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Cover Page Footnote (optional)

Jakob Kotas would like to acknowledge the students that took part in this study. He is also deeply grateful to University of Portland Professors Hannah Highlander, Gregory Hill, and Herbert Medina for distributing surveys to their students which served as control data. Mel Henriksen and Mami Wentworth would like to thank their students as well as the students of Professor Cesar Barreto for agreeing to participate in this project and for providing control data.

Specifications-Based Grading Reduces Anxiety for Students of Ordinary Differential Equations

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Abstract: Specifications-based grading (SBG) is an assessment scheme in which student grades are based on demonstrated understanding of known specifications which are tied to course learning outcomes. Typically with SBG, students are given multiple opportunities to demonstrate such understanding. In undergraduate-level introductory ordinary differential equations courses at two institutions, SBG has been found to markedly decrease students' self-reported anxiety related to the course as compared to traditionally graded courses.

1 Introduction

Specifications-based grading (SBG) is a type of grading system in which the course grade, or a part of the course grade, is based on the number of predetermined specifications that the student demonstrates learning during the semester. SBG overlaps heavily with standards-, competency-, proficiency-, and mastery-based grading; indeed, these terms are not rigorously defined and are often used interchangeably. There are many variations of SBG, but a hallmark of all is that reassessment opportunities are given after specifications are learned. These can take place through an in-class assessment, similar to a regular midterm exam; during office hours; or through other means.

Widespread interest in SBG began with standards-based education reform at the K-12 level, which was enacted in the United States through the 1994 re-authorization of the Elementary and Secondary Education Act (ESEA) [7]. Among the supposed benefits of SBG is a decrease in assessment anxiety for students. Furner and Gonzalez-DeHass have argued for mastery-based grading in mathematics at the pre-college level, saying that grades should be based upon "pre-defined standards" in order to limit assessment-related anxiety [3].

In the intervening decades, interest in SBG has percolated to the university level. Many authors have considered SBG at the undergraduate level for specific mathematics courses including introduction to abstract math [9] and introduction to proofs [10]. There has also been work on general guides and ideas for implementing SBG for undergraduate mathematics courses [1]. In an AMS blog post, Owens explains that since grades continue to improve over time, and therefore the stakes are lower on any single assessment, test anxiety is lessened [6].

Elsinger and Lewis describe the implementation of SBG in several undergraduate mathematics courses, including ordinary differential equations (ODEs), at four institutions [2]. Their implementation for ODEs classified each learning objective as either "core", of which there were 18, or "supplementary," of which there were 13. Grades were assigned based on the numbers of core standards mastered once, core standards mastered twice, and supplementary standards mastered. The authors mention that qualitative student comments indicate a decrease in anxiety amongst students in their SBG classes, including ODEs, but did not perform a detailed quantitative study.

In another study, Lewis states that his students reported lower anxiety for his SBG class, in which students were allowed reassessments during office hours [4]. Most recently, Lewis performed a study on student anxiety in SBG courses including differential equations and linear algebra [5]. The author points out that the issue of student anxiety is complicated and difficult to measure. Test anxiety is a component of, but not the only source of, math anxiety, and SBG is not a silver bullet. For example, receiving a constantly updated list of standards not yet achieved could in fact produce additional anxiety for students. Overall, this study showed that SBG did result in students' self-reported lower anxiety, but that a validated State-Trait Anxiety Inventory (STAI) actually showed slightly *increased* anxiety. A possible explanation for this discrepancy was that students' knowledge of reassessment opportunities resulted in a lowering of their self-reported anxiety. It is noted here that Lewis's study allowed for reassessments during office hours, which is different from our implementation of SBG as we will discuss later in Section 2.

In this paper, we describe our process in employing SBG in introductory ordinary differential equations courses taught at the University of Portland and Wentworth Institute of Technology. We also describe a study wherein students' self-reported anxiety was found to decrease over the course of the semester. The data from students in our classes using SBG are compared to that from students in control classes in which a traditional grading scheme is used by their instructors. The body of research on the topic has not definitively determined the relationship between SBG and anxiety in college mathematics courses. However, our study finds that SBG largely decreases self-reported student anxiety in undergraduate ODEs while retaining the same learning outcomes covered prior to implementing SBG.

