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Some time, in 1979, I talked to Gail Hirsch about my work with NYU undergraduates, and my curiosity about their mathematics schooling before college. Gail asked me if I knew that Hassler Whitney, in recent years, had been doing important work in mathematics education with young children and their teachers; she had met him and seen him work at Bank Street College for Teachers, was impressed with his ideas, and suggested that I contact him.

I knew, of course, of Hassler Whitney, one of the creators of modern topology and also well known as mountaineer and Alpinist, member of the Princeton Institute for Advanced Studies; but neither Peter nor I knew that he had been working in mathematics education, mainly with elementary school children and their teachers.

Hassler's second career occupied him during the last two decades of his life. Many of his mathematical, first-career colleagues and admirers wondered why such an incisive, original researcher would abandon his seminal work to tackle the complex intractable problems of education. And many of his second-career collaborators wondered why he gave up his prestigious academic position for activities that seemed lacking in scientific stimulation and challenge, full of potentially frustrating bureaucratic and political impediments to meaningful changes. The mathematical community bemoaned the "loss" of its stellar members, while the educational community feared that the rough realities would dampen—perhaps burn out—the new spirit who, some thought, had descended from the ivory tower to join their ranks. Members of each of these communities wanted to know what had led Hassler Whitney to change careers, what accounted for such a transformation or conversion in Hassler. These questions are still being asked, even after his death.

Since I did not know Hassler before or during his career change, I shall leave descriptions of that period to those who knew him at the time; his mathematical biographers, who will report on his scientific contributions, and his educational biographers who witnessed his second career from its beginnings, while I note here the answers he himself gave when people asked him about his transition within my ear shot. These answers, together with pedagogical views and their mathematical illustrations—as well as his personal actions and reactions I am about to recall—may well throw some light on how his two careers can be perceived: not antithetical and conflicting, but the second as being a quite natural outgrowth of the first, resolving rather than creating conflict in Hassler, the whole person.

To the question "Why did you leave Harvard?", he replied that he had some trouble thinking of good dissertation problems for his Ph.D. students. To the question "What made you become interested in education, especially working with elementary school children and their teachers?" I heard him give several responses on various occasions, among them "I wanted to become human" and "For the sake of our children."

On October 8, 1979, I addressed my first letter to Hassler Whitney, asking him if he were willing to discuss some of his pedagogical views in connection with a course (Mathematical Thinking) we had designed at NYU for incoming students. He must have responded by telephone; he was always glad to be consulted about mathematics education. He came to New York and had lunch with Peter and me. This was the first of a long series of occasional visits Hassler paid to NYU, often inserting them between other business he had in New York on the same day—see somebody in The Little Red School House, attend a meeting at Columbia, talk to Debbie Meyer, look something up in a big library, renew his passport, or attend a reunion of alpinists. Yet, he never seemed to be in a hurry. He would sit in my office, listen carefully to our accounts of what happened in a Mathink class, to our questions and doubts, and respond with examples; for instance of how one group of children derived a long division algorithm by sharing a given amount of play money equally, or how different children used a string of colored beads to get from 18 to 42 and invented various subtraction schemes to find 42-18 without having to use a new "rule" for taking the units' digit 8 away from the smaller digit 2, or other "taught" schemes. We exchanged examples of how a group of learners might "act out" a familiar situation, observe, reflect and eventual-

ly pursue various mathematical activities to sort out its sundry aspects and take it further. We exchanged articles we had noticed, disagreed with or liked; he, it turned out, knew almost all the people I was just discovering: David Hawkins (whom I had met in Boulder and whose work at the Mountain View Center intrigued me); Stella Baruk, the French advocate for children defeated by school mathematics; and a host of others.

Hass would telephone, tell me when he planned to be in New York, we would arrange a time, and he would arrive, usually with a small back pack containing papers and his lunch. He was unperturbed by goings on in my office—students who came and went, telephone calls, etc.—content to read something while our conversations were interrupted. He met several of my collaborators and colleagues, snared Erika Duncan's and my interest in the role of language in learning mathematics and visited our workshop. He had immediate personal rapport with each individual he met, said little, listened intently, even "between the lines."

