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Detecting Fraud in Bankrupt Municipalities Using Benford's Law

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DETECTING FRAUD IN BANKRUPT MUNICIPALITIES USING BENFORD'S LAW

by

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Abstract

This thesis explores if fraud or mismanagement in municipal governments can be diagnosed or detected in advance of their bankruptcies by financial statement analysis using Benford's Law. Benford's Law essentially states that the distribution of first digits from real world observations would not be uniform, but instead follow a trend where numbers with lower first digits (1, 2…) occur more frequently than those with higher first digits (…8,9). If a data set does not follow Benford's distribution, it is likely that the data has been manipulated. This widespread phenomenon has been used as a tool to detect anomalies in data sets. The annual financial statements of Jefferson County, Vallejo City, and Orange County were analyzed. All the data sets showed overall nonconformity to Benford's Law and therefore indicated that there was the possibility of fraud occurring. I find that Benford's Law, had it been applied in real time to those financial statements, would have been able to detect that something was amiss. That would have been very useful because each of those jurisdictions subsequently went bankrupt. This paper demonstrates that Benford's Law may in some cases be useful as an early indicator to detect the possibility of fraud in municipal governments' financial data.

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1. Introduction

This thesis is going to explore if fraud and mismanagement in municipal governments that subsequently went bankrupt could have been diagnosed or detected in advance of their bankruptcies by financial statement analysis using Benford's Law. Benford's Law essentially states that the distribution of first digits from real world observations would not be uniform, but instead follow a trend where numbers with lower first digits (1, 2…) occur more frequently than those with higher first digits (…8,9). If a data set does not follow this pattern, it is likely that the data has been manipulated. With the municipalities of Jefferson County, Vallejo City, and Orange County going bankrupt amidst controversial circumstances, the question arises: Did these municipalities ever intentionally manipulate their financial data? This paper aims to answer the question whether or not Benford's Law could be useful as an early indicator to detect the possibility of fraud in municipal governments' financial data.

In order to investigate this question, I will analyze the financial statements of Jefferson County, Vallejo City, and Orange County and apply Benford's Law to them. Since we know (thanks to the luxury of hindsight) that these three municipalities have committed fraud or outrageously mismanaged their finances, by retrospectively testing Benford's Law on these municipalities' financial data, we might be able to determine whether or not Benford's Law is a good indicator of fraud and could have been used to provide a warning sign for regulators and investors.

My analysis will be done by conducting two different tests. First, I will analyze the financial statements from five years before filing for bankruptcy and the year of

filing, and apply Benford's Law to each fiscal year, resulting in a total of eighteen tests. In the second test, I will compile all of the six years' data for each municipality into one data set, so there are a total of three data sets, one for each municipality. If the data sets do not follow Benford's distribution, I can then conclude there was possibly a manipulation of the data and further analysis of potential fraud would have been warranted.

Along with the economic downturn of the U.S. economy since 2008, there has been increasing press publicizing the dire financial conditions of various municipal governments. This is because from 2002 to 2008, "the states had piled up debts right alongside their citizens: their level of indebtedness, as a group, had almost doubled, and state spending had grown by two-thirds. In that time the states had also systematically underfunded their pension plans and other future liabilities by a total of nearly $1.5 trillion. In response, perhaps, the pension money that they had set aside was invested in ever riskier assets. In 1980 only 23 percent of state pension money had been invested in the stock market; by 2008 the number had risen to 60 percent. To top it off, these pension funds were pretty much all assuming they could earn 8 percent on the money they had to invest, at a time when the Federal Reserve was promising to keep interest rates at zero. Toss in underfunded health-care plans, a reduction in federal dollars available to the states, and the depression in tax revenues caused by a soft economy, and the states were looking at multi-trillion-dollar holes that could be dealt with in only one of two ways:

massive cutbacks in public services or a default—or both."[1] Poor management, irresponsible governments, inflated economic expectations, whatever the cause, there have been increasing calls for bankruptcy and reorganization.

Municipal bankruptcies weigh on both those who rely on municipal services as well as municipal employees. Since municipalities are legally obligated to meet the costs of funding pension plans, a major competent of its financial obligations, they can only respond to a fiscal crisis by cutting elsewhere. For example, San Jose, California, once run by 7,450 workers, had to reduce the number of city workers to 5,400, which is back to the staffing levels of 1988 when there were a quarter million fewer residents. The remaining workers then had to take a ten percent pay cut, yet even this was not enough to offset the city's pension liabilities. Other municipal services of the city were also forced to cut back. The city closed its libraries three days a week and had to cut back servicing its parks. It also refrained from opening a brand-new community center, built before the housing bust, because it couldn't pay to staff it. And on top of this, for the first time in history, it laid off police officers and firefighters. At that point, if not before, the city was nothing more than a vehicle to pay the retirement costs of its former workers.[2] This is just one example of how municipalities' funding of burdensome pension plans are causing other municipal services to suffer.

Municipal employees are also exposed by the financial strain pension plans are putting on municipalities. Employees face the possibility of losing their jobs and health

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[1] Lewis, Michael. "California and Bust." *Vanity Fair*. Nov. 2011. Web. 10 Apr. 2012. <http://www.vanityfair.com/business/features/2011/11/michael-lewis-201111>.
[2] Ibid.

benefits if a municipality goes bankrupt because of the financial stresses it is under. Employees worry they could lose the health benefits promised in retirement due to a municipality's bankruptcy, and many are even afraid of getting sick because they know they may not have health benefits in the future. It is crucial to try to prevent municipalities from going bankrupt because it harms not only themselves, but also the people it is supposed to serve.

Filing for bankruptcy gives a municipality a chance to start fresh by relieving debt obligations and offering creditors a chance to gain some measure of repayment. Municipalities across the U.S. have become so financially distressed that they are seeking protection from creditors by filing under Chapter 9 of the Federal Bankruptcy Code. According to the Administrative Office of the United States Courts, Chapter 9 bankruptcy is meant to "provide a financially distressed municipality protection from its creditors while it develops and negotiates a plan for adjusting its debts".[3] A municipality can adjust its debts either by "extending debt maturities, reducing the amount of principal or interest, or refinancing the debt by obtaining a new loan".[4]

Although Chapter 9 is similar to other chapters, there are some significant differences. Chapter 9 is much like Chapter 11 bankruptcy, in that it provides the opportunity for reorganization, however, only municipalities such as cities, townships, counties, public improvement districts and school districts may file for a Chapter 9 proceeding. There is also no stipulation in the Law for the liquidation of the

[3] "Chapter 9." *United States Courts*. Web. 01 Dec. 2011. <http://www.uscourts.gov/FederalCourts/Bankruptcy/BankruptcyBasics/Chapter9aspx>.
[4] Ibid.

municipality's assets and distribution of the proceeds to creditors. This is because "such a liquidation or dissolution like this would violate the Tenth Amendment to the Constitution and the reservation to the states of sovereignty over their internal affairs."[5] Also due to the severe limitations placed upon the bankruptcy court's power in Chapter 9 cases, it is generally not as active in managing a municipal bankruptcy case as it is in corporate reorganizations under Chapter 11. Some of these limitations include not interfering with – "(1) any of the political or governmental powers of the debtor; (2) any of the property or revenues of the debtor; or (3) the debtor's use or enjoyment of any income-producing property unless the debtor consents or the plan so provides".[6] The provision makes it clear that the debtor's day-to-day activities are not subject to the court's approval. The bankruptcy court's functions include approving the petition for bankruptcy, confirming a plan of debt adjustment, and ensuring the plan's implementation.[7]

The nationwide dramatic decline in U.S. commercial, industrial, and residential property values over the past three years has led to disastrous declines in tax receipts for most municipalities. Declining revenues have recently been a major source of municipality bankruptcies, although typically, the main cause is stress to the system that occurs from managing the expense side, whether it is undisciplined financial management, out of control spending without correlating revenues, or lack of accountability for responsible financial management.

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[5] Ibid.
[6] Ibid.
[7] Ibid.

Three municipalities of note that have captured financial headlines in recent years are Jefferson County, Alabama, Vallejo City, California, and Orange County, California because of their controversial bankruptcies.

The bankruptcy of Jefferson County, Alabama is an instructive model. Jefferson County filed the largest bankruptcy in U.S. history in 2011 with $3.14 billion in debt because of two controversial projects which accounted for a majority of the municipality's debt. The first project involved overhauling the city's sewer system. The second transaction involved entering into a series of risky bond-swap agreements. Both deals were scrutinized in retrospect because of the widespread corruption, bribery and fraud charges against Jefferson County's officials and government workers. Larry Langford, the former mayor of Birmingham, was sentenced to fifteen years of imprisonment for his role in the bribery scheme and corrupt business deals that fueled the multi-billion dollar sewer debt that led to Jefferson Country's bankruptcy.[8] Experts expect years of litigation to determine how much debt was used for the legitimate sewer system improvements versus enriching former county officials. Litigations include Wall Street bondholders who dispute the repayments and this "could have far-reaching implications for the $3.7 trillion municipal bond market" and investor confidence in the marketplace.[9]

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[8] United States of America versus Larry P. Langford. United States Court of Appeals for the Eleventh Circuit. U.S. Court of Appeals Eleventh Circuit. 5 Aug. 2011.*United States Court of Appeals for the Eleventh District*. United States Court of Appeals for the Eleventh District, 5 Aug. 2011. Web. 22 Mar. 2012. <http://www.ca11.uscourts.gov/opinions/ops/201011076.pdf>.

[9] Corkery, Michael, and Katy Stech. "Alabama Bond Fight Begins a New Round." *Wall Street Journal*. 16 Apr. 2012. Web. 19 Apr. 2012. <http://online.wsj.com/article/SB10001424052702303624004577340312976464358.html?mod=googlenews_wsj>.

Another model is Orange County, California, which filed for the second largest municipal bankruptcy in U.S. history in 1994. The county treasurer, Bob Citron, gambled with public funds by investing in risky Wall Street securities. In the beginning of 1994, the Orange County investment pool had about $7.6 billion in deposits. Citron then borrowed $2 for every $1 on deposit and increased the size of the investment pool to $20.6 billion.[10] In essence, he was "borrowing short to go long and investing the dollars in exotic securities whose yields were inversely related to interest rates. Unfortunately, also during 1994, the Federal Reserve Board began and kept on raising interest rates, and Citron kept buying in the hope they would decline."[11] Almost no one was paying attention to what the treasurer was doing and even fewer understood it. It wasn't until November of 1994 when the auditors informed the Board of Supervisors that Citron had lost nearly $1.7 billion.[12]

Vallejo City filed for bankruptcy in 2008, becoming the largest municipality ever to do so in California. The primary reason for the municipality's financial turmoil was due to the municipality's labor contracts.[13] Stephanie Gones, Vallejo City Councilwoman, blamed the bankruptcy largely on the inflated salaries and benefits for

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[10] "The Orange County Bankruptcy: Who's Next?" *Research Brief* 11 (1998). *PPIC*. The Public Policy Institute of California, Apr. 1998. Web. 22 Mar. 2012. <http://www.ppic.org/content/pubs/rb/RB_498MBRB.pdf>.
[11] Ibid.
[12] Ibid.
[13] Jones, Carolyn. "Vallejo Votes to Declare Chapter 9 Bankruptcy." *SFGate*. San Francisco Chronicle, 7 May 2008. Web. 3 Apr. 2012. <http://www.sfgate.com/cgibin/article.cgi?f=/c/a/2008/05/06/BACH10HUK6.DTL>.

Vallejo firefighters and police officers.[14] In Vallejo, the public safety employees were some of the highest paid in the San Francisco Bay Area. The municipality spent about "74% of its $80 million general fund budget on public safety salaries, which is significantly higher than the state average".[15] This was a result of the generous contracts from deals made in the 1970s following police strikes. While officials wanted to file for bankruptcy, "police, fire and other unions…were outraged" and "accused the council of poor leadership".[16] Following bankruptcy, the unions challenged Vallejo's solvency, questioning whether they were hiding money, demanded they pay them from restricted funds, and ultimately fought them on whether they could tear up their CBAs (collective bargaining agreements)".[17] The municipality's bankruptcy was caused by a major mismanagement of funds, and the employees and taxpayers whose services have been reduced are paying the price.

Since municipal bankruptcies have such severe repercussions on the lives of the taxpayers, municipal employees, and the entire bond market, it's important to try to prevent them. Municipal bankruptcies, like the ones above, could have been prevented. One method of prevention is to have better oversight and accountability over the use of funds. The Board of Supervisors or City Council could more actively supervise the

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[14] "Re: Speech before the Contra Costa Taxpayers Association." Web log comment.*Http://www.stephaniegomes.com/*. 24 June 2011. Web. 3 Apr. 2012. <http://www.stephaniegomes.com/2011/06/speech-to-contra-costa-taxpayers.html>.
[15] Jones, Carolyn. "Vallejo Votes to Declare Chapter 9 Bankruptcy." *SFGate*. San Francisco Cronicle, 7 May 2008. Web. 3 Apr. 2012. <http://www.sfgate.com/cgi-bin/article.cgi?f=/c/a/2008/05/06/BACH10HUK6.DTL>.
[16] Ibid.
[17] Nessi, Ted. "A Vallejo Councilwoman Warns Central Falls about Chapter 9." Web log post. *WPRI Eyewitness News*. WPRI, 15 Aug. 2011. Web. 3 Apr. 2012. <http://blogs.wpri.com/2011/08/15/a-vallejo-councilwoman-warns-central-falls-about-chapter-9/>.

municipality's investment decisions and business transactions. Another method would be for state governments to monitor the fiscal condition of its local governments more closely instead of waiting for problems to become more serious or catastrophic. The state's controller could analyze which counties show signs of distress and address the issue before it reaches a critical level.[18]

A method the over sight bodies and auditors could apply is a theory called Benford's Law, which can detect fraud in financial statements. Financial reporting fraud occurs when a company, government agency, or other organization, intentionally misstates or omits information from its financial statements. Many articles and research have promoted the use of Benford's Law as a tool for auditors to uncover fraud in financial statements. This law could have been applied to these three municipalities as a method of early detection within each municipality and possibly could have caught the financial problems early enough to prevent their eventual bankruptcies.

I find below that if Benford's Law had been applied to the financial statements of all three municipalities, it would have given warning that something was amiss. It will also help promote Benford's Law in providing auditors with a tool that is effective in detecting fraud.

Background and an overview of relevant research and papers about Benford's Law and applying it to financial statements can be found in section II. In section III, my methodology and data are explained and in section IV, my results. Section V and VI

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[18] "The Orange County Bankruptcy: Who's Next?" *Research Brief* 11 (1998). *PPIC*. The Public Policy Institute of California, Apr. 1998. Web. 22 Mar. 2012. <http://www.ppic.org/content/pubs/rb/RB_498MBRB.pdf>.

include my discussion and conclusion. And finally, all the tables are found in the appendix.

II. Benford's Law

In 1881, Simon Newcomb published an article, *Note on the Frequency of Use of the Different Digits in Natural Numbers,* describing what has become known as Benford's Law. He noticed in the library's logarithmic books, the beginning pages with lower digits were more worn out than the pages with higher digits. He concluded from this pattern that "the ten digits do not occur with an equal frequency" and the "first significant figure is oftener 1 than any other digit, and the frequency diminishes up to 9".[19] The conclusion was that more numbers begin with the digit 1 than with larger numbers like 9.

In 1938, Frank Benford, published *The Law of Anomalous Numbers,* which describes his findings, now referred to as Benford's Law. Benford, like Newcomb, also observed in logarithmic books that the first pages showed more wear than the last pages, indicating that the more numbers begin with the digit 1 than the digit 9. He then empirically tested the first digit frequencies of about 20,000 numbers from a wide variety of sources, like areas of rivers, death rates, and atomic weights of elements.[20] His results showed that an average of 30.6% of the numbers had the leading digit 1, while in contrast, only 4.7% of the numbers had the first digit 9. From this, Benford then hypothesized that naturally occurring data should form a geometric sequence. Through

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[19] Newcomb, Simon. "Note on the Frequency of Use of the Different Digits in Natural Numbers." *American Journal of Mathematics* 4.1 (1881): 39-40. *JSTOR*. Web. 27 Feb. 2012. <http://www.jstor.org/>.
[20] Benford, Frank. "The Law of Anomalous Numbers." *The American Philosophical Society* 78.4 (1938): 551-72. *JSTOR*. Web. 4 Dec. 2011. <http://www.jstor.org>.

calculus, he formulated the expected frequencies for the first and second digits in numbers as well as the two digit combinations of the first and second digits. The formulas using base 10 logarithms are as follows, where P is the probability of observing the event in parentheses, D_1 is the first digit of a number, D_2 is the second digit of the same number, and d_1 and d_2 denote the possible digits, 0 through 9.

$$P(D_1 = d_1) = \log(1+1/d_1)$$
$$d_1 \in \{1,2\ldots9\}$$

$$P(D_2 = d_2) = \sum_{d1=1}^{9} \log(1+(1/d_1d_2))$$
$$d_2 \in \{0,1\ldots9\}$$

$$P(D_1D_2 = d_1d_2) = \log(1+1/d_1d_2))$$
$$d_1d_2 \in \{10,11\ldots99\}$$

Table 1 in the appendix shows the expected digital frequencies under Benford's Law (and the above formulas) for all digits 0 through 9 in the first two places in a number. The digit 1 has the highest expected frequency of 30.1%, while the expected frequency of the digit 9 is just 4.6%. The second digits are less skewed than the first, as seen in Table 1. The expected frequencies of first and second digit combinations can be found by using the above formula, where for example,

$$P(D_1D_2 = 99)$$
$$= \log(1+(1/99)) \approx 0.44\%$$

Benford's Law is an empirically observable phenomenon and applies to many data sets like lists of numbers, populations of cities, market values, and net incomes.

Benford's Law has also been used in empirical studies as a way to help determine

a data set's lack of authenticity. If a data set does not conform to Benford's distribution, then the data has likely been manipulated. Following this idea, many have applied Benford's Law in accounting as a way of detecting fraud.

The first accounting application of Benford's Law was in a study done by Carslaw (1988). He observed that earnings numbers from New Zealand firms departed significantly from expectations. According to Benford's distribution, the earnings numbers contained a higher frequency of zeros in the second digit position than expected, and fewer nines.[21] This implied, for example, that when a firm had earnings such as 1,900,000 they rounded up to 2,000,000.

Thomas (1989) was inspired by Carslaw's work and decided to conduct a study to see if reported earnings for U.S. firms followed similar patterns as New Zealand firms. He discovered there was a similar pattern in the earnings of U.S. firms. However, U.S. firms' numbers deviated less from expectations than New Zealand firms, and reflected an opposite pattern, in that there were fewer zeros and more nines than expected. The results show that managers of U.S. firms that reported losses avoid round numbers. Therefore, for example, they would round 2,000,000 down to 1,900,000.[22]

Nigrini (1996) is one of the first researchers to apply Benford's Law extensively in an accounting context with the goal of detecting fraud. His dissertation used digital analysis to help identify tax evaders. His research indicated that low-income taxpayers practiced unplanned tax evasion to a greater extent than high-income taxpayers.

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[21] Carslaw, Charles A.P.N. "Anomolies in Income Numbers: Evidence of Goal Oriented Behavior." *The Accounting Review* LXIII.2 (1988): 321-27. *JSTOR*. Web. 5 Dec. 2011. <http://www.jstor.org>.
[22] Thomas, Jacob K. "Unusual Patterns in Reported Earnings." *The Accounting Review* LXIV.4 (1989): 773-87. *JSTOR*. Web. 5 Dec. 2011. <http://www.jstor.org>.

Unplanned tax evasion is a blatant manipulation by the taxpayer at filing time by inventing numbers for line items. High-income taxpayers practiced unplanned tax evasion by understating income items and overstating deduction items.[23]

Nigrini and Mittermaier (1997) did another study examining accounting data for their conformity to the digital frequencies to Benford's Law. They concluded that auditors could test the authenticity of lists of numbers by comparing the actual and expected digital frequencies.[24]

However, there is some academic literature that is cautious about promoting the effectiveness of Benford's Law in detecting fraud. In particular, such work, like one by Etteridge and Srivastava (1999), cautions that a data set which, when tested, does not conform to Benford's Law, may only show operating inefficiencies or flaws in accounting and reporting systems rather than fraud.[25]

In Durtschi, Hillison and Pacini (2004), the authors expand on the findings of Etteridge and Srivastava by discussing why certain data sets are appropriate for digital analysis and others not. They explain the effective use of Benford's Law in assisting in the detection of fraud in accounting data by specifically identifying which data sets can be expected to follow Benford's Law and the types of fraud that can be detected. The types of data sets that can be used are sets of numbers that result from the mathematical combination of numbers. For example, accounts payable, which is a combination of the

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[23] Nigrini, Mark J. "A Taxpayer Compliance Application of Benford's Law." *Jata* 18.1 (1996): 72-91. *EBSCOhost*. Web. 5 Dec. 2011. <http://web.ebscohost.com/>.
[24] Nigrini, Mark J. "The Use of Benford's Law as an Aid in Analytical Procedures." *Auditing: A Journal of Practice & Theory*16.2 (1997): 52-67. Web. 5 Dec. 2011. <fabweb.cityu.edu>.
[25] Michael L. Ettredge and Rajendra P. Srivastava (*1999*) Using Digital Analysis to Enhance Data Integrity. Issues in Accounting Education: November 1999, Vol. 14, No. 4, pp. 675-690.

number of items bought multiplied by the price. On the other hand, Benford analysis would not be useful on data sets comprised of assigned numbers, like check numbers or invoice numbers. The authors also address potential problems that may arise, specifically, “Benford analysis should only be applied to accounts which conform to the Benford distribution, and the auditor must be cognizant of the fact that certain types of fraud will not be found with this analysis”.[26] Nevertheless, Durtschi, Hillison, and Pacini demonstrate in their paper that when Benford’s Law is used correctly, it is proves to be successful in identifying fraud in a population of accounting data sets.

In *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection,* Nigrini illustrates other limitations of the application of Benford’s Law. One limitation is that a data set can conform to Benford’s Law and still contain some fraud or errors. The problem is that “when many data points are combined in one data table, the data can still contain fraud and errors, and these will be overwhelmed by the rest of the data. A controller needs to overstate only one number to overstate income.”[27] Changing one number, however, will not cause non-conformity to Benford’s Law. Fraud in just one unit is difficult to detect by just using Benford’s Law. Therefore, Benford’s Law is very useful in detecting macro issues in a data table, like multiple campaign donations to a single candidate that are at the limit allowed by the law.

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[26] Durtschi, Cindy, William Hillison, and Carl Pacini. "The Effective Use of Benford's Law to Assist in Detective Fraud in Accounting Data." V.1524-5586 (2004): 17-34. *UIC*. Journal of Forensic Accounting. Web. 4 Dec. 2011. <http://www.uic.edu/>.

[27] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 214. Print.

Another major limitation Nigrini explains is that there might be too few numbers in a set of financial statements to accurately detect fraud using Benford's Law. Detecting fraud "would be easy if we could simply compare the patterns of a single set of financial statements to Benford's Law and then conclude that nonconformity means that the financial statements are misstated. This would make Benford's Law similar to a lie detector test. Unfortunately, fraudulent financial statements are rarely identified only by analyzing the numbers of financial statements."[28] This is because there are simply too few numbers in a set of financial statements for Benford's Law to accurately signal fraud or irregularities. Because there are "just a few hundred numbers, any set of financial statements could deviate substantially from Benford's Law [merely by chance]."[29] Benford's distribution might not be a valid expectation for small data sets. For example, the IRS could then run Benford's Law on all of the millions of income tax returns submitted in a particular year and expect the numbers to conform to Benford's distribution. However, since there are only a "few numbers on an individual tax return, the return when analyzed could have a large deviation from Benford's Law and still be accurate and compliant".[30] Although Benford's Law might not accurately prove there is fraud, it can still be an indicator of the possibility of fraud. Nonconformity to Benford's Law is an indicator of possible irregularities and therefore directs an auditor's attention to the financial statements that merit further attention.

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[28] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 215. Print.
[29] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 221. Print.
[30] Ibid.

Although Benford's Law has its limitations, it is legally admissible in U.S. criminal cases at the federal, state, local levels, and even internationally. For example, it was invoked as evidence of fraud in the 2009 Iranian elections. Boudewjin Roukema in *Benford's Law Anomalies in the 2009 Iranian Presidential Election* (2009) used Benford's Law to test the results from the Iranian election. Roukema noticed a strange anomaly in the votes for Mehdi Karroubi, who came in third place; he found that the number 7 occurred as a first digit more often than would be expected in Benford's Law.[31] He then discovered that this anomaly also occurred in three of the six largest voting areas, and that the winner, Mahoud Ahmedinejad, had a greater portion of the votes in those three areas than the others. Roukema thus concluded there was an error in the official count of votes.

Benford's Law is also one of the techniques used within the accounting profession to detect the possibility of fraud. In Saville's *Using Benford's Law to Detect Data Error and Fraud: An Examination of Companies Listed on the Johannesburg Stock Exchange* (2006), his goal is to explore the relevance of Benford's Law in the detection of anomalies in data in companies listed on the Johannesburg Stock Exchange. He concludes that although Benford's Law has limitations, his findings suggest that Benford's Law still has the capacity to play a helpful role in assisting users of accounting data to detect error or fraud in financial information. He found all 17 of the 'errant' companies failed the test and only 3 of the 'compliant' companies failed the test, giving

[31] Roukema, Bouewjin. "Benford's Law Anomalies in the 2009 Iranian Presidential Election." *Annals of Applied Statistics* (2009): 1-14. Annals of Supplied Statistics, 16 June 2000. Web. 4 Apr. 2012. <http://arxiv.org/PS_cache/arxiv/pdf/0906/0906.2789v1.pdf>.

the accuracy rate of 88.8%.[32] His final paragraph notes: "Benford's Law has the capacity to serve as an effective indicator of data problems in accounting information" and also "Benford's Law has the potential to act as a highly effective detector of data error of fraud in accounting information".[33]

In addition to this, the Big 4 accounting firms use Benford's Law to conform to the fraud detection recommendations of the Statement of Auditing Standards No. 99. This statement requires accounting firms to assess the possibility of material misstatement as it relates to fraud in the audit of financial statements. Accounting firms' computer-assisted audit tools, like IDEA and ACL, have a command for Benford's Law enabling them to use Benford's Law as an initial "smell test". Although analytic methods by using Benford's Law alone usually cannot detect fraudulent financial reporting, deviations from Benford's Law should "cause an analyst to question the validity, accuracy, or the completeness of numbers".[34] Therefore, Benford's Law can still be an appropriate method to detect the possibility of fraud. The purpose of this paper is to examine if Benford's Law may be useful as an early indicator to detect the possibility of fraud in municipal governments' financial data. I find that Benford's Law, had it been applied in real time to Jefferson County, Vallejo City, and Orange County's financial statements,

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[32] Saville, AD. "Using Benford's Law to Detect Data Error and Fraud: An Examination of Companies Listed on the Johannesburg Stock Exchange." *SAJEMS* 9.3 (2006): 341-54. *Scribd*. Scribd. Web. 4 Apr. 2012. <http://www.scribd.com/doc/47789223/Saville-Using-282006-29>.
[33] Ibid.
[34] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 213. Print.

would have been able to detect that something was amiss and further investigation was warranted.

III. Methods

I first obtained the annual financial statements for Jefferson County, Vallejo City, and Orange County. For each municipality, this included each of the five years before filing for bankruptcy and the year of filling. The first test was done by applying Benford's Law to each of the six annual financial statements for each municipality, which resulted in eighteen statements total. For the second test, all six years' of data were compiled for each municipality, which created three sets of data for analysis, one for each municipality. For Jefferson County, I examined the annual financial statements for 2005, 2006, 2007, 2008, 2009, and 2010; Vallejo City 2003, 2004, 2005, 2006, 2007, and 2008; and Orange County 1989, 1990, 1991, 1992, 1993, and 1994.

The financial statement numbers were analyzed by applying the following rules which are based on Nigrini's rules in *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*:

1. The numbers were analyzed from the municipalities' Comprehensive Annual Financial Reports (CAFR). This included from the financial statements the statement of net assets, statement of activities, and the balance sheet.
2. In the first test, the numbers analyzed were only those for each fiscal year, while in the second test, the numbers included all six years compiled.

3. Numbers such as page numbers, dates, the number of notes, numbers in product descriptions (e.g., Concerta 18 or 87 octane fuel), and other general references to time (e.g., depreciation over 10 years or 90-day notes) were omitted.[35]
4. Numbers that are subtotaled or totals that do not convey any new information were omitted. For example, subtotals such as total current assets or total current liabilities were omitted. Since these subtotals and totals do not reflect any new information, they cannot be manipulated because they are the sums or differences between items.[36]
5. Also omitted were obvious duplications of information. For example, the statement of net assets usually includes cash and cash investments and these numbers are also reported on the balance sheet. These numbers on the balance sheet are omitted because they do not convey any new information.[37]
6. Zeros were omitted because zero is not a number with analyzable digits.[38]
7. Negative numbers were analyzed as if they were positive numbers.[39]

The digital analysis on each data set was done using a program called *NigriniCycle.xlsx*, which is a software program in Excel created by Mark J. Nigrini. In Excel, the columns A, B, and C contain any descriptive data or labels for the account. All of the data was entered into Column D. The spreadsheet analyzed the data from column D and generated Column E with the first digit of the number, column F with the second

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[35] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 218-219. Print.
[36] Ibid.
[37] Ibid.
[38] Ibid.
[39] Ibid.

digit, and column G with the first-two digits. The spreadsheet then automatically generated the digit tests with graphs that compared the actual frequency to the expected frequency from Benford's Law.

