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Mathematics and Its Application

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GOALS FOR MATHEMATICS EDUCATION

1. To give the educated person a fund of mathematical analogies and the ability to use them.

2. To enable the educated person to further create mathematical analogies for his own use when needed.

3. To enable the educated person to understand how and to what effect mathematical analogies have been used significantly for our civilization and our culture.

4. To enable the educated person to intelligently anticipate directions, areas, ways in which mathematical analogies, new or old, might have further significant effect for our civilization and our culture.

The latter two are rexpression of Peter Hilton's dictum, that we should "strive to awaken in as many people as possible, irrespective of their chosen vocation, an awareness of the nature of our science, and its significance for our civilization, material and spiritual."

The 'analogies' mentioned in 1 through 4 above need not be applications in the traditional sense; they may often have more to do with the relationships between ideas than with sensed reality; they may in fact be pure mathematics.

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When mathematics is taught using lots of applications and the applicability of mathematics is emphasized (perhaps to the point of becoming the guide and justification of mathematics education), students are likely to come to believe that the mathematics is "in" the worldly circumstances which are the focus of the application at hand. Such a belief does not encourage the investigation of the limitations of the application of mathematics to the situation. Further, it is ground for coming to see worldly reality as a mathematical edifice which must be conquered, and oneself as feeble before it. It would be better to teach that mathematics is the study of an independent, extant reality; and that any application of mathematics is the postulating of an analogy between a set of worldly circumstances and a set of mathematical circumstances. Then the coincidence of pattern is a cause for delight, and the limitations are expected, even if they are not seen.

I wish here to argue that we mathematics teachers should not feel that it is necessary to justify whatever mathematics we teach by its applicability, and further that even the most applicable mathematics should be presented as being worthy of study independent of its applicability, not least because its application will be done best when its independence is accepted. By 'application' I mean not only the use of mathematical patterns to increase control or predictive power in practical situations, but also the use of mathematical patterns to increase understanding in practical or impractical or even fantastical situations.

When I was learning to fly, my instructors told me, "The throttle controls your altitude, the elevator controls your speed, not the other way around." I now believe that is a distortion, but it was a very useful one. When one begins flying an airplane, he has a natural tendency to believe that the throttle controls the airplane's speed and the elevator makes the plane go up or down. In fact, the situation is much more complicated than that; and my instructors' dictum was instrumental in my learning to use the two controls effectively. I would use it today if I were to take the controls of an airplane again.

I don't think the phenomenon of an idea being of good effect even if it is a distortion is a particularly unusual one. Another example might be the dictum, "There is no important difference between men and women." Or, "When acting in a play by Shakespeare, one should never give emphasis to a personal

pronoun." If those sentences are not true, acting as if they were might nevertheless have good effects because they counteract some deeply ingrained erroneous biases.

The idea I want to offer here is, I suspect, correct. Even if it is not, however, acting as if it were would have good effects. And so I want to present the idea here, examine some of its consequences, and defend that complex consisting of the idea and its consequences.

The idea is this: first, that mathematics is an account of an extant reality that is independent of physical reality or social reality; and second, that it is therefore appropriate to understand applications of mathematics in various arenas as analogies, analogies which often appear as metaphors.

I don't know, nor do I care, just what philosophical position I am taking when I say that mathematics is an account of an extant reality. The important point is that mathematical truth is independent of human judgment. 29 is prime. 27 is not prime. Those sentences are true not because I or any expert says so, but because 29 <u>is</u> prime and 27 is <u>not</u> prime. That is just the way they are. I don't know offhand whether forty-three times sixty-seven is equal to fifty-one times sixty-one, but there is no doubt that it either is or it isn't, independent of anything anyone does or says. Mathematical truth is about the least contingent truth around. Mathematical reality is there to be discovered or observed.

