to each group’s particular second project. As already mentioned, I also wanted to reinforce what students had been learning about *Mathematica*, so part of their project score involved making a significant

Mathematica demonstration (e.g., animations of solution curves traveling through a phase plane).

The projects were mildly successful. The students produced good reports on average, but they were not quite as creative in their work as I was encouraging them to aim for. In retrospect, that's not too surprising. I wanted them to be creative, but it was a challenging enough task for them to just answer the questions, make their *Mathematica* content, and make sure the content was delivered well. I had also hoped that video reports would be more enjoyable and a bit less work for me to grade than written reports. They were enjoyable, but just as much work, if not more. Mathematical content often went by too quickly when it was in video format to fully assess its validity, so I asked them to send me any written notes as well. This took as much time to sift through as a full written report would have, and it was less organized.

As you might expect with this course, I needed to be flexible in scheduling student exam times. Because of the necessity of this flexibility, and to reduce possible cheating, I provided students with copies of old exams, told them the types of problems to expect ahead of time, allowed them to use notecards, gave them more time for exams than I typically did for traditional courses, and created two basically equivalent versions of each exam (along with the extra one or two questions tailored to project 2 topics on the final exam). Because of my own schedule, it was also essential for me to make use of trusted teaching assistants for proctoring. I highly recommend that others do the same.

4 Outcomes and a Transition to Flipped Classrooms

I taught online differential equations courses of the same form in the spring semesters of 2009 and 2010 as well. As you might expect, my role shifted to be more of a course-manager rather than a course-developer. I didn't like this as much, but I thought it was worth doing for at least a couple years to make use of all the effort I had put forth in 2007 and 2008. In 2009, I did have the benefit of an extra-engaged core group of about seven or eight students, out of thirty-two overall, who attended office hours consistently. This group turned out to be one of my favorite cohorts leading up to their graduations in 2011 and 2012, and I've continued to stay in contact with a few of them to this day. Twenty students took the course in 2010, and they turned out to be my least engaged and lowest achieving group. I was ready for a change after that semester.

Before going on to describing my transitions in more recent years, let me make a brief and informal assessment of student evaluations and achievement overall for these three classes (2008, 2009, and 2010). Bethel University uses the IDEA company (<http://ideaedu.org>) for student evaluations of faculty teaching. They use a 5-point scale for most of their questions (5 being the highest and 1 being the lowest), where instructors are asked to identify various learning objectives as "minor", "important", or "essential" ahead of time, and where scores are reported in "raw" form and "normed" form. The "normed" form is an adjustment to compare with similar courses at similar schools.

Figure 5 shows my evaluation scores on the most important part of the report, "Progress on (Learning) Objectives" (which reflects students' assessments of their own learning), both in raw and normed form, for the four times I taught the traditional course after the

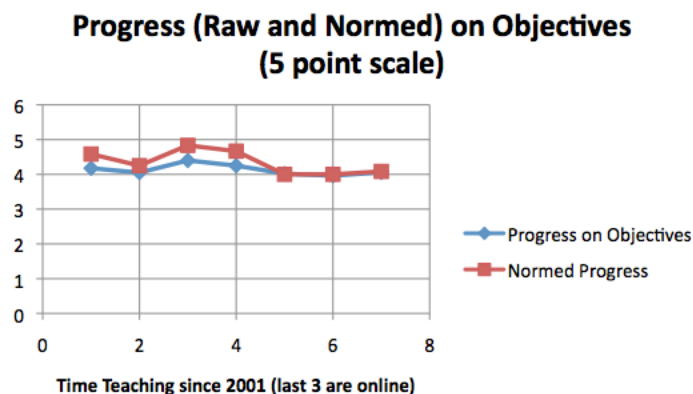


Figure 5: IDEA evaluations for “Progress on (Learning) Objectives”. The first four are traditional classes and the last three are online. The vertical axis corresponds to a 5-point scale with 5 being the highest and 1 being the lowest.