2 Our Approach

SBG can take many forms. Typically, the list of specifications (specs) that are directly tied to learning outcomes is provided to the students at the beginning of the term. Individual specifications can be graded on a simple binary scale (pass/no pass) or a finer scale, such

as a 4-point scale. Specs can be weighted evenly toward the final grade, or some can be worth more than others. For example, there could be a "list A" of specs that must be mastered to pass the course and a "list B" of specs that will determine the cutoffs for an A, B, C, etc. This approach could be appropriate for courses that are prerequisites for later courses, ensuring all students passing on to the subsequent course meet a minimum level of understanding of prerequisite material. Other course performance issues such as homework completion, attendance and participation can be taken into account by including them as specifications themselves, or by taking a weighted average for the final course grade (for example, 80% specifications passed, 20% homework assignments.)

Across all these different implementations, the commonality in SBG is that students are given multiple opportunities to demonstrate mastery of the specifications. This stands in contrast to a traditional midterm exam approach where students who do not learn a topic by the time of the midterm are penalized. One SBG approach is to allow students to take specs during designated times outside of class, such as office hours, or other weekly hours set aside for that purpose. In practice, this approach works best for small classes due to the need for individualized student meetings. Another approach is that the instructor could designate several in-class testing days where students may select as many specs as they wish to work on. The key is that students who do not pass a specification on the first try are motivated to learn the material and try again.

Two implementations of SBG were employed in the spring 2019 semester for Differential Equations courses at the authors' two institutions, the University of Portland (UP) and Wentworth Institute of Technology (WIT). UP is a private, mid-sized Catholic university located in Portland, Oregon, USA. WIT is a private, mid-sized technical design and engineering university located in Boston, Massachusetts, USA. At both schools, the majority of students enrolled in introductory ODEs are engineering majors at the sophomore level; the remainder are largely mathematics and physical science majors.

2.1 University of Portland

At UP, SBG was implemented for one section with 20 undergraduate students, while two other sections taught by other instructors and using traditional grading schemes were used as controls. Students' final grades were calculated from a weighted average of 20% homework assignments and 80% demonstrated proficiency of 26 specifications. A full list of specifications is given in Appendix A.

Students were given four opportunities to demonstrate proficiency: three during 85minute in-class periods, and one during the university-scheduled 120-minute final exam period. Each opportunity was cumulative in the sense that all specs up to and including the most recent material covered in class were included on each assessment day. Out of 26 specs, students could attempt the first 11 on the first assessment, 21 on the second, and all 26 on both the third and fourth assessments. Thus, students had four opportunities to demonstrate proficiency of the earliest 11 topics covered in the term, three opportunities for the middle ten topics, and two opportunities for the final five topics. An effort was made to push the exams back as far as reasonably possible in the semester so that more opportunities were allowed for the largest number of specs. While the time constraints of the allotted classes prevented most students from completing all specs from start to finish, specs were numbered so that a student wishing to perform better on a specific outcome would know which problem(s) he/she wishes to rework. Problems were intentionally designed to be of a similar type and difficulty from assessment to assessment, but were not identical to previous iterations. One of the known dangers of a grading scheme where retakes are allowed is that students do not take their multiple retakes seriously until the final opportunity. The fact that students would likely not have time to complete all specs from start to finish, and thus would have to prioritize which specs to focus on during each assessment day, had the effect of minimizing procrastination among students.

Specifications were graded on a 4-point scale according to the following rubric:

4: Work is completely correct and clearly written.

3: Solution contains small errors but demonstrates understanding of a correct approach to the problem.

2: Solution contains more serious errors but demonstrates some understanding of a correct approach to the problem.

1: Solution is incomplete, misses key steps, or does not demonstrate understanding of a correct approach to the problem.

o: Solution does not demonstrate any understanding of an approach to the problem or is missing.

The highest score across all attempts for a particular spec was used in tabulating the final grade.

2.2 Wentworth Institute of Technology

At WIT, SBG was implemented in four sections of ODEs. The enrollment in the four sections were 26, 26, 26 and 25 students. An additional section with 19 students was used as a control.

The course grade had two factors. One was the baseline grade, which was computed based on the number of specifications passed, and the other was the final exam grade. The final exam was cumulative and common to all sections of ODEs.

There were six fundamental theorems and concepts. Students must have passed all to receive a passing grade in the course. Additionally, there were 25 specifications that were closely tied to the learning outcomes. A full list of the specifications is given in Appendix A.