The three digit tests consisted of (a) first digits, (b) second digits, and (c) the first-two digits. The first-digit test was an overall test of the reasonableness of the data. The general rule is that if the first digit test was a weak fit to Benford's Law, it was a signal the data set contained abnormal duplications and anomalies. Therefore, the first digit test was a predictor test in that it was the first indicator of data issues. If there was a weak fit in the second digit test, it indicated issues with rounding numbers, such as an excess of 0s or 5s due to rounding numbers to 75, 100, and so on. The first-two digit test was the test that highlighted possible bias in the data. For example, if a company was analyzing company expenses, and there was a spike in the number 24, it could indicate that at a firm where employees are required to submit vouchers for expenses that are $25 and higher, people are submitting more expenses for $24 than in reality.[40]

In order to assess each digit test's conformity to Benford's Law, a test called the mean absolute deviation (MAD) was used. The formula is as follows:

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{K} \left|AP - EP\right|}{K},$$

where K represents the number of digits, AP denotes the actual proportion of the number of times a particular digit occurs within the data, and EP is the expected proportion

[40] Nigrini, Mark J. *Datas 2009 for Excel 2007 Program Details*. Web. <http://www.nigrini.com/datas_software.htm>

according to Benford's Law. Here, a "digit" may refer to a first-two-digit combination; so a "digit" may refer not only to any of the single digits from 1 to 9 but also to a two-digit number like 78, even though 78 is a first-two-digit combination. The deviation in the numerator measured the difference between the actual proportion and the expected proportion for each digit. For example, if there was an actual proportion of 0.0412 and an expected proportion of 0.0414 for the number 10, the deviation was the difference between the two numbers, which was -0.0002. The absolute symbol means that the deviation was given a positive sign irrespective of whether it was positive or negative. The summation sign in the numerator summed up the deviations from all the accounts and the denominator denoted dividing the summation by the total number of digits in the data set. This then equated to the mean absolute deviation.[41]

The MAD was the answer to whether or not the data set conformed to Benford's Law and to what extent it did or did not. The higher the MAD, the larger the average difference between the actual and expected proportions. Drake and Nigrini (2000) developed guidelines based on personal experience with testing everyday data tables with Benford's Law. Table 2 shows the MAD critical value ranges for the first, second, and first-two digits.[42] The *NigriniCycle.xlsx* template calculated the MAD in column N.

Although the MAD showed whether the data followed Benford's distribution, the Z-statistic test analyzed whether the actual proportion for a digit statistically differed from the expected proportion from Benford's Law. Again here, a digit also refers to not

[41] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 158-160. Print.
[42] Ibid.

only 1 or 9, but also to two-digit combinations such as 45. The Z-statistic formula measured the absolute magnitude of the difference between the actual and expected proportions, the size of the data set, and the expected proportion. The formula is:

$$Z = \frac{|AP - EP| - (1/2N)}{\sqrt{\frac{EP(1-EP)}{N}}}$$

where *AP* denotes the actual proportion of the digit, *EP* the expected proportion according to Benford's Law, and *N* the number of digits. The last term in the numerator (1/2N) is a continuity correction term and was only used when it was smaller than the first term in the numerator, but this had little impact on the calculated Z-statistic.

As the difference between the actual proportion, *AP*, and the expected proportion, *EP*, gets larger, the Z-statistic becomes larger. For example, if there was a spike in the data set at 32, this means there was a difference between the expected proportion of 0.0134 and actual proportion of 0.0152. "The size of the spike is then 0.0019 (rounded). If, for example, there were 19,2482 records greater than or equal to 10 in the data set, the Z-statistic is calculated to be 2.260. For a significance level of 5 percent, the cutoff level is 1.96. Our calculated Z of 2.260 exceeds this cutoff level, leading us to conclude that the actual proportion differs significantly from the expected proportion. For a 1 percent significance level, our cutoff score would be 2.57, and for this case, the calculated Z is not significant at the 1 percent level. The significance level is calculated by using Excel's *NORMSDIST* function. The result of *NORMSDIST* (2.260) is 0.988. The significance level is calculated by:

Significance = 2 x (1- *NORMDIST*(x))

A *NORMDIST* result of 0.988 gives a significance level of 0.024. Therefore, the result is significant at the 5 percent and the 2.4 percent level, but not significant at the 1 percent level."[43]

The effect of *N* in the denominator is that as the data set gets larger, the Z-statistic for any spike becomes larger. This means that a larger spike for a smaller data set might not be significant, and a smaller spike for a larger data set might be significant. For example, the "spike of 0.0019 becomes more significant as the data increases in size. Using the same actual proportion, if the data had 100,000 records, the calculated Z-statistic would be 5.178. With a fivefold increase in the number of records, the Z-statistic would only double in size because *N* is inside the square root sign."[44]

The *EP*, the expected proportion, is used twice in the denominator because for any given difference, a smaller expected proportion gives a larger Z-statistic. "In the preceding example, there is a 0.19 percent difference (0.0152-0.0134). If the expected proportion was (say) 5.0 percent and the actual was 5.19 percent, we would still have a 0.19 percent difference but the Z-statistic would be lower and insignificant at 1.200. This is logical because the 0.19 percent difference is a bigger difference for an expected percentage of 1.34 percent than it is for an expected percentage of 5.00 percent. The general rule is that a difference is more significant for the higher digits (which have lower

[43] Nigrini, Mark J. "Fraudulent Financial Statements, Part II." *Benford's Law: Applications for Forensic Accounting, Auditing, and Fraud Detection*. Hoboken, NJ: Wiley, 2012. 150. Online.
[44] Ibid.

expected proportions) than for the lower digits (which have higher expected proportions)."[45]

The Z-statistic tells us whether the actual proportion for a digit deviates significantly (in the statistical sense of the word) from the expected proportion. For Benford's Law, a significance level of 5 percent was used. This means if there was a Z-statistic for a digit of 1.96 or higher, it was significant. The *NigriniCycle.xlsx* template calculated the Z-statistic in column G for the first and second digits test and column 0 for the first-two digits test.

The MAD test concluded whether or not the data set overall conformed to Benford's Law, while the Z-statistic measured whether a specific digits' deviation was significant. If the data did not conform to Benford's Law according to the MAD, then, I could infer the municipality was possibly manipulating its data, and could then conclude where the manipulation was occurring according to the Z-statistic. Further investigation could then be focused.

IV. Results

My analysis was be done by conducting two different tests for each municipality. For the first test, I analyzed the financial statements from five years before filing for bankruptcy and the year of filing, and applied Benford's Law to each fiscal year. In the second test, all of the six years of data were aggregated into one data set. If the data sets do not follow Benford's distribution, I can then conclude there was possibly a

[45] Ibid.

manipulation of the data and further analysis of potential fraud would have been warranted.

A. Jefferson County

1. First tests

The first digits for Jefferson County's 2005 annual financial data show that the accounting data does not conform to Benford's Law. The mean absolute deviation (MAD) is 0.05687, which exceeds the 0.015 critical value for nonconformity by a wide margin. Figure 1 shows the difference in the actual proportions and the expected proportion of first digits from Benford's Law. The bars represent the actual proportions of the digits and the line represents the expected proportions. Since there is overall nonconformity to Benford's Law in the first digit test, this signals the data set may have abnormal duplications and anomalies. However, none of the first digits have a Z-statistic higher like 1.96, meaning the individual differences in the actual and expected frequencies are not significant.

The second digits also show nonconformity to Benford's Law. The MAD of 0.06656 exceeds the critical value of 0.012 for overall nonconformity. Figure 2 shows the proportions of all the digits' deviation from Benford's Law. However, according to the Z-statistics, the digit 0 is the only one that deviates significantly. It has a Z-statistic of 2.162, which is above the 1.96 threshold for 5% conformity. This implies that there were issues within the data set like rounding numbers.

The first-two digits show nonconformity to Benford's Law because it has an MAD of 0.01713, which exceeds the critical level of 0.0022. The second order results in

Figure 3 show large spikes throughout the data set at many different first-two digits. According to the Z-statistics, the significant differences are at the digit 20, with a Z-statistic of 2.736, and 60, with a Z-statistic of 3.126. The accounting data then indicates fraud is possibly occurring and a closer look is warranted.

Jefferson County's 2006 annual financial data also shows an overall nonconformity to Benford's Law. The first digits do not conform to Benford's Law with an MAD of 0.04186. Figure 4 shows the differences between the expected and actual frequencies of the first digits. The actual frequency of the digit 5 significantly differs from the expected frequency because of the Z-statistic of 2.174. The second digits also show nonconformity with an MAD of 0.05350. In Figure 5, how the actual frequencies of the second digits differ from the expected frequencies is shown. However, none of them differ significantly because they all have Z-statistics lower that 1.96. With an MAD of 0.01289, the first-two digits also do not conform to Benford's Law. Figure 6 shows spikes throughout the data first-two digits data. There is significant deviation at the digits 80 and 95 because of their Z-statistics of 2.918 and 3.254. Like the results from 2005, Jefferson County's 2006 results indicate there is a possibility of fraud occurring in the data and a further look is indicated.

For Jefferson County's 2007 annual financial data, the first digits do not conform to Benford's Law with an MAD of 0.07533. The digits 3 and 8 actual frequencies significantly differ from their expected frequencies, with Z-statistics of 2.042 and 2.017 respectively. The second digits also show nonconformity with an MAD of 0.06335. The difference between the actual and expected frequencies of the digit 4 is significant with a

Z-statistic of 1.992. The differences in frequencies are shown in Figures 7 and 8. The first-two digits have an MAD of 0.01611, which also demonstrates nonconformity to Benford's Law. Figure 9 shows the fist-two digits all occur more often than expected. However, the significant differences occur at the digits 34, 39, and 84, with a Z-statistics of 2.126, 2.350, and 3.836. The evidence from the years of 2005, 2006 and 2007 show a trend that Jefferson County may have been practicing fraud.

The annual financial accounting data for Jefferson County in 2008 also illustrates nonconformity to Benford's Law. The first digits have an MAD of 0.04090, the second digits an MAD of 0.03801, and the first-two digits, an MAD of 0.01464. However, none of the digits in all three tests have significant differences between their actual and expected frequencies. Figure 10, 11, and 12 show the overall data differences in the actual and expected frequencies of the digits. There was a trend of increasing disparity from the years 2005 to 2007, followed by a year of no specific disparity in 2008.

The first digits, second digits, and first-two digits for Jefferson County's 2009 annual financial data demonstrate overall nonconformity to Benford's Law with MADs of 0.04646, 0.04701, and 0.01376 respectively. As seen in Figure 13 and 14, the first digits and second digits do not conform to Benford's distribution. However, none of the digits' frequency differences are significant with Z-statistics lower than 1.96. Figure 15 shows the occurrences of all the first-two digits, with spikes at 13, 33, and 55. However, only the digits 33 and 55 are significant with Z-statistics of 2.079 and 2.961,

The 2010 annual financial data for Jefferson County also implies there is the possibility of data manipulation. The MAD for the first digits is 0.03482, which is excess

of the 0.015 critical value for nonconformity, as seen in Figure 16. The second digits also do not conform to Benford's Law with an MAD of 0.04930. As seen in Figure 17, the digit 0 differs significantly with a Z-statistic of 2.162, which indicates that rounding was occurring. The first-two digits have an MAD of 0.01446, which is also in excess of the critical value of 0.0022 for nonconformity. In Figure 18, there are spikes at every first-two digits combination, however none of them are statistically significant.

For each of the annual financial statements for Jefferson County, there has been ongoing overall nonconformity to Benford's Law implicating there is a possibility of fraud and a closer review is necessary.

2. Second Test

To apply the second test, the annual financial annual data from 2005-2010 for Jefferson County were compiled. The first digits have an MAD of 0.05687. Figure 19 shows this is because there are very few occurrences of the digit 1 and over occurrences of the digit 8. However, neither of these are significant with Z-statistics of 1.755 and 1.109, which is lower that the 1.96 significance level. The MAD of 0.06656 for the second digits is warranted because of the excess frequency of the 0 digit and the lack of the 1, 5 and 7 digits, as seen in Figure 20. The digit 0 is the only digit that differs significantly with a Z-statistic of 2.162. As seen in Figure 21, the fist-two digits have an MAD of 0.01713 because of the spikes at all digit combinations. This is especially because of the digits 20 and 60 have Z-statistics of 2.736 and 3.128. This implies that Jefferson County manipulated their financial data and if the accountants had known, then should have done further analytical tests.

B. Vallejo City

1. First Tests

The results from the tests of the annual financial data for Vallejo City imply there is the possibility of fraud. For Vallejo City's 2003 annual financial data, the first digit test and second digit tests have MADs of 0.05132 and 0.03881, which is above the 0.015 and 0.012 critical values for nonconformity, respectively. Figure 22 illustrates that there are too many occurrences of the digit 6 in the first digit place, with a Z-statistic of 3.062. Figure 23 shows the differences of the actual and expected frequencies of the second digits, however none of the differences are significant according to their Z-statistics. The first-two digits test also does not have overall conformity to Benford's Law, as seen in Figure 24, and exceeds the critical value of 0.0022 for nonconformity with an MAD of 0.01479. This is because of the spikes at all the digit combinations, especially at 60, 67, with a Z-statistics of 3.128 and 3.350.

Vallejo City's 2004 annual financial data also implies there may have been fraud. The first digits have an MAD of 0.05288 because of their overall nonconformity, as seen in Figure 25. However, none of the first digits actual frequencies significantly differ from the expected frequencies of Benford's Law. As seen in Figure 26, the second digits have an MAD of 0.03925 because of the lack of the frequency of the digit 3 and the digit 4 occurring too frequently. However, the digits Z-statistics are not high enough to be significant. The first-two digit test has a MAD of 0.01461 due to larger than expected frequencies of all the digit combinations, especially the digit 47, which is significant due to its Z-statistic of 2.672, as seen in Figure 27.

The first digits test, second digits, and first-two digits tests for Vallejo City's 2005 annual financial data also have MADs higher than the critical value for nonconformity; 0.05064 for the first digits, 0.04237 for the second digits, and 0.01449 for the first-two digits. As seen in Figure 28, this is due to the excess frequency of the digit 6 in the first digit place, although it is not statistically significant. Figure 29 shows that in the second digit place, the even digits are favored more than the odd digits with higher than expected frequencies of the digits 2, 4, 6, and 8. However, the differences in the actual and expected frequencies of each digit are not significant either. There are spikes in Figure 30 at all of the digit combinations in the first-two digits test, but especially at 31 and 68, which are significant because of their Z-statistics of 1.981 and 5.900.

The data from Vallejo City's 2006 annual financial data also points to the possibility of fraud. This is because of the first, second, and first-two MADs are higher than the critical values of overall nonconformity; 0.05379, 0.05465, and 0.01535, respectively. Figure 31 and 32 show the overall differences in the actual and expected frequencies of the first and second digits, although none of them are significant with lower than 1.96 Z-statistics. On the other hand, the first-two digits actual frequencies vary significantly from the expected frequencies, especially at the digits 18 and 36 with Z-statistics of 2.527, 4.062, as seen in Figure 33.

The first digits test for Vallejo City's 2007 annual financial data has an MAD of 0.04356 and the second digits test has an MAD of 0.04408. Therefore the data does not conform to Benford's Law. However, none of the digits actual and expected frequencies differences are significant, but are shown in Figures 34 and 35. The first-two digits have

an MAD of 0.01498, which confirm an overall nonconformity, especially at the digit 31, which has a Z-statistic of 1.981.

Vallejo City's 2008 annual financial data shows overall nonconformity with Benford's Law as seen in Figure 37 and 38. The first digits test has an MAD of 0.02893 and the second digits test, an MAD of 0.04381. Although there is overall nonconformity, none of the digits differ significantly according to their Z-statistics. The first-two digits test also shows nonconformity in Figure 39 with an MAD of 0.01483. This is due to the digits 18, 35, and 46, which are significant because of their Z-statistics of 2.527, 2.173, and 2.633.

Each of the annual financial statements for 2003-2008 for Vallejo City showed overall nonconformity, and all according to specific digits. This indicates that there was a possibility of fraud and further analysis should be done on the financial statements.

2. Second Test

To conduct the second test, I compiled the data of the annual financial statements for 2003-2008 for Vallejo City. The first digits test has an MAD of 0.05132, which is above the critical value for nonconformity of 0.015. Figure 40 shows the difference between the actual and expected frequencies of the first digits. There is a significant excess frequency of the digit 6 because of its Z-statistic of 3.062, which is above the 1.96 significance level. The second digits test has an MAD of 0.03881, which is also above the critical value for nonconformity of 0.012. Figure 41 shows the data's nonconformity to Benford's Law, although none of them are significant because their Z-statistics are all below 1.96. The first-two digits test has an MAD of 0.01479, which is in excess of the

0.0022 critical value for nonconformity. This is because of the digit 60 and 67 have Z-statistics of 3.128 and 3.350, as seen in Figure 42. These are also the two digits that were statistically significant from Vallejo City's 2003 annual financial data. All of the tests conducted on Vallejo City's financial data point to the possibility of fraud occurring and a close look was necessary.

C. Orange County

1. First Tests

For the first test, each of Orange County's annual financial data from 1989-1994 was analyzed. For 1989, both the first and second digits tests show there is an overall nonconformity to Benford's Law with MADs of 0.04998 and 0.04185. However, no digits' actual frequencies deviate significantly from the expected frequencies because all of them Z-statistics lower than 1.96. However, the first-two digits test not only demonstrates overall nonconformity with an MAD of 0.01513, but significantly at the digit 17, with a Z-statistic of 2.416. The differences in the actual and expected frequencies of the first, second, and first-two digits can be found in Figures 43, 44, and 45.

Orange County's 1990 annual financial data also illustrates overall nonconformity to Benford's Law. The first and second digits have MADs of 0.04798 and 0.06822. Figure 46 and 47 show the differences between the first and second digits actual and expected frequencies. However, their Z-statistics demonstrate that no particular digits' actual frequencies significantly differ than the expected frequencies. The first-two digits

test has an MAD of 0.01440 because of the excess frequency of the digit 49 with a Z-statistic of 2.746, as seen in Figure 48.

The annual financial data for 1991 for Orange County is very similar to that of 1990 because the first and second digits tests show overall nonconformity to Benford's Law, as seen in Figure 49 and 50, with MADs of 0.03534 and 0.03867. However, there are no digits with Z-statistic high enough to be significant within the test. The first-two digits test has an MAD of 0.01608, which also shows there is overall nonconformity, as seen in Figure 51. This is especially due to the over occurrences of the digits 21 and 39 because of their Z-statistics of 2.836 and 2.350.

Orange County's 1992 annual financial data did not conform to Benford's Law in the first, second, or first-two digits tests. Figure 52 shows the first digits overall nonconformity because of an MAD of 0.04696. However, no particular digits' Z-statistic was high enough to be significant. For the second digit test, there is also nonconformity with a MAD of 0.05636. This is due to the excess frequency of the digit 0 with a Z-statistic of 2.162, as seen in Figure 53. The first-two digits test has an overall nonconformity to Benford's Law, as seen in Figure 54, because of its MAD of 0.01530. This is due to the higher than expected frequencies of the digits 34 and 89 with Z-statistics of 2.126 and 3.994.

The annual financial data for 1993 also lacks conformity to Benford's Law. The first digit does not have overall conformity, as seen in Figure 55, with an MAD of 0.05217. However, no particular digit's frequency significantly differs from expected because all have Z-statistics lower than 1.96. With an MAD of 0.05246, the second digit

test also does not conform to Benford's distribution in Figure 56. The digit 4's actual frequency is significantly higher than expected, with a Z-statistic of 2.658. The first-two digits have an MAD of 0.01480 and lack conformity, as seen in Figure 57, especially because the digit 84 occurs more frequently than expected with a Z-statistic of 3.836.

Orange County's 1994 first, second, and first-two digit test show nonconformity to Benford's Law with MADs of 0.03262, 0.04300 and 0.01464, as seen in Figure 58, 59 and 60. Although all three tests show overall nonconformity, none of the digits' actual frequencies differ significantly from their expected frequencies. For the years 1989-1993, specific digits showed nonconformity, while for 1994, the year Orange County filed for bankruptcy, no specific digits raised red flags.

2. Second Test

To perform the second test, all the annual financial data for 1989-1994 was compiled. The first and second digits tests demonstrate nonconformity with MADs of 0.04998 and 0.04185, which are above the critical values for nonconformity of 0.015 and 0.012. However, none of the digits actual frequencies significantly differ from the expected frequencies because their Z-statistics are all lower than 1.96. The first-two digits test had an MAD of 0.01513, which is also above the critical value of 0.0022 for nonconformity, significantly the digit 17 with a Z-statistic of 2.416.

All of the tests for Orange County had overall nonconformity with Benford's Law, and in most cases, specific digits raised red flags. There is a high likelihood that Orange County was committing fraud and these results warrant that further investigation was needed.

V. Discussion

The tests conducted on the annual financial statements of Jefferson County, Vallejo City and Orange County all showed overall nonconformity to Benford's Law.

It is important to take note that Benford's Law is designed to be more useful when applied to much large data sets. When looking at the graphs in the appendix, some of the obvious gaps could have been caused by the limited amount of data that was being used. With this sort of analysis, one wishes that one had about 10 times more data, but this is not possible because financial statements inherently do not provide a robust set of data. However, one can only work with the information that is available and Benford's Law is still proven to be useful in these situations.

The nature of statistical analysis includes the risk of false positives and false negatives just by random chance. This possibility applies to my tests too, in that violations of Benford's Law from all the two digit combinations from 10 to 99 might occur by chance due to the limited nature of the data. For example, if the first-two digit test rejected the null for, say, 26, 48, and 68, this could occur randomly and not actually indicate fraud. While the data from one annual financial statement may pass or fail Benford's Law, a series of data sets from the same municipality over several consecutive years would reduce that risk substantially. Because I did in fact use a series of years, the analysis is still valid and Benford's Law is a good leading indicator. For example, Jefferson County showed a pattern from 2005 to 2007 of specific two-digit combinations that showed nonconformity and then in 2008, had a year with none. Even with that year of more conformity, the trend analysis would still create suspicion of fraud. It must be

noted that one year of conformity could also be caused by something other than a false negative, like a change in the personnel overseeing the accounting.

Because of these possible errors, we cannot and reach a firm conclusion that fraud actually occurred in some cases. However, we have the benefit of being able to look retrospectively at these situations of where we know where fraud existed. If a professional had reviewed the financial statements using Benford's Law, there would have been at least a strong inference of fraud and that would compel that profession to investigate further and possibly where to investigate. Even that has value.

VI. Conclusion

This thesis explored if fraud or mismanagement in municipal governments could have been diagnosed or detected in advance of their bankruptcies by financial statement analysis using Benford's Law. The annual financial data for Jefferson County, Vallejo City, and Orange County were analyzed because we know these three municipalities committed fraud or grossly mismanaged their finances. By conducting two different tests, I was able to determine that the data did not conform to Benford's distribution, and therefore it had possibly been manipulated and a closer look at the financial statements was necessary. Benford's Law therefore can be useful as an early indicator to detect the possibility of fraud in municipal governments' financial data. It could have even been used as a warning sign to have prevented these bankruptcies.

Although Benford's Law wasn't used to prevent these bankruptcies, it can still be used in resolving disputes in the aftermath of the bankruptcies. For example, the current litigation in bankruptcy court between the bondholders and other constituents in Jefferson

County over the available capital, could be assisted by using Benford’s Law. It is likely that evidence of financial fraud could tip the balance between those parties in that litigation. Additionally, Benford’s Law might be able to be applied now to mitigate bankruptcies and decrease the amount damage of them. With the increasing rate of municipalities going bankrupt, Benford’s Law can be a very valuable tool for the municipalities about to consider bankruptcy, and even prevent others from entering into it.

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VIII. Appendix

Table 1. Benford's Law: Expected Digital Frequencies

Digit	**1st Place**	**2nd Place**
0		.11968
1	.30103	.11389
2	.17609	.19882
3	.12494	.10433
4	.09691	.10031
5	.07918	.09668
6	.06695	.09337
7	.05799	.09035
8	.05115	.08757
9	.04576	.08500

Source: Nigrini, 1996

Table 2. Digits and Conformity Range

Digits	**Range**	**Conclusion**
First Digits	0.000 to 0.006	Close conformity
	0.006 to 0.012	Acceptable conformity
	0.012 to 0.015	Marginally acceptable conformity
	Above 0.015	Nonconformity
Second Digits	0.000 to 0.008	Close conformity
	0.008 to 0.010	Acceptable conformity
	0.010 to 0.012	Marginally acceptable conformity
	Above 0.012	Nonconformity
First-Two Digits	0.0000 to 0.0012	Close conformity
	0.0012 to 0.0018	Acceptable conformity
	0.0018 to 0.0022	Marginally acceptable conformity
	Above 0.0022	Nonconformity

