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HUMANISTIC MATHEMATICS NETWORK

Newsletter

Number 3

December 1988

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HARVEY
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DEPARTMENT
OF
MATHEMATICS

714-621-8023

December 1988

Dear Colleagues:

Mathematics as a Humanistic Discipline began modestly, but the concept and the Humanistic Mathematics Network have grown rapidly. The initial conference of about thirty has become a network of about four hundred. A network list is included in this newsletter. The November 12 Southern California regional MAA meeting included a contributed paper session on Humanistic Mathematics with six presentations. The January national meeting in Phoenix will have three contributed paper sessions on Humanistic Mathematics with a total of thirty presentations. A book Humanistic Mathematics will be published as one of the MAA Notes.

The Network and Newsletter are arriving at a situation where decisions about the future should be made. Interested people may be able to gather in Phoenix to discuss future directions of the Network and Newsletter. At each of the three contributed paper sessions a time and place for such a gathering will be announced. If you would like to contribute to the discussion by having your remarks printed in a future newsletter send your remarks to me (single spaced). Include your BITNET address if you have one. A steering committee will discuss various ideas and suggestions and try to chart the future.

The initial Exxon grant for the newsletter will run out soon. Should the Network do more than publish the Newsletter? The Network and Cal State University, Northridge, are sponsoring a workshop February 3, 4, at which Sherman Stein will lead us in creating non routine problems. Should other workshops or conferences be part of the Network activities? Should contributed paper sessions at regional and national meetings be regularly organized?

The essays that have been printed in the Newsletter have been of high quality. Many have been reprinted in more established refereed journals. Several have been mentioned in other journals and many requests for copies have come to me. Should the informal submission and solicitation of essays be continued?

Among the requestors for a copy of an essay was a senior editor from Freeman and Company, Publishers. A senior editor from Random House Publishers is very interested in the Network and Newsletters and would like to encourage authors of texts and other math books to write "humanistically." The Random House editor sent me some copies of the Random House Dictionary, Second Edition Unabridged as possible prizes for outstanding essays printed in the Newsletter, or any other use. Suggestions are welcome.

If we pay at least $5 a year dues, the Network can become a constituent organization of the AAAS. Should we become a Special Interest Group (SIG) of the MAA if that is possible? Should we charge dues to finance the Newsletter and other activities? Should we apply to foundations, publishers, corporations, etc. for funding? Should the Network be guided by a "Mission Statement" or equivalent?

Newsletter #4 will include presentations to the MAA contributed paper sessions, essays about our future directions, and perhaps other essays which are contributed. I hope that the time between #3 and #4 will be shorter than the time between #2 and #3.

Sincerely,

Al White

Alvin White

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AW/bg

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Newsletter #1

August 3, 1987

DEPARTMENT
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714-621-8023

Dear Colleague,

This newsletter follows a three-day **Conference to Examine Mathematics as a Humanistic Discipline** in Claremont 1986 supported by The Exxon Education Foundation, and a special session at the AMS-MAA meeting in San Antonio January 1987. A common response of the thirty-six mathematicians at the conference was, "I was startled to see so many who shared my feelings".

Two related themes that emerged from the conference were 1) teaching mathematics humanistically, and 2) teaching humanistic mathematics. The first theme sought to place the student more centrally in the position of inquirer than is generally the case, while at the same time acknowledging the emotional climate of the activity of learning mathematics. What students could learn from each other, and how they might better come to understand mathematics as a meaningful rather than an arbitrary discipline were among the idea of the first theme.

The second theme was focused less upon the nature of the teaching and learning environment and more upon the need to reconstruct the curriculum and the discipline of mathematics itself. The reconstruction would relate mathematical discoveries to personal courage, relate discovery to verification, mathematics to science, truth to utility, and in general, to relate mathematics to the culture in which it is embedded.

Humanistic dimensions of mathematics discussed at the conference included:

a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished versions to be "merely technical".

b) An appreciation for the human dimensions that motivate discovery- competition, cooperation, the urge for holistic pictures.

c) An understanding of the value judgements implied in the growth of any discipline. Logic alone never completely accounts for *what* is investigated, *how* it is investigated, and *why* it is investigated.

d) There is a need for new teaching, learning formats that will help wean our students from a view of knowledge as certain, to-be-received.

e) The opportunity for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems and to participate in controversy over mathematical issues.

f) Opportunities for faculty to do research on issues relating to teaching, and to be respected for that area of research.

This newsletter, also supported by Exxon, is part of an effort to fulfill the hopes of the participants. Others who have heard about the conferences have enthusiastically joined the effort. The newsletter will help create a network of mathematicians and others who are interested in sharing their ideas and experiences related to the conference themes. The network will be a community of support extending over many campuses that will end the isolation that individuals may feel. There are lots of good ideas, lots of experimentation, and lots of frustration because of isolation and lack of support. In addition to informally

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sharing bibliographic references, syllabi, accounts of successes and failures, . . ., the network might formally support writing, team-teaching, exchanges, conferences, . . .