Specs were tested in nine separate assessments given in class. Each spec was graded "pass", "in progress" or "not yet passed". Although the grade of "in progress" did not count as a pass, students were allowed to submit corrected work to receive a passing grade without retaking the spec. For the specifications that they did not pass on in-class assessments, students were given retake opportunities outside of class time. Students could take up to three retakes for each of the specs, having a total of four opportunities to pass a spec. Students were given until the day of the final exam for retakes. Prior to

each retake session, students indicated which specs they were retaking. Because the same retakes were given to students at different retake sessions, students were not allowed to discuss any aspect of the retake except with the instructors.

3 Survey Description

In this study, we administered a survey at various times throughout the course. The purpose of this survey was to measure the level of anxiety and other attributes students experienced about the class, and to understand the factors that contribute to these attributes. By conducting the survey multiple times, we wanted to see not only how much anxiety students were experiencing at the time the survey was given, but also how the anxiety level changed throughout the course.

The survey asked students to rank their anxiety, motivation to succeed, and ability to manage time on a Likert scale in relation to their other courses. They were also invited to share factors that contributed toward increasing or decreasing each of these dimensions. The full text of the survey can be found in Appendix B.

4 Survey Results

In classes employing SBG (experimental group), the survey was given four times. At UP, the survey was given on weeks 4, 5, 12, and 14 in a 14-week long course. At WIT, the survey was given on weeks 2, 4, 10 and 13 in a 15-week long course. In both schools, the first and second surveys were administered right before and right after the first in-class assessment, respectively. In classes employing traditional grading schemes at both schools (the control group), students were given the survey twice in the semester: once before their first exam and once at the end of the semester.

Of the three factors we measured, we see a noticeable divergence of the self-reported anxiety levels for students in the experimental group and students in the control group. The levels of two other self-reported attributes by students, motivation and time-management, were relatively flat over time, and were not appreciably different across the two groups. See Figure 1.

Figure 2 shows the difference in the trend for the experimental group and the control group. In Figure 2 (a), we see a symmetric distribution of anxiety level from much more to much less anxiety for the control group. The distribution was essentially unchanged by the end of the semester. In Figure 2 (b), for the experimental group, the distribution of self-reported anxiety level shifted from a symmetric distribution to a monotonically decreasing distribution.

In the last survey, students in the control group commented that their anxiety was primarily caused by the difficulty of the material; students in the experimental group largely commented that their anxiety was lower than that in other courses due to retakes. Even for those students who still felt anxiety in the SBG classes, the vast majority felt anxiety due to the limited number of test dates remaining, which can be remedied through aspects of the course design, as we will discuss in Section **6**.



Figure 1: Average of students' self-reported (a) anxiety, (b) motivation, and (c) timemanagement skills over the course of a semester. Likert score options ranged from 1 to 5 with 1 very low and 5 very high.

5 Grade Results

5.1 Results by Specification - UP

In Figure 3, we show the proportion of students achieving a 4/4 score for each of 26 specifications at UP. Because we always choose the highest grade for multiple attempts at any single learning outcome, these proportions are monotonically increasing with respect to number of attempts. For graphical clarity, the specifications are referred to by their number as listed in Appendix A.

Next we explore in detail some of the specifications with low initial pass rates.

#1: Check general solutions. Students were asked to determined whether a given function is a general solution to an ODE. Students tended to be good at the mechanics of taking derivatives and plugging functions in. Many students would check whether a given solution satisfied the DE, but struggled with the distinction between a general solution and a particular solution. Quantitatively, on the first attempt, only 5% of students mastered this concept, but by the fourth attempt, 53% did.



Figure 2: Students' self-reported anxiety scores over the course of a semester for (a) a traditionally graded class versus (b) SBG class. Survey 1 is taken before the first assessment opportunity, and survey 4 just before the final exam period.

#12: Rewrite a linear second-order equation as a system of first-order equations. Like spec #1, this spec is more conceptual. When transitioning into systems of equations, we discuss how a second-order equation, y'' = f(y, y', t), can be written as a system of two coupled first-order equations through a change of variables, w = y', where w = w(t) is a new variable. Like #1, the proportion of students mastering this spec jumped dramatically, from 21% with one attempt to 79% with three attempts.