Figure 1. Jefferson County 2005 First Digits

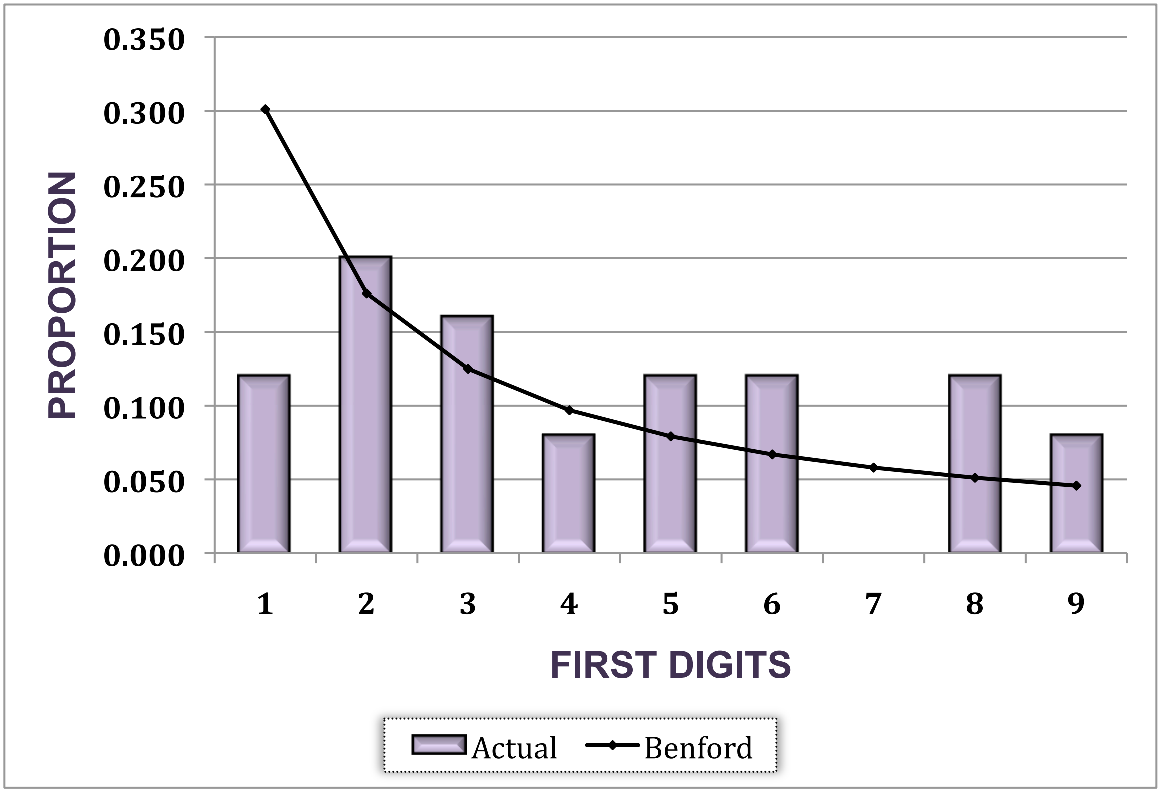


Figure 2. Jefferson County 2005 Second Digits

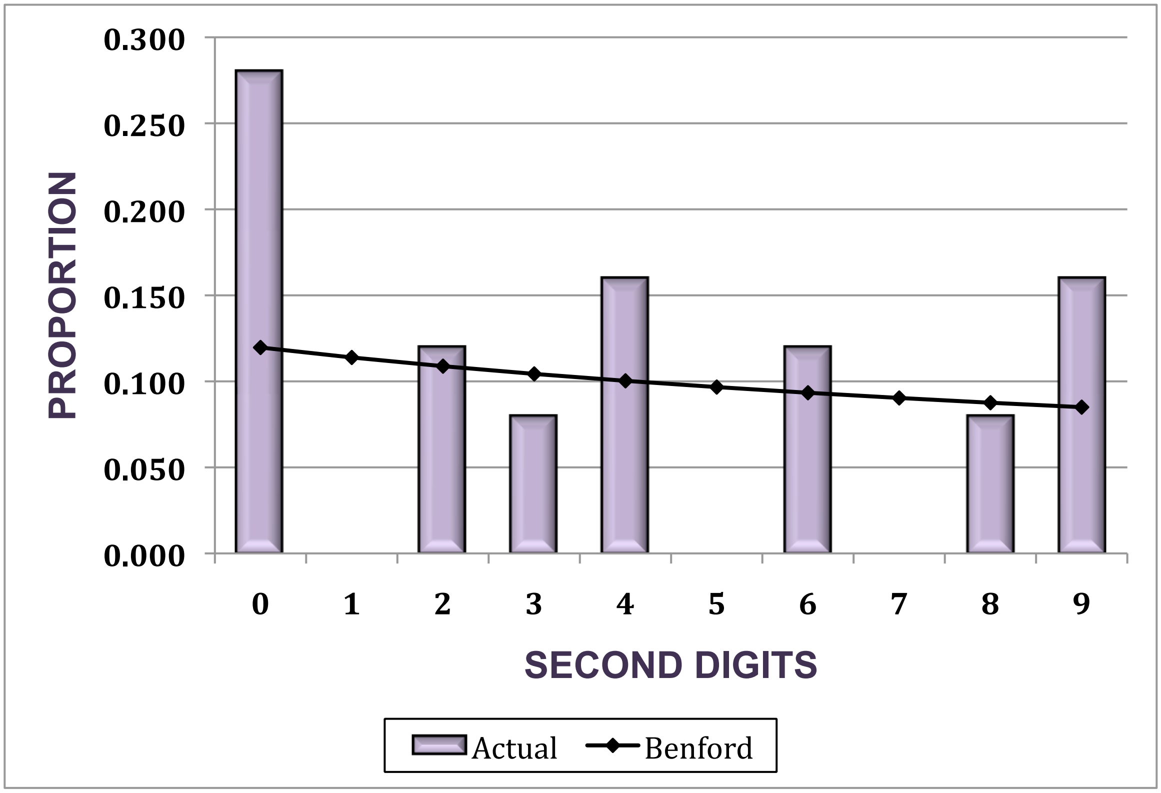


Figure 3. Jefferson County 2005 First-Two Digits

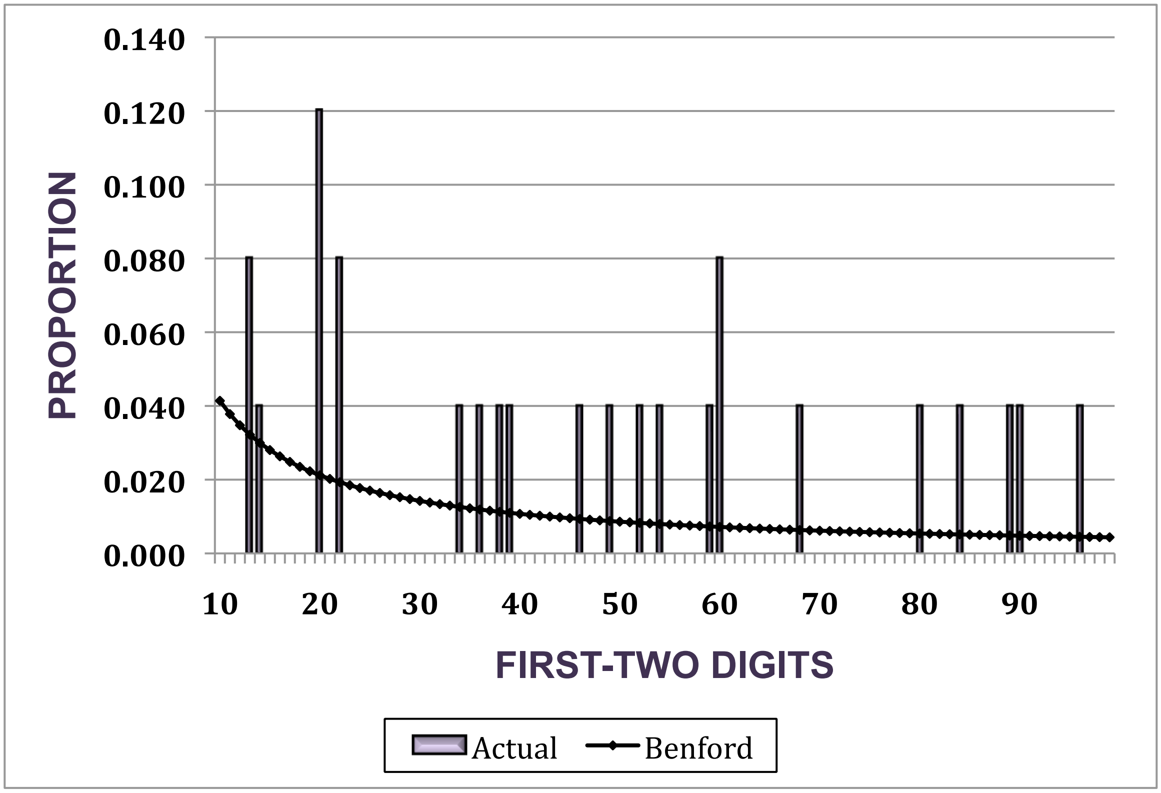


Figure 4. Jefferson County 2006 First Digits

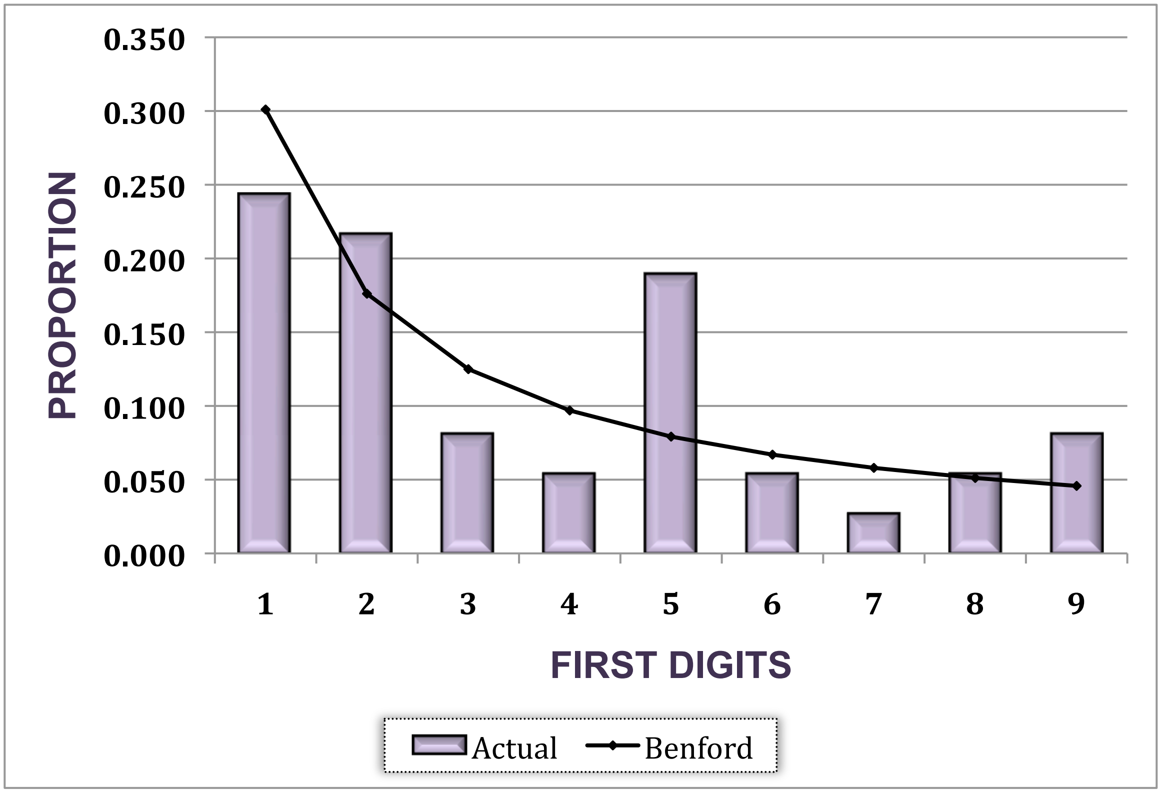


Figure 5. Jefferson County 2006 Second Digits

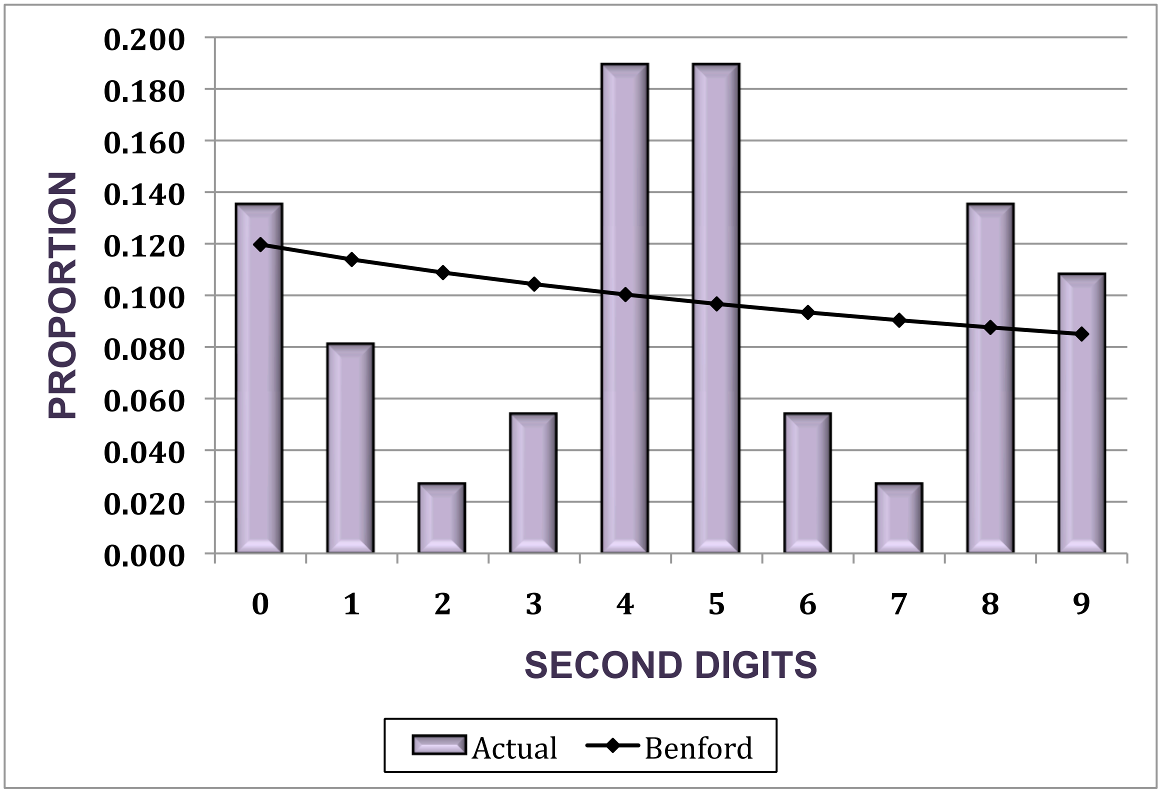


Figure 6. Jefferson County 2006 First-Two Digits

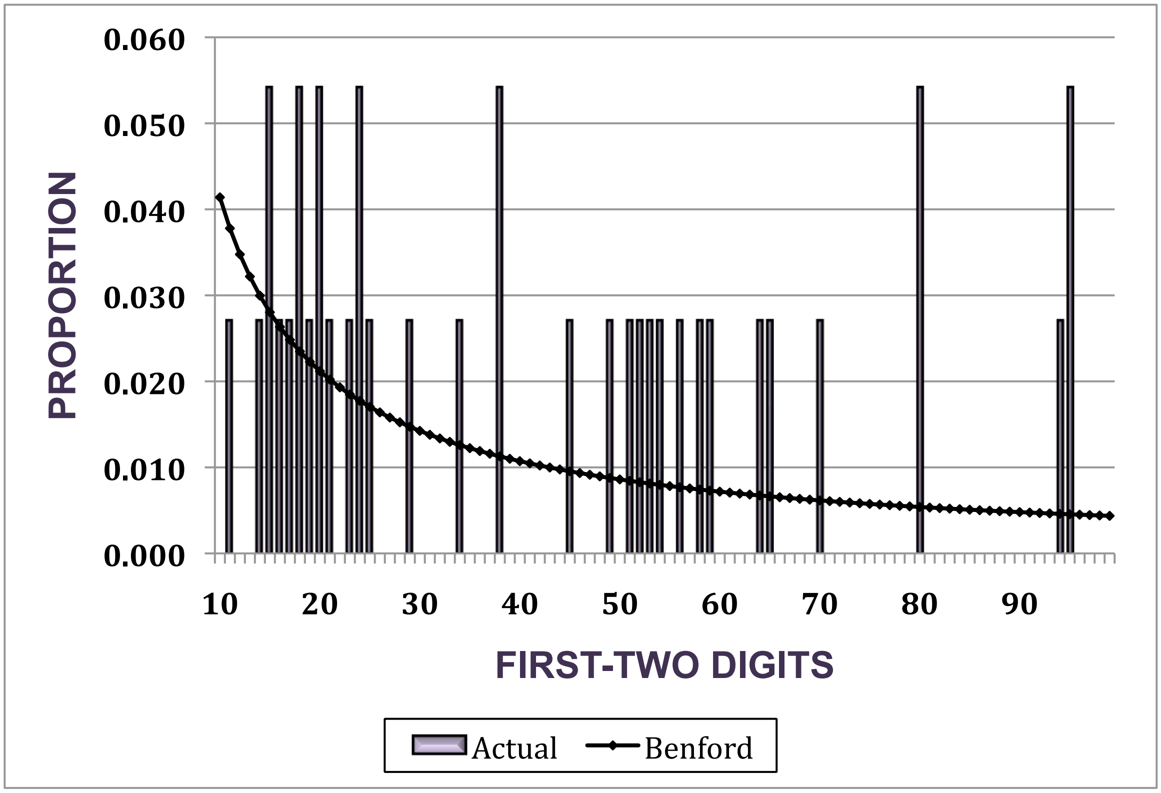


Figure 7. Jefferson County 2007 First Digits

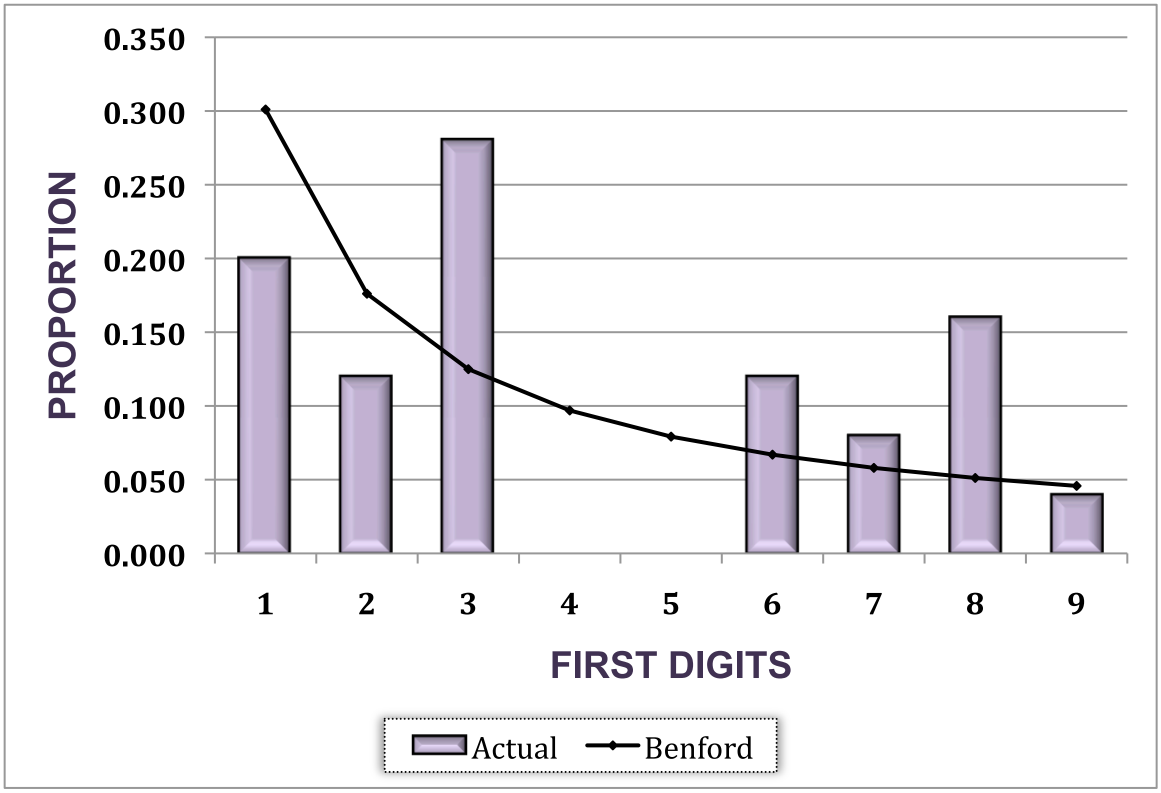


Figure 8. Jefferson County 2007 Second Digits

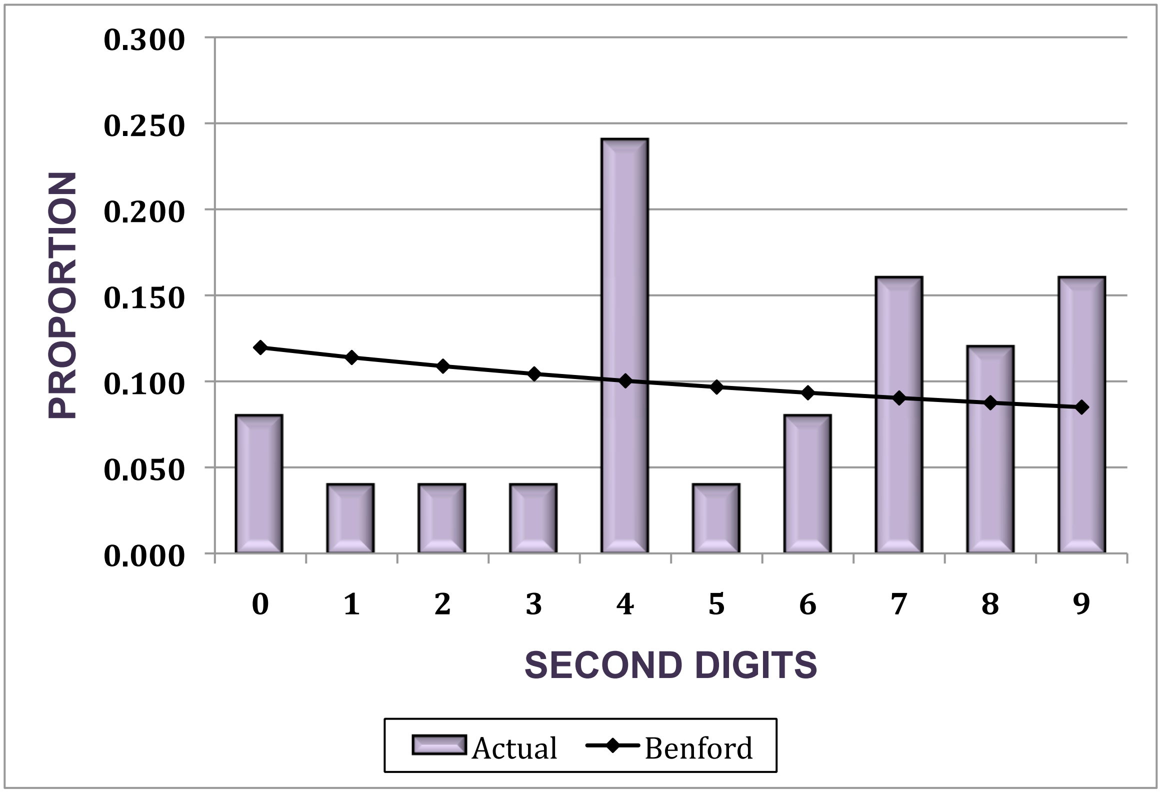


Figure 9. Jefferson County 2007 First-Two Digits

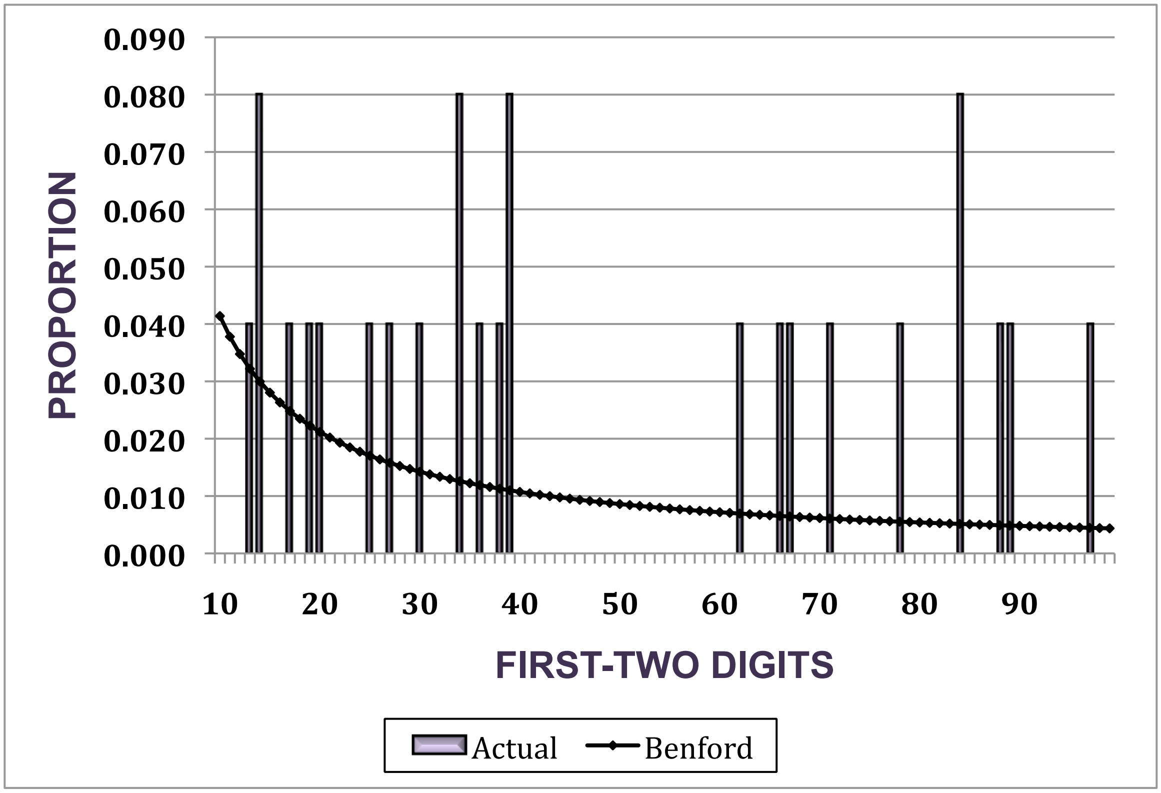


Figure 10. Jefferson County 2008 First Digits

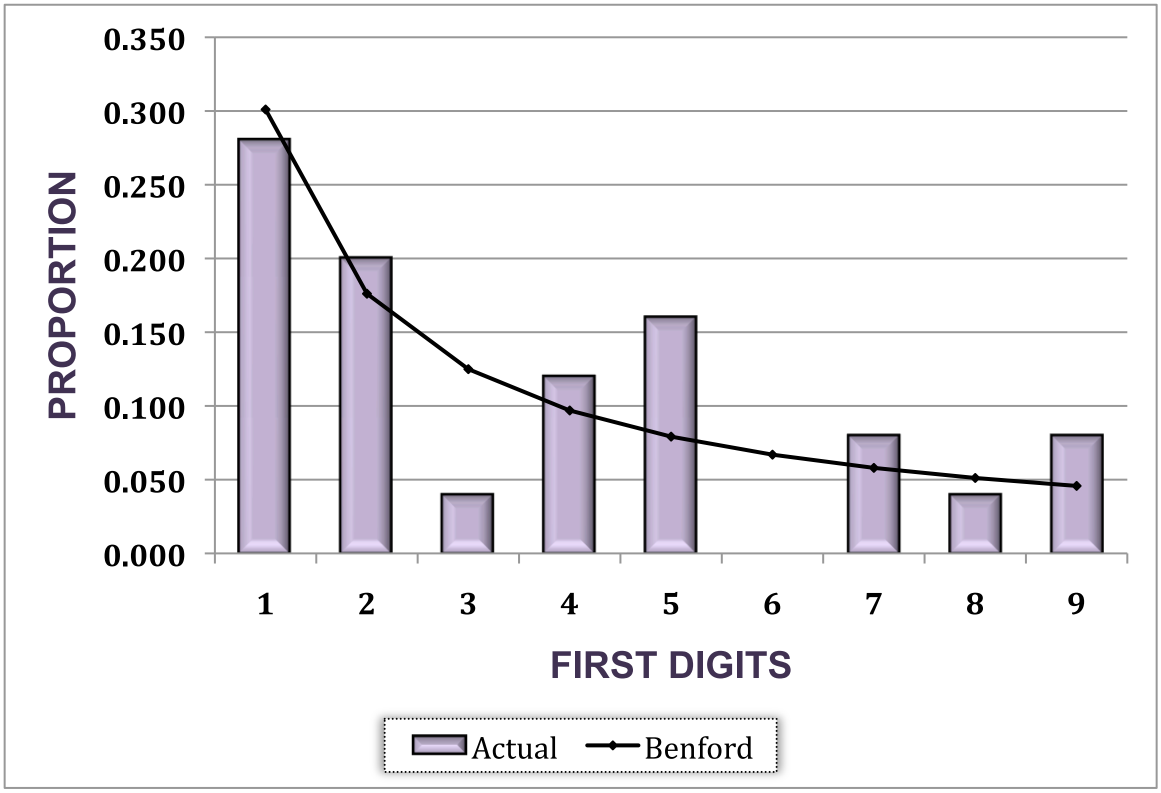


Figure 11. Jefferson County 2008 Second Digits

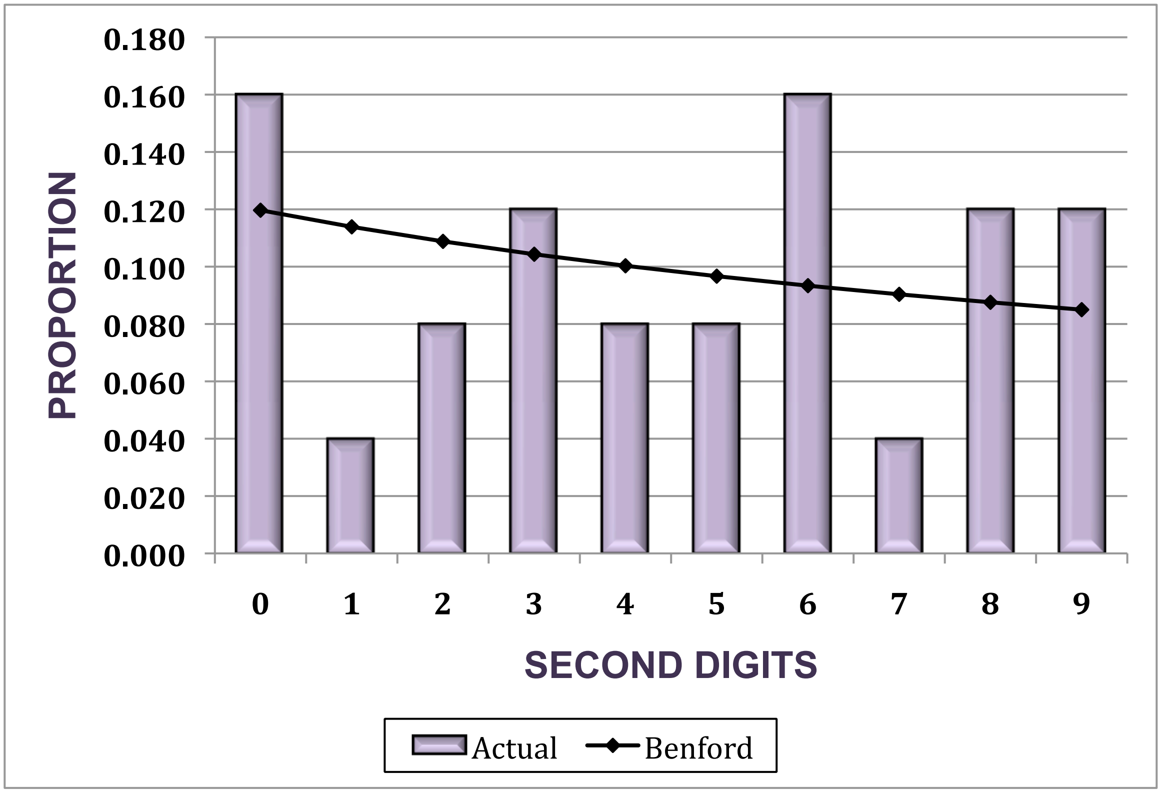


Figure 12. Jefferson County 2008 First-Two Digits