His pedagogical views might best be expressed by quotes from his papers:

> A primary aim of studying mathematics must be to grow in your own natural reasoning powers, especially in domains where precise reasoning is valid. The growth must include creative and critical thinking, increasing control over one's work, seeing connections with related matters, and raising communication skills. Simply stated, the individual must take increasing responsibility over the work being done..."(1)

He almost always illustrates such statements with little mathematical examples serving as implicit advice to any teacher who might want to try out similar ideas. He obviously did not believe in separating pedagogy from "content", nor intellectual from emotional aspects of learning. He did, however, separate school mathematics from the spontaneous mathematical explorations children pursue out of school and as pre-schoolers. He wrote, for example,

> "...the 'mathematical' experience of most children these days, especially in the inner city, is one of trying to learn the rules of the day; they give up seeing meanings somewhere in the early grades. The great need for the children is to return to their wonderful preschool learning, when they were full of vitality and curiosity, exploring their environment, observing a myriad of inter-connections, and learning complex concepts and skills like communicating verbally and nonverbally, beyond what any of us adults do, and without any formal teaching." (2, p.1)

Later, in the same paper (2, p.8) he follows an example of a simple "subtraction through addition" scheme that mirrors a child's thoughts with the following:

> "Warning: DO NOT try to teach this! for they must understand, i.e. get it into their thoughts, in their way. The superiority in other countries is largely because they teach less, knowing the children are intelligent. We say we know this, but act otherwise, through ... teaching each little step, destroying the kids' thought processes in the school situation ..."

His opposition to the formal teaching of arithmetic in the early grades was strengthened by his discovery of a 1935 report by L.P. Benezet (3) on an educational experiment carried out in New Hampshire. Hassler distributed copies and called attention to it at every opportunity. In case you have not seen it, let me just say that the experiment, as I recall it, was run in the public schools of some New Hampshire mill towns with large immigrant populations and consisted in selecting classes in which formal arithmetic instruction was suspended during grades 1-3. These children and a "control" group of their traditionally taught contemporaries were later examined and interviewed. Those who had not received formal arithmetic instruction performed better than their counterparts not only in solving mathematical problems; they also reasoned more maturely and responsibly, were more involved with their studies and more "alive."

Hassler's curiosity about this experiment and the reasons for its abandonment led him to search for people who remembered Benezet, and learn about political repercussions of the experiment. (All this will be reported elsewhere (4).) In any case, I asked Hassler what, in the experiment, had replaced formal arithmetic instruction. He said more time was devoted to communication skills. I thought Hassler's condemnation of teaching was a bit extreme and questioned him about what role he thought teachers sould play. I found some of my questions in a

letter I had written to him on September 13, 1984, after he had visited us in our Adirondack retreat. This and some other letters in my file refer to "unfinished conversations", evidence of seeds of thoughts—sometimes mathematical, sometimes educational—that Hassler planted in people with whom he chatted, and which they then mulled over and often developed further.

About his warning against teaching, I asked how teachers might initiate the kinds of fruitful explorations he advocates. I have since learned a bit about that, partly by following Hassler's example, and partly by listening to students and pointing out connections between their concerns and some piece of mathematics that they might find helpful. I also pointed out to Hassler that his ideal non-teaching facilitator would need to ask the right questions to help the exploring novices get unstuck or take a next step before they get too frustrated or discouraged in their investigations. I reminded Hassler of the questions he had asked me when I tried to figure out why sailboats, when you let go of the mainsheet and rudder, turn into the wind—questions that were rooted in a deep understanding of scientific principles which most teachers/facilitators rarely have.

The sailboat conversation was, as the reader will discover in the next page a very hands-on, real world problem, right up Hassler's pedagogical alley.

In the summer of 1934, Peter and I invited Hass to spend a few days with us at our Adirondack retreat on Loon Lake. He had planned to be at the Bennington music camp (he played violin) and decided to combine visiting us with finding a place called Hurricane, where he had spent some time as a boy on a family vacation. Characteristically, he arranged his drive from Vermont so as to allocate a prescribed time interval to his search and still reach his destination as planned; and sure enough, although Hurricane Village no longer existed, he found the area by orienting himself with the help of Hurricane Mountain, which he had climbed more than six decades earlier.