#18: Solve a system of two linear, constant-coefficient, nonhomogeneous first-order equations using the method of undetermined coefficients. Students were provided the solution to the homogeneous equation and needed to use the method of undetermined coefficients to find the nonhomogeneous solution. Mastering this spec requires students both to be comfortable with matrix/vector arithmetic as well as solving systems of algebraic equations. Most of the students in the class had not taken a full linear algebra course, but were merely introduced to the linear algebra concepts needed for systems of DEs as we went through. As a result, it took time for these techniques to settle in. Thus we see only 21% of students mastered spec #18 on the first attempt while 58% did by the third attempt.

#19: Solve a second-order differential equation of the form y'' = f(t, y') or y'' = f(y, y'). Second-order nonlinear equations of one of these forms can be reduced to first-order via a change of variables. If the resulting first-order equation is separable, it can then be solved, and finally the change of variables is undone. These problems require students to be comfortable with both the notion of a change of variables, as well as integrating by hand and solving separable first-order equations. The author notes that one common point of confusion is having two arbitrary constants that appear: one in doing separation on the first-order equation, and then another when undoing the change of variables to recover y(t). Probably because of the many skills required to master this spec, *not one* student mastered it on the first try and 11% had mastered it after the third attempt.

#26: Find the power series solution to a second-order, linear, variable-coefficient differential equation. Here we considered second-order linear DEs with polynomial



Figure 3: Percent of students achieving a 4/4 score for each of 26 specifications at UP. 1 student who dropped the course is not included.

coefficients. The process requires students to be adept at manipulating infinite series. Perhaps because this was the last topic covered in the term, the proportion of students mastering this spec was *zero* on the first try, but increased to 16% by the second.

Overall, across all the specs at UP, the average proportion of students achieving a 4/4 after the first attempt was 44.1%; after two attempts 51.2%; after three attempts 58.7%; and after four attempts 61.1%. This shows the increase of understanding of specs after multiple attempts albeit with diminishing returns as the number of attempts increases.

5.2 Results by Specification - WIT

In Figure 4, we show the percent of students that mastered each of 28 specifications. The percentages are broken down into the 4 attempts that were provided for mastery. Selected specs which resulted in low initial pass rates are discussed below.

#6: Solve first order linear equations and IVPs by using an integration factor. Empirically speaking, solving linear equations of the form y' + P(t)y = Q(t) is a struggle for many students. Some resort to memorizing the formula for the final form of the solution, $y = \frac{1}{\mu} \int Q\mu \, dt$, with $\mu = \exp(\int P \, dt)$. Many others struggle with integration techniques. It is noted that while spec #6 had only an initial pass rate of 18%, the overall pass rate



Figure 4: Percent of students achieving a "pass" for each of 28 specifications at WIT. Students who dropped the course are not included.

after four attempts was 72%. This potentially reflects students' persistence in learning the concept.

#12: Solve a homogeneous first order, constant coefficient, 2 x 2 matrix equation, with initial conditions, with complex eigenvalues and eigenvectors. Many students are introduced to matrix algebra for the first time in the DE class. Dealing with complex numbers is also an unfamiliar task for them. The low pass rate potentially indicates the lack of conceptual understanding of the solution process. Many students struggle with solving complex eigenvalue and eigenvector problems because they see the process as a series of steps they need to memorize. Similarly to spec #6, students struggle with specs that require multiple steps to solve a problem.

#17: Find the form of a solution to a non-homogeneous differential equation using the method of undetermined coefficients. For this problem, one must write the form of the particular solution (but not solve for the coefficients) to five different forcing functions for a given second order DE (e.g. y'' + 5y' + 6y = f(t).) Some f(t) result in a linearly dependent initial guess for the particular solution that must be modified to make it linearly independent. This spec had a very high pass rate in the second attempt.

#24: Use Laplace transforms to solve constant coefficient linear differential equations that have forcing terms that are continuous. Students tend to feel rushed for assessments involving specs #21-25 since Laplace Transforms is the last unit in the course. Although students did fairly well in forward transformation (spec #22) and inverse transformation (spec #23) separately, students struggled to solve an IVP completely, as less than 40% of students ended up passing the spec. Prioritizing other courses over ODEs at the end of semester may also have influenced students' performance.

#25: Use Laplace transforms to solve constant coefficient linear differential equations that have forcing terms that are discontinuous. We didn't have enough time to cover the material for this spec in class. Spec **#**25 was made optional, and had two possible attempts.