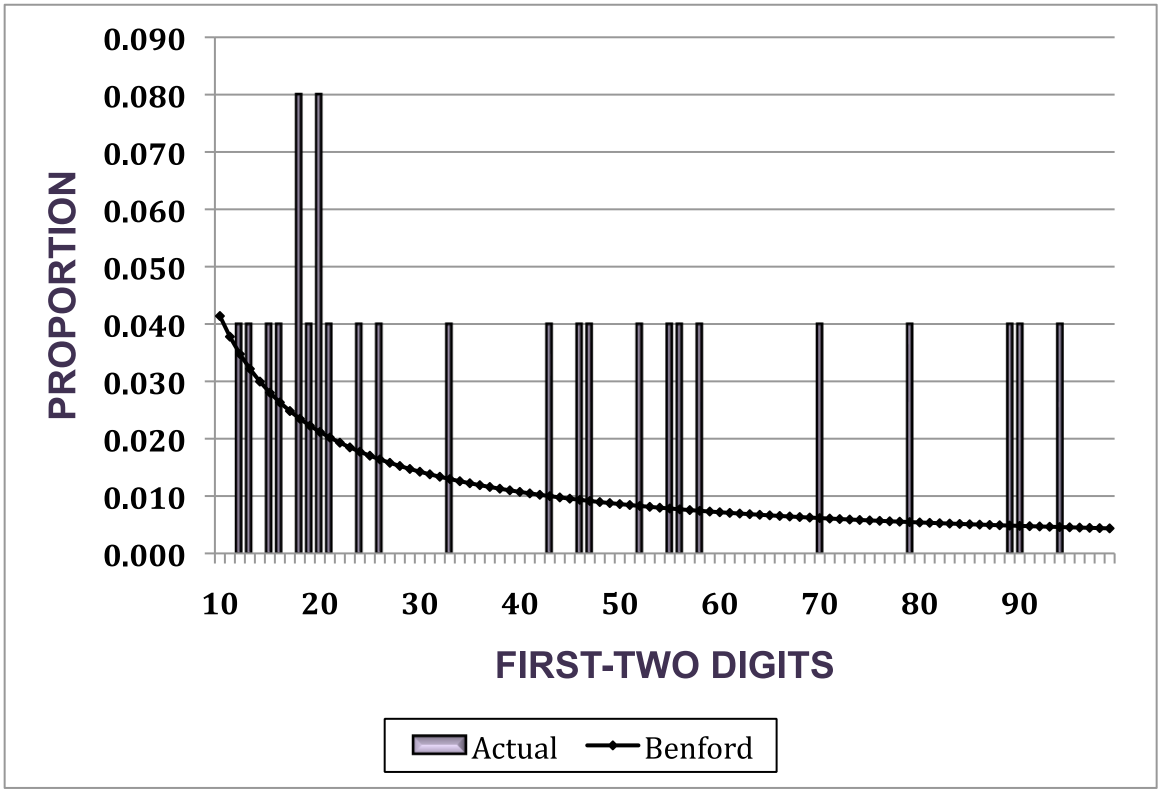


Figure 13. Jefferson County 2009 First Digits

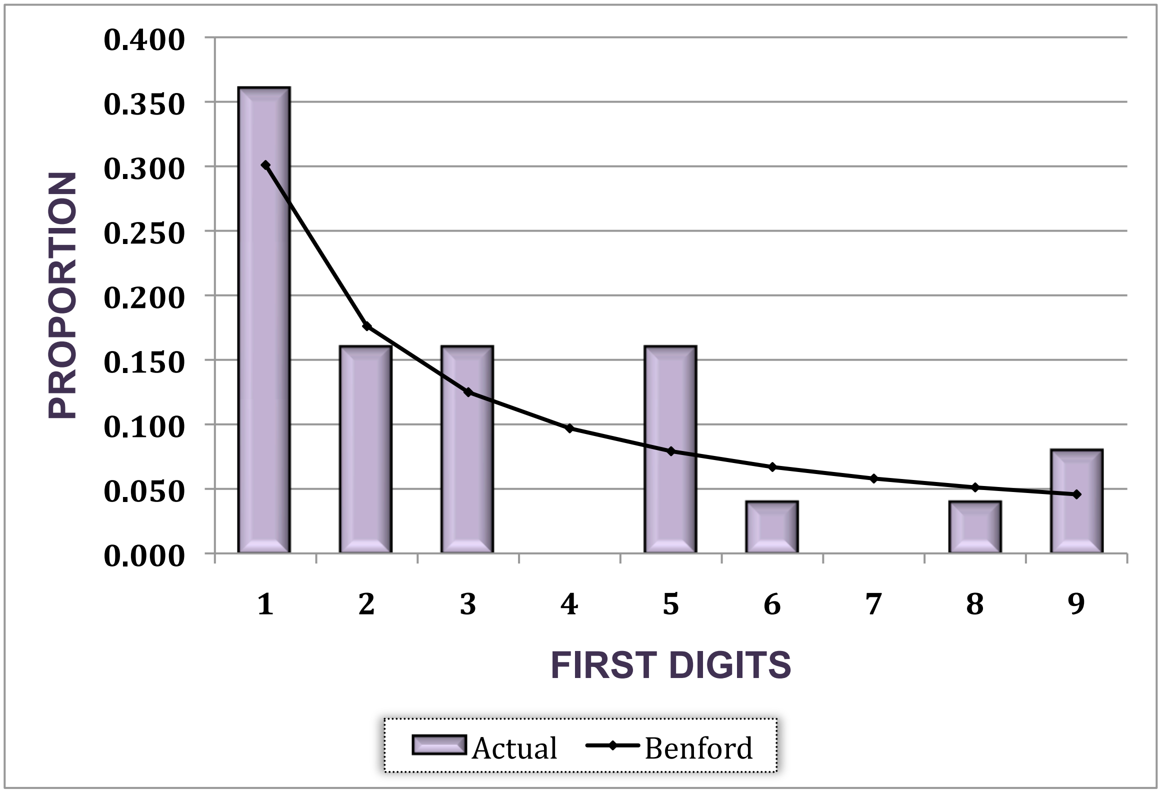


Figure 14. Jefferson County 2009 Second Digits

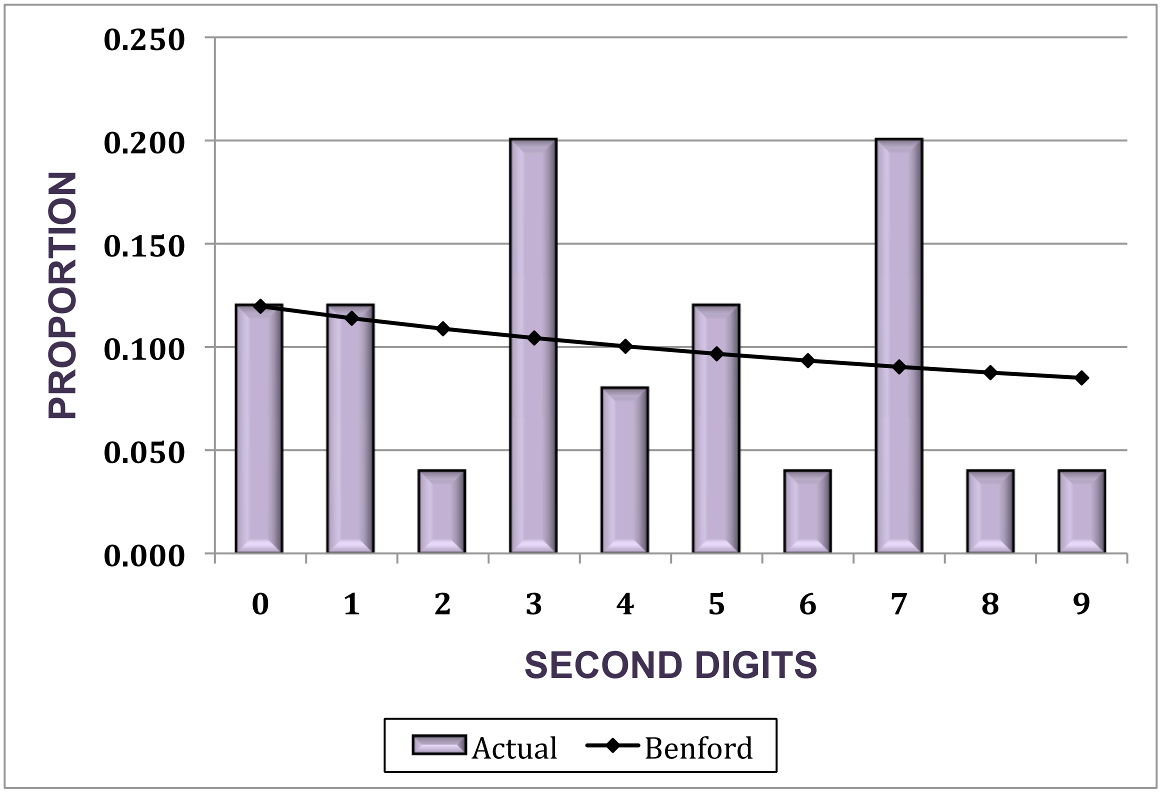


Figure 15. Jefferson County 2009 First-Two Digits

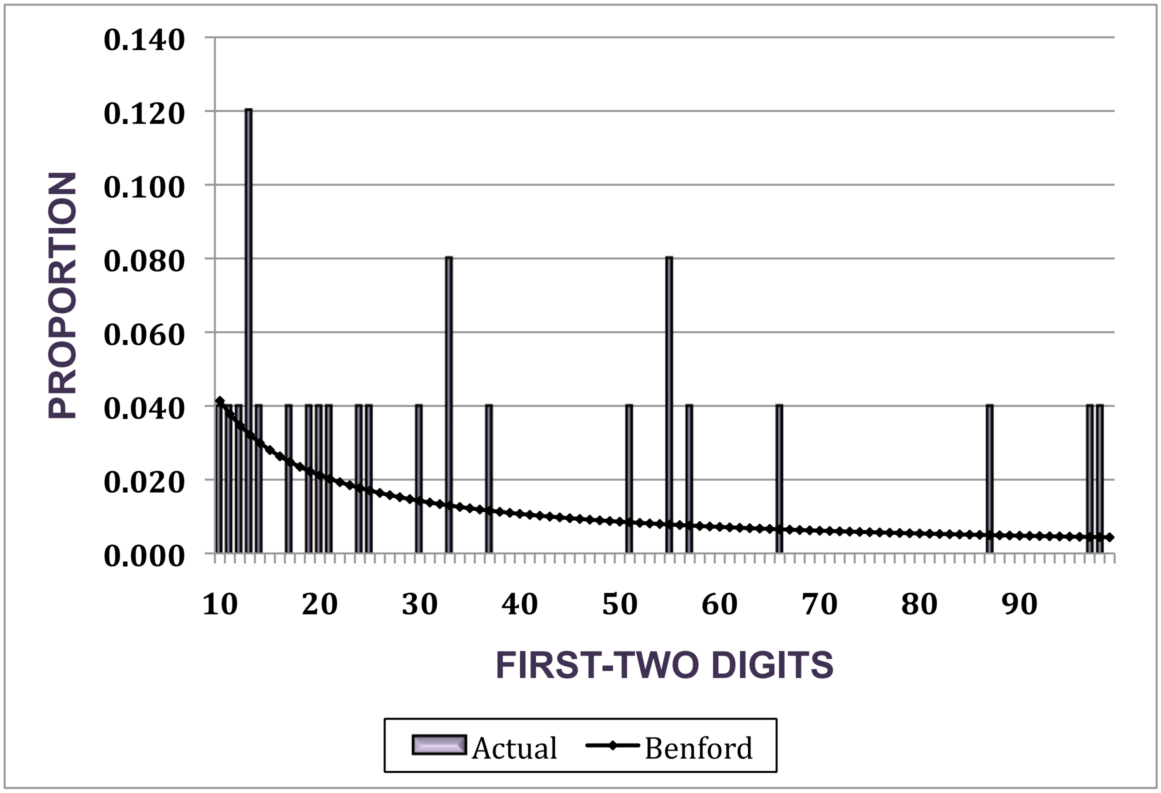


Figure 16. Jefferson County 2010 First Digits

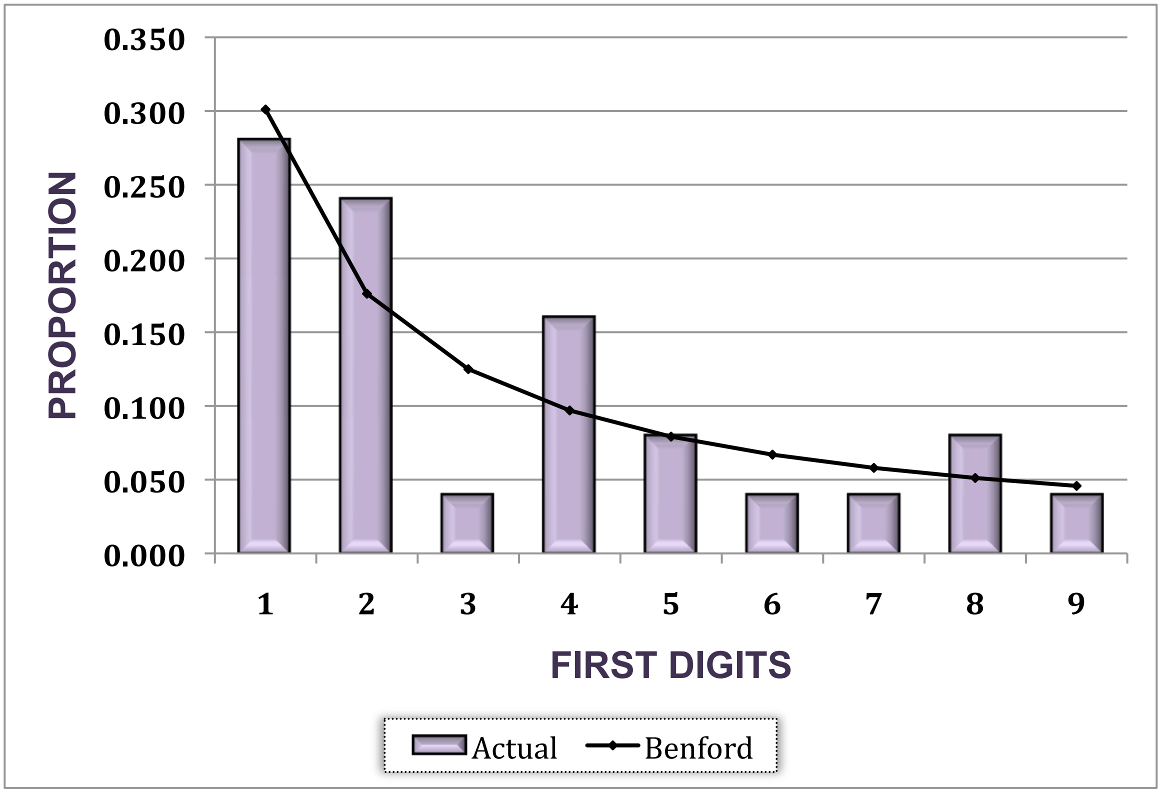


Figure 17. Jefferson County 2010 Second Digits

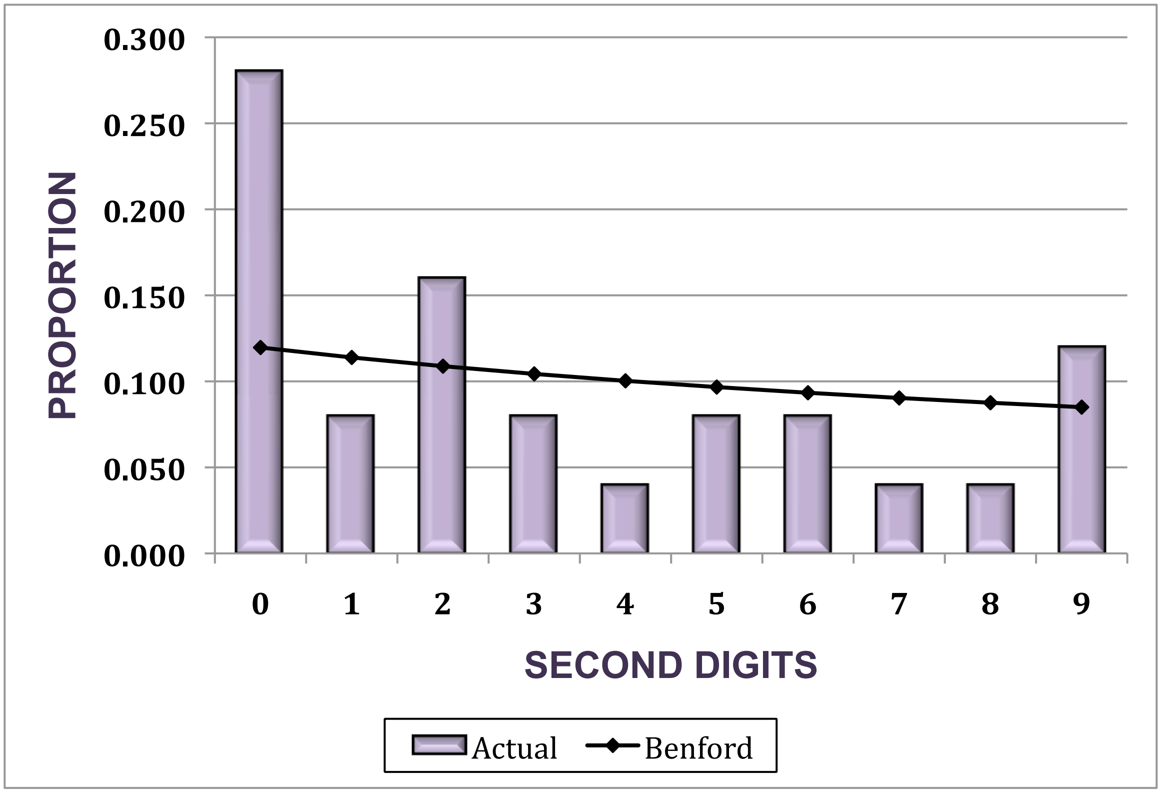


Figure 18. Jefferson County 2010 First-Two Digits

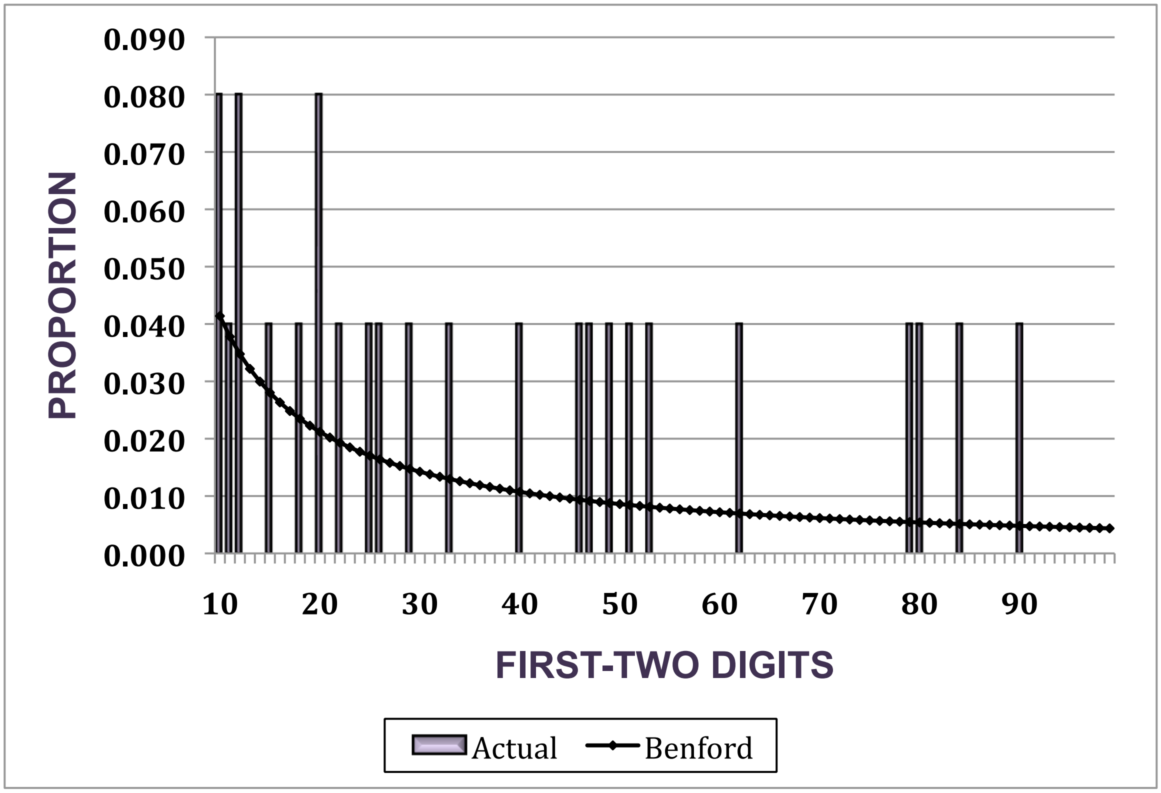


Figure 19. Jefferson County 2005-2010 First Digits

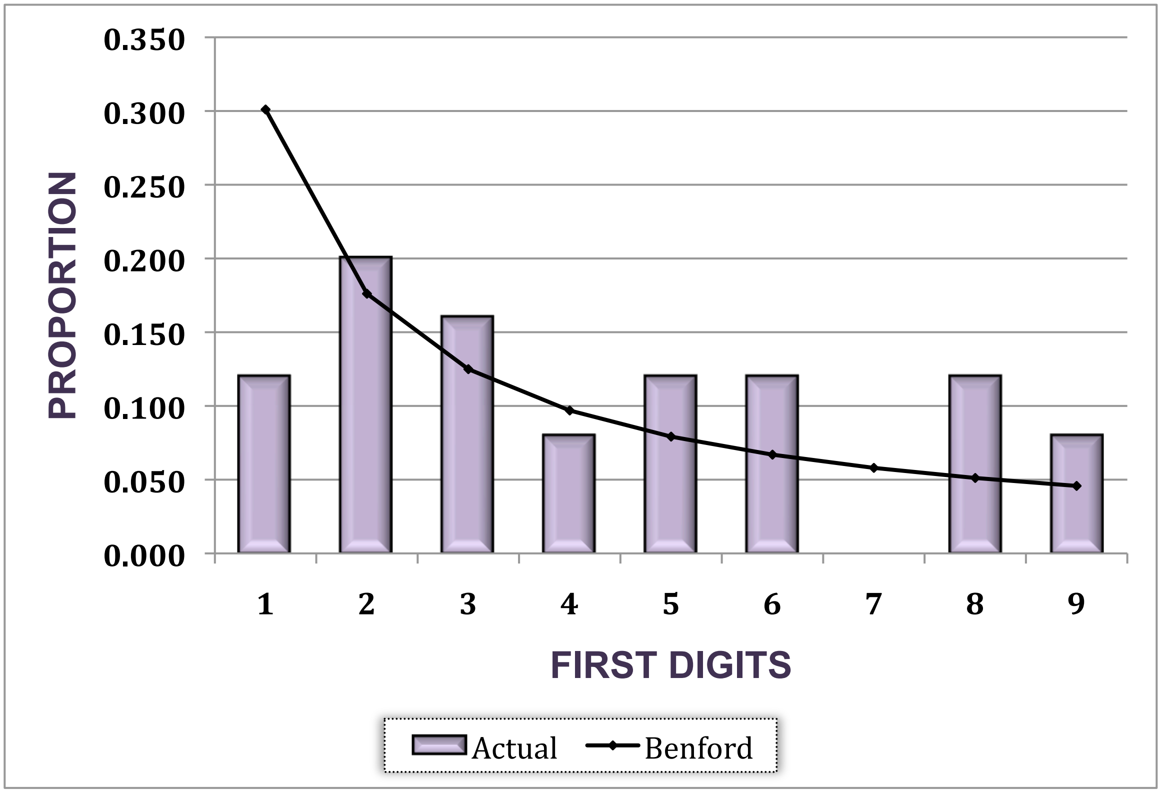


Figure 20. Jefferson County 2005-2010 Second Digits

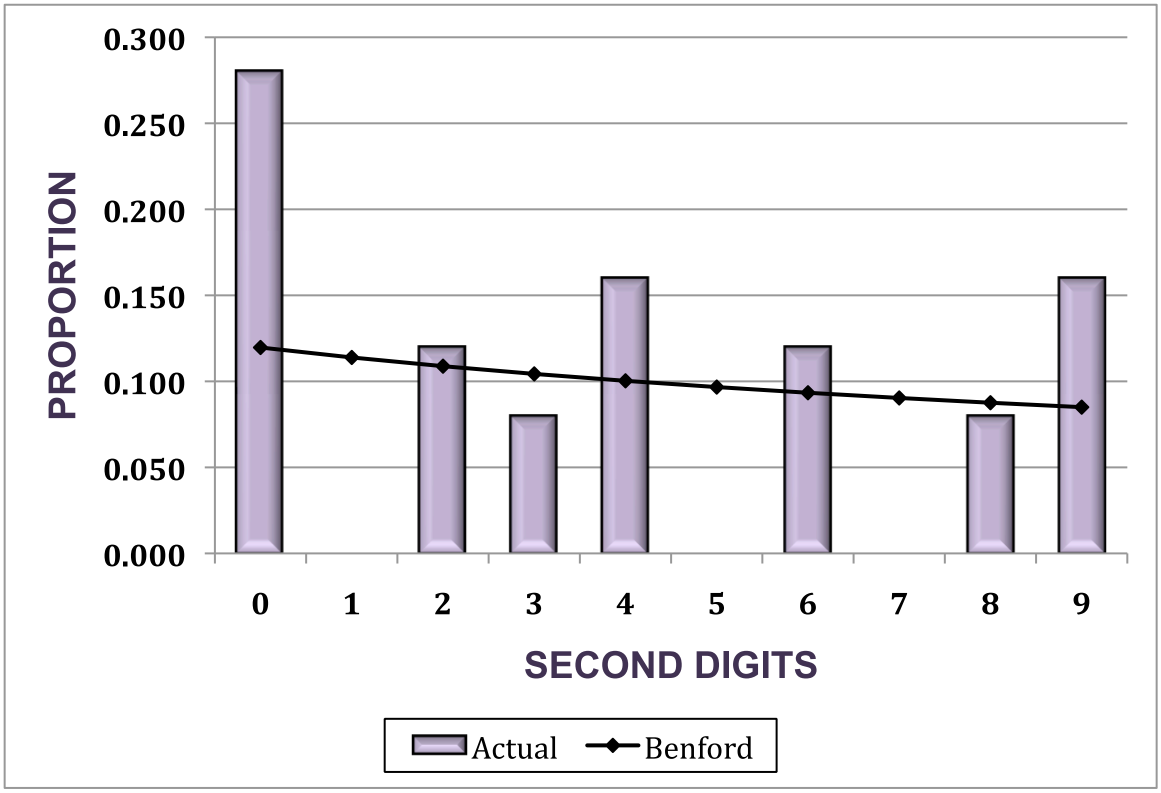


Figure 21. Jefferson County 2005-2010 First-Two Digits

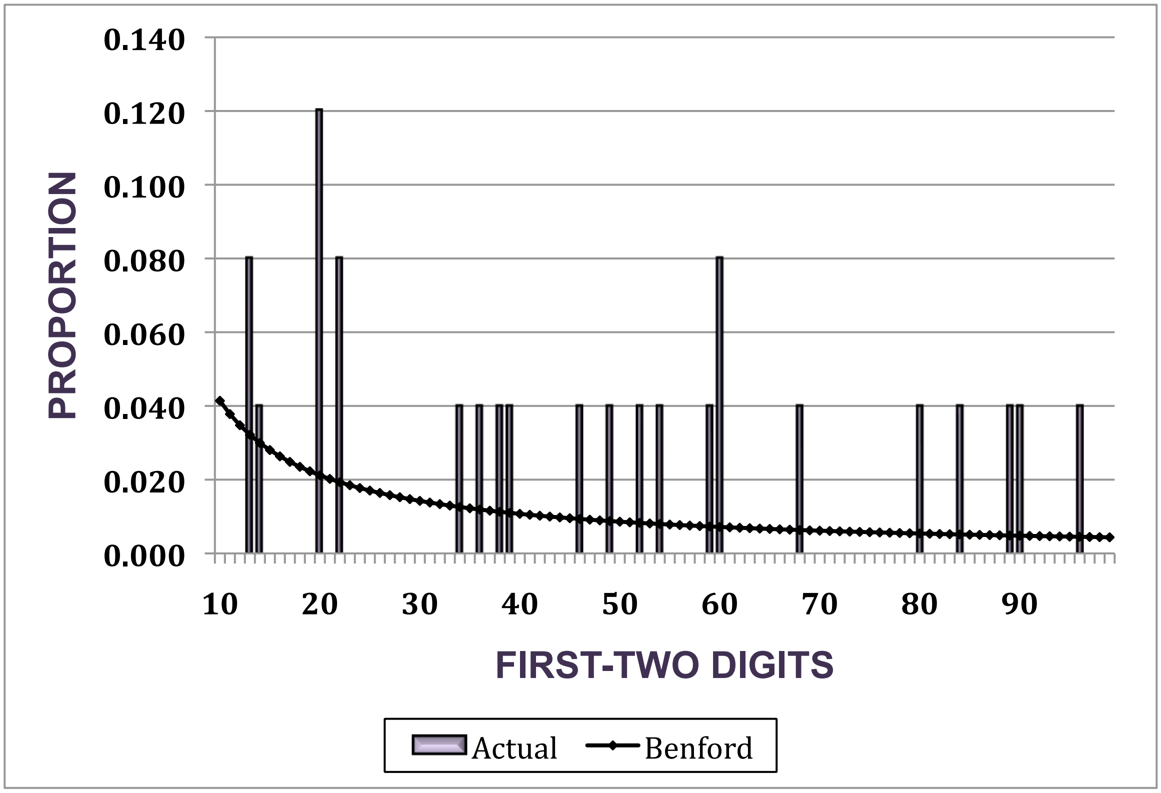


Figure 22. Vallejo City 2003 First Digits

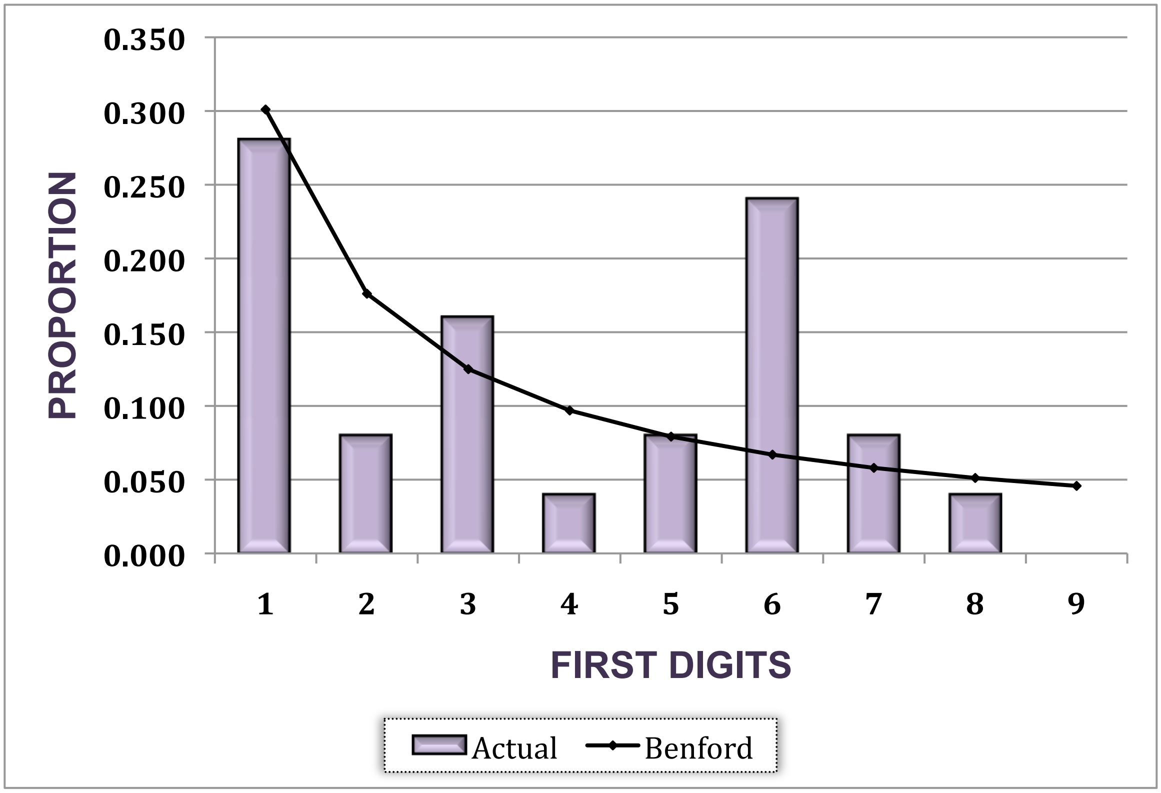
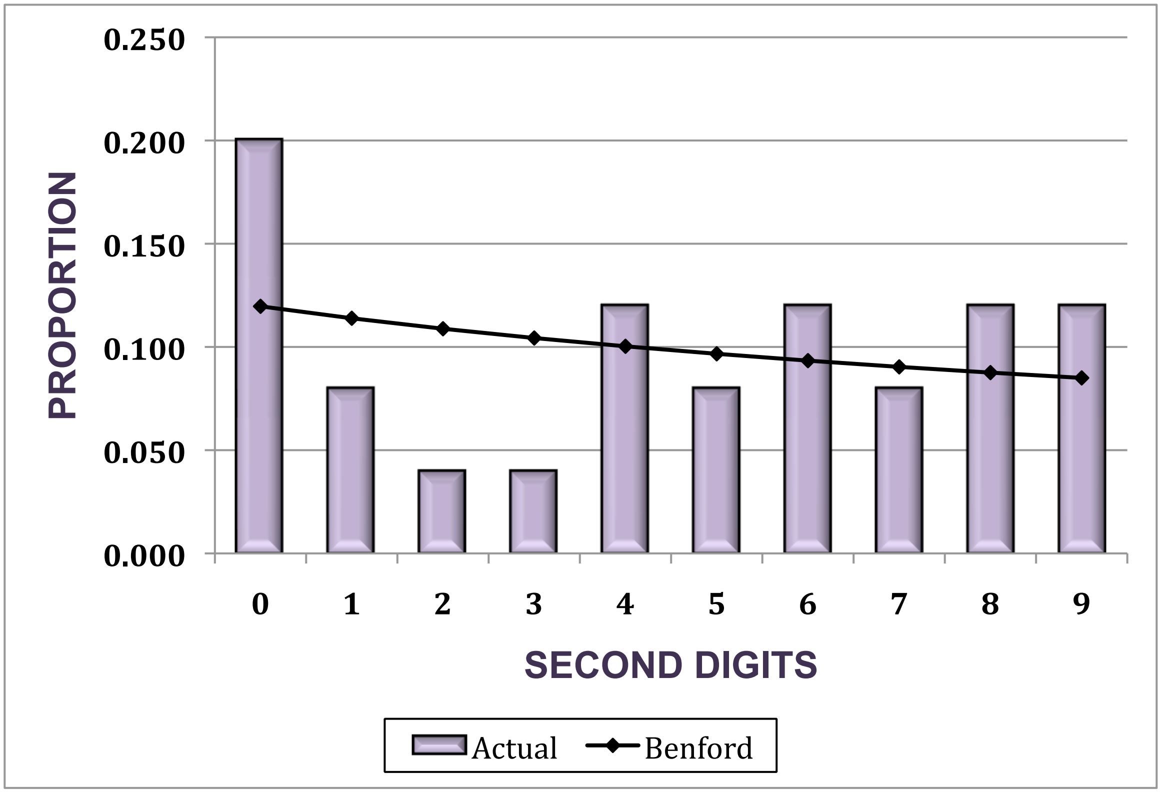