At the same time, mathematics gives an account of logical inevitability, not physical inevitability. "There is one thing...of which a geometry is <u>not</u> a picture, and...that is the so-called real world," says the famous mathematician, G. H. Hardy. (1) No one can mathematically prove that the sun will rise tomorrow or that it won't, that the stone will fall when released, that closing the circuit will cause the light bulb to come on, or whatever; physical reality may more or less coincide with mathematical pattern, but it is not constrained by it. This fact gives substance to the word 'unreasonable' in the title of Eugene Wigner's essay, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." (2)

What, then, is the relationship between mathematics and, say, physics? Is mathematics a language in which physics is expressed, as is suggested by the title of Tobias Dantzig's book, <u>Number: The Language of Science</u>? (3) If I am right that mathematics is an account of an extant reality, then it is certainly more than a language. Perhaps mathematical symbolism is a language, but the mathematical ideas, the mathematical facts, expressed by the symbols are not mere language. Rather, physics is a science which, among other things, makes analogies

between mathematical circumstances and physical circumstances. If extent of spatial separation is like number, and this particular spatial separation has an extent which we take to be like the number 5, then that one has an extent which is like the number 7, and furthermore, the two of them together constitute a spatial separation the extent of which is like the number 12. This is not a particularly unusual idea; mathematicians and scientists often refer to mathematical "models" of worldly circumstances, by which they don't mean anything too different from what I mean when I use the word 'analogy.'

Why, then, do I choose to use that somewhat unusual word? Because the word 'analogy' tends to remind us of certain things that the word 'model' does not. The important points are these: any analogy has two parts or sides, and the extent of validity of an analogy is always in question. (Note Wendell Berry's words, quoted later in this paper.)

There are, roughly speaking, two kinds of analogy: explicit and implicit, simile and metaphor. The literary critic Northrop Frye observes,

> In descriptive writing you have to be careful of associative language. You'll find that [simile], or likeness to something else, is very tricky to handle in description, because the differences are as important as the resemblances. As for metaphor, where you're really saying "this _is_ that," you're turning your back on logic and reason completely, because logically two things can never be the same thing and still remain two things....The motive for metaphor, according to Wallace Stevens, is a desire to associate, and finally to identify, the human mind with what goes on outside it, because the only genuine joy you can have is in those rare moments when you feel that although we may know in part, as Paul says, we are also a part of what we know. (4)

If application of mathematics is the making of analogies, then in much of human knowledge, mathematics is (ironically, for if mathematics is not logical, then nothing is) not a simile but a metaphor, in fact an unconcious metaphor, a culturally subconcious metaphor, for other sorts of reality.

Digression

The words left out by the ellipsis in the quote by Northrop Frye are,

> The poet, however, uses these two crude, primitive, archaic forms of thought in the most unhibited way, because his job is not to describe nature, but to show

> you a world completely absorbed and possessed by the human mind. So he produces what Baudelaire called a "suggestive magic including at the same time object and subject, the world outside the artist and the artist himself." (5)

And when Frye mentions Wallace Stevens, he is referring to the following poem by Stevens:

The Motive for Metaphor

You like it under the trees in autumn,
Because everything is half dead.
The wind moves like a cripple among the leaves
And repeats words without meaning.

In the same way, you were happy in spring,
With the half colors of quarter-things,
The slightly brighter sky, the melting clouds,
The single bird, the obscure moon--

The obscure moon lighting an obscure world
Of things that would never be quite expressed,
Where you yourself were never quite yourself
And did not want nor have to be,

Desiring the exhilarations of changes:
The motive for metaphor, shrinking from
The weight of primary noon,
The A B C of being,

The ruddy temper, the hammer
Of red and blue, the hard sound--
Steel against intimation--the sharp flash,
The vital, arrogant, fatal, dominant X.

Frye's contrast between "describing nature" and "showing a world possessed by the human mind," and Stevens's use of technical symbols (A B C, X, weight, primary noon, temper, steel) for what is shrunk from when one is motivated to metaphor, suggest that I am off base, that the whole point of what they are talking about is the contrast between poetry on the one hand and the "objective" worlds described by math and science on the other. But remember that Stevens's title is "The <u>Motive</u> for Metaphor." Is not science an (in some ways quite successful) attempt by the human mind to absorb and possess a world?

It is one of the ironies of Man's present condition that a motive of identification should have been a driving force behind what seems to so many to be the source of so much alienation, i.e., the application of mathematics to worldly circumstances.

