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In 1983 the CUPM Panel on Mathematics Appreciation Courses issued a report which articulated the goal of such courses. "Students must come to understand the historical and contemporary role of mathematics, and to place the discipline properly in the context of other human achievements." [4]. The statement of the CUPM Panel is an assertion of the place of mathematics in a liberal arts education. Liberally educated people have a broad base of knowledge in a variety of aspects of human culture and society. They are able to assess their own experience and opinions in the context of human cultural tradition. The mathematical component of a liberal education should help students see mathematics as an integral part of human culture, not as an isolated discipline.

In this paper I describe a course developed at Villanova University, which offers three perspectives on the connection of mathematics to other aspects of human culture: the fact that mathematics has always been part of our culture, the Pythagorean view that number is the basis of all creation, and the tradition of using geometry to structure our visual perceptions of the world.

Synopsis of the Course

The course begins by recognizing that mathematics has been an integral part of human society from the beginning. Bones and sticks containing tally marks can be found among the artifacts of human species from the very beginning of the species [2]. Examples of such bones are shown and the speculation about more advanced mathematical ideas implicit in the Ishango Bone is discussed [1,3]. We point out that every human society has had some concept of number and some language for speaking of numbers. The numeral systems and computational techniques of ancient Egypt and Mesopotamia are discussed. This included the duplication method of multiplication as well as the use of unit fractions in Egypt and sexigesimal fractions in Mesopotamia. Note is taken of the fact that astronomers used the sexigesimal system until the Renaissance.

A discussion of place value systems in general naturally follows. Base 60 has been seen. Base 10 should be familiar. One or two other bases are discussed to emphasize the society's choice of the decimal system is arbitrary, at least from the mathematical standpoint. Students sometimes have difficulty with other bases than ten because our language for numbers as well as our symbolism is based on ten. The words one thousand, three hundred eighty-five mimic the numeral 1385. If we write this in base eight, 2551 cannot be pronounced. The problem can be addressed by working in base twelve, in which there is a language. Our number is 975, and is read as nine gross, seven dozen five. It may also be helpful to remind students of phrases such as, "four score and seven years ago", which has a base of twenty and of the representation of fractional parts of angles in minutes and seconds, using the Babylonian base, sixty.

It is important to discuss the binary system and its relevance to modern computing.

The discussion of positional numeration culminates with the theorem that the radix representation of every rational number either terminates or repeats; for a reduced fraction the representation terminates iff each prime factor of the denominator is a factor of the base.

The mathematics at this stage is easy. The material is important for helping students appreciate that math is an inseparable component of human culture not only in modern technological society but throughout the history of culture. The question is raised, why is this the case and how did it come to be so. The customary answer is that as society grew in complexity, the practical requirements of society demanded ever more sophisticated mathematics. The evidence put forward in support of this view is first that it seems reasonable. Secondly one can cite problems of an applied nature in ancient mathematical writings such as the Rhind Mathematical Papyrus [5]. A completely different opinion is found in the very interesting theory of A. Seidenberg that mathematics grew up with ritual practice [6].

One criterion for knowing if we understand a natural phenomenon is being able to express our understanding in mathematical terms. In the words of Galileo, "Philosophy is written in that book which lies ever before our eyes—I mean the universe—but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in the mathematical language . . ." [2, pp. 328, 329]. This view originated early in the development of Greek rational philosophy when Pythagoras is supposed to have considered that, "all is number," by which he meant that we can understand all of creation in terms of relationships among whole numbers. In the next part of the course we explore these ideas. Some aspects of number theory are studied on the assumption that Pythagoras initiated number theory to lay a foundation for understanding the universe. The subject is treated frankly as an intellectual exercise in furtherance of a philosophical position.

Triangular, square, and oblong numbers are discussed. It is shown by partitioning the figures that a triangular number is a sum of consecutive integers, a square is a sum of odds and an oblong is a sum of evens. Obviously a square number is a product of a number with itself, an oblong is a product of two consecutive numbers. One sees by drawing a diagonal that a triangular number is half an oblong number from which we get the traditional formula for a sum of integers

$$1 + 2 + 3 + \ldots + n = 1/2\, n(n + 1).$$

The Fundamental Theorem of Arithmetic is stated with emphasis on the uniqueness of the representation of a number as a product of primes. Because the factorization is unique one can extract from the product of primes a complete list of divisors of a number. This enables one to check if a number is perfect.

Moreover the uniqueness of factorization can be used as a basis for proving that the square root of every natural number is either integral or irrational. This is related to the Pythagorean discovery of incommensurable quantities and the consequent collapse of the Pythagorean philosophical program.