Figure 23. Vallejo City 2003 Second Digits



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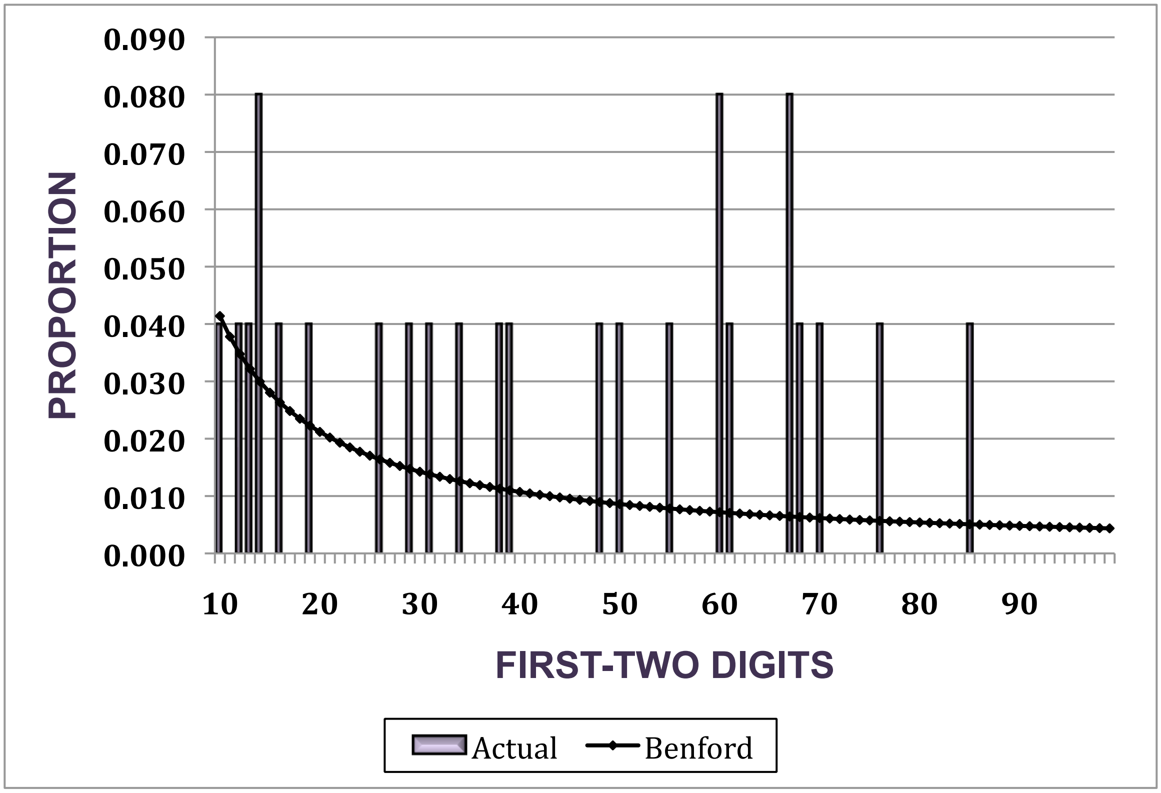

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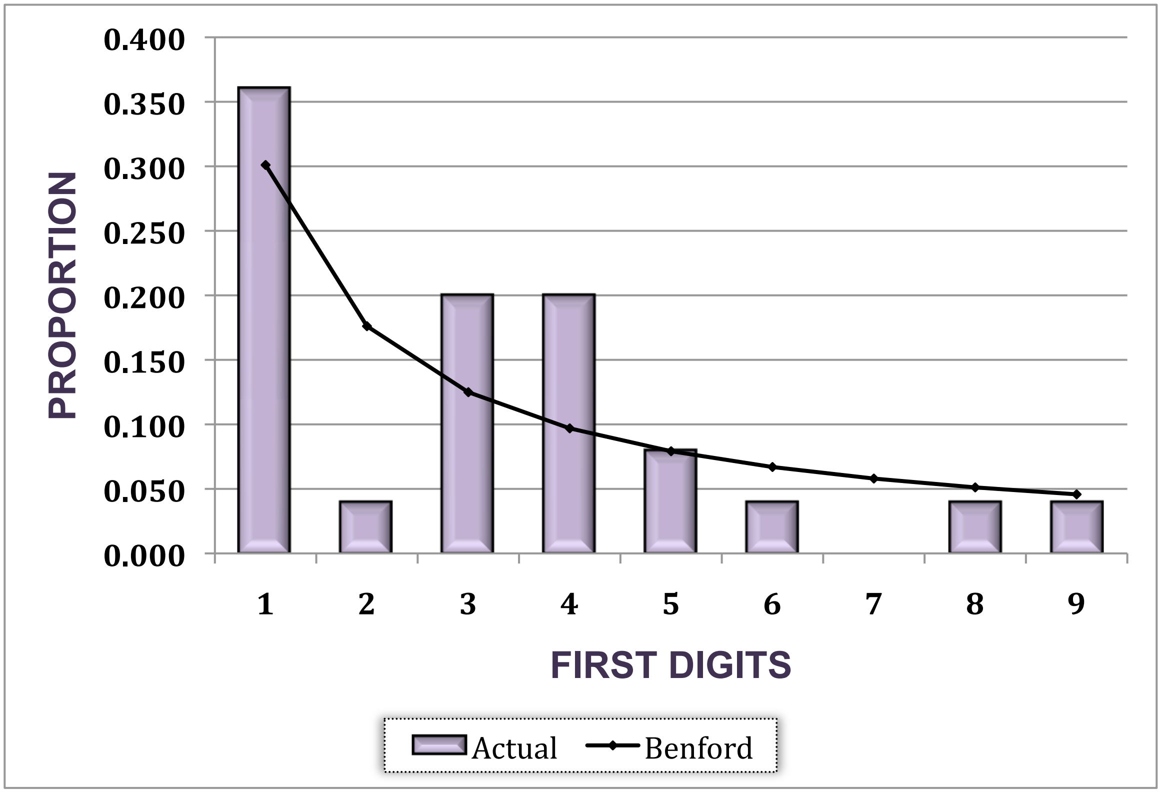

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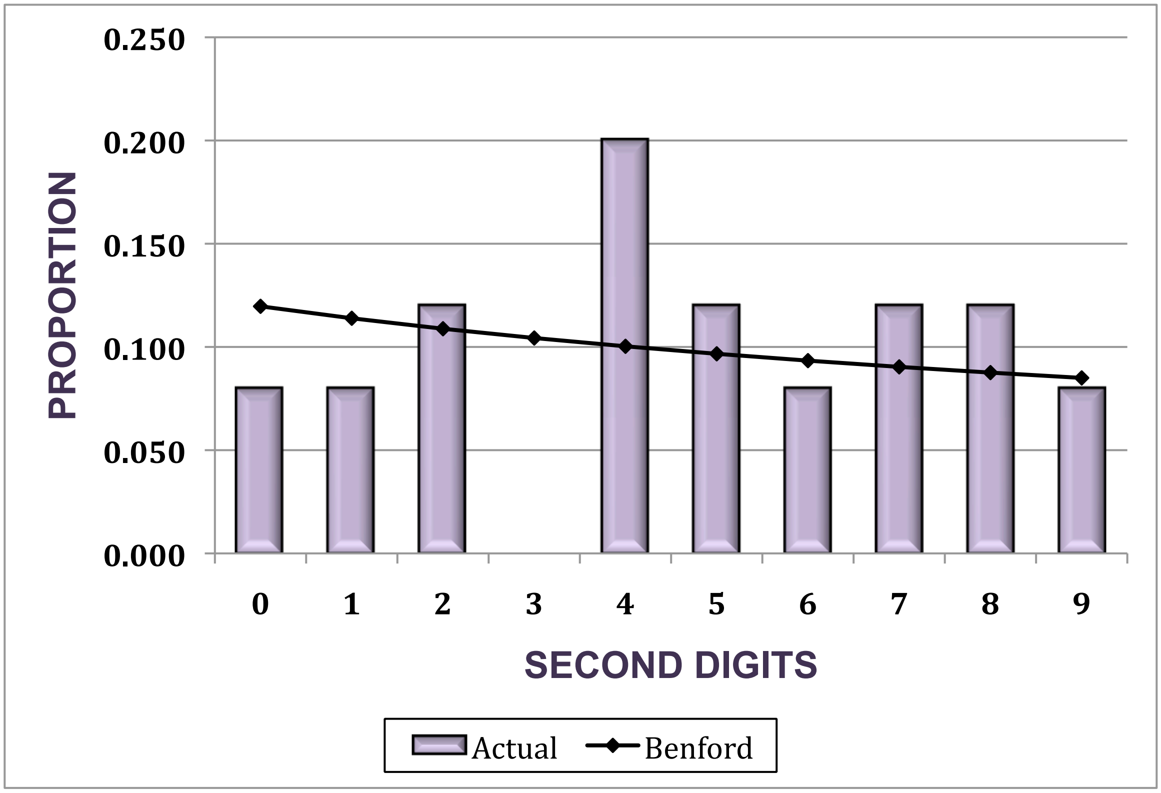
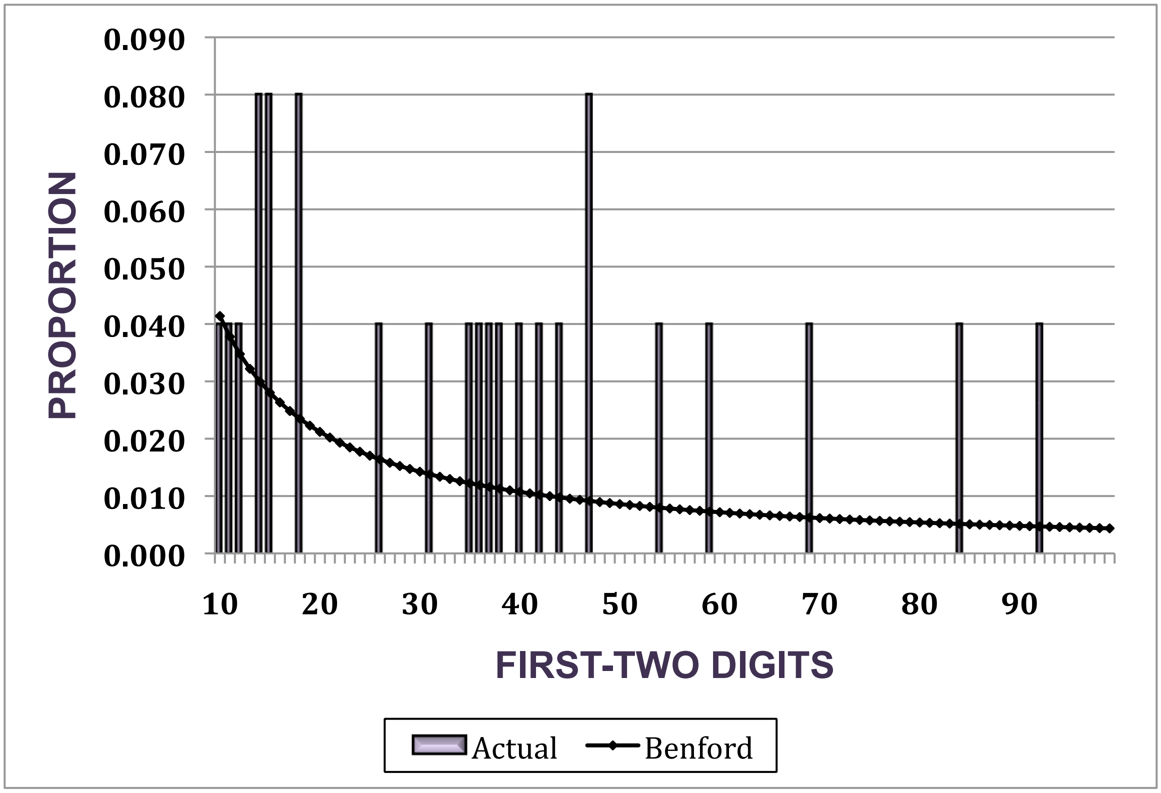


Figure 27. Vallejo City 2004 First-Two Digits



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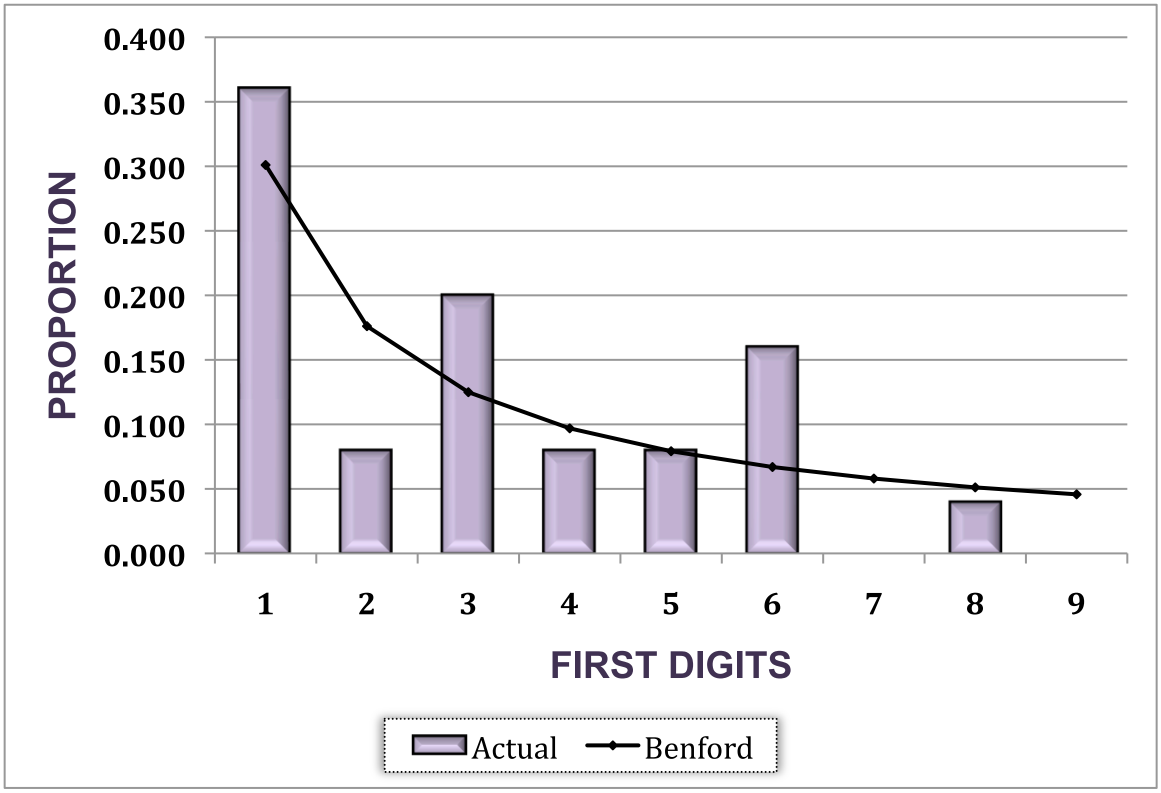

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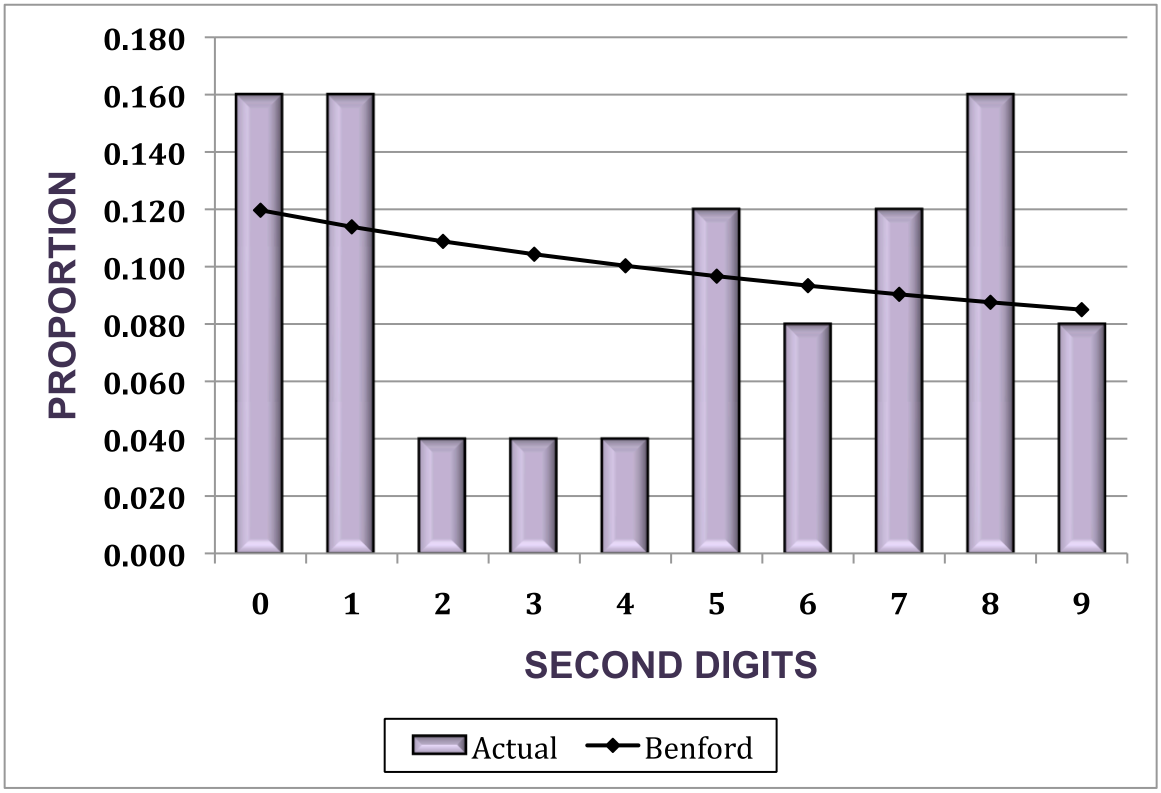

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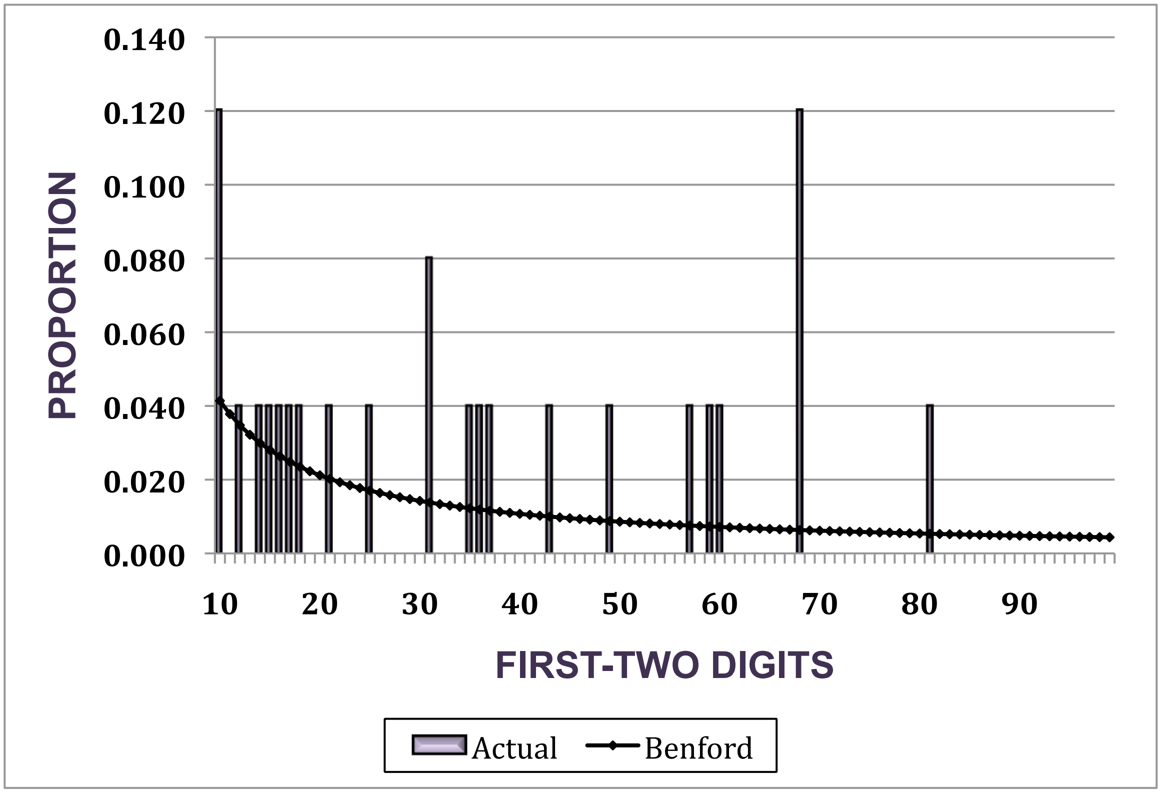

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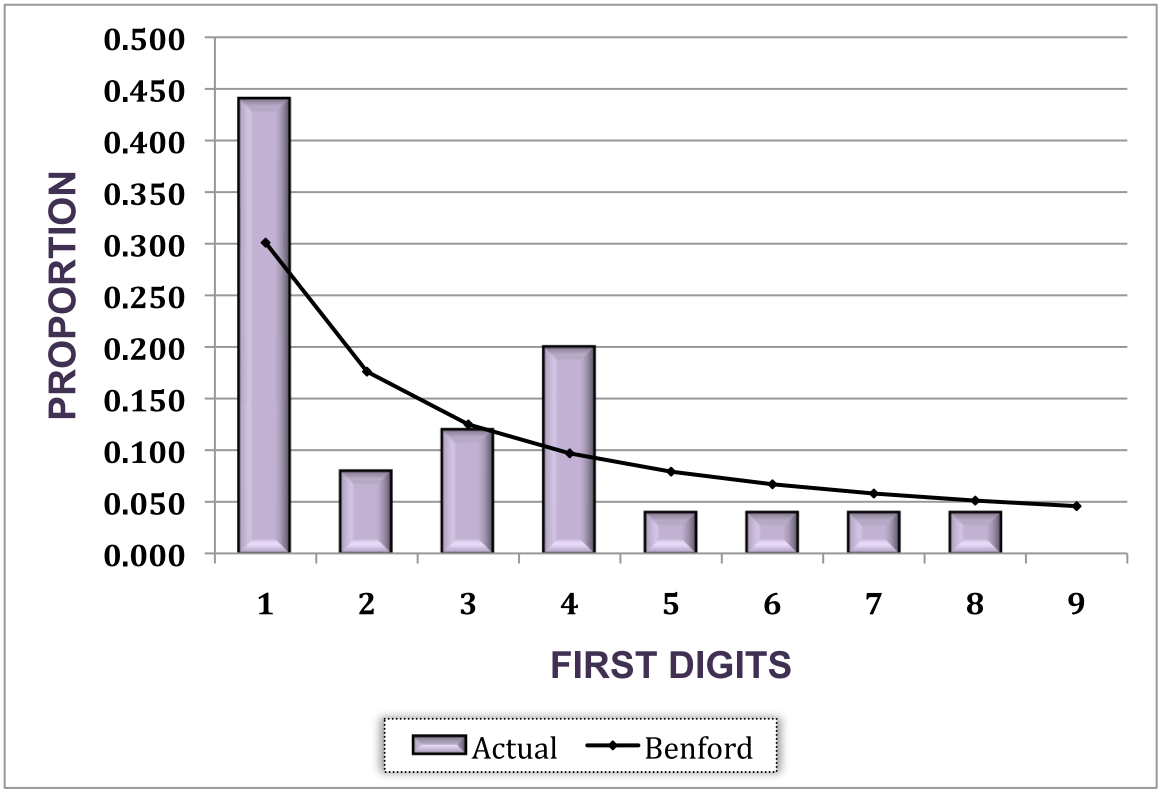

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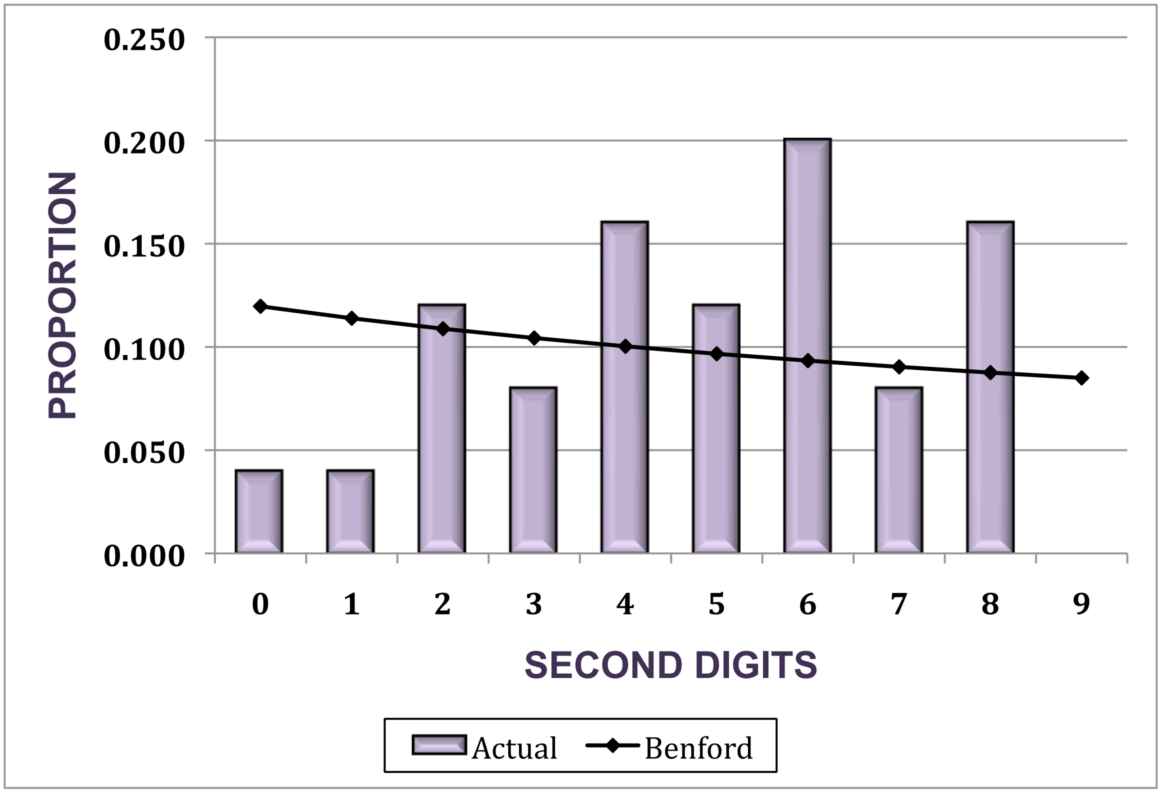

