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An Innovative Curriculum Idea for the First Two Years of College Mathematics

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Introduction

> It was a late Friday afternoon. Two of our department's faculty members were lounging in the mathematics study/tutor room, talking with a bright mathematics student who was trying to make an argument. "Just grant me this," Sandy was saying. "Let x be a rational number, and let y be the next larger rational number." We were not willing to do such letting. Our questions about the nature of the number $(x+y)/2$ were greeted with puzzlement. Yes, it is a rational number. Yes, it is bigger than x but less than y. But so what?

Sandy was "filtering out" the contradiction that was so apparent to us as mathematicians. We were both reminded again of how "discrete" college freshmen's ideas of numbers are. Over the years in school they learn about integers first, then scatter in some fractions, and eventually throw in a few irrationals such as pi and square roots. But the notion of a continuum is beyond most of them when they enter college.

The disturbing realization on that Friday afternoon was that Sandy was not a bright freshman mathematics student. Sandy was a bright junior mathematics major, enrolled in our advanced calculus course. Sandy's previous 2-1/2 years of college mathematics had done little to develop a sense of the continuum. Calculus texts universally ignore this conceptual problem, and our calculus courses are virtually defined by the chosen text. So the Sandy phenomenon should not surprise us so much as it distresses us.

Sandy's limitations are broader than a superficial concept of the continuum, however. In reality, very few junior mathematics majors are well prepared to do well in the junior level courses we wish to teach. Our complaints include 1) lack of skill at the formal reasoning needed to understand and create proofs and other arguments; 2) lack of visualization ability; 3) inability to read mathematical writing, even expository writing; 4) poor skills at communicating mathematically; and 5) a lack of awareness of what we, and our colleagues in science, think is important about mathematics.

Perhaps our overriding frustration, however, is that our sophomores do not have a good basis on which to make a decision about majoring in mathematics or the sciences. Those who decide to enroll in junior level mathematics courses have been successful at the mechanical calculations learned in calculus and linear algebra - the same mechanics that have driven away many other students, especially women and minority students. Not only have they encountered precious little of the logical reasoning that makes up so much of the discipline; most of them have not experienced the elegance of beautiful mathematical relationships, the tension between intuition and logic, the breadth of exciting mathematical ideas beyond analysis, or the thrill of <u>doing mathematics</u>: asking questions, investigating special cases, making conjectures, trying to prove them, revising conjectures, and successful proofs.

Although we don´t pretend that college freshmen and sophomores can engage in research on the forefront of mathematics, we have seen students in our non-majors courses acquire a better feel for mathematical processes than our own majors do after two years of our courses. Students in our "Mathematics for the Liberal Arts" course understand the important role that mathematics has played in the development of civilization better than those who intend to do or use mathematics in their careers.

Our collective frustrations have led my department faculty to do some intensive thinking together (and with our other colleagues in the science division) about these problems since the fall of 1986. What I say here is my own organization of those thoughts. At least one other member of our department is here today to give other perspectives.

Our situation is very typical. Augsburg College is a liberal arts college of about 1500 students. As in most colleges, the first two years for students majoring in mathematics or the sciences include three semesters of calculus (two for biology majors), and - for mathematics, physics, and computer science majors - a semester of linear algebra. A few students outside the science division take Calculus I to satisfy a distribution requirement.

Occasionally I hear from mathematical colleagues at other schools the opinion that "If they can´t get it through these courses then they should major in something else." At Augsburg, however, we are asking if "these courses" really show students a true picture of mathematics, if they prepare students to become mathematicians or users of mathematics as well as other courses might. To get a handle on these questions, we made a list of goals that we want all of our mathematics and science majors to accomplish, at least eventually.

Our goals

Our goals can be stated rather simply. They are

The student should be able to

A. learn mathematics independently;

B. do mathematics; and

C. communicate mathematically.

D. The student should understand the importance of mathematics.

Let´s look at these in turn.

Learning mathematics, as we all have been trained to say, is not a spectator sport. It requires practice. Usually when we say this we

mean working lots of mechanical exercises. We also might include "problem solving," in at least one of its many interpretations.