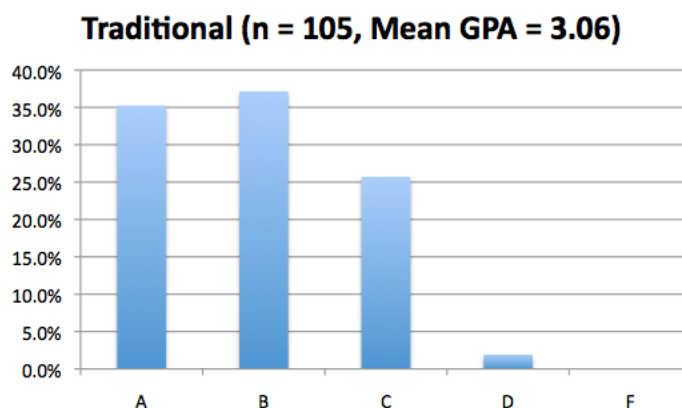


Figure 6: Grade Distribution for Four Traditional Classes

year 2001 and the first three times I taught it online. The two clear patterns are: 1) the online normed scores are lower than the traditional normed scores and 2) raw and normed scores converged online. A plausible explanation for the second observation is the fact that there were very few sophomore-level mathematics classes that were offered online, at least at the time.

As for student achievement, in the form of grades, a comparison of traditional versus online grades shown in Figures 6 and 7 shows that the online students got better grades. Plausible theories for this include: 1) I succumbed to unconscious grade inflation, 2) the students’ abilities were higher and/or they learned more, or 3) I gave them exam question-types ahead of time as well as more time for exams. My hunch is that (3) is the main reason, in addition to the fact that it’s the only one of these three options that definitely happened.

I also personally surveyed students on whether they liked the course online or not. Opinions were very mixed (at about 50-50 liking it versus not) and often extreme. Some

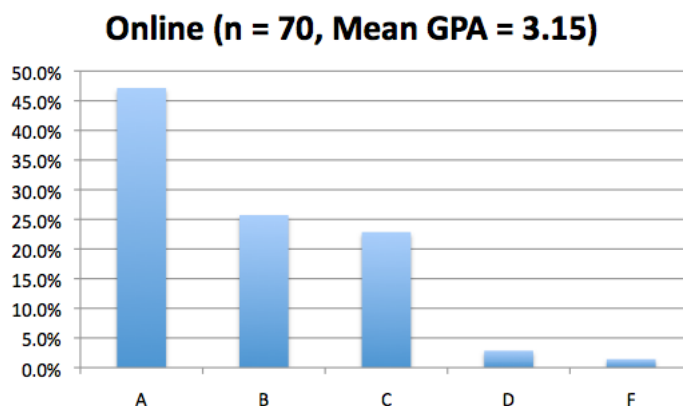


Figure 7: Grade Distribution for Three Online Classes

students absolutely loved it and felt it was one of the best educational experiences of their lives, while others absolutely hated it. Student opinions were most typically correlated with the following aspects of the course: 1) how well they learned from video lectures, 2) their time-management skills, 3) how successful they were with *Mathematica*, and 4) whether being forced to take the course online bothered them or not (it was the only form of the course offered at Bethel for those years). The main evidence I have for helping some students develop an exploratory mindset as mentioned in the introduction comes from the students who loved the course in the spring 2009 section. Three or four of those who consistently came to office hours seemed to be continually amazed at what they were learning, especially with regard to using *Mathematica* to model phenomena and understand topics more deeply. In fact, one of these students went on to become a *Mathematica* developer at Wolfram Research before going on to graduate school in physics. A bit of tangential evidence that the overall mathematics and computer science department at Bethel University has helped our students develop good exploration and problem-solving skills has been the success of many of our teams in the Consortium for Mathematics and Its Applications Mathematical Contest in Modeling (COMAP MCM).¹

I took time off from teaching differential equations, both traditionally and online, in the spring semesters of 2011 and 2012. In 2013 and 2014, I decided I wanted to go back to teaching it traditionally, in large part because I didn't want to be an online course manager again. When students' needs pressed me to offer it online in the summer of 2014, I used all my old video lectures to deliver content, but I ran it more like a traditional course and didn't make as much of an emphasis on *Mathematica* since the students would be doing it distance-ed. In the spring semester of 2015, I planned to just teach a traditional face-to-face course, but I got requests from a number of students to teach it online as well.

¹See Bethel University's ("College" before 2005) teams in the submissions classified as "Outstanding*" (99th percentile) or "Meritorious*" (85th to 90th percentile or so), as well as special awards for the following years: 2001 (Problem B** and B*), 2002 (Problem B** and A*), 2003 (Problem B*), 2004 (Problem A*), 2005 (Problem B*), 2006 (Problem A*), 2008 (Problem B*), 2009 (Problem A* and A*), 2010 (Problem A*), 2011 (Problem B*), 2012 (Problem B**), 2013 (Problem A** and MAA award), 2014 (Problem A*), 2015 (Problem B**, MAA Award and Frank Giordano Award) <http://www.comap.com/undergraduate/contests/matrix/index.html>

I did so, but I also decided to make new videos of me teaching the traditional course that semester, and the online students were expected to learn from these videos rather than the old ones.