5.3 Course Grades

Prior to calculating the course grade, baseline grades associated with specs were calculated as shown in Table 1. At both institutions, the baseline grade represented 80% of the final course grade.

Baseline Grade	Criteria: UP	Criteria: WIT
А	94/104 Points	22/24 Specs
В	84/104 Points	20/24 Specs
С	73/104 Points	16/24 Specs
D	63/104 Points	11/24 Specs
F	<63/104 Points	<11/24 Specs

The proportion of students achieving each of the baseline grades after varying numbers of attempts are shown in Figure 5. We see that at both institutions, grades significantly improved after multiple attempts.



Figure 5: Percentage of students achieving each baseline grade with varying number of attempts at (a) UP and (b) WIT.

Some caution should be exercised when interpreting Figure 5. It may be tempting to think of the "1 Attempt" data as a control, or an indication of what the grade distribution would have been had this been a traditionally graded course. The fact that these grade distributions are quite low can be at least partially attributed to some students not taking attempts seriously at first, knowing that they would have retake opportunities; this theory is confirmed by student feedback. Thus the very thing that reduced anxiety in some students also caused procrastination among other students. The relationship between these two seemingly opposed trends and the prevalence of this procrastination bears further investigation.

6 Instructional Observations

6.1 Instructor Workload

In terms of instructor workload, the need to write multiple versions of spec questions for various retakes takes increased time as compared to the traditional approach. On the other hand, while cumulative assessments and multiple retakes result in more problems to grade, the grading scheme is easier than a traditional grading approach. It is much easier to tell quickly whether a solution sits on a 2- or 4-point scale than it is to search through lines of algebra looking for missing minus signs. At WIT, the retake system the authors used, however, was quite laborious due to both its volume and the decision to allow retakes up to the final day. We feel that if we simplify the testing logistics, then we will be able to better attend to students, helping with their anxiety. Potential changes to the SBG implementation is discussed in Section 7.

6.2 Student Behavior

At UP, it was anecdotally noted that students would ask questions about old topics in office hours, typically before each exam date, whereas this generally did not happen in previous semesters the author had taught this class. This seems to indicate that students remained committed to learning earlier material on subsequent attempts, as opposed to letting the topic fall by the wayside until the cumulative final, as would occur with non-cumulative midterms. It was clear in the conversations with students that they appreciated what, in their mind, were "do-overs" on the exams.

Similar observations were made at WIT. Because students were allowed to schedule retakes up to the day of the final exam, some students procrastinated significantly, resulting in a situation where passing all of the specs became logistically impossible for them. Anecdotally, the realization of an overwhelming number of specs students wanted to pass seemed to increase the students' anxiety level on the last few days before the final exam.

6.3 Student Feedback

At UP, end-of-semester course evaluations reflected students' preference of SBG to a traditional grading scheme. Selected student comments involving SBG are below:

- "The test structure with multiple chances helped me actually learn the material better than a traditional test structure, while also having less test anxiety."
- "[SBG] was such a large stress relief for me. I loved the way that we were able to show at different times what we learned because I often struggle with learning things fast."
- "The new grading system was a bit confusing at first, but ended up being really cool and really helped me learn."

At WIT, a course survey was conducted at the end of the semester in addition to an anonymous, school-wide course evaluation. In both surveys, not surprisingly, the most popular aspect of SBG was the opportunity for multiple retakes. The second most popular comment was the clarity of expectations that SBG provided.

More than 50% of students responded positively to the retake aspects of the SBG. Some also indicated that SBG reduced anxiety and provided comments such as the following:

- "Test taking was much less stressful and having the ability to pick retake times to fit into your schedule and other course work was very nice and helpful."
- "[I liked] the stress reduction around assessments. The high stakes test taking in college is too high [SBG] helps to reduce that."

Some feedback on SBG in response to the question "What aspects of SBG do you wish were different?" included more lenient grading (20%), retake scheduling (14%) and the number of specs per assessment (10%). It is noted that many of these items are the exact items that some others liked about SBG. Additionally, 38% of students responded that nothing should be done differently.

7 Future Changes

Reflecting on our own and our students' experiences during our first term of SBG implementation, we recognized several areas of possible improvement.

One such improvement would be to rate each learning objective as "primary" or "secondary". Final grades could then depend on some number of primary and secondary objectives being met. For example, one may consider solving a separable first-order equation as a primary objective if a student who has not mastered this skill should not pass the course; whereas something like Laplace transforms, which are arguably less central in an introductory ODE course, may be categorized as a secondary objective. By using this two-tier categorization of objectives, we may be able to further reduce student anxiety by helping students focus on key objectives.

