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A Note on Equity Within Differential Equations Education by Visualization

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Abstract: The growing importance of education equity is partly based on the premise that an individual's level of education directly correlates to future quality of life. Educational equity for differential equations (DEs) is related to achievement, fairness, and opportunity. Therefore, a pedagogy that practices DE educational equity gives a strong foundation of social justice. However, linguistic barriers pose a challenge to equity education in DEs. For example, I found myself teaching DEs either in classrooms with a low proficiency in the language of instruction or in multilingual classrooms. I grappled with a way to create an equity educational environment that supported students both with social justice mathematics and satisfaction for the students. According to the 2000 NCTM (National Council of Teachers of Mathematics) *Principles and Standards for School Mathematics*, mathematics instructors should strive to meet the demands of Principles and Standards. However, meeting the standards such as the Equity Principle can be a daunting task. DE student diversity in language proficiency might enrich the educational environment, but it also raises challenges. This brief report summarizes a way to build an equity learning environment by providing visualization support.

1 Linguistic barriers in DEs education

For the past decade, I have taught DEs with an eye toward promoting multiple representations [8]. In line with [3, 11], I encouraged my students to access notation by mathematical communication. Mathematical communication, verbally or in writing, is challenging for many students, especially multilingual students, and poses barriers on exams ([4, 5]). I found that students who were strong in mathematics in their native language (such as Azari) often find themselves handicapped in using a second language (such as Farsi) to communicate their mathematical thoughts. I noticed that those who passed my DE exams (in Farsi, the official language for instruction) were mostly native speakers of Farsi, even though the student population was mostly multilingual, with different native languages (such as Azari, Farsi, and Kurdish). The main problem for multilingual students was in language proficiency. In the same vein, Martiniello, in a study comparing students' performance, showed that nonnative speaking students performed significantly lower than their native speaking peers despite the fact that they had the same level of mathematical knowledge [9]. So nonnative speaking students were challenged by lacking the language skills ([1, 2]). In fact, it was hard to distinguish whether my DE exams measured a student's language proficiency or his or her knowledge of differential equations. For multilingual students who may have appropriate DE knowledge but who had difficulty explaining their mathematical thinking, my exams were a gatekeeper. Thus, it required me to be knowledgeable about potential linguistic bias.

2 Translating verbalization into visualization

I found that my multilingual students' ways of thinking, although different from that of native speakers, were correct. Therefore I asked all students to translate verbal expressions into visual aspects. It engaged my students in thinking and communicating in any language. Using visualization to express their mathematical thinking helped them feel empowered and excited by mathematical communication ([6, 10, 12, 13]). Even though some students' visualization may not be perfect, the mathematical thinking and mathematical communication derived from this exercise was very helpful ([7, 8]). One of the aims of the paper reported in [8] was to determine the extent to which visualization might enable multilingual students to improve themselves. The following briefly describes how visualization could be useful, for example, to better understand autonomous DEs.

KarimiFardinpour and Gooya [8] highlighted two students' difficulties: long-term prediction and coordinating graphs in different coordinate planes. The former concerns students' attempts to visualize the graphs of solution curves of autonomous differential equations for predicting the long-term behavior for various initial conditions. The latter involves the coordination of two or more graphical representations in different coordinate planes and is the focus of this report. In this same report, we outlined three different geometric approaches for visualizing autonomous differential equations and their effectiveness for overcoming student difficulties. The three approaches are referred to as Standard, Traditional, and Dynamic. The authors came to the conclusion that the positive effect of the Traditional method was more than the Standard method, and the Dynamic method had better results than the Traditional method. In the following, these three methods are briefly described for the autonomous differential equation

$$dy/dt = -r(1 - y/T)(1 - y/K)y$$

where r, T and K are constants.

With the Standard geometrical approach, students are provided with an autonomous differential equation in the form $dy/dt = f(y)$ and the corresponding graph of $f(y)$ vs. y. Then they are requested to draw sample solution curves in the $[t, y(t)]$ plane that are coordinated with that graph. For example, Fig. 1(a) is the graph of $f(y) = -r(1 - y/T)(1 - y/K)y$ versus y that is given to the students, and Fig. 1(b) is what the students are asked to draw.