Please send references, essays, half-baked ideas, proposals, suggestions, and whatever you think appropriate for this quarterly newsletter. Also send names of colleagues who should be added to the mailing list. All mail should be added to the mailing list. All mail should be addressed to

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This issue contains some papers and excerpts of papers that were presented at the conferences.

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A Humanistic Academic Environment for Learning
Undergraduate Mathematics

by

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A Humanistic Academic Environment for Learning Undergraduate Mathematics

Teachers of undergraduate mathematics work under conflicting professional responsibilities. In strong Ph.D. granting mathematics departments undergraduate enrollment in mathematics forms the major support for graduate students as well as the regular mathematics faculty. At these universtities much undergraduate mathematics is taught by graduate assistants who have their primary obligation to their graduate studies and research. Most regular mathematics faculty at these universities have no interest in teaching undergraduate mathematics. (It is rare for undergraduates to understand or to participate in the research of their mathematics teachers.). If regular mathematics faculty teach undergraduate mathematics, the lecture method is used most often with very large classes so that the lecturers have almost no knowledge of the hopes, anxiety, or growth in mathematical maturity of their students. Mathematics faculty at these universities expect a large number of their best graduate students to be foreign students and very few will be selected from the undergraduates they teach at their own university. Research grants and fellowships are sought in order to relieve a faculty member from teaching, and in particular undergraduate teaching.

In group M and B Departments (Departments granting a Master's or Bachelor's degree as the highest degree) the teaching loads, support for graduate students and research imply that the primary responsibility for the mathematics faculty is to teach undergraduate mathematics. However, research publications compose an important part of the qualifications for tenure and promotion, although the academic climate for high quality research is not very favorable at these colleges and universities.

One estimate is that more than 50% of the current enrollment in mathematics courses at public four-year colleges is for remedial courses and pre-calculus courses. Also on average fewer than 10% of undergraduate credits in the mathematical sciences are in post-calculus level courses. Often students are assigned to remedial and pre-calculus courses by placement examinations which may not correlate very well with the preparation and ability of students to learn college mathematics, but may correlate with the academic environment for learning undergraduate mathematics at a particular college or university. In general, students do not enjoy studying these courses and teachers do not enjoy teaching them. Frequently, part-time teachers are employed to teach these courses. Perhaps the failure rate in regular calculus courses is an indication of an unfavorable academic environment for learning undergraduate mathematics, as well as the lack of effectiveness in teaching remedial and pre-calculus courses in such an environment.

In large group M and B Departments with few mathematics majors many regular faculty members may have an opportunity very rarely to teach a post-calculus course for mathematics majors. Some faculty members believe that only those students who have the ability to continue their studies in mathematics to the graduate level and to become research mathematicians should be encouraged to major in mathematics. Since research in mathematics is very competitive, mathematics majors should be limited to an elite class of geniuses, and a particular college or university may have

very few or no members of this class. Thus many mathematics faculty members are encouraged to process students through undergraduate mathematics courses for supply departments and/or general college mathematics course requirements, and this processing helps to establish an unfavorable academic environment for learning undergraduate mathematics.

At two-year technical colleges and community colleges most of current enrollment in mathematics courses is for remedial courses and pre-calculus courses. Very limited or no opportunity is provided for many faculty members at these colleges to teach across the mathematics curriculum and rarely is there provided a humanistic academic environment for learning undergraduate mathematics.

Some talented and dedicated teachers at each type of school described above are able to obtain some good results in teaching students undergraduate mathematics. However, the writer contends that most students study undergraduate mathematics in academic environments which are dehumanizing for both students and teachers. Too many future elementary and secondary school teachers study mathematics in such environments. It is important for mathematicians and mathematics educators to discuss such things as the content of the calculus course and how calculus is taught, concrete vs. abstract in mathematics education, as well as the role of problem-solving. The writer challenges chairs of Mathematics Departments and other responsible academic administrative officers to provide a humanistic academic environment for learning undergraduate mathematics. If this can be done, the lay public may view mathematics more favorably, and give mathematics education and research in mathematics more support.

During the past 20 years, the Mathematics faculty at SUNY Potsdam has made a determined effort to establish a humanistic academic environment for learning undergraduate mathematics, and some unfavorable national trends in mathematics education have been reversed. Our average number of bachelor's in mathematics during the last three years is 193 and the average percent is 24% while the national average is about 1%. The gender imbalance in mathematics seen nationally is not a factor at Potsdam College. A little more than 54% of our college graduates during the last 18 years have been women and little more than 55% of our Bachelor's in mathematics have been women during the same period. At SUNY Potsdam the completion of mathematics as a major is gender independent.

Most students enroll in mathematics courses on a voluntary basis and not as a requirement for a major or minor in some other subject. Our college has no mathematics requirement as a condition for graduation. For example, one year with a freshman class of less than 1000 students, more than 600 students enrolled in beginning calculus. No more than 100 of these students came from supply departments. The issue of teaching algorithms vs. teaching thinking or concrete vs. abstract in mathematics education is not a problem at Potsdam College. Students consider the study of mathematics as an important part of a liberal arts education and not necessarily as a way of making a living using mathematics primarily. For example, some bachelor's in mathematics in the class of 1987 completed a second major in the following subjects: Anthropology, Biology, Chemistry, Computer and Information Sciences, Education, Economics, English, French, Geology, History, Political Science, Psychology and Physics. Also, they completed minors in the following subjects: American Politics, Business

Economics, Business of Music, Directing, and Health Science.