But learning mathematics also involves a number of other skills. For one thing, to learn mathematics effectively a student must be able to read and listen to mathematics with understanding. Without getting into a discussion of understanding, I´ll say that understanding what is read and heard seems to have several prerequisites:

a knowledge of the shorthand used in mathematical communication;

an ability to follow logical arguments;

understanding of a representative set of mathematical concepts (so that the distance from a new idea to an old one is fairly small); and

familiarity with the processes the author or speaker may have used to arrive at the results being described.

These processes are what I mean by **doing mathematics**, our second goal. They include

modeling problems in mathematical terms, whether the problems arise out of "real world" situations or from questions about extending other mathematics;

generating data to search for patterns, including data from special cases;

making conjectures based on seeing patterns in data;

trying to prove or disprove those conjectures, reformulating them, and looking at specific cases, pitting intuition against logic, with each being informed by the other; and

asking exploratory questions that will lead to new problems.

All of these activities involve, of course, understanding a representative set of mathematical concepts.

Our third goal, **communicating mathematically** in an effective way, can be summarized as "writing and speaking well." But what´s really involved here? Writing and speech teachers can tell you in much more detail, of course, but I list a few ideas. Besides what must be rather standard for all fields - the necessity for an awareness of audience level and the ability to organize ideas - I can think of three requirements especially important to good mathematical writing:

knowledge of the shorthand of mathematical communication - the subtleties of quantifiers, and the careful use of English connectives to communicate the direction of an argument;

a sensitivity to the power and limitations of mathematical symbols and a feeling for how to achieve an effective balance between symbols and words; and, of course,

an understanding of a representative set of mathematical concepts.

By the **importance** of mathematics I certainly include the successes of mathematical models for solving practical problems - and their limitations, too. But I also refer to the effects of mathematics on the development of civilization, about which our non-majors learn something but mathematicians learn about only by accident if at all. Especially at a liberal arts college like Augsburg, with an awareness that the most creative contributors to a field are those who can look at problems from many different perspectives, it's important for mathematics graduates to have seen the many faces of mathematics that history has exposed.

Experiences to achieve these goals

Given that these are our major goals, what kinds of experiences are most likely to help students achieve them? Just restating what I have said will give us some major categories for these experiences; after doing so I shall expand upon them briefly.

We should provide students with <u>coached practice</u> at

reading mathematics,
listening to mathematics,
following logical arguments,
mathematical modeling,
generating mathematical data,
making conjectures,
creating mathematical proofs,
posing mathematical problems,
writing mathematics, and
speaking about mathematics;

our students should study the effects of mathematics on civilization;

and our students should encounter a representative set of mathematical concepts.

Let me expand on these three categories.

The term **coached practice** reminds me of football training or perhaps music lessons. Coaching involves demonstrating or describing how to do something, and then (even in team sports) giving a good bit of advice to individuals, based on their performance. It goes well beyond telling a group of people how to do something, asking them to do it on their own, and then testing them to see if they can do it. But isn't that what most of us do when we try to teach mathematics? Our approach works

fairly well for teaching skills at working exercises; but teaching more complex skills and concepts requires giving a good bit more individual attention. For whatever reason, instead of giving that attention we abandon the teaching of higher level skills, and Sandy arrives at the junior year without having obtained them.

In discussing the **influence of mathematics on civilization** we have meant the ways mathematics has influenced - and, naturally enough, been affected by - areas of life that, only in the last few hundred years, have come to be seen as different from mathematics. Alphabetically, some of these areas are

architecture,
literature,
music,
mysticism,
painting,
philosophy,
science, and
sculpture.

And here we are again at **understanding a representative set of mathematical concepts.** In one sense, the exact nature of these concepts diminishes in importance if our other goals are met. Does it matter whether or not we've spent a class period integrating by partial fractions if students can learn such techniques and even the ideas behind them on their own? On the other hand, the sample of ideas the student has learned must be representative. With no understanding of the concept of integration, learning about integration by partial fractions would be left to the few Ramanujans of the world.

But what must the set of ideas represent? Mathematics, of course. And what all does mathematics include? Think, now: geometry. Number theory. Algebra. Topology. Probability. Statistics. Graph theory. Combinatorics. Yes, even analysis. Can we get a representative sample of mathematical ideas by teaching calculus and linear algebra? No, not even if we throw in a little "discrete mathematics." We need a much broader sample to have a collection of ideas that are more-or-less dense in the continuum of ideas that make up contemporary mathematics.

