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Experiences Using Inquiry-Oriented Instruction in Differential Equations

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Abstract: Inquiry-Oriented Differential Equations (IO-DE) (Rasmussen, 2002) is a curriculum that fosters mathematical modeling, thinking about the associated mathematics, and understanding the structural qualities of DEs. When implemented, it is not uncommon for students to invent vocabulary terms or use metaphorical reasoning to grasp new ideas. In this article, I share two tasks from the IO-DE curriculum, examples of student work on these tasks, anecdotes from whole class discussions, and student feedback regarding the course as a whole. The article closes with some of the challenges and affordances of inquiry in general mathematics environments.

1 Introduction

The subject of differential equations (DE) was introduced to me in a traditional, cut-and-dried fashion. I remember learning specific theory, recognizing and classifying equations, and solving equations through the lens of the theory. Having learned the class in exactly this manner, why would I ever question the effectiveness of such teaching practices? I passed on this tradition to my students (and future teachers) by teaching in the same way I was taught. However, after teaching DE three times, I became concerned with not only what my students knew but how they knew it. For example, I will never forget the A student who admitted to me he had no idea how to generate a slope field nor any idea how a differential equation might be derived (both were discussed in class). However, he could solve any differential equation you presented to him and he thought most DEs could be solved with a mathematical formula. I found his views troubling and I attributed this mainly to my teaching. A bigger concern was the direct impression I had on him and his classmates: they praised me for being an “excellent teacher.”

Like anyone aiming to improve their practice, I sought alternatives to teaching undergraduate differential equations. Beginning around 2007 and up until 2015, I consumed the corpus of research on inquiry methods in the teaching and learning of differential equations (Rasmussen, 2001; Rasmussen & King, 2000; Rasmussen & Kwon, 2007; Rasmussen & Marrongelle, 2006; Rasmussen, Zandieh, King, & Teppo, 2005; Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). The materials and philosophy were new to me at the time,
so I reached out to Chris Rasmussen in an effort to learn how to use and possibly modify the Inquiry-Oriented Differential Equations (IO-DE) curriculum to best suit the needs of my students. In this paper, I first give a brief outline of how I taught the differential equations class and how the assumptions therein contrast with lecture environments. Second, I report on two classroom activities and share student work from our collective classroom engagement. Third, I share some anonymous feedback from the students on how they felt the class was progressing. Finally, I offer some reflective thoughts on having taught IO-DE three times and adapting to a more student-centered environment.

2 What Does Inquiry Look Like?

Since every teacher is familiar with lecture-based instruction, I will use this space to answer the question, “What is Inquiry?” from the point of view of IO-DE. In short, inquiry starts with rich, demanding tasks that require a conversation. This conversation can be with the teacher or with classmates. Teacher responsibilities include guiding students with questioning, prompting, and constructive feedback. From a teaching angle, inquiry’s telltale characteristics are (1) listening to students, and (2) using their ideas to teach the mathematics. Students are responsible for making sense of the material but the teacher is there to help, should this prove problematic. It is important to mention that inquiry is not “sending students down aimless pathways” or the “teacher not teaching.” Like any nonstandard method of instruction, there are clearly defined goals and objectives. However, these goals are achieved in a different way from lecture-based models. A typical DE class for me can be described in what follows. Students were in groups of three or four.

- **Assign a recorder.** One student in the room was assigned “recorder” duty for the day. While this individual was relieved of “thinking” with his/her group, his/her responsibility was to keep track of what happened in class and take detailed notes on what was learned, the big picture, small mishaps, etc. I would examine his/her written work that evening and recommend edits, if need be. The purpose of this written record was two-fold: (a) Class members remained focused and attentive on the task at hand, and they were less concerned with “taking notes” since they knew they would receive a detailed summary at a later time. (b) A written record proved to be an excellent way to keep absent students attuned to what we were doing in class. The recorder changed with every class meeting.

- **Task Management.** I would provide an IO-DE task and make sure all groups understood what they were charged with doing. While the students worked together to make sense of the problem, I would navigate from group to group, offering assistance as needed. I rarely gave answers per se but I would steer groups back on course if they were out at sea.

- **Classroom Discourse.** Near the end of the activity, I would ask a group (sometimes two) to present their work. Classmates were encouraged to critique the presenta-

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1Visit [https://IO-DE.wordpress.ncsu.edu/](https://IO-DE.wordpress.ncsu.edu/) for the most up-to-date IO-DE materials.
tion(s), offer different perspectives, or ask questions. Oftentimes, I would chime in as teacher and either talk about important connections across multiple groups’ work or tie student idiosyncrasies to mathematical convention. It was during this stage where much of the “learning” occurred.