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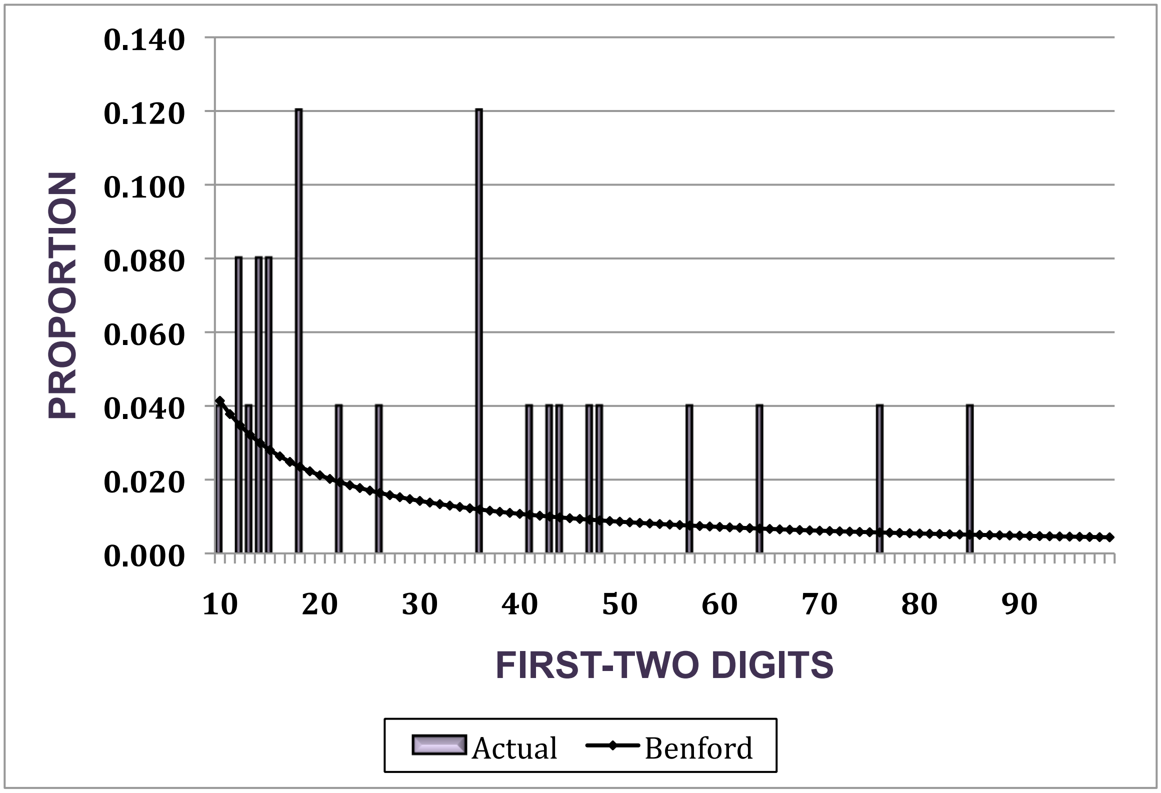

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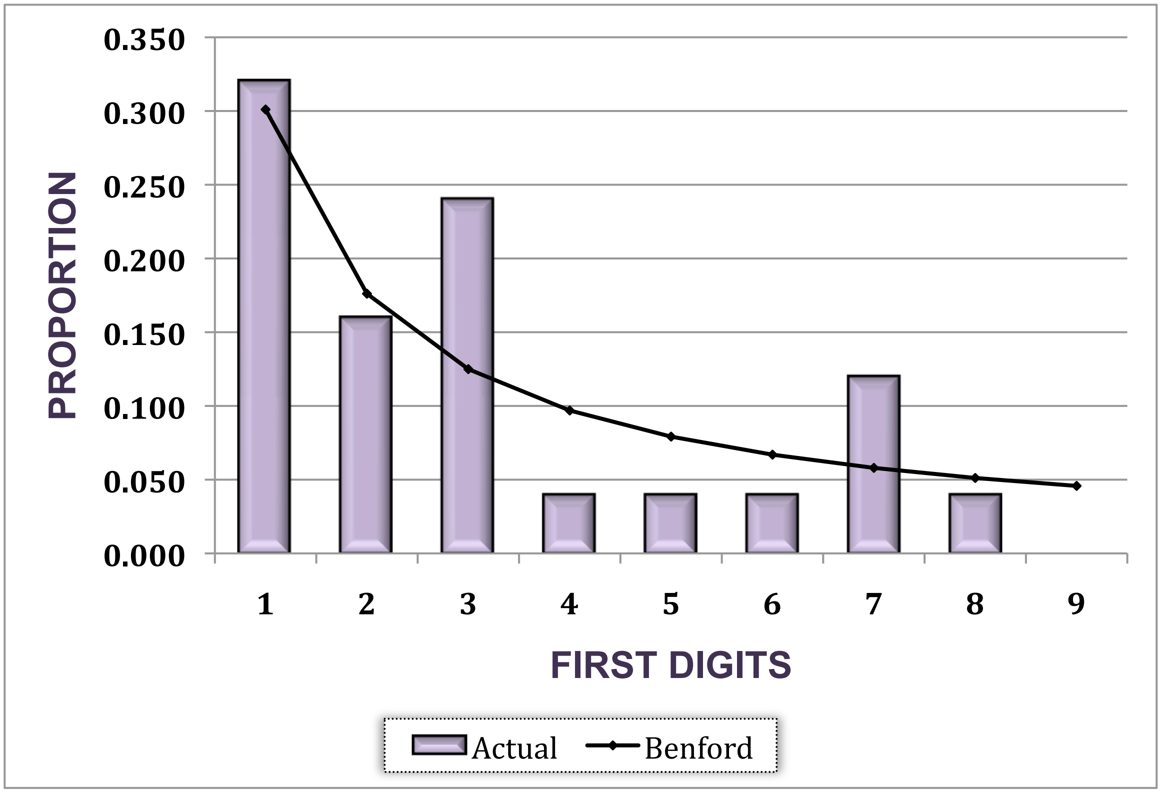


Figure 35. Vallejo City 2007 Second Digits

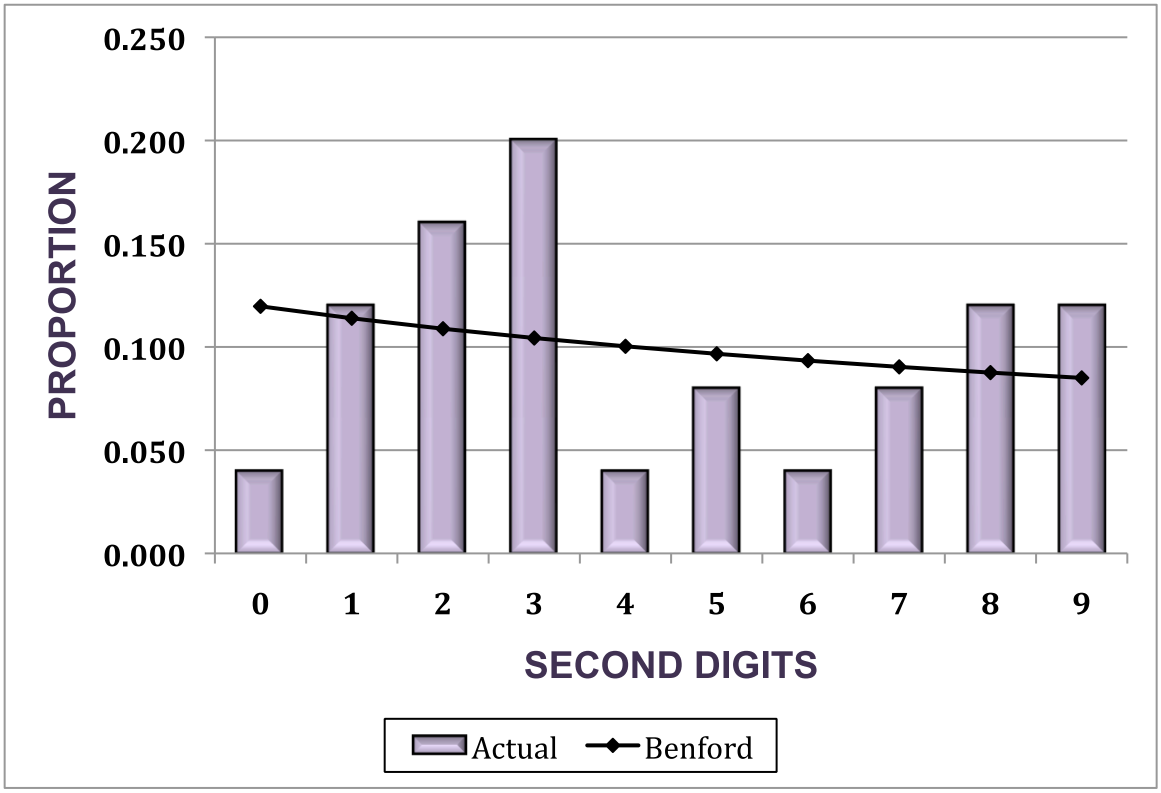


Figure 36. Vallejo City 2007 First-Two Digits

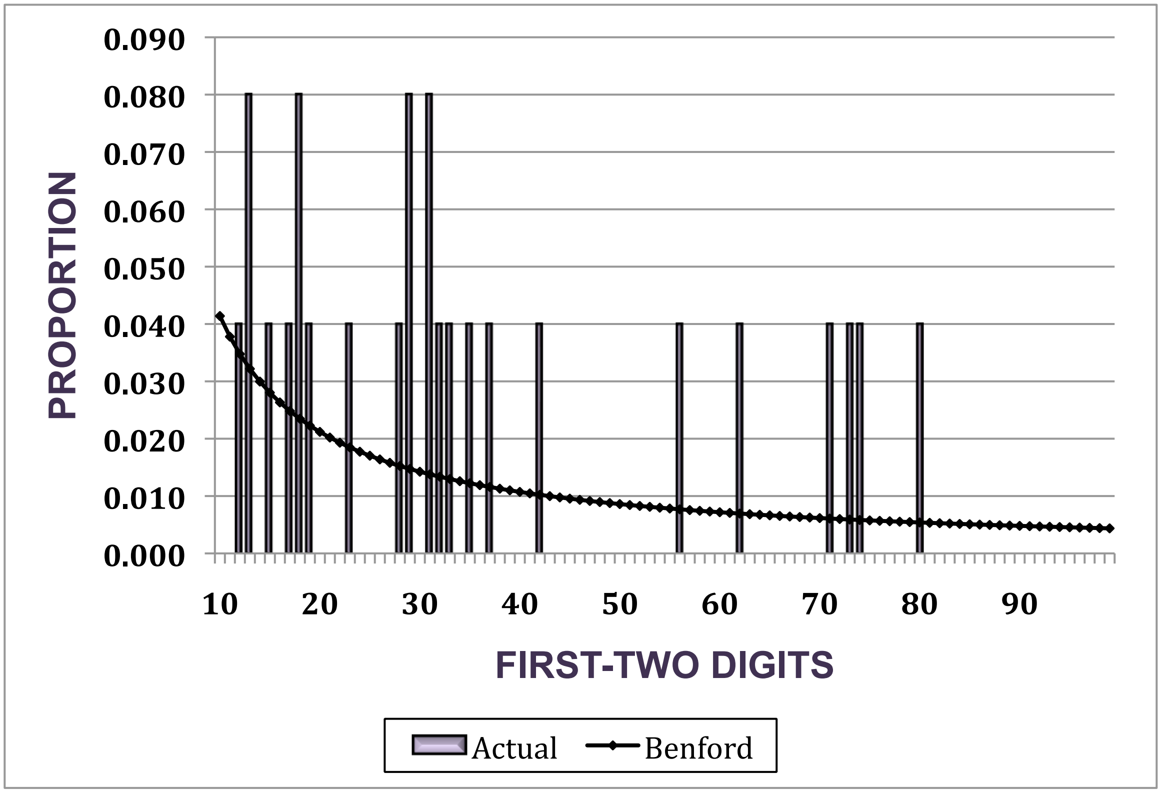


Figure 37. Vallejo City 2008 First Digits

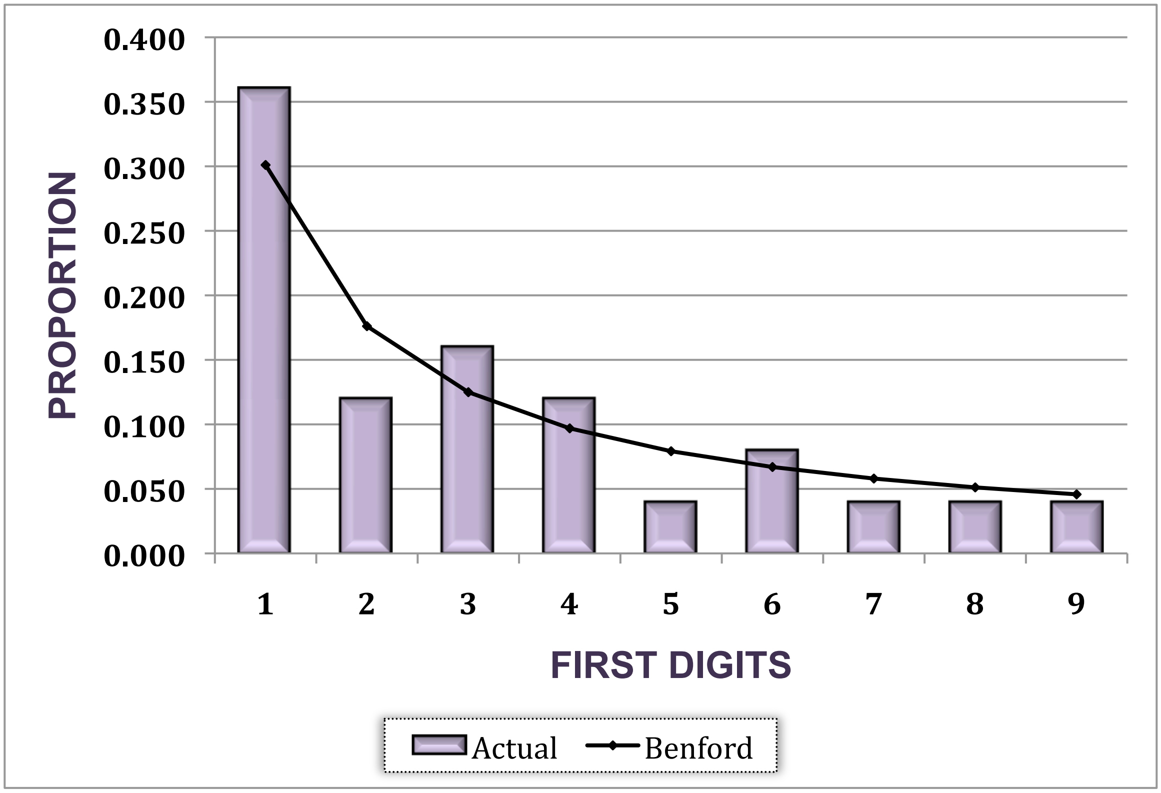


Figure 38. Vallejo City 2008 Second Digits

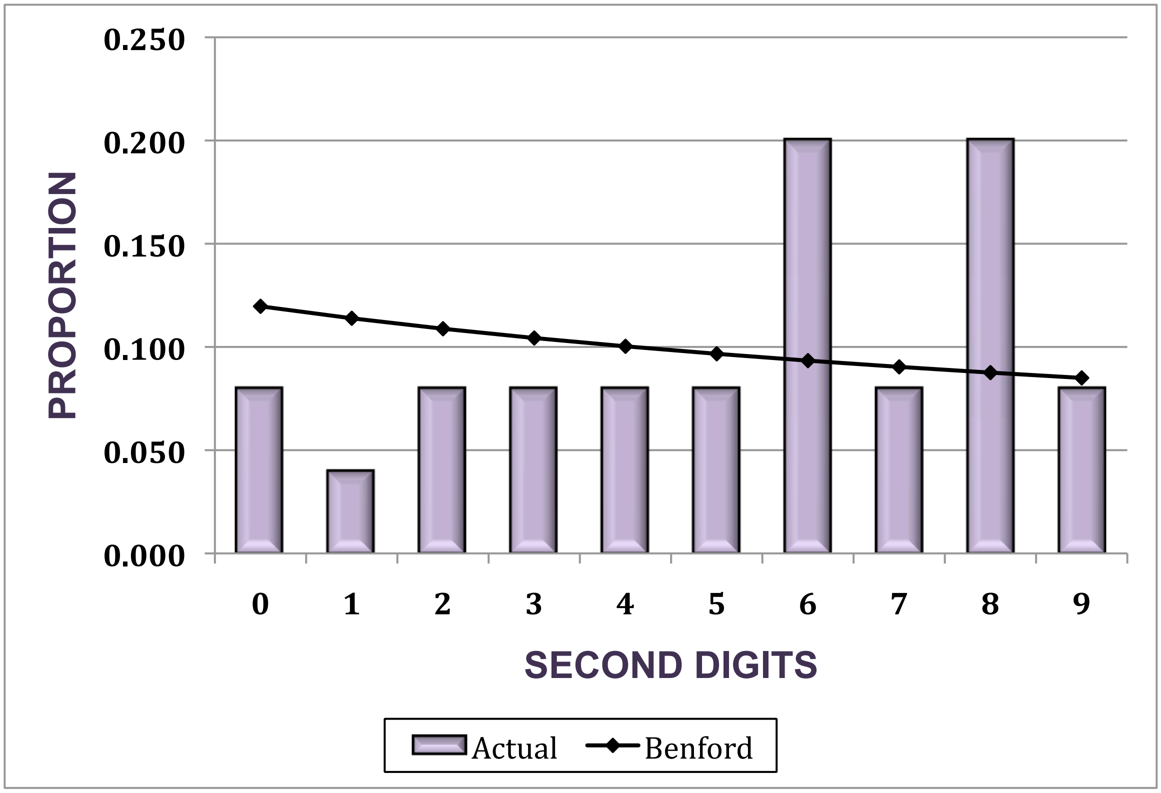


Figure 39. Vallejo City 2008 First-Two Digits

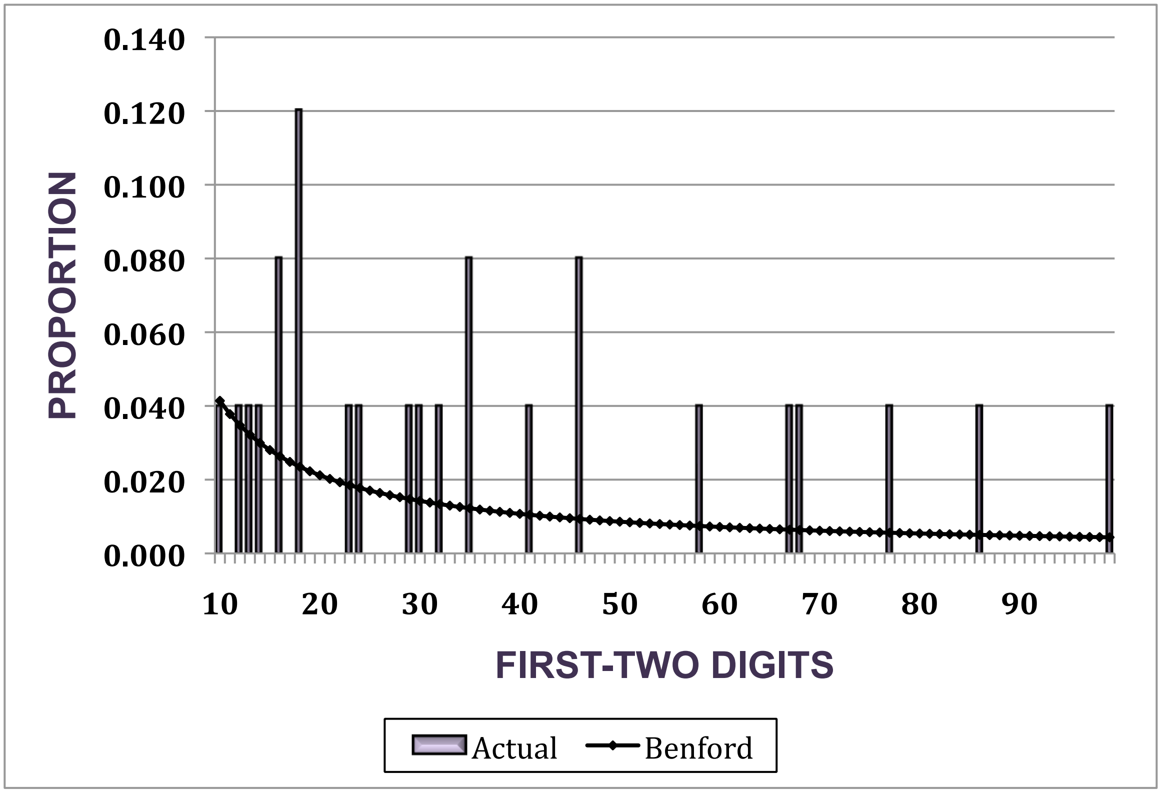
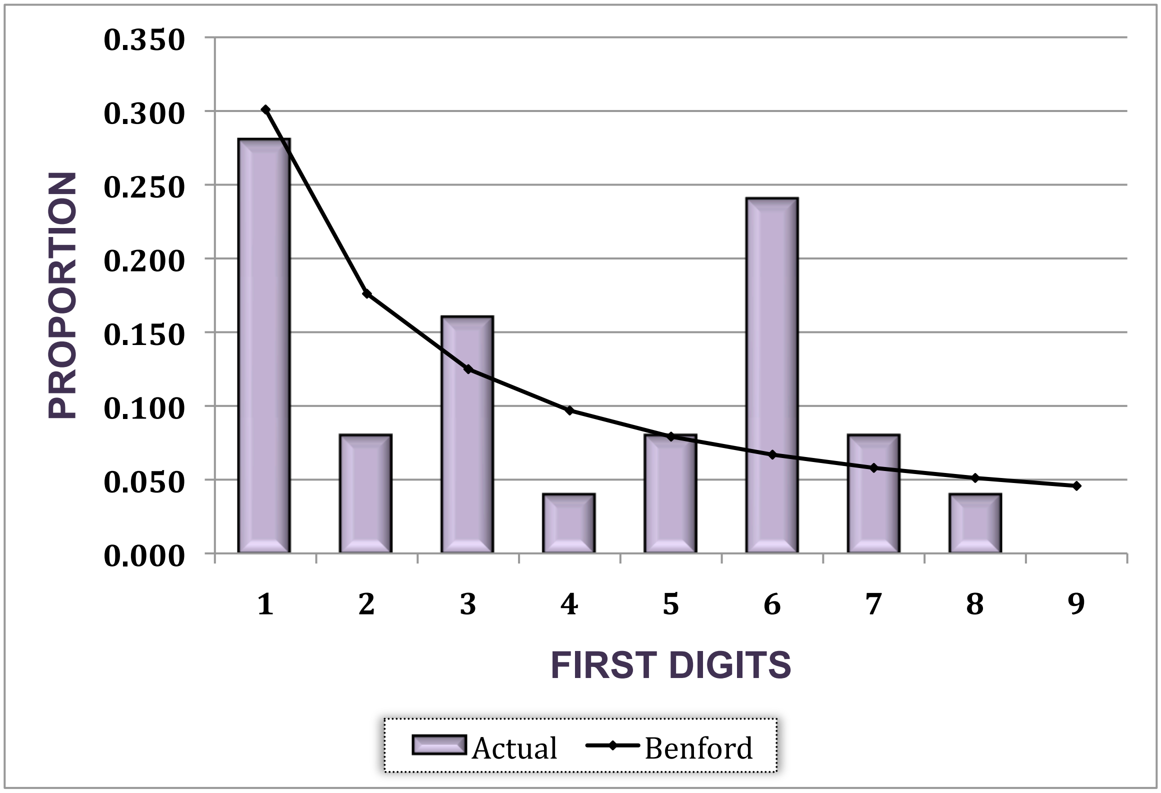


Figure 40. Vallejo City 2003-2008 First Digits



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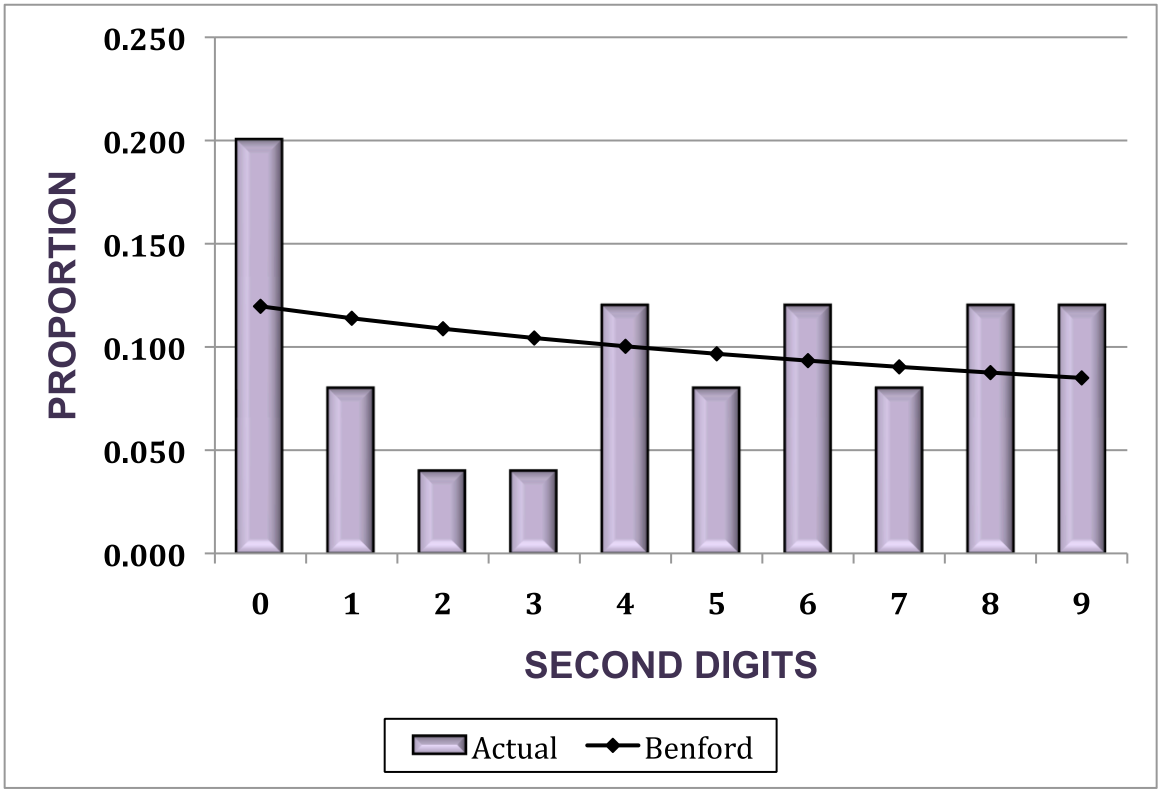