Another possible area of change is what portion of the final grade, if any, should be allocated to the aspects of students' performance aside from the specifications, such as attendance, participation and homework assignment scores. In its perhaps purest form, a SBG grade would be determined only on the learning objective benchmarks; in practice, one may wish to assign a portion of the final grade to other factors. At WIT, we counted class participation and homework as specs, where mastery was based on a student's level of participation in the in-class activities and successful completion of a certain percentage of the homework. We believe such "rewards" for positive behavior provided an incentive to be more engaged, resulting in higher confidence and lower anxiety.

Additionally, we struggled to determine the number of attempts students should have to master each learning objective as well as how to optimally schedule those assessments. Ideally, the number of attempts provided should be as large as possible, but this is precluded by practical considerations on time in the academic term, plus the time of the instructor(s). At WIT, we settled on four attempts; at UP, it was two to four, depending on when during the semester that particular spec was introduced. Having the number of attempts to be in this range still led most students to take each attempt seriously; we assume that a much greater number of possible attempts could foster more procrastination in addressing the material.

At WIT, several changes were made to the SBG scheme in the following semester. The authors initially limited the retake opportunities to two, all of which were in class. Each retake assessment was used only once, allowing students to freely discuss any of the assessment instruments with their peers. Recognizing early in the term that students may become discouraged with much of the term in front of them if they failed to master a spec after the second retake assessment, we added a single end-of-term retake session. This revised assessment structure has significantly simplified the process, reduced the level of logistics required and given us back the opportunity to meet with students during our office hours. We believe that streamlining the assessment retake process led to less procrastination and more clarity of the retake process, resulting in reduced student anxiety.

8 Conclusion

In undergraduate-level introductory ODE classes at two different institutions, we have found both qualitative and quantitative evidence of reduced student anxiety using a specifications-based grading approach. When asked to rank their anxiety surrounding the course on a Likert scale at multiple time points throughout the semester, we found a monotonically decreasing level of anxiety in SBG classes. Qualitative feedback reinforces our claim that the reduction in testing anxiety is, at least in part, the result of the grading scheme.

Appendix A: Specifications

Specifications used at UP

- 1. Check general solutions.
- 2. Check solutions to initial value problems.
- 3. Find and classify equilibrium points.
- 4. Plot solutions based on a slope field.
- 5. Draw phase line based on a slope field.
- 6. Draw a bifurcation diagram.
- 7. Find bifurcation points.
- 8. Nondimensionalize an equation given a change of variables.
- 9. Classify first-order equations as separable and linear.
- 10. Solve separable first-order equations.
- 11. Solve linear first-order equations.
- 12. Rewrite a linear second-order equation as a system of first-order equations.
- 13. Find equilibrium points and nullclines for a system of two first-order differential equations.
- 14. Draw a direction field.
- 15. Draw time-series plots using a phase plane with a given trajectory.
- 16. Solve a system of two linear, constant-coefficient, homogeneous first-order equations.
- 17. Match phase planes with their corresponding systems of equations.
- 18. Solve a system of two linear, constant-coefficient, nonhomogeneous first-order equations using the method of undetermined coefficients.
- 19. Solve a second-order differential equation of the form y'' = f(t, y') or y'' = f(y, y').
- 20. Solve a second-order, linear, constant-coefficient, homogeneous differential equation.
- 21. Solve a second-order, linear, constant-coefficient, nonhomogeneous differential equation.
- 22. Find the Laplace transform of a piecewise-defined function.
- 23. Find the inverse Laplace transform of a given function.
- 24. Solve an initial value problem using the Laplace transform.
- 25. Solve a Cauchy-Euler differential equation.
- 26. Find the power series solution to a second-order, linear, variable-coefficient differential equation.