At the moment, I plan to teach both online and traditional courses in the future, using the videos from spring 2015 for *both* classes. Because of this, I will end up teaching the traditional class with more of a “flipped” teaching approach [2], where students watch lectures outside of class and work on activities and/or homework during class. This is something I have been working towards in some of my other courses as well. I’m enjoying making course material for active learning in the classroom while leaning on lectures I’ve put on my YouTube channel to deliver content. Ironically, I foresee my online differential equations class becoming more traditional since I won’t be there for an active-learning component. The main benefit of the online course is that it will be there for those students who need flexibility in their schedules and are self-motivated to stay on task. The opinions of such students have been consistently positive because of self-selection. In both courses, the *Mathematica* supplements will be emphasized less because of the increased lecture time. I do plan to use them to some extent, however. I also plan to make them available on the Internet for others.

If you want to do online teaching, hybrid teaching, or flipped-teaching in similar ways as me, be aware that it does take a lot of time. But I hope you will find it as enjoyable as I have.

5 Personal Advice

I’ll end the main part of this article with some personal advice if you are just starting to think about or are working on creating an online course. Written and online appendices with *Mathematica* content follow this section.

1. Keep it simple and try to save yourself work wherever possible (I didn’t always follow my own advice here).
 - (a) Don’t be too ambitious in topic coverage, at least the first time you teach it. Make sure that the topics you do get to are done well. Personally, I covered about five sections fewer from the textbook for the first time teaching the online course than I did with the traditional course.
 - (b) Make use of teaching assistants for proctoring exams and to help with grading.
2. Be flexible, systematic, and redundant.
 - (a) Be flexible on content coverage (see above), due dates, technical (computer) difficulties, individual vs. group work on assignments, and on how groups are formed.
 - (b) Give homework “sabbaticals” and/or “passes” once in a while (or maybe drop their two or three lowest assignment scores, especially if you have strict due dates, as more students will forget about due dates than traditional classes).

- (c) Structure activities in terms of weekly schedules and use a lot of variety for the type of workload each week to get a nice “flow”.
 - (d) Consider letting the students know the types of problems on exams ahead of time. Write out solutions to (at least some) similar problems.
 - (e) Carefully think through your communication plans to make sure key messages are communicated at appropriate times, giving lots of (friendly) reminders.
3. Be an encourager.
- (a) Use a lot of positive words in any written correspondence (especially when you are trying to point out errors), giving them gentle nudges to stay on task and to help motivate them.
 - (b) Listen to and empathize with their math and technology complaints. Be as understanding and positive as you can in helping them work through their complaints and difficulties.

6 *Mathematica* Content

A good example of a project where dynamic *Mathematica* demonstrations can aid student experimentation, conjecturing, and understanding comes from Lab 5.3 in the fourth edition of [1], which students can analyze to some degree with technology after chapter 2 has been completed. The goal of this project is to find bifurcation points of a nonlinear system of predator-prey equations. The relevant system is

$$\begin{aligned}\frac{dx}{dt} &= 9x - \alpha x^2 - 3xy \\ \frac{dy}{dt} &= -2y + xy,\end{aligned}$$

where y is the predator, x is the prey, and $\alpha \geq 0$ is a parameter related to carrying capacity for the prey. A screenshot of the *Mathematica* plot of the phase plane when $\alpha = 2$ is shown in Figure 8, which can be animated by moving (or “playing”) the slider.

A bifurcation in the number of equilibria in the first quadrant occurs at $\alpha = 4.5$, which can be found with purely algebraic methods. A more subtle bifurcation from a spiral sink to a non-spiral sink (which would not even be a bifurcation from a topological point of view) for the equilibrium point with positive coordinates when $0 \leq \alpha < 4.5$ can be found to occur, by analyzing the Jacobian matrix, at $\alpha = \sqrt{22} - 2 \approx 2.7$. For a student with no knowledge of linearization methods, it would take a great deal of experimentation with *Mathematica* to approximate this bifurcation point to one decimal place, but it could probably be done with enough patience and zooming.