In the next section, I discuss two tasks from the IO-DE curriculum, the work generated by two groups, and how their learning aligns with current research in active learning environments.

3 Design Tasks and Student Work

On any given day, my DE class would either complete one large task (including debriefing, problem solving, presentation, and discussion) or solve two smaller tasks. In this section, I present student work on two such tasks. The first task, How Many Bugs?, exemplifies nonstandard notation and vocabulary generated from students; the reader will recognize the conventional notations as repellers and attractors. The second task, Fish Harvesting, resulted in a group of three students reinventing a bifurcation diagram completely from scratch. Both of these are detailed below.

Repellers and Attractors

The task provided to the class was the following (see Figure 1):

![Sample Task](image)

Figure 1: Sample Task How Many Bugs? from IO-DE (Rasmussen, 2002)
As indicated in the problem, students were familiar with graphing a population on one axis and its rate of change on the other. Content experts will recognize this task as one that could potentially introduce the ideas of stationary points, repellers, and attractors. Below are the contents of what one group presented as a way to make sense of “an impossible number of different scenarios.” They made this statement because they recognized that situations a)-f) presented a small number of possible starting points for the population.

![Figure 2: Sample Student Work](image)

Because you could “literally start anywhere,” the group members decided it would make sense to have a graph that showed the continuum of choices (a number line) and to categorize their results in one of two ways – i.e., “Are we moving toward this value or away from it?” One group member suggested the terminology “push” and “pull” to reflect the behavior near the stationary point. Moreover, we see the decision to represent the number line horizontally (not vertically, as is customarily shown in texts). This is because, at this developmental stage in their thinking, there is no reason to construct a number line in any way different from what is most familiar.

This way of documenting and organizing one’s thinking would be commonly referred to as a transformational record (Rasmussen & Marrongelle, 2006). One of the characteristics of a transformational record is “some form of notation (typically informal or unconventional notation) . . . used by a student in whole-class discussion or introduced by the teacher to record or notate student reasoning” (Rasmussen & Marrongelle, 2006, p. 4).
In this situation, we regularly used this notation in discussions and it wasn’t until a few lessons later that we formally introduced repellers, attractors and a vertical phase line, alongside the reasoning to support it. Thus, in this case, the groups’ reasoning was used as a teaching tool to orchestrate discussions and adopt mathematical conventions. Speer and Wagner (2009) reference this as a form of analytic scaffolding since the teacher moves guide “students further toward the desired mathematical goal(s) by using selected student contributions” (p. 536). As previously mentioned, the idea of using student work to advance one’s mathematical agenda is easily the most essential feature of inquiry-based learning. Since student thinking is used as both a starting point and as a reference to connect to standard mathematical content, it is often more memorable to students.² I still remember the sheepish look on the group member’s faces when they were introduced to the conventions of repeller and attractor – “That’s the stuff where we used the magnet analogy!” one of them shouted. It is clear that when students own the ideas, deeper and more meaningful learning is likely to occur.

Bifurcation

The first two times I taught with the IO-DE curriculum, I had limited time to cover some of the topics. However, in my third attempt, I gave the following task as a 50 minute activity:

![Figure 3: Sample Task Fish Harvesting from IO-DE (Rasmussen, 2002)](image)

Similar to the How Many Bugs? task, expert eyes will recognize ample opportunity to explore the qualitative features of differential equations including slope fields, long term behavior of solutions, phase plane analysis, etc. What I was not expecting was a group of three to reinvent the rudiments of a bifurcation diagram from scratch. While the group had other supporting work on scratch paper, Figure 4 shows the end result of their mathematical work (not the report) that was shared with the class.

²See select student comments in the Student Feedback section.
Initially, one can see that we agreed on the convention of a vertical phase line as built from the horizontal phase line first used in the *How Many Bugs?* task. Once this group discovered the effect of changing the value of $k$ (the constant harvesting rate), they let $k$ vary along the $x$-axis and compared this with the population of fish, $P$. The students here engaged in what some might call *horizontal mathematizing* (Rasmussen, Zandieh, King, & Teppo, 2005). This behavior can be described as “experimenting, pattern snooping, classifying, conjecturing, and organizing” (p. 54). Upon realizing the absurdity of repeatedly re-solving the same quadratic equation – plainly visible in Figure 4 – the students generalized this result and plotted only what was important and meaningful to the problem: $k$ versus $P$. At a later point in the class, we connected this work to a bifurcation diagram and the group members knew they were on to something special.