As I put the finishing touches on this essay, I am starting to read a new book by the philosopher William Barrett, in which I found this:

> We note the extraordinary power and constructivity of the human mind in producing the great edifice of modern science. And yet, precisely here occurs one of the supreme ironies of modern history: The structure that most emphatically exhibits the power of mind nevertheless leads to the denigration of the human mind. The success of the physical sciences leads to the attitude of scientific materialism, according to which the mind becomes, in one way or another, merely the passive plaything of material forces. The offspring turns against its parent. We forget what we should have learned from Kant: that the imprint of mind is everywhere on the body of this science, and without the founding power of mind it would not exist.
>
> The irony here is not one that we can merely sit back and enjoy aesthetically. This doubt of the mind, in its actual consequences, in the lives of individuals and societies, provides one of the ordeals that modern civilization will have to go through. (6)

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Frye suggests earlier in his essay that as science moves from data towards laws, it "moves toward imagination," it tends to invoke mathematics, which, along with literature and music, is (he says) a language of the imagination. "A highly developed science and a highly developed art are very close together, psychologically and otherwise." (7) Pursuit of mathematics is the pursuit of understanding. (Henry Pollack has said that the essence of science is the right to repeat an experiment, while the essence of mathematics is the right to understand.(8)) Surely understanding is, in Frye's terms, the identification of the human mind with something outside it, the attribution of meaning to the coincidence of pattern. In the case of applications of mathematics, meaning is often attributed to the coincidence of mathematical patterns with patterns of worldly circumstances. Carried along by the logical inevitability of mathematics, undaunted by obscurities in the coincidence of the patterns, users of mathematical applications can come to believe that the mathematics is "in" the worldly circumstances. (9) Understanding becomes "knowledge,"

Digression

Scott Buchanan gives a more thorough account of this in his book Poetry and Mathematics. Here are some relevant excerpts:

"Belief is the natural attitude of a thwarted mind. It

> arises from fatigue and confusion....For the most part confusion is of two sorts, one involving symbols, and the other metaphysical nostalgia, the tendency of thought toward the absolute." (10)

For Buchanan, symbols are things (aesthetic objects) which point to ideas (intellectual objects). He explains the confusion involving symbols as follows:

> The aesthetic properties of ceremony, formula, natural processes are intimations of complex and profound intellectual objects, but the difficulties of intellectual clarification and discrimination leave the mind in various attitudes of belief. For every intellectual object, half-comprehended, there is an aesthetic object before which we bow in more or less deep reverence. Pure aesthetic contemplation and complete intellectual clarity are seldom found in human beings, and any middle ground is touched with credulity and idolatry. (11)

As to the metaphysical nostalgia,

> [a stage of mathematical discovery] results when we can see the <u>relations</u> holding between <u>qualities</u>.... Mathematical functions find elementary values in qualities. Qualities find their relations in the functions of mathematics. Whenever this happens, a system is recognized, and it takes on a quasi-independence and reality. Often the effect on the thinker is a conviction. Belief attaches itself only to such systems. The further expansions and the wider assumptions are ignored and there is a resting point for thought in a mathematico-poetic allegory. (12)

Of course, we are dealing here not with a disorder or aberration in human thought processes, but with the very nature of thought itself. Buchanan's account has some similarities with this description by Ernst Cassirer:

> [For the religious genius,] the power of his belief first proves itself in being made public. He must communicate his belief to others, he must fill them with his own religious passion and fervor, in order to be certain of his belief. This is possible only by means of religious constructs--constructs which begin as symbols and end as dogmas. Thus, even here, every initial expression of feeling is already the beginning of an alienation. It is the destiny and, in a sense, the immanent tragedy of every spiritual form that it can never overcome this inner tension; to extinguish it is to extinguish the life of the spirit. For the life

> of the spirit consists is this very act of severing what is whole in order that what has been severed may be even more securely united. (13)

And, in Cassirer's view, this pattern runs deep. Later, but talking about the same example, he says,

> And so it is that here, too, we find the same oscillation which sets in within all forms of culture as they begin to take shape[,]...the ceaseless and irresistible rhythm of life itself. (14)

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and that knowledge is passed from one human being to another, often as subject matter in academic courses. And so we can see how mathematical circumstances, which are intrinsically logical, become illogically (but perhaps more or less appropriately) identified with things they are not, namely worldly circumstances.