The relevant *Mathematica* code to produce the dynamic output related to the screenshot shown in Figure 8 is in Figure 9 below. `StreamPlot` and `VectorPlot` make the streamlines and direction field, respectively. `NSolve` and `ListPlot` can be used to first find and then plot the equilibrium points. `Show` combines all the graphic objects. Finally, `Manipulate` makes it all dynamic as α varies.

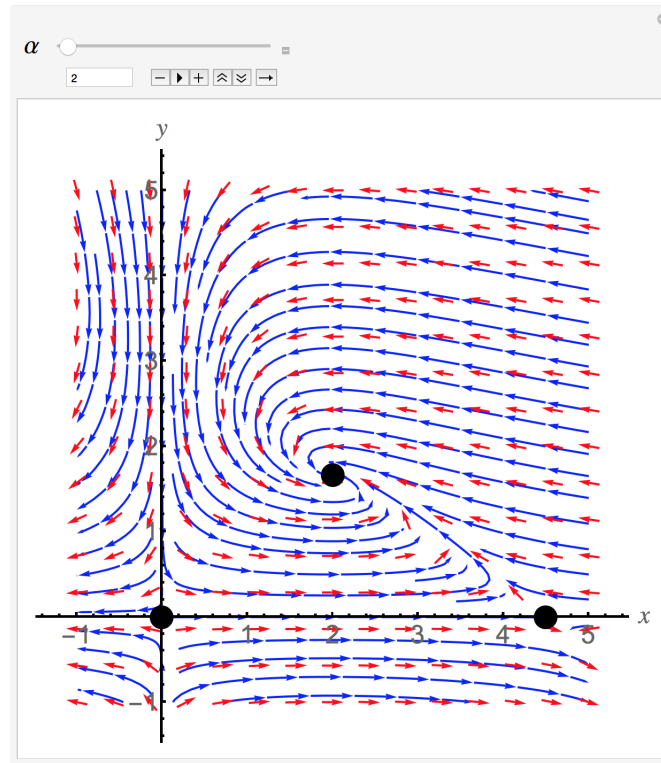


Figure 8: ScreenShot of *Mathematica*-produced dynamic phase plane of the predator-prey system when $\alpha = 2$

```

fα[x_, y_] := 9 x - α * x^2 - 3 * x * y; g[x_, y_] := -2 y + x * y;
EqPts[α_] := {x, y} /. NSolve[{fα[x, y] == 0, g[x, y] == 0}, {x, y}];
Manipulate[
  Show[StreamPlot[{fα[x, y], g[x, y]}, {x, -1, 5}, {y, -1, 5},
    StreamStyle → {{Thick, Blue}}],
    VectorPlot[{fα[x, y], g[x, y]}, {x, -1, 5}, {y, -1, 5},
    VectorScale → {.03, 1, None}, VectorStyle → {{Thick, Red}}],
    ListPlot[EqPts[α], PlotStyle → {Black, PointSize[.04]}, Frame → False,
    Axes → True, AxesStyle → Thick, TicksStyle → Large,
    AxesLabel → {Text[Style["x", Large, Italic]],
      Text[Style["y", Large, Italic]]}, ImageSize → Large], {α, 2, 5},
    LabelStyle → Large]

```

Figure 9: *Mathematica* code to produce the dynamic output related to Figure 8

References

- [1] Paul Blanchard, Robert L. Devaney, and Glen R. Hall. *Differential Equations*. Brooks/Cole Cengage Learning, 4 edition, 2012.
- [2] Sarah Evelyn Harvey. Flipped classrooms turn learning on its head. *Techonomy*, available online at <http://techonomy.com/2012/08/flipped-classrooms-turn-learning-on-its-head/>.
- [3] William M. Kinney. An application of conley index techniques to a model of bursting in excitable membranes. *Journal of Differential Equations*, 162(2):451–472, 2000.
- [4] William M. Kinney. Applying the conley index to fast-slow systems with one slow variable and an attractor. *Rocky Mountain Journal of Mathematics*, 38(4):1177–1214, 2008.
- [5] William M. Kinney. Teaching complex analysis as a lab-type course with a focus on geometric interpretations using *mathematica*. In *Nineteenth Biennial Conference of the Association of Christians in the Mathematical Sciences*, 2013.
- [6] William M. Kinney. A smorgasbord of problems in actuarial mathematics and reliability modeling with an emphasis on *mathematica* usage. submitted to *Mathematics Magazine*, July 2015.