We can learn something, we think, from other disciplines. In the first year of a physics major, the student takes "General Physics." In the first year of a sociology major, students study "Principles of Sociology." These courses introduce students to the big ideas and techniques of the field. To design a real "principles of mathematics" course we must do more than redesign Sandy's calculus course for a new century.

Implementation ideas

So how do we accomplish all this? We at Augsburg have been experimenting with a number of teaching ideas in our current courses, and some of them show promise for accomplishing some of the goals we have set above. For example:

Some of us assign the students to read a section of the text in advance, and we spend the class time answering their questions about it. We try to avoid just repeating the book in our own words. When the students have no more questions, we talk about an application or work some problems involving the ideas they have read about. They leave with an assignment that includes not only working more problems but also reading another part of the text.

Instead of introducing a new topic with "It´s very important to know this," we sometimes start with a problem. It may be a practical problem, but it might also be a problem that intrigues students through its mystery or even nonsense. In analyzing the problem together we develop some mathematics ideas, and, eventually - perhaps after several days or even weeks - solve the problem.

Encouraging discussion among students works well for creating an atmosphere of doing mathematics. As we implement a program that focuses on learning and process goals, we should take the opportunity to substitute quality for quantity.

If we want students to learn to learn by listening, they must not be deprived of lectures. They need to be coached, however, in how to listen effectively to mathematical talks.

Several of us require our students to write. I, for example, insist on a careful presentation, using complete, grammatical sentences, of each solution. I critique the presentation as well as the content, and if it´s not done well I ask for revisions. Others use journal writing or summary term papers. We ask students to explain concepts in their own words - a humbling experience for us all, but one that allows us to coach more individually.

Some of us ask students to prepare and deliver short oral presentations in class, even in the freshman year. Again, these are critiqued for style as well as content.

In our "Mathematics for the Liberal Arts course" we have found that a powerful way to demonstrate the influence of mathematics on civilization is to arrange the topics chronologically. We frequently use a "pseudo-historical" approach, choosing mathematical topics most appropriate to the historical period in question. This organization scheme has the additional advantage of gradually increasing the level of abstraction and thus the level of difficulty.

The notion of encountering ideas repeatedly, at increasing levels of difficulty, is known as a "spiral approach." We find that method very

effective within our courses as well as through the curriculum as a whole. Our thoughts about curriculum design are that during the first year we might move chronologically through a representative set of mathematical ideas, and then spiral through the main ideas another time during the second year, with less emphasis on cultural influences and with more detail and a higher level of rigor. In our standard junior and senior courses the students would encounter the same ideas a third time (at least), with even more details and an even higher level of rigor.

Some side benefits

We anticipate two important positive side effects of an approach like this. First, we hope that this approach will "hook" those students who in the past have been discouraged by the tedium and apparent irrelevance of the current calculus sequence. Research indicates that, for example, many women and minority students have been "turned off" by an apparently non-humanistic discipline. If the students can experience some of the ecstasy of doing mathematics, they will come to regard the discipline as less cold and inflexible.

Second, those students who are taking a mathematics course at this level purely to satisfy a liberal arts requirement will achieve a much better understanding of the breadth of mathematics and its importance to the development of society than they do in their current calculus course.

Augsburg's agenda

We see the five steps in our curriculum development process:

In the first step, we need to add to our collection of stimulating problems whose exploration will help us (with the students) "create" many ideas of mathematics. At the same time, we need to be addressing some of the logistical questions, such as making sure that our program meshes with others well enough so that transfer students are not handicapped too greatly.

Then we need to decide which problems are most likely to lead to discussions of particular ideas, and in which order they might be posed.

Some teaching methods are more appropriate for some problems than for others. At times we might want use lectures, at other times emphasize reading, and for other topics "invent" the mathematical ideas together through class discussion.

Since reading plays such an important role in any college course, students need to have written materials concerning a wide variety of mathematical topics at an appropriate level of difficulty. Of all the books we know of that address a variety of topics, none has both the

scope we desire and a level appropriate for students who are currently taking the calculus sequence. So we need to create our own text, using already-written materials as much as possible and writing our own to fill in gaps.

This brings me to where you come in. We need help. What stimulating problems have you used? What excellent reading do you know of about these various topics? Would you be interested in joining us in this new venture? in helping us design courses? in class testing ideas?

I´d be glad to hear from anyone willing to help us address the Sandy phenomenon.