Mathematics is often a tremendously effective metaphor, "unreasonably effective," in Eugene Wigner's words. But there is danger here. Essayist Wendell Berry, in his volume on culture and agriculture in the United States, refers in passing to

> ...the model of the scientists and planners:... an exclusive, narrowly defined ideal which affects destructively whatever it does not include. (15)

Joseph Weizenbaum, a computer scientist and teacher of computer science, speaks this message to his fellow computer science teachers:

> I...affirm that the computer is a powerful new metaphor for helping us to understand many aspects of the world, but that it enslaves the mind that has no other metaphors and few other resources to call on. The world is many things, and no single framework is large enough to contain them all, neither that of man's science nor that of his poetry, neither that of calculating reason nor that of pure intuition. And just as love of music does not suffice to enable one to play the violin--one must also master the craft of the instrument and of music itself--so is it not enough to love humanity in order to help it survive. The teacher's calling to teach his craft is therefore an honorable one. But he must do more than that: he must teach more than one metaphor, and he must teach more by the example of his conduct than by what he writes on the blackboard. He must teach the limitations of his tools as well as their power. (16)

Mathematics is, like the computer, a powerful metaphor. Unlike the computer, it is not a new metaphor. Any metaphor can take on excessive significance in a human imagination. Perhaps because mathematics is not a new metaphor, we are inured to it a bit; but that may just mean that we have lost the nervous uncertainty that comes with venturing into new territory, but not seen enough failure of the metaphor to undermine our credulity. Metaphor is a wonderful thing, not at all to be scorned. Even if it were not wonderful, it would probably be inevitable. (17) The motive for metaphor is a glorious aspect of human nature. G. Spencer-Brown observes, "That mathematics, in common with other art forms, can lead us beyond ordinary existence, and can show us something of the structure in which all creation hangs together, is no new idea." (18) But we need not take away from the value of the metaphor, or from the metaphorical experience, indeed we will enhance them, if we keep in mind the question, "What are the limits of this metaphor?"

If the application of mathematics is the construction of analogy, whether simile or metaphor; and if we are to effectively understand the limits of the analogy; then we must have some understanding of each of the two things being compared, and that understanding must be wider than what is immediately relevant to the analogy. Hearing Macbeth say of Duncan,

After life's fitful fever he sleeps well, (19)

we bring to bear all our experience of life, fever, sleep, and death; and if we did not know enough of sleep and death, of life and fever, to know many ways in which they are not alike, as well as ways in which they are, then the metaphors would not be as rich and effective, the sentence not as beautiful, as they are. Of course, at the same time, analogies broaden our experience, deepen our understanding, give us new insights into the things being compared; indeed, that is their very function.

What does all this mean for the teaching and learning of mathematics? It means we should teach and learn mathematics beyond that which is "relevant," that which appears explicitly in applications of practical importance. The perennial students' question, "When will we ever use this?" is a misguided question, one to which we should not succumb. That is not to say that we should refuse to answer it; but we should deny that the question is determinative of what is important in a person's study of mathematics. Mathematics is one side of a myriad of important analogies; if we are to understand that side, then we must understand, we must teach and learn, mathematics itself. (And, of course, the study of mathematics itself has its own rewards, quite apart from applicability. Alfred North Whitehead describes the pursuit of mathematics as "a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings." (20) Hardy says, "Real mathematics...must be

justified as art if it can be justified at all." (21) Scott Buchanan observes, "The structures with which mathematics deals are more like lace, the leaves of trees, and the play of light and shadow on a meadow or a human face, than they are like buildings and machines, the least of their representatives." (22)) Knowledge of calculus has vocational value for engineers and others, but its value is increased by deep understanding of the mathematical theory of calculus; knowledge of statistics is perhaps necessary for informed citizenship; knowledge of arithmetic has survival value; knowledge of number theory may have none of these, but it strengthens one's understanding of mathematics as an independent reality which is in some of its facets analogous to some facets of other kinds of reality. (23)

It also means that we should resist, and teach our students to resist, any tendency to neglect those aspects of other sorts of reality that do not fit into those analogies with mathematical reality that we call applications. We should in fact look for them. We should seek to understand the limitations of our analogies, and we will understand them better if we know what is beyond them, on both sides. We will be the richer for knowing what of mathematical reality does not fit the physical circumstances, and what of physical reality does not fit the mathematical circumstances, of whatever mathematical application we are dealing with.