We have a Sailfish (a tiny sailboat). He had not sailed before and was eager to try. After sailing with me for a while, he wanted to handle the boat alone. I got off and swam to our dock, remembering how, a few years earlier, I had capsized repeatedly in the process of learning how to sail, and how tiring it was to capsize, right the boat, get back on, capsize again, several times in a row. I watched Hass from the dock, saw him capsize, right the boat and get back on only to capsize again. Hass was then about 77, and I became concerned that he, too, might find the struggle tiring. I then noticed that he was holding the mainsheet at a wrong place and shouted instructions to him. He revised his grip and had no further trouble. However, as he brought the boat back to the dock, his eyeglasses dropped into the water, out of his sight. We looked for them, took turns diving for them in the hope of seeing them or feeling them in the sand, but to no avail. Hass had a square pair and was, usual, not at all upset by the mishap.

The next morning was sunny and clear, not windy, so I decided to make my 6:30 am swim near the dock and make another attempt at retrieving the glasses. When I got to the dock, there was Hass already in the water, evidently with the same plan. He clearly enjoyed the challenge though the object to be retrieved was of little important to him. The glasses were not recovered during that season, nor to this day.

From the Adirondacks, Hassler returned to Princeton, I to New York. I took him up on his offer to drive me to New York and drop me at my office. The conversation about sailboats occurred during that trip. He asked me also if I had ever wondered in what ways tightrope walkers were helped in maintaining their balance by the long rods they hold in their hands. I had no idea, and he again asked some incisive questions that started my thinking. Months later, when I mentioned the tightrope problem in another context, Hass asked if I had had a chance to think about it and take it further. I sheepishly admitted that I had not. In retrospect, I wonder if his Harvard graduate students similarly got distracted or were insufficiently involved with their dissertation problems to take them further, and if this had made Hassler feel that his lecturing and teaching at Harvard had been too remote from his students, and that he had to find ways of helping them gain the necessary confidence and control to get the kind of joy out of doing mathematics that he was experiencing in his research.

Hassler deposited me and my duffle bag at my office before going on to Princeton. Just as I was settling down to work, he appeared at my door asking if he might use my telephone. He explained that he had locked his car keys into his trunk and was investigating ways of getting himself out of this dilemma. These included telephoning to see if his second set of keys might be brought from his house to New York; if not, telephoning his Princeton garage; and as last resort, calling a local locksmith to open his trunk. His equanimity, even slight amusement at having done such a silly thing, immediately stopped me from thinking "this would not have happened if he had gone directly to Princeton, nor if I had helped get my duffle

bag out of his trunk" and other guilty musings I am ordinarily prone to. Hassler solved his problem via a local locksmith who arrived sooner and charged less than expected.

One day, when Hassler was to drop by before going to a meeting at Columbia, he displayed the same amazing equanimity. He telephoned, saying he would be a little late because his wallet had been stolen by a subway pick pocket, and he needed to do something about important stolen papers. When he arrived, only a little late, he showed no signs of annoyance, let alone anger, or any kind of stress. He seemed to take things as they came, to stay on top of all situations.

During the period when Erika and I led workshops for teachers, we worked in two high schools and one elementary school in Brooklyn, under a Ford Foundation grant. We told Hassler how much we enjoyed the hospitable atmosphere created by a supportive principal and an enthusiastic bunch of teachers of grades K-4; and that this was the least pressured, most receptive school we regularly visited. Hassler said he was interested in Brooklyn elementary schools and decided to join us at the next workshop there. As usual, he listened hard and said little. However, he left some sheets of paper (see below and p. 5b) for any child or teacher who might want to play with them. I noticed them only by chance at the end of the session and stuffed them into my briefcase. We learned in subsequent weeks that a couple of teachers had also seen them, had fiddled around with them, used them in classes, and told us what their kids had done with them.