Specifications used at WIT

- FCP 1. Explain what it means that a function is a solution to an ordinary differential equation.
- FCP 2. Verify by direct substitution that a function is a solution to an ordinary differential equation.
- FCP 3. Verify by direct substitution that a vector function is a solution to a matrix ordinary differential equation.
- FCP 4^{*}. Attend classes and engage in mathematics.
- FCP 5^{*}. Write a paper on a specified project.
- FCP 6^{*}. Successfully complete a significant amount of the homework.
 - 1. Identify the following characteristics of an ordinary differential equation: Order, linear or non-linear, separable or linear, the dependent variable and the independent variable.
 - 2. Understand, and create or identify slope fields for first order ODEs.
 - 3. Plot and explain solutions to first order IVPs on slope fields.
 - 4. Calculate and explain a piecewise linear approximation of a solution to a first order IVP (Euler's Method).
 - 5. Solve first order separable equations and IVPs by "separating" the dependent and independent variable terms and integrating.
 - 6. Solve first order linear equations and IVPs by using an "integration factor."
 - 7. Correctly formulate an appropriate first order equation and initial condition to model each of the following types of problems, and solve an IVP for one of the following types of problems: Growth or decay.
 - 8. Correctly formulate an appropriate first order equation and initial condition to model each of the following types of problems, and solve an IVP for one of the following types of problems: Newton's law of cooling.
 - 9. Correctly formulate an appropriate first order equation and initial condition to model each of the following types of problems, and solve an IVP for one of the following types of problems: Mixtures.
 - 10. Rewrite systems of constant coefficient, first order, ordinary differential equations as matrix equations for systems of homogeneous and/or non-homogeneous equations.
 - 11. Solve a homogeneous first order, constant coefficient, 2x2 matrix equation, with initial conditions, with real eigenvalues and eigenvectors and without computer assistance.

- 12. Solve a homogeneous first order, constant coefficient, 2x2 matrix equation, with initial conditions, with complex eigenvalues and eigenvectors and without computer assistance.
- 13. Describe the physical meaning of the vector and/or scalar solutions in terms of the original system of equations and the physical system that it is modeling.
- 14. Directly solve a second order, constant coefficient, homogeneous differential equation with distinct real roots.
- 15. Directly solve a second or higher order, constant coefficient, homogeneous differential equation with real and repeated roots.
- 16. Directly solve a second or higher order, constant coefficient, homogeneous differential equation with complex roots.
- 17. Find the form of a solution to a non-homogeneous differential equation using the method of undetermined coefficients.
- 18. Solve a non-homogeneous IVP.
- 19. Directly solve a constant coefficient, homogeneous or non-homogeneous differential equation that is higher than second order.
- 20. Describe the physical meaning of a solution function in terms of the original equation and the physical system that it is modeling.
- 21. Find the Laplace transform of a function using the integral definition of the transform.
- 22. Find the Laplace transform of functions using a Laplace transform table.
- 23. Find the inverse Laplace transform of functions using a Laplace transform table.
- 24. Use Laplace transforms to solve constant coefficient linear differential equations that have forcing terms that are continuous.
- 25. Use Laplace transforms to solve constant coefficient linear differential equations that have forcing terms that are discontinuous.

*Fundamental Theorems and Concepts (FTC) 4-6 did not have multiple retakes, and are excluded from certain analyses in this paper.

Appendix B: Anxiety Survey

Q1 (a) Rank the amount of anxiety you feel surrounding test-taking in this class compared to other math classes you have taken or are currently taking.

- I feel much more anxiety in this class.
- I feel a little more anxiety in this class.
- I feel about the same anxiety in this class.
- I feel a little less anxiety in this class.
- I feel less anxiety in this class.

Q1(b) List any factors that are contributing toward increasing your anxiety surrounding test-taking in this class.

Q1(c) List any factors that are contributing toward decreasing your anxiety surrounding test-taking in this class.

Q2(a) Rank the motivation you feel to succeed on examinations in this class compared to other math classes you have taken or are currently taking.

- I feel much motivated.
- I feel a little motivated.
- I feel motivated about the same.
- I feel less motivated.
- I feel much less motivated.

Q2(b) List any factors that are contributing toward increasing your motivation to succeed on examinations in this class.

Q₂(c) List any factors that are contributing toward decreasing your motivation to succeed on examinations in this class.

Q₃(a) Please rank your time-management skills in preparing for examinations in this class compared to other math classes you have taken or are currently taking.

- I managed my prep time much better.
- I managed my prep time better.
- I managed my prep time about the same.
- I managed my prep time worse.
- I managed my prep time much worse.

Q₃(b) List any factors that are contributing toward increasing your time-management skills in preparing for examinations in this class.

Q₃(c) List any factors that are contributing toward decreasing your time-management skills in preparing for examinations in this class.

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