We should, in short, let mathematics be, just as other disciplines are, the pursuit of ways of seeing, the pursuit of visions. We should teach our students to look for mathematical analogies, to delight in them when they find them, to stretch them and test them and savor them, but not to be consumed by them lest they, we, suffer the fate of all tragic heroes. (24) We will lay the proper foundation if we teach them mathematics itself, as independent, extant reality, whose "applications" are in fact analogies which often appear as metaphors.

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Notes

(1) G. H. Hardy, "What is Geometry?," Presidential Address to the Mathematical Association, 1925.

(2) Eugene P. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," Communications on Pure and Applied Mathematics, Vol. XIII, 001-14 (1960).

(3) Tobias Dantzig, Number: The Language of Science (Garden City, NY: Doubleday & Company, Inc., 1954)

(4) Northrop Frye, The Educated Imagination (Bloomington: Indiana University Press, 1971), pp. 32-3.

(5) Ibid, pp. 32-3.

(6) William Barrett, Death of the Soul (Garden City, NY: Anchor Press/Doubleday, 1986), p. 75.

(7) Frye, The Educated Imagination, pp. 23-4. Perhaps one of the more important differences has to do with belief. See the second digression.

(8) This was in a talk that Dr. Pollack gave on "The History of an Application" at the 1986 Woodrow Wilson Summer Institute on High School Mathematics in Princeton, New Jersey.

(9) As part of its promotion of "Math Education Month" (April, 1987), the National Council of Teachers of Mathematics is offering for sale a bumper sticker which reads, "Math Keeps the World in Motion." Of course, as is proper for a bumper sticker, the sentence is ambiguous, suggestive, a play on words. It points to the central role of mathematics in modern technology, government, economics, etc. But it also suggests that math causes the earth to rotate on its axis and revolve around the sun, the automobiles and telephones to work, and so on. It can be read as ironic (in a couple of subtly different ways), but for one who cannot see the irony, it could be an intimidating and depressing, an unfortunate, message.

(10) Scott Buchanan, Poetry and Mathematics (Chicago: The University of Chicago Press, 1975), p. 135.

(11) Ibid, p. 140.

(12) Ibid, pp. 146-7.

(13) Ernst Cassirer, The Logic of the Humanities (New Haven: Yale University Press, 1974), pp. 115-6.

(14) Ibid, p. 214.

(15) Wendell Berry, The Unsettling of America: Culture and Agriculture (San Francisco: Sierra Club Books, 1986), p. 112.

(16) Joseph Weizenbaum, Computer Power and Human Reason (San Francisco: W. H. Freeman and Company, 1976), p. 277.

(17) When looking up the sentence about belief and the thwarted mind, I discovered how much my thinking in this essay had been affected by Buchanan's book. For example, the last sentence in one of his chapters is, "Any history of thought might begin and end with the statement that man is an analogical animal." Buchanan, Poetry and Mathematics, p. 141.

(18) G. Spencer-Brown, Laws of Form (New York: E. P. Dutton, 1979), p. xxix.

(19) This example will be recognized as borrowed from Hardy.

(20) Alfred North Whitehead, Science and the Modern World (New York: The New American Library of World Literature, 1954), p. 22.

(21) G. H. Hardy, A Mathematician's Apology (New York: Cambridge University Press, 1977), p. 139.

(22) Buchanan, Poetry and Mathematics, p. 36.

(23) Buchanan makes several comments relevant to mathematical pedagogy. Here is one that relates to the use of mathematical applications in the classroom:

> Mathematics is not a compendium of memorizable formula and magically manipulated figures. Sometimes it uses formulae and manipulates figures, but it does this because it is concerned with ideas already familiar to the ordinary mind, but needing special sets of words or symbols for the sake of precise expression and efficient communication. Further, the abstraction thus signalized, which most people from bad emotional habits fear, is actually much more familiar to the untrained mind than any observed facts could possibly be. Abstract ideas are of the very tissue of the human mind. For this reason and for many others, illustration of mathematics by concrete event, fact, or object is never as effective as illustration by equally abstract analogous ideas.

Ibid, pp. 35-6. Of course, in this passage Buchanan is speaking of the use of the concrete to illustrate mathematics, rather than the use of mathematics to explain the concrete.

(24) I have heard attributed to Mark Twain the remark, "It ain't what people don't know that causes all the trouble, it's what they do know that ain't so."