Before leaving the elementary school that day, Hassler had a chat with the principal. I learned later that she had put him in touch with another, extremely troubled Brooklyn elementary school. Hassler was grateful to her and enthusiastic about a new challenge. It was quite usual for him to be pleased when things were going well,

EXPLORATIONS

Small and large numbers

We read in the papers about the federal government paying millions for this, billions for that. It seems really too much to pay for such things. Millions, billions, is there any difference? We have a very little sense of this. On the other hand, no househusband would mix up ten dollars with a hundred dollars. We just are not used to larger numbers.

Is there some way we can see all the way from the small to the large?

So, let us put one and a million both on the same line. What are some other numbers you would like to put between them? Where would you put those numbers? That is what I ask you to explore.

And the beautiful thing is that the further you explore, the further you may wish to go. For instance, have you put in 2, and 3? Different people will surely make different choices. But some amy be more satisfying than others. Here is a chance for sharing of views! One group might come to somewhat of a consensus, with another group having different ideas.

Certainly an arbitrary, wiggling, choice, will not be very useful. So might there be some meaning to "useful," which could guide us in placing numbers?

A word about several people or groups working on a question or situation: If it is intriguing, and you want to share, it might be better to share some vague generality than specifics; the latter might "gives things away" and take away some basic joy of discovery. Of course brilliant thoughts might come out and prove keys to answers, without even the realization that this is happening. Spreading success can also be illuminating and pleasurable to all or most concerned.

To end, I do mention that some channels of this exploration can lead to very basic and beautiful parts of mathematics: and you don't at all need to be brilliant mathematically to get into these directions.

Where's your pencil?

					71
		42		56	63
		37	43		55
			37		
		27		35	39
16	19	22		28	

but to feel that his presence was not needed in such places. I suspect he planned to use the occasion to gain entry into a "difficult" school where he would be needed and that he just ascertained, *en passant*, that things were going well enough at our school, well enough to leave behind these little gems—not your ordinary educational "materials", but more seeds for thoughts that might fall on fertile ground.

I saw Hassler at the January meetings in Phoenix talking to a group of people including U. D'Ambrosio in the lobby one evening. As I stopped to greet this little group, one member (I cannot recall who) asked if any of us had sen V.G. Paley's article "On Listening to What the Children Say" (5). I quickly said "yes, in fact I sent a copy of it to Hassler," and I heard Hassler say "This is the most important article I have seen since Benezet's!" I found this rather refreshing, because at many previous meetings Hass had seemed so preoccupied with Benezet's experiment that most people who wanted to hear about some of his other ideas and concerns began to feel he was a bit hung-up on Benezet. But his most recent activities and writings show that he was forging ahead on many educational fronts, was gaining breadth and balance in is expression, and kept growing.

During our mathematical or educational conversations, Hassler would often make some personal remarks. At some point he began to speak fondly and admiringly of Barbara, a painter. Some time later he told us that he and Barbara had decided to get married. Eventually we met Barbara, a warm, delightful person, who seemed to support and supplement his educational interests and to add an artistic, aesthetic component, helping him towards his goal of becoming even more "human".

On April 28, 1989, Shirley Hill was scheduled to speak at NYU on "The Future of Education in Urban America", one of several regional talks she was giving about the report *Everybody Counts*. I had called it to Hassler's attention. He phoned to say that he planned to come to New York, also to take care of some other matters in preparation for a trip to Europe he and Barbara were planning to take in the summer. They were to spend the night at our house and go back to Princeton on April 29, the next day. On April 27 Barbara called to let me know that Hassler had had a severe stroke. He did not recover.

Barbara was in touch with me during her sad time since April 27. Hassler's family gathered in Princeton; he died on May 10. I visited the following Sunday. His daughter Moll, who had come from New Zealand, had called me, wanting to talk about her father's work in education. She had developed some explorations of pentominoes in her father's spirit. She also thought she had alerted him to the importance of keeping track of units (dimensional analysis) when setting up and solving equations that describe scientific phenomena or, for that matter, any situation. (Hassler had published a beautiful little article on the motion of a simple pendulum using only the units to derive the fundamental relations, see (6). I have never understood why high school and college teachers so rarely use this essential tool in the classroom to keep the meaning of formulas in mind and/or to check that algebraic manipulations and results make sense, i.e., are dimensionally correct.)