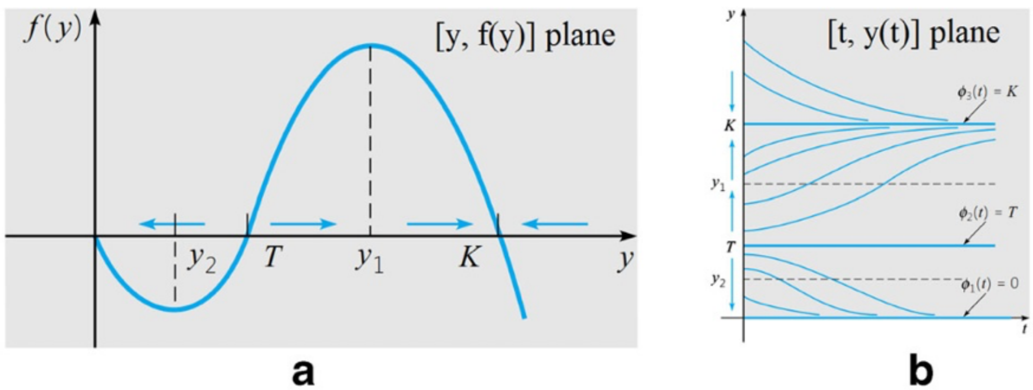

Figure 1: The Standard geometrical approach for $dy/dt = -r(1 - y/T)(1 - y/K)y$. Graph **a** is given, graph **b** is the task for students.

In the Standard method, I preferred to use $[y, f(y)]$ and $[t, y(t)]$ instead of the yy'-plane and ty-plane in order to emphasize the dual role of y. It is very important that students be aware of this dual role, as the y is meant to represent the function of t in the $[t, y(t)]$ plane, as well as to act as a variable in the $[y, f(y)]$ plane.

In the Traditional geometrical approach (as shown in Fig. 2), the phase-line has also been drawn (part c), coordinated with the given $[y, f(y)]$ plane.[1] Now students are requested to draw the sample solution curves in the $[t, y(t)]$ plane, coordinated with the phase-line, which has been isolated from the y-axis in the $[t, y(t)]$ plane. This visual connection helps students to understand the phase space of y as $t \to +\infty$.

The new Dynamic method (shown in Fig. 3) has the following unique features:

- the $[y, f(y)]$ plane is rotated counterclockwise 90 degrees;
- a sign-chart is put between $[y, f(y)]$ and $[t, y(t)]$ giving the positive/negative values of $f(y)$,$f'(y)$ and $f(y) \cdot f'(y)$;
- horizontal dotted lines partition the whole solution space at the equilibria and inflection points;
- small solid circles and small hollow circles respectively represent equilibrium solutions and inflection points of solutions.

In the Dynamic method, the horizontal dotted lines play two different roles. Those connecting solid circles have partitioned the whole solution space according to initial conditions, to help students understand the structure of the solution space. Thus the critical points, the zeros of $f(y)$ in the $[y, f(y)]$ plane, are linked to the equilibrium solutions (as

[1] Fig. 1 is taken from the 7th edition of the Boyce and DiPrima's textbook *Elementary Differential Equations* and Fig. 2 is taken from the 9th edition. Although diagrams similar to Fig. 1 appeared earlier in the text by Blanchard, Devaney, Hall, it is unclear who first isolated the phase-line from the y-axis in the $[t, y(t)]$ plane.