4 Disclaimer

Whenever I show these examples to teachers, they often ask, “Does this happen in every class?” The safe answer is certainly not. I show these examples because they illustrate the potential of an open-ended, discourse-oriented classroom. Research supports that what students do is a function of many variables – the quality of the curriculum, the specific task on hand, attitudes toward the content, group dynamic, quality of the discussions that take place, students’ prerequisite knowledge, teacher skill, teacher feedback, and teacher specialized content knowledge, just to name a few. Several scholars have pointed to the
need to refine what precisely precipitates these classroom victories (Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007). However, interestingly enough, the students rarely see a difference in the day-to-day classroom activity. From a student perspective, every day is a day to think about the problems, have conversations, and figure things out. Some of this is visible in the feedback in the next section.

5 Student Feedback

At the conclusion of my third iteration of teaching IO-DE, I felt an increased level of comfort with the goals of inquiry, and facilitating the curriculum came easier to me. Naturally, I was interested in what the students thought about the class. Thus, I asked each student to anonymously provide some feedback about the class. I simply asked them, How is the class going for you? Please provide some specifics. Below are five of the responses that were most representative of the class as a whole:

Figure 5: Sample Student Feedback
My favorite part of asking for feedback is the unvarnished responses. While the overall feeling is one of promise and optimism, this is enveloped with some degree of uncertainty. At the end of the term, I discovered this was each students’ first time learning mathematics in a nonlecture environment. This was analogous to my first time teaching IO-DE a few years prior – I felt hope, optimism, and skepticism all wrapped in one. As a teacher, I needed time to buy into the methods and explore this new pedagogy. Similarly, students need time to adapt – to see mathematics as a collective byproduct of exploration and engagement.

6 Reflections

I close this article with some assertions I have come to believe as a result of teaching IO-DE. I make no claim that these apply to all inquiry-based environments but I suspect they do. For example, many of these assertions apply to other courses I have taught using active learning pedagogy (Nabb, Hofacker, Ernie, & Ahrendt, 2018; Nabb, 2018).

- **Assertion 1.** Inquiry is hard!

- **Assertion 2.** Inquiry requires deeper content knowledge on the part of the teacher.

- **Assertion 3.** Inquiry is messy and often slow to unfold.

- **Assertion 4.** Assessment is more challenging in an inquiry environment.

- **Assertion 5.** Inquiry-oriented classrooms cover qualitatively different content but this is often with equal or better results than traditional instruction.

- **Assertion 6.** Both teacher and students may be slow to acclimate to this environment.

Although I have reached the above conclusions based on my classroom experiences alone, research supports many of these claims. For example, with regard to Assertion 1 and 2, there is convincing research pointing to the specialized knowledge post-secondary teachers need to have to be effective teachers (Speer, King, & Howell, 2015; Speer, Smith, & Horvath, 2010). Two prominent examples elaborate on differential equations and inquiry environments (Speer & Wagner, 2009; Wagner, Speer, & Rossa, 2007) and build on the work of teachers using student reasoning to teach in the IO-DE environment (Rasmussen & Marrongelle, 2006).

With respect to Assertion 5, reports in post-secondary environments in the sciences (CBMS, 2016; Freeman et al., 2014) and IO-DE specifically (Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006) support that teaching fewer topics with deeper conversations have long-term benefits for students. Moreover, the communication of ideas in team environments is a valuable asset outside of the academy. In support of some of the other assertions (e.g., 3, 4, and 6), just an honest conversation with anyone who has used POGIL (www.pogil.org), the Five Practices (Smith & Stein,
or some other means of student-centered instruction should easily convince the reader. My hope is that this article will encourage some teachers to take the initial steps toward active learning. Research largely shows positive results and this pedagogy better suits our current world in which content is so readily accessible (e.g., YouTube). With the proper supports in place, deep conversations around important mathematical ideas are primed to occur. Students can and will produce important mathematics that the teacher can connect to lesson objectives. Many of the leading mathematics organizations – AMATYC, AMS, AMTE, MAA, NCTM, and SIAM – have recently endorsed this change in our classrooms (CBMS, 2016) so it is time we support one another in making this a reality. It is not an easy change but one that needs to happen.

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References