Molly told about her own scientific training and her present work in New Zealand. She recited the words of a round her father had written about a butterfly struggling out of its cocoon, and I hope to see a written version also of the melody. The words symbolize his educational philosophy.

I briefly met some of Hassler's other children, their spouses and his grandchildren; they would all disperse the next day. Barbara, still a bit overwhelmed, would stay in Princeton.

After talking with Molly, I thought Hassler's "for the sake of our children" undoubtedly included his own. He was one of the rare people who, in discussing a mathematical problem with others, did not let his own agenda get in the way of listening, thus allowing himself to learn from others, including young students. He would often report about such experiences with delight. Molly may well have initiated his emphasis on units in elementary mathematics instruction.

In the present climate of educational reform, the question "What is the impact (of this project or that)?" is often raised, especially by program officers of funding agencies who want to maximize the effects of the grants they award. "What has been the impact of Hassler's work in mathematics education?" will undoubtedly be asked by many people. As others tell their reminiscences, very different Hasslers may appear; he touched different individuals in different ways. He did not teach or preach. I conjecture that his impact will grow as the seeds of thought and reflection he planted subtly continue to transform people he touched, and who, in turn, will have their educational ripple effects.

Hassler enjoyed solving problems—in mathematics, in mountain climbing, in education. He took in the whole situation, formulated simple goals and enjoyed the process—the means of attaining them—at least as much

as getting there. I have heard him comment on how good it feels to negotiate a path through thick woods, even in the dark, to be physically fit; and though I did not hear him say so, I am sure he felt that same delight in blazing his mathematical trails and in feeling intellectually fit and in control. He did not see why these struggles and pleasures of growth should belong only to a privileged few. Everybody, he thought, should have a chance to grow, gain control, become responsible, and that schools should help rather than hinder this process. After attending some mathematics education policy committee of a school system, he said "the goals are so simple, but in the discussions people lose sight of them and clutter up the path wit short term 'objectives'".

Hassler attacked problems with his eyes wide open, tolerant and patient with impediments to be overcome, deeply involved. He said to Louise Raphael, when she was a rotator at the N.S.F. "people think I am naive; I am not naive." Indeed, he was not naive, but neither was he *blasse*.

Hassler was not a martyr who, from a sense of duty, sacrificed his time and mathematical talent for the sake of education. On the contrary, he did precisely what he wanted to do in both his careers, his mountaineering, his music, and in his interactions with others. May his example convince the mathematics research and education communities that there is, in fact, no real schism between them. What he advocated in education—using children's curiosity to help them explore, individually and in groups; have them communicate and revise their findings, learn to justify and defend their reasoning methods; struggle and feel good when they gain ground and control; become so involved that they no longer need either approval from authorities or external incentives—is, after all, what good researchers do, albeit at a different level. This is what human learning is all about, not just for mathematicians and scientists, but for all who want to understand, help modify, and find their places in our complex world.

Notes and references

1. H. Whitney. *Coming Alive in School Math and Beyond* , June, 1985

2. H. Whitney. *Mathematical Reasoning, Early Grades*. Growth through involvement, curriculum outline. May, 1988

3. L. P. Benezet. The Story of an Experiment, *Journal of the National Education Association*, 1935, 24(8) pp. 241-244, 24(9) pp. 301-305, and 1936, 25(1) pp. 7-8.

4. Whitney's educational papers and correspondence is being collected with the help of his widow, Barbara. She was established a Fund for a Mathematical Education, c/o W. Guthrie, Rider College School of Education and Human Services, Lawrenceville, N.J. 08648, so that Whitney's educational works can be catalogued and made available for study and biographical information. W. Guthrie and other colleagues are helping with this task; further details will be announced probably in Focus (MAA), The Mathematics Teacher (NCTM) and other publications.

5. V.G. Paley On Listening to What the Children Say, *Harvard Educational Review*, 1986, 56(2) pp. 121-131.

6. H. Whitney The Mathematics of Physical Quantities, Part II, Quantity Structures and Dimensional Analysis, *American Mathematical Monthly*, March 1968, p.254.