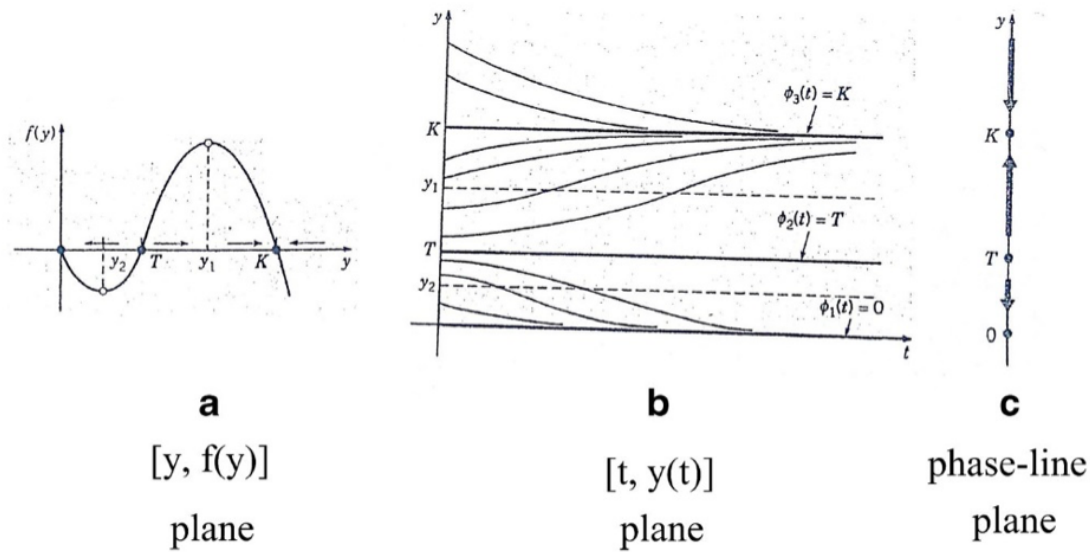

Figure 2: The Traditional geometrical approach for $dy/dt = -r(1 - y/T)(1 - y/K)y$. Here graphs **a** and **c** are given, graph **b** is the task for students.

fixed value functions in the $[t, y(t)]$ plane), which enhances students' understanding of the concept of equilibrium solution.

The dotted lines between open circles connect the extreme points in the $[y, f(y)]$ plane to the inflection points in the $[t, y(t)]$ plane. Seeing these inflection points helps students to understand the concavity of the sample solution curves.

The originality of the Dynamic method is its ability to connect planes, and make a visual coordination between them. These visualization components of the Dynamic

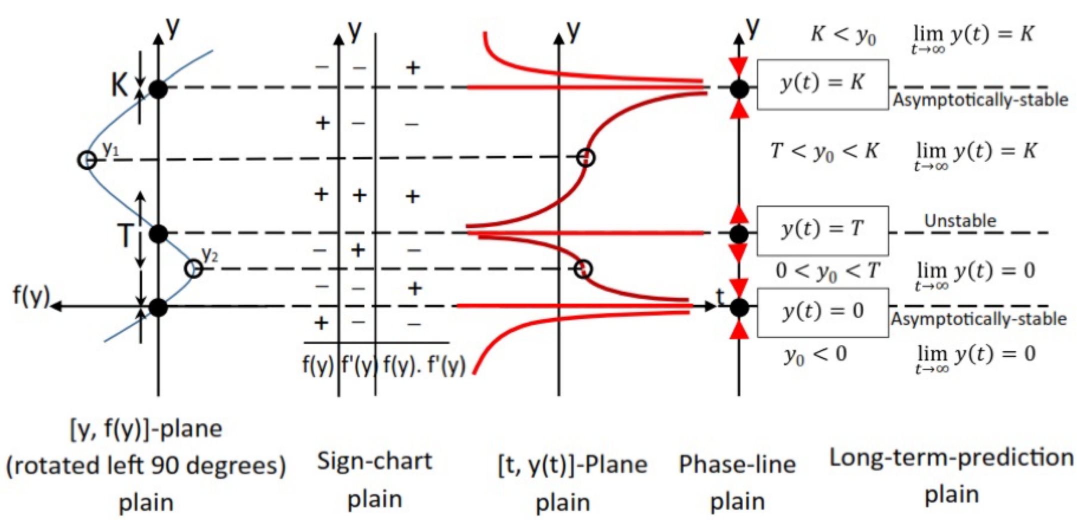

Figure 3: The Dynamic geometrical approach for $dy/dt = -r(1 - y/T)(1 - y/K)y$. The graph at far left is given; the students are asked to add the dotted lines, circles, sign chart, and solution curves, and arrows on the phase line.

method help students to make sense of the connections between the differential equation and the long term behavior of its solutions.

3 The Last Word

I have found that the Dynamic method supports my students who have a low language proficiency in Farsi, so that all of my students could find themselves belonging to the classroom group and achieving understanding of the mathematics. That is a foundation of social justice. I also turned the mirror back on myself to examine my pedagogy and my exams. I asked myself what I did about the equity lens, and this note summarizes the research results described in [8]. I continue my attempt to observe social justice regarding equity within differential equations education by visualization.

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