Figure 42. Vallejo City 2003-2008 First-Two Digits

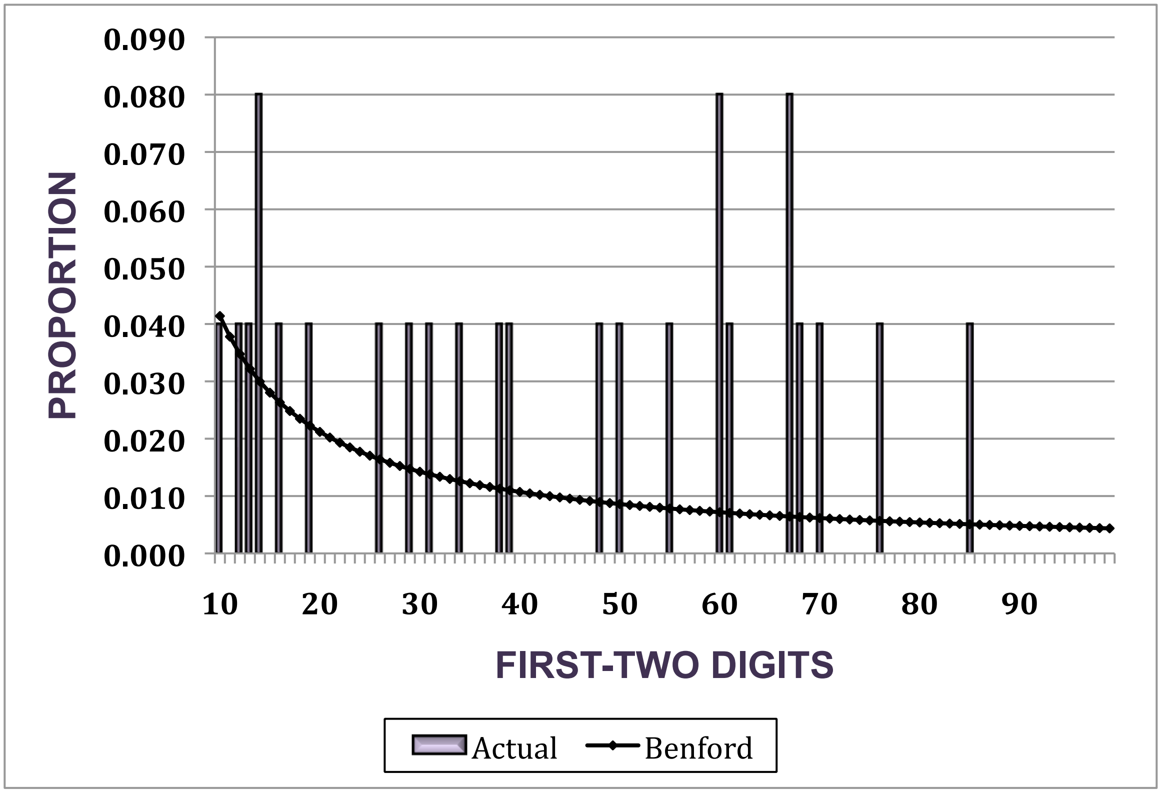

Figure 43. Orange County 1989 First Digits

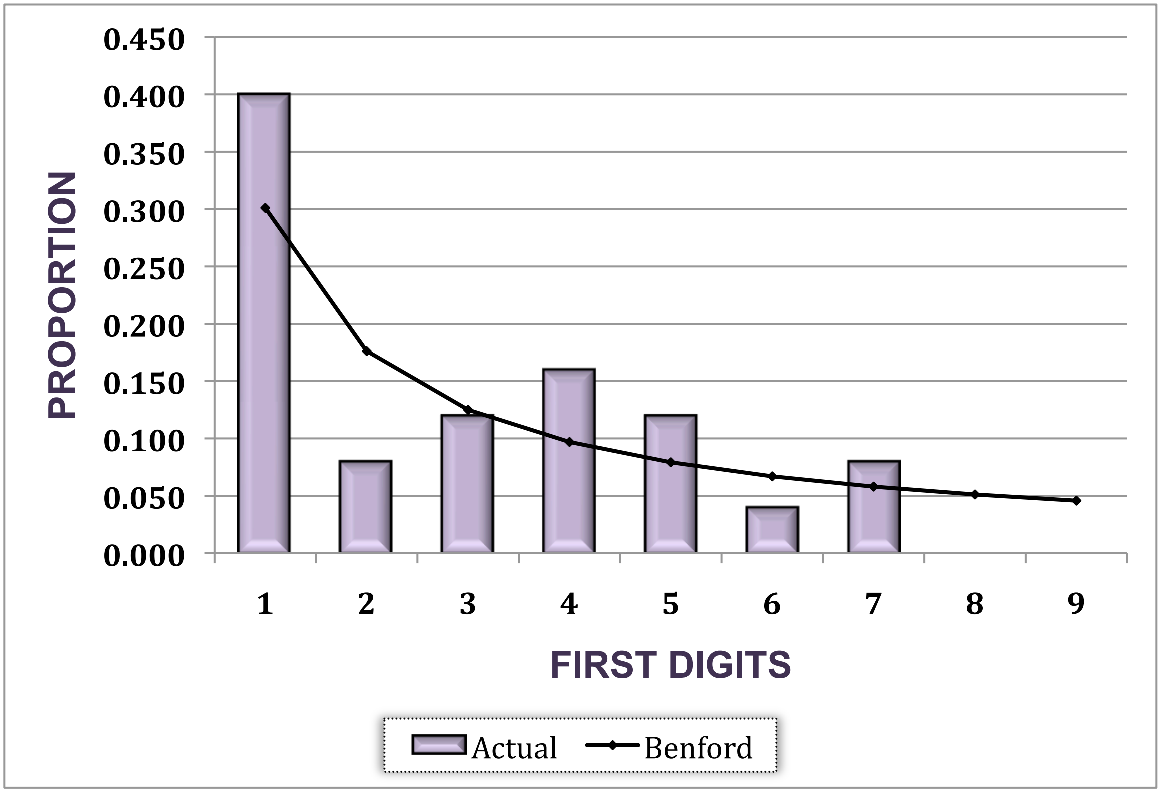

Figure 44. Orange County 1989 Second Digits

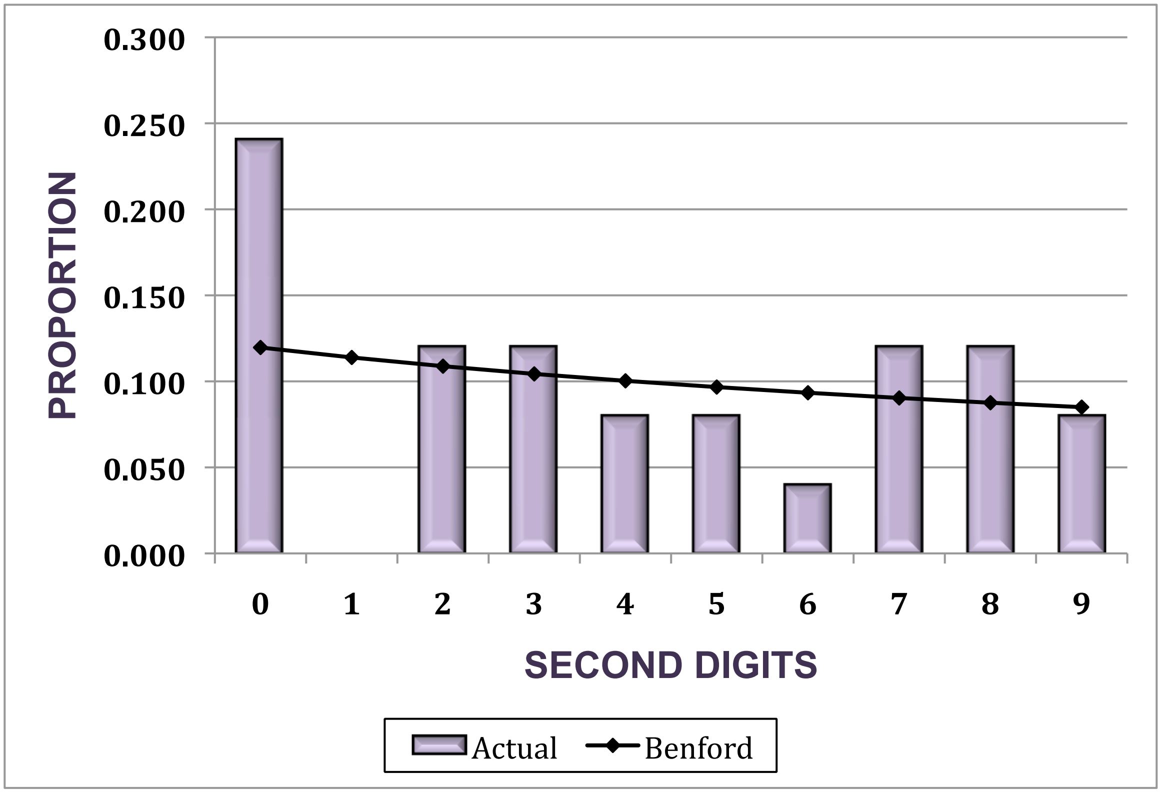

Figure 45. Orange County 1989 First-Two Digits

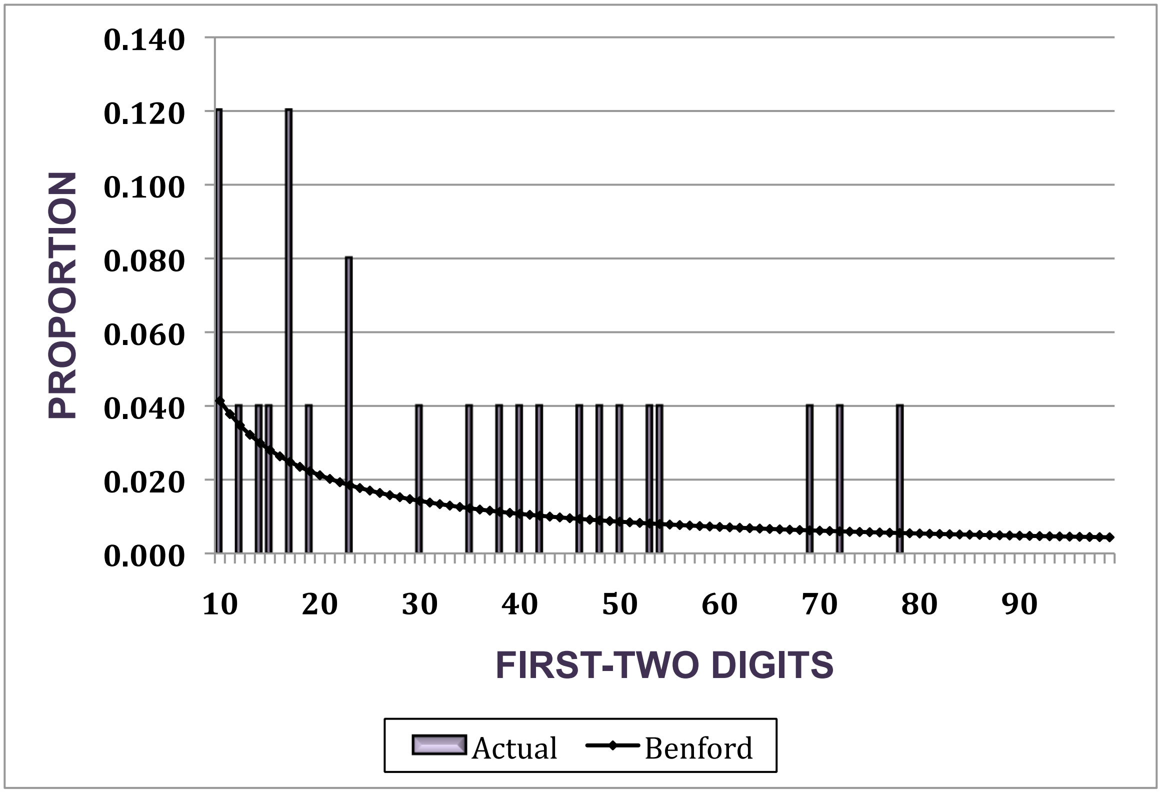

Figure 46. Orange County 1990 First Digits

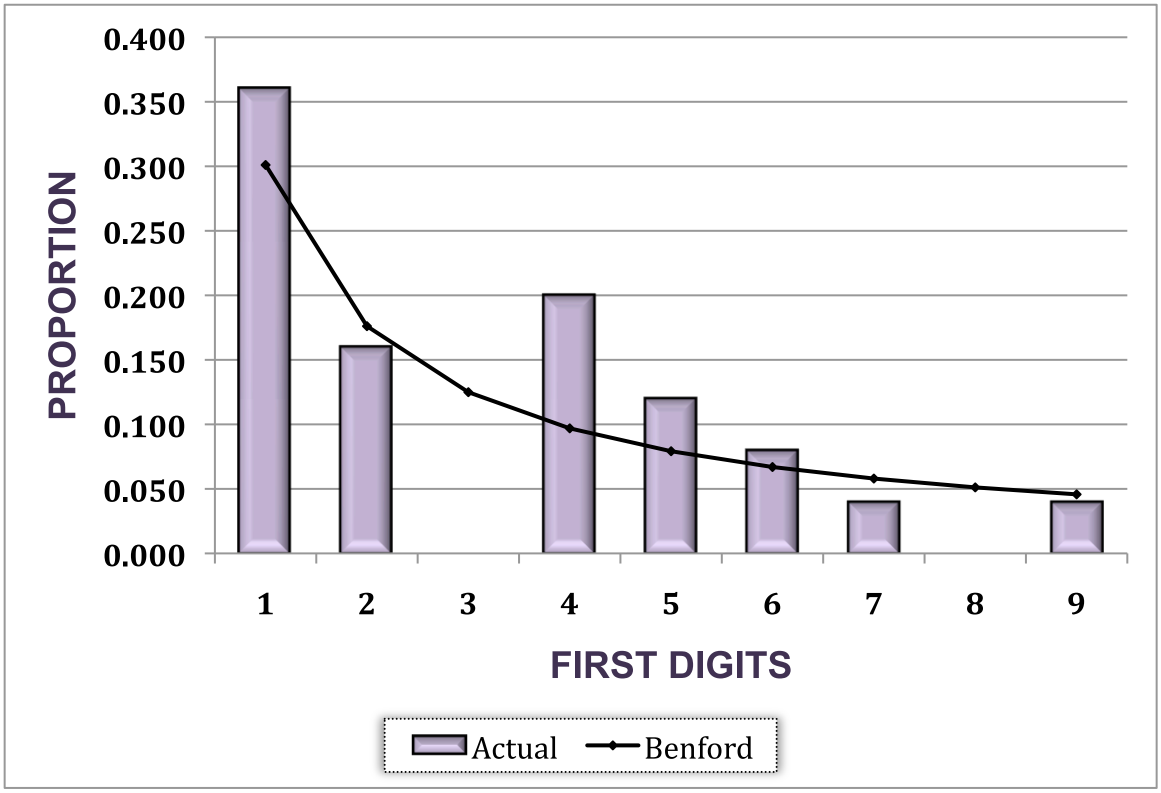

Figure 47. Orange County 1990 Second Digits

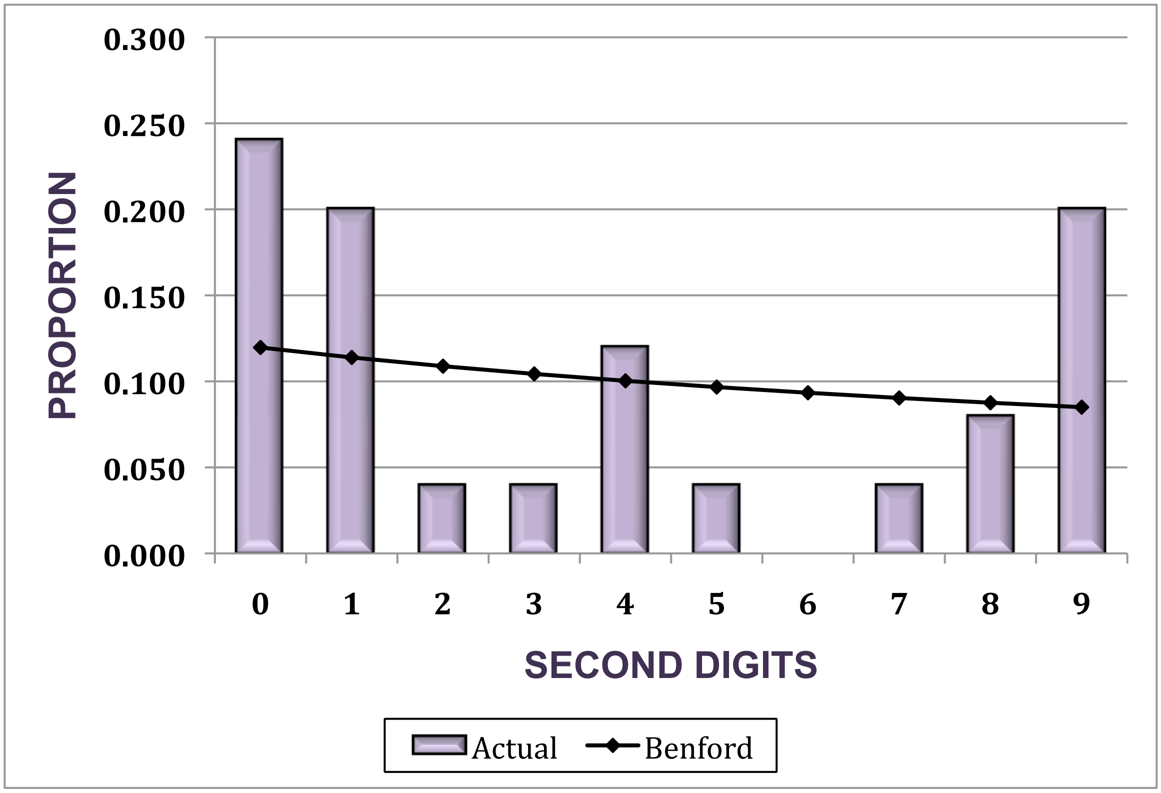

Figure 48. Orange County 1990 First-Two Digits

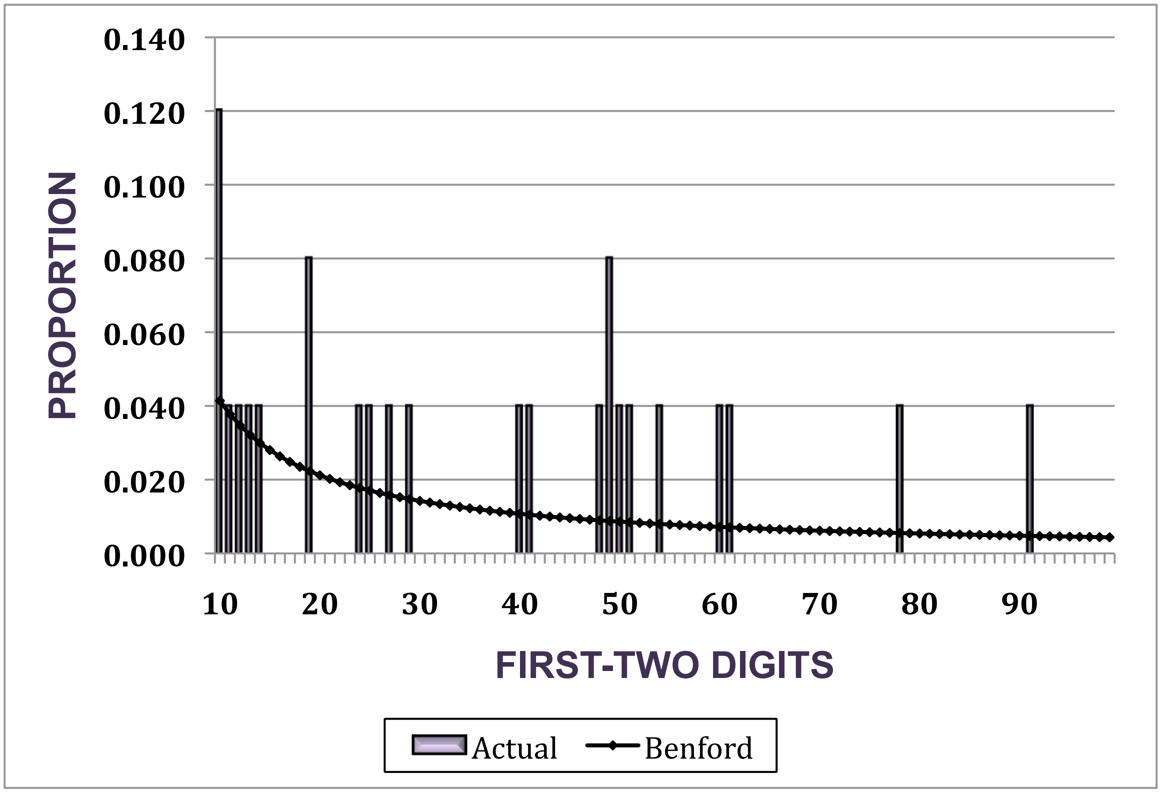

Figure 49. Orange County 1991 First Digits

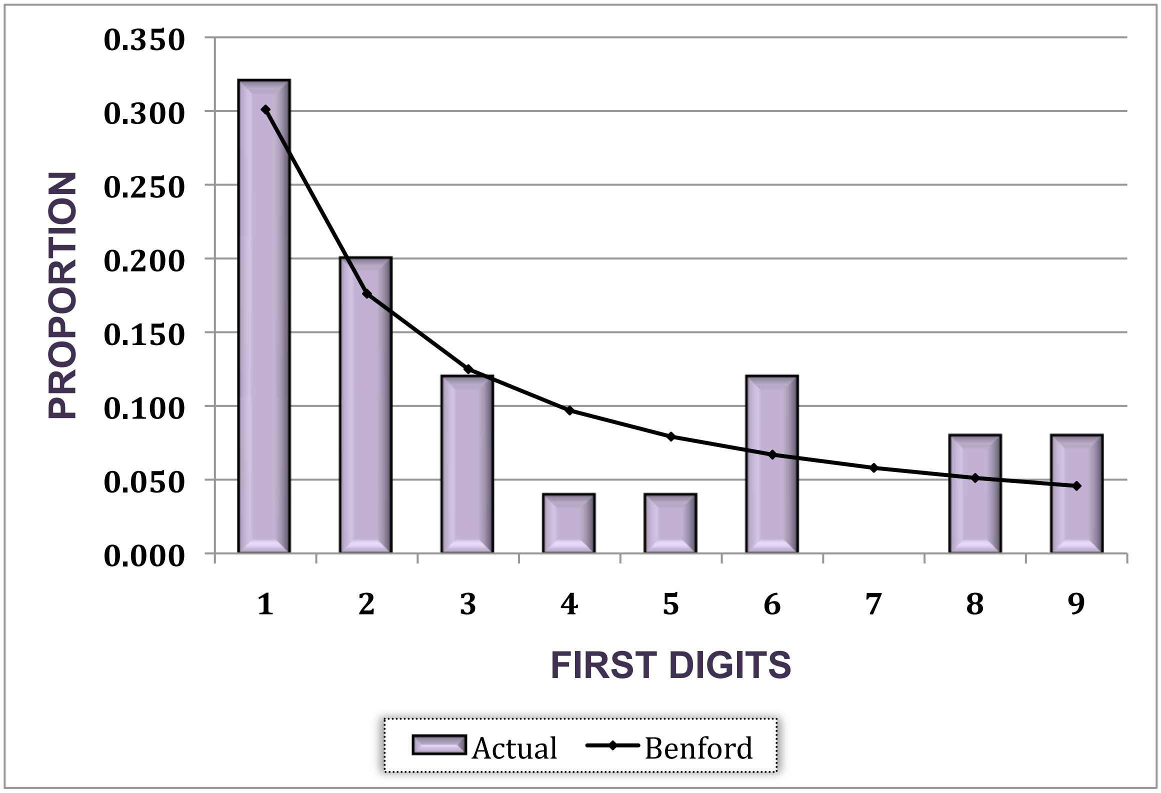

Figure 50. Orange County 1991 Second Digits

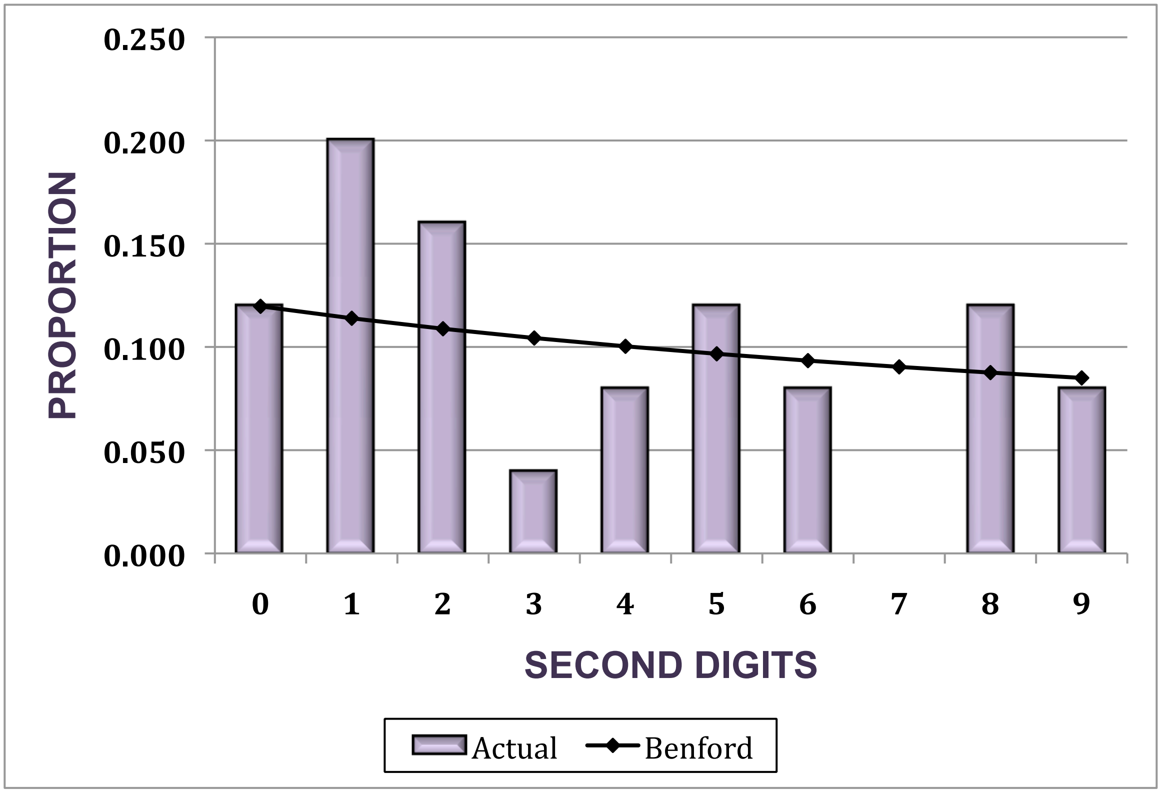

Figure 51. Orange County 1991 First-Two Digits

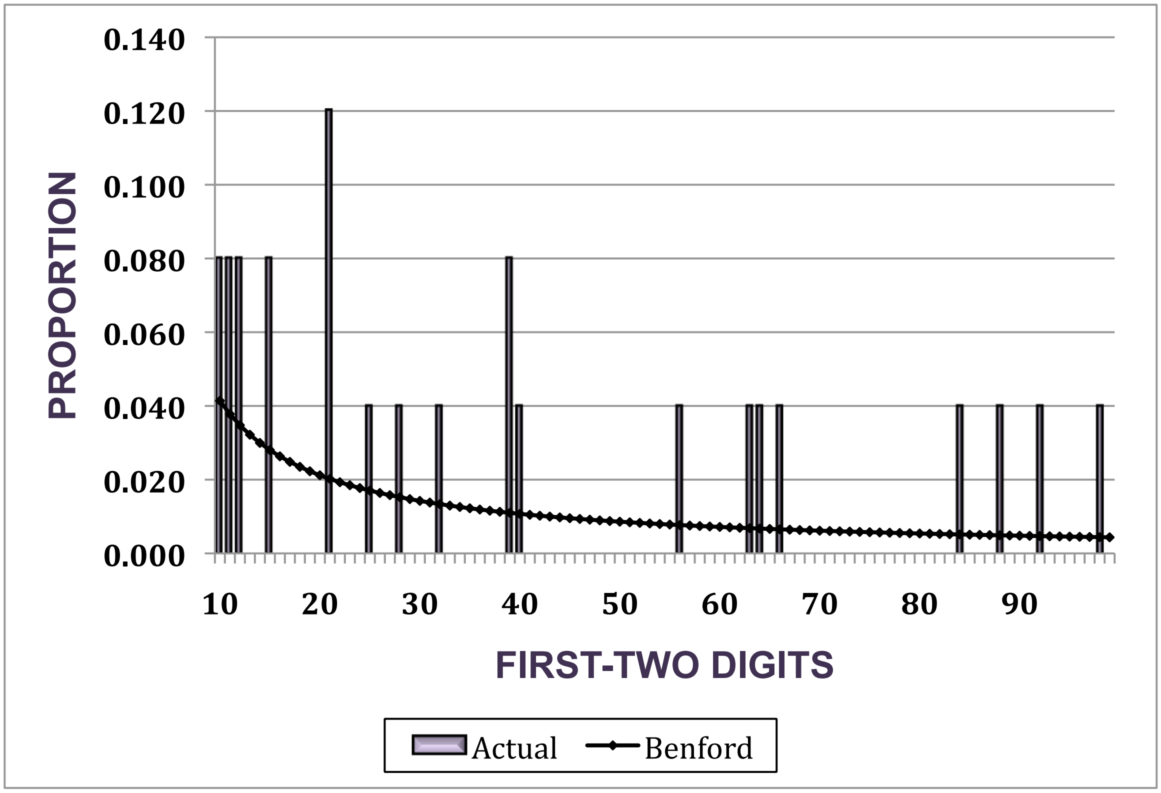

Figure 52. Orange County 1992 First Digits

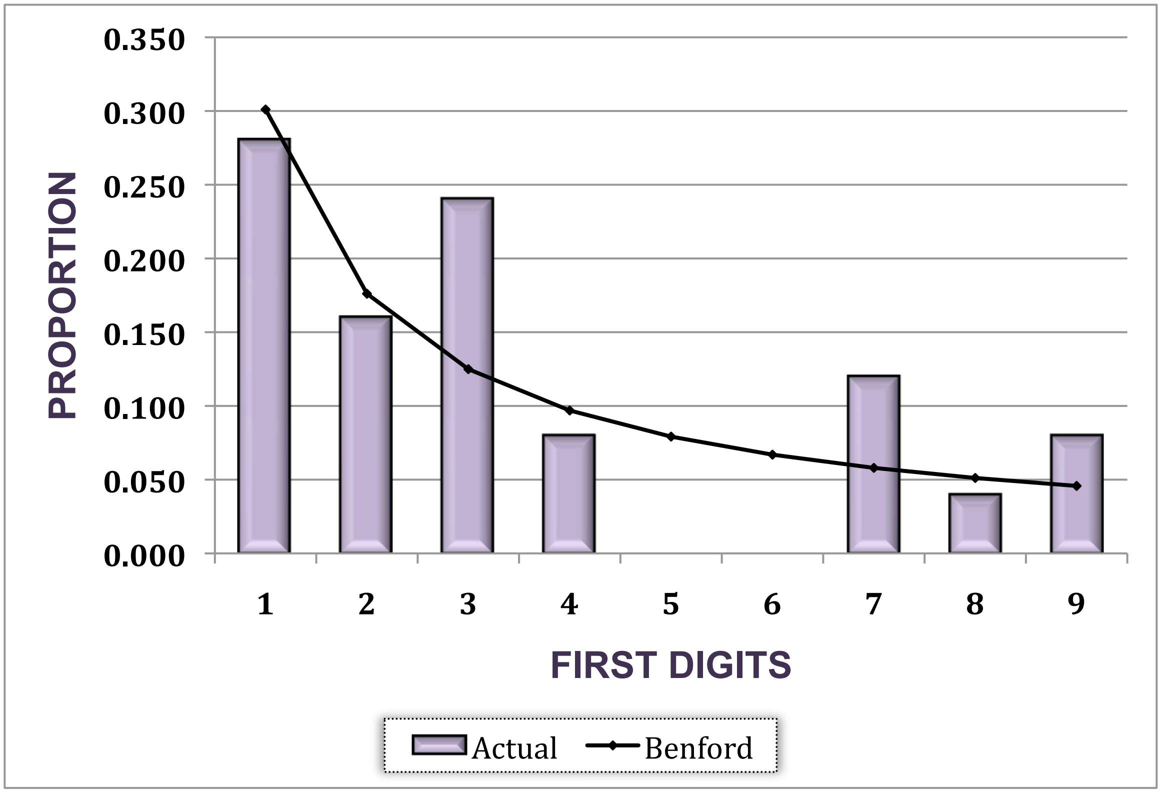