Although the Pythagorean formulation of the mathematical foundation of all things could not be maintained, its residue survives in the attitude common in the sciences that the best explanation is one which can be expressed in mathematical terms. This view was one of the main themes of the Scientific Revolution and is illustrated by Galileo's mathematical theory of motion.

Galileo begins with uniform motion, motion at constant speed, which exists only as an ideal but is mathematically simple. He then discusses the motion of free fall for which he takes $v = at$ as the basic formula for uniform motion, $d = rt$. From $v = at$ he uses mathematics to deduce $d = 1/2\ vt$ and $d = 1/2\ at^2$. Then projectile motion is analyzed as a combination of uniform motion and free fall.

Many believe that the Greeks made such significant contributions to geometry because Pythagoras' emphasis on number proved untenable. Plato is said to have insisted on a knowledge of geometry as a prerequisite to entry to his academy. Over the centuries Euclidean geometry came to be the dominant model of the physical world. Therefore Euclidean geometry is reviewed with an emphasis on parallelism. Then it is pointed out what Euclidean geometry is not logically necessary and a brief introduction to non-Euclidean geometry is presented.

An intuitive discussion of spherical geometry brings out the points that a line is a great circle, parallel lines do not exist, and the sum of the angles in a triangle is greater that 180 degrees and varies with the area of the triangle. There is a brief and superficial discussion of Lobachevskian geometry and its potential for modeling the universe.

Course Objectives

The primary goal of this course is to help students understand how mathematics has always been a primary tool in human efforts to understand our world. Mathematics has been with us since the beginning of our species. At least since the time of Pythagoras mathematics has been seen as a language for expressing abstract models of natural phenomena. Because the models are abstract they gloss over some details but they may also yield deeper insights. The models may be developed out of experience, observation or intuition of nature. But it often happens that mathematics developed for its own sake is seen later to model some phenomenon (for example, the parabola as the path of a projectile). The insistence in mathematics on precise formulation and the deductive proof provides a discipline for checking a scientific theory.

Another goal of the course is to give students insight into the nature of mathematics as one of the many areas of human activity. Mathematics has an integrity of its own and provides impetus for its own development apart from any utility. There is a beauty in a body of mathematical theory that can be seen for example in Pythagoras' work on figurate numbers, the theory of conic sections, or Galileo's theory of motion. Mathematics has its own standards of truth, the criteria for what constitutes a valid proof. We point out how these standards are different from standards in other realms of human activity - law, science, everyday affairs, religion, etc. A few selected results in the course are proved (irrationality of some square roots, some theorems in geometry) and the opportunity is taken to discuss the nature of proof in mathematics and how it is different from other systems of evidence.

General Comments

This is a one semester course. There is more than enough material for one semester and decisions have to be made about what to omit. On the other hand there is not enough material for two semesters. In any case a course of this kind extending over two semesters would probably tax the patience of students who take the course to satisfy a requirement rather that out of genuine interest. A second semester course should concentrate on the uses of mathematics in the twentieth century, treating such topics as statistical thinking, linear, polynomial and exponential models, and modern computing.

An obvious difficulty with this course is the choice of a textbook. Nothing suitable is available and I have written notes for my course.

Some people with whom I've discussed these ideas have raised the objection that mathematicians are accustomed to teaching strictly mathematical material and either can't teach non-technical material or do not want to. It is asserted that we don't know how to lead a classroom discussion of Pythagorean philosophy or that we do not know how to formulate exercises and test questions on the cultural implications of non-Euclidean geometry. This objection seems to me a luxury we can ill afford. If it is true we must seek resources that will help us. The center section of [4] contains an excellent list of resources. We might seek assistance from our colleagues in the humanities. A common view of mathematics is that it is an essential subject for modern society, but one which few can appreciate; ignorance of mathematics is not only permissible but something of a distinction. The popularity of this view indicates not only the cynicism of our academic colleagues but the failure of our mathematics appreciation courses. We must learn to discuss mathematics in a broad cultural contest.

Bibliography

1. De Heinzelin, Jean, Ishango, *Scientific American* 206 1962, 105-106

2. Kline, Morris, *Mathematical Thought from Ancient to Modern Times*, New York: Oxford University Press, 1972.

3. Marschak, Alexander, *The Roots of Civilization*, New York: McGraw-Hill, 1972.

4. Mathematics Appreciation Courses: The Report of the CUPM Panel, *Amer. Math. Monthly* 90 (1983), 44-51, Center Section.

5. *Papyrus Rhind*, trans. by A. Chace, Mathematical Association of America, 1929.

6. A. Seidenberg, The Origin of Mathematics, *Archive for History of Exact Sciences*, 18 (1978), 301-342.