Figure 53. Orange County 1992 Second Digits

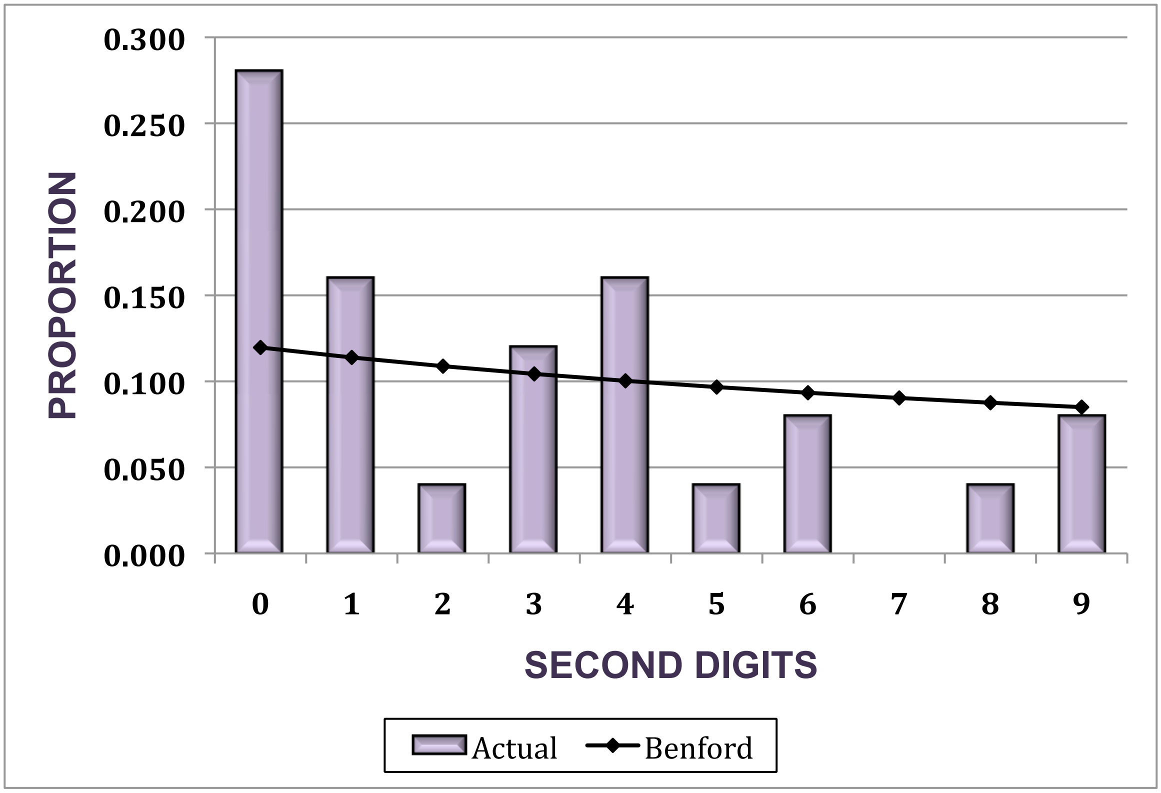

Figure 54. Orange County 1992 First-Two Digits

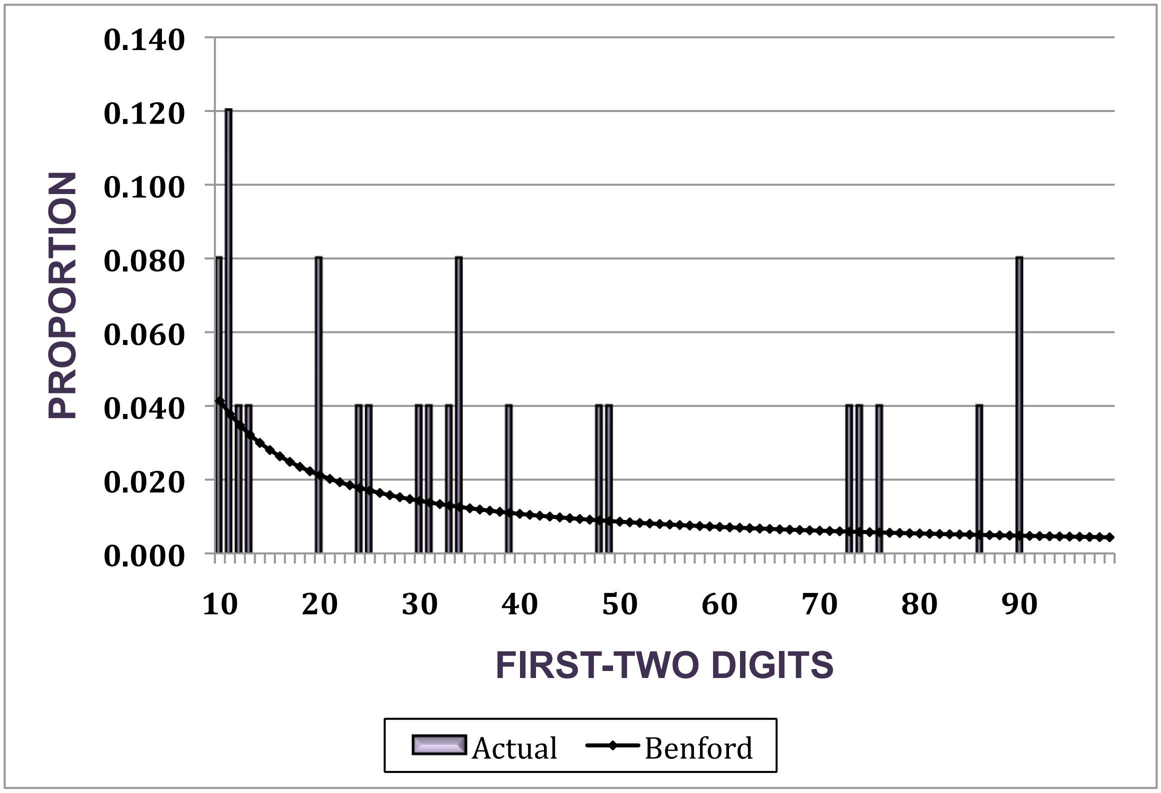

Figure 55. Orange County 1993 First Digits

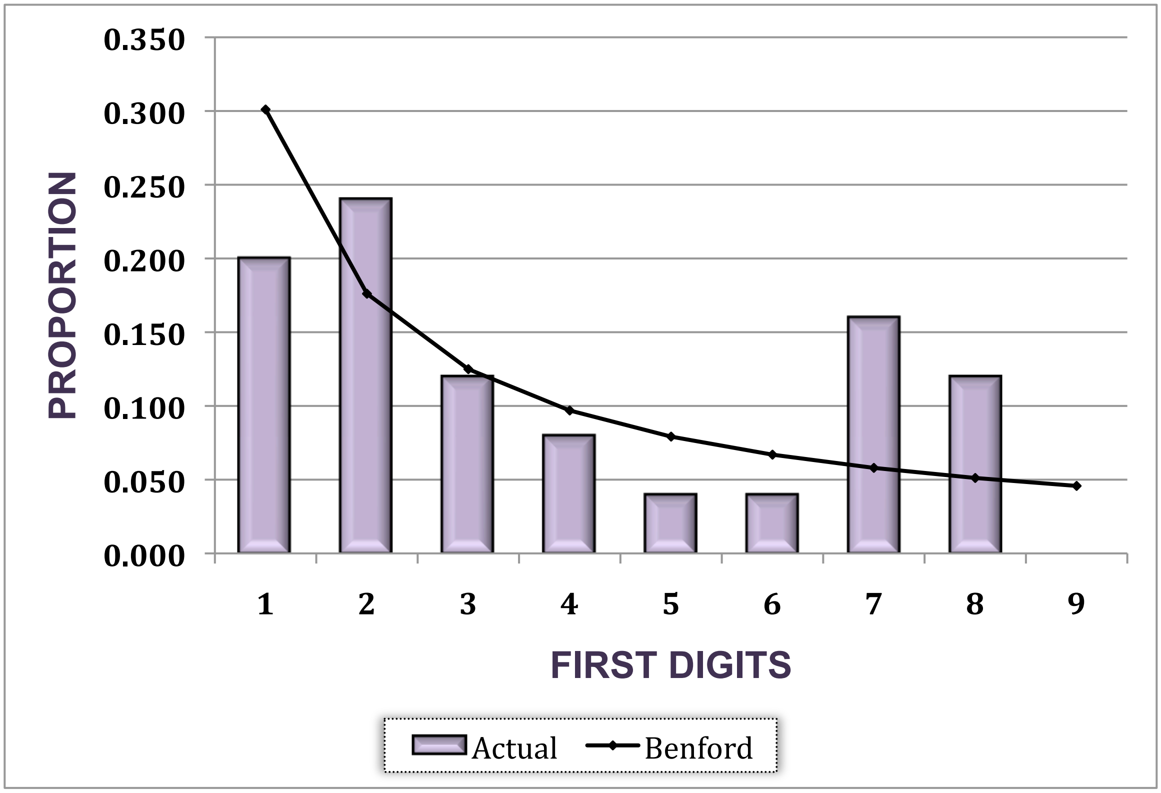

Figure 56. Orange County 1993 Second Digits

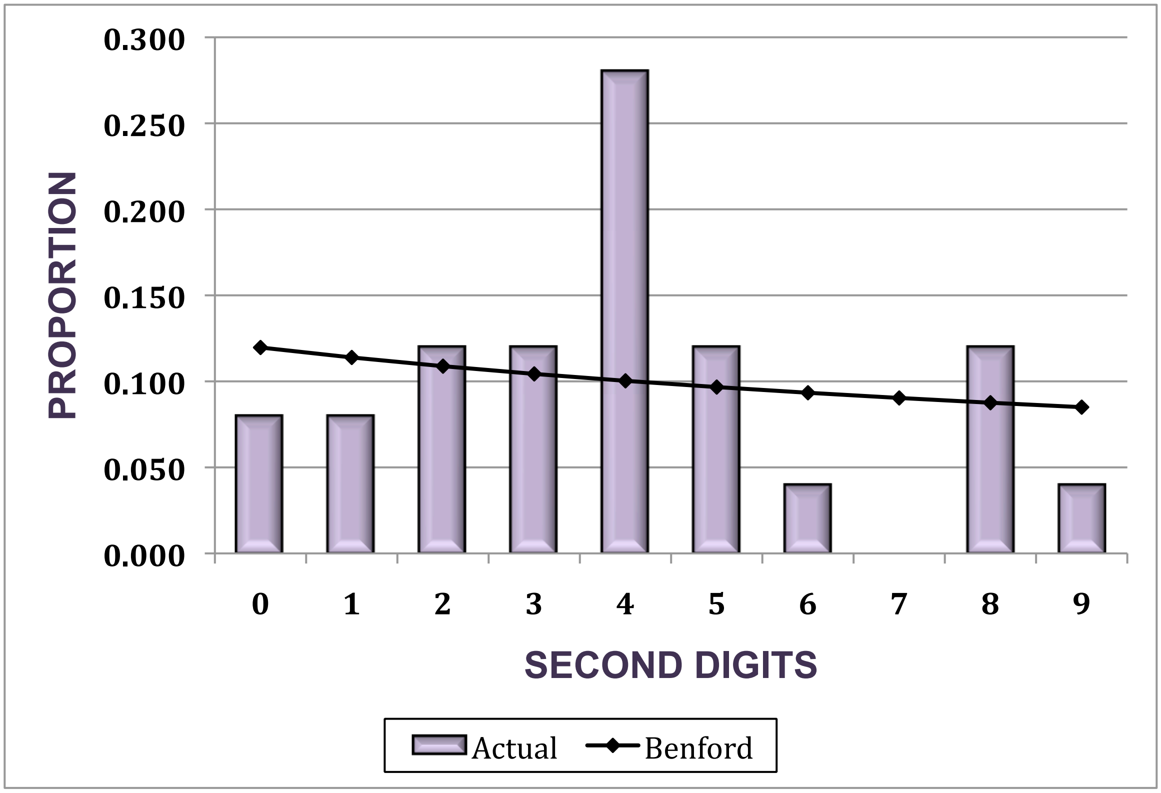

Figure 57. Orange County 1993 First-Two Digits

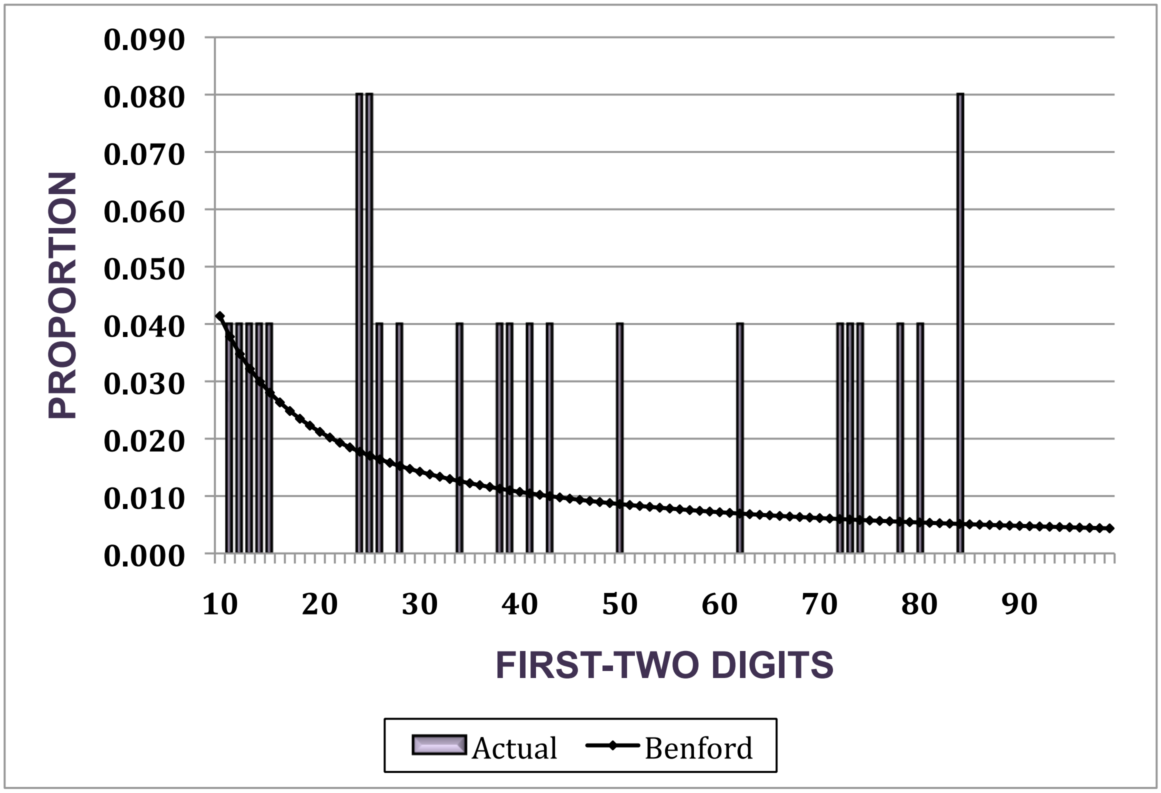

Figure 58. Orange County 1994 First Digits

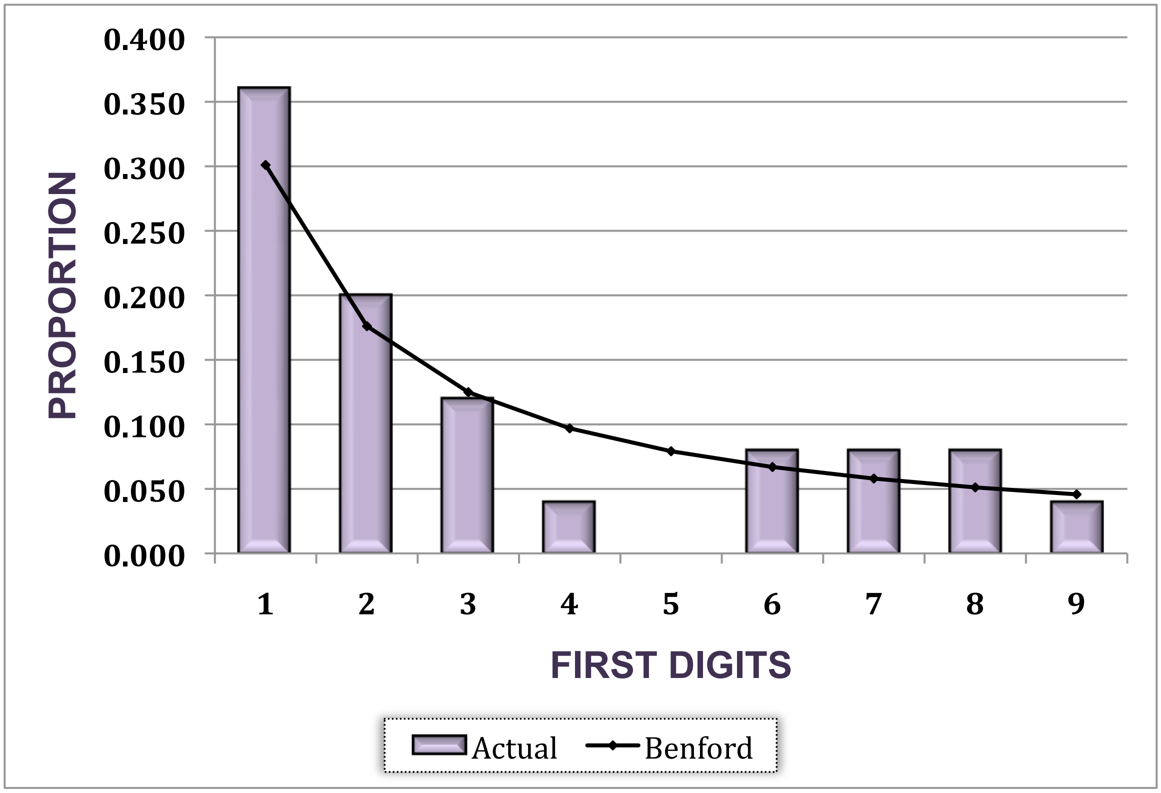

Figure 59. Orange County 1994 Second Digits

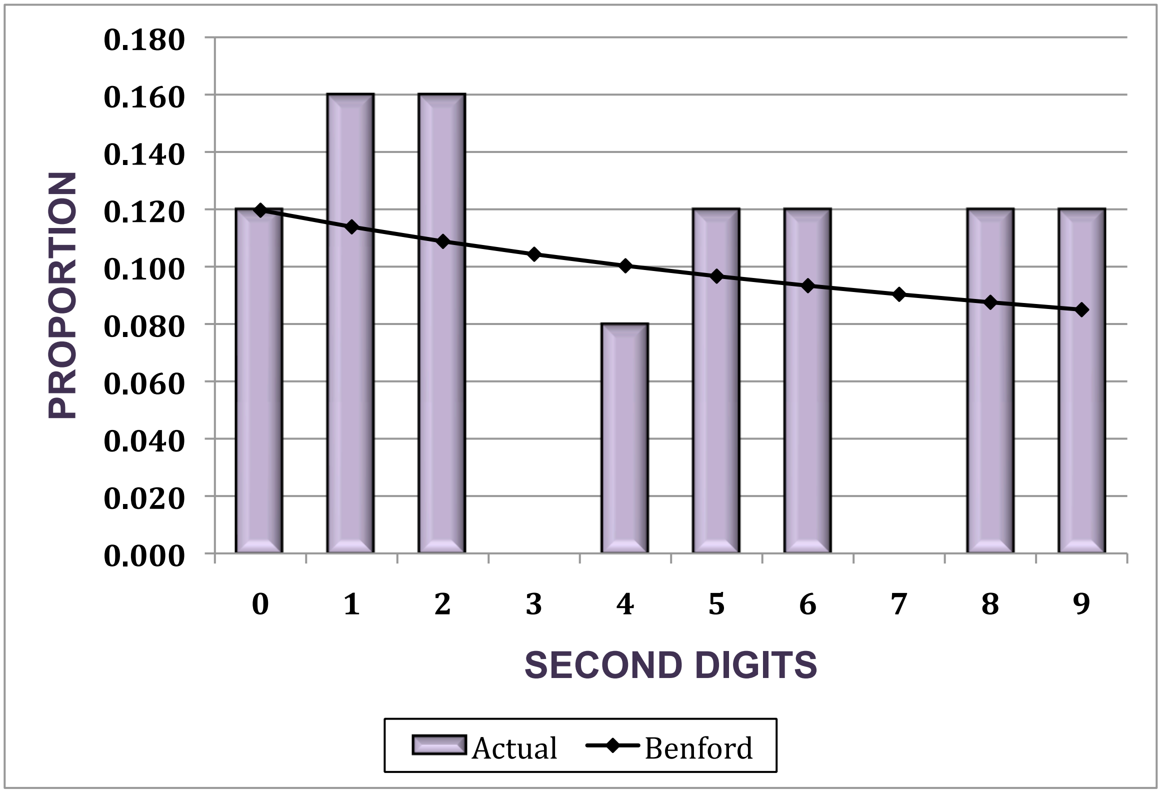

Figure 60. Orange County 1994 First-Two Digits

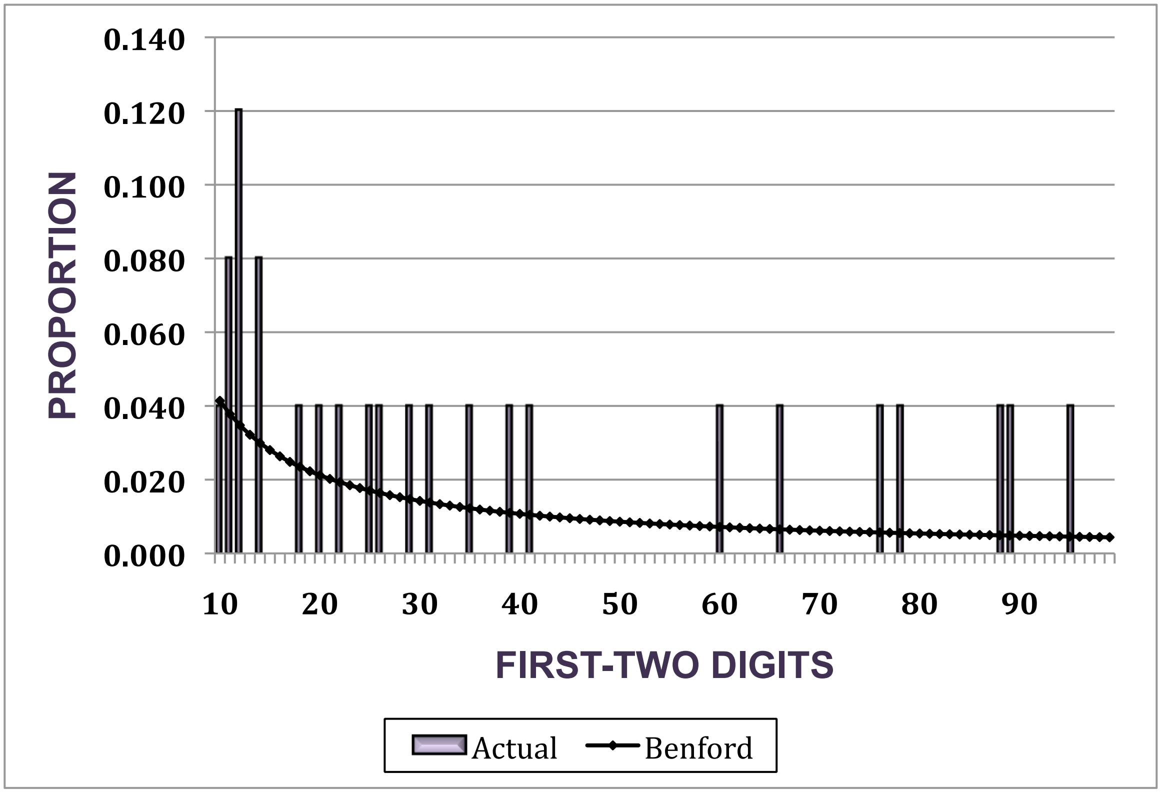

Figure 61. Orange County 1989-1994 First Digits

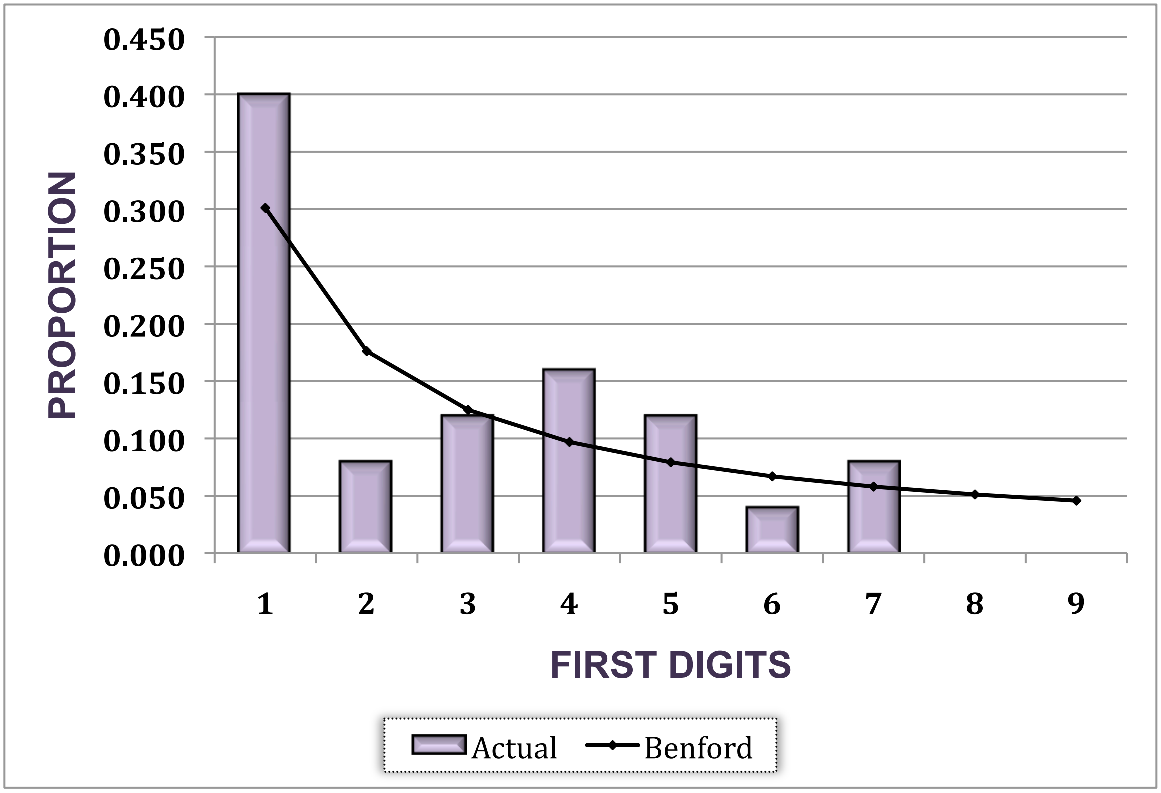

Figure 62. Orange County 1989-1994 Second Digits

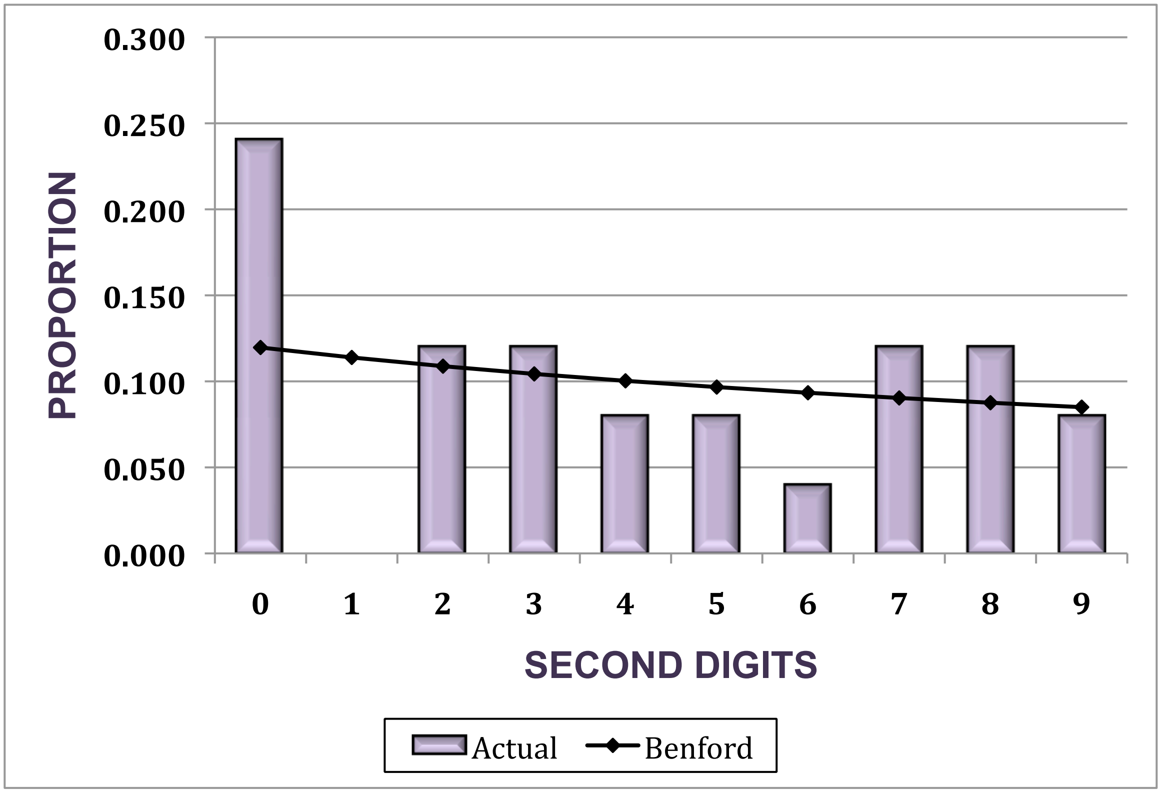

Figure 63. Orange County 1989-1994 First-Two Digits