The number of bachelor's in mathematics who entered Potsdam College with high school averages of 90 and above increased more than 9 times during the past 18 years. In our graduating class of 1987 more than 40% of the students who graduated summa cum laude or magna cum laude were bachelor's in mathematics. More than 50% of our undergraduate credits in mathematics are in post-calculus courses, while the national average is less than 10%. We have good cooperation from supply departments, our bachelor's in mathematics choose many different career options, and they are repeatedly hired by the same company or government agency as industrial mathematicians. They make many professional choices for graduate study.

A brief outline of the steps we took to establish our academic environment is given below.

Entering freshmen students with high school averages in mathematics of 90 and above, quantitative SAT scores of 550 and above, good general high school averages and aptitude test scores are invited to elect our honors calculus course during the fall semester of their freshman year without regard to their intended major in college. Each student is sent a personal letter of invitation. We make clear to students invited that they will not be penalized by the grades they will receive as a result of electing the honors calculus course rather than the regular calculus course. We explain that all students enrolled in calculus will be given the same final examination. Also, the teachers who teach an honors calculus section teach a regular calculus section and are well able to compare the achievement of students in different calculus sections.

From an entering freshman class of 1,000 students or less, we usually invite about 130 to 150 students. A little more than one-half of the students invited elect the honors calculus section. In the fall of each year, we usually offer two sections of honors calculus with an enrollment of 35 to 45 students. Some of the students invited who do not elect the honors calculus course do elect the regular calculus course. Many of our best mathematics majors complete our honors calculus course, although our mathematics faculty never discussed or made any particular effort to increase the number of mathematics majors.

We established in 1970 a BA/MA program in which able students can complete both the bachelor's and master's degrees in four years without attending summer school. Some of our most able students with advanced placement credit in calculus complete our courses in Linear Algebra I and Set Theory and Logic during the first semester of their freshman year. Students who do well in these courses are invited to apply for admission to our BA/MA program. We believe that a long period of preparation is not necessary in order to be successful in advanced courses in mathematics.

We do not give placement mathematics examinations in order to assign students to mathematics courses.

The Honors Calculus courses and our BA/MA Program give us an opportunity to recognize early our entering students, as well as their high school teachers, for their excellent academic achievement as high school students. We use our love and respect for the students we invite to lead

them to an enjoyment of the study of mathematics, to understand the meaning of a mathematical proof and respect for a mathematical proof, to learn how to learn mathematics, to develop the ability to read a mathematics textbook for pure enjoyment, and to study indpendently. These students serve as role models to help us provide an intellectual climate where the mathematical potential of all students who elect to take mathematics courses in the Department can be identified and nurtured. Some of these students tutor in our mathematics lab and provide leadership in our large chapter of Pi Mu Epsilon. An opportunity to teach students in our BA/MA Program aids faculty members to teach mathematics courses close to their research interest.

Faculty members choose the teaching methods which have been most effective in maximizing the development of the mathematical potential of their students. While some teachers use the lecture method as their primary teaching method, others use many different methods of teaching which include the active learning method. Each teacher has an opportunity to teach across the mathematics curriculum. Every effort is made to provide the most favorable working conditions possible for faculty members so that they can maximize their teaching effectiveness and professional growth.

The writer helped to develop a similar humanistic academic environment for learning undergraduate mathematics during the 15 years he served as chair of the Mathematics Department at Morgan State University, Baltimore, Maryland with similar success in the achievement of students in mathematics. Therefore, the writer conjectures that similar environments can be established in many colleges and universities. If such environments are established, perhaps over time, most of the lay public will no longer regard mathematics as its most hated and feared subject.

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Increasing Learning by Decreasing Math Anxiety

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Not long ago, teaching a remedial pre-(very pre) Calculus sequence at a small liberal arts college, I encountered what has to be one of the more bizarre manifestations of math freak-out. An unusually articulate student, engaged and attentive throughout the first few weeks of the semester, soon appeared withdrawn and tense. Suddenly overcome with newfound nervous energy, she continuously played with her hair and seemed distracted and unfocused. Review of circular functions and periodic behavior apparently overloaded already sensitized math neurotransmitters upstairs ("why tangent repeats every π while sine and cosine do it every 2π just blows me away!"). In some weird dehydrated frenzy, she began to consume copious quantities of "grape juice" (later verified true: g.j. it was) from a large thermos brought to class with text and notes. This soon evolved into a well-defined ritual and (from my end) the visual fixture of a front-row student feverishly washing down blackboard notes with cold juice proved somewhat disarming. My curiosity quickly peaked and following one particularly, how should we put it, thirsty afternoon, I asked if there might be some physiological problem. "No," she mused almost wistfully. "It's just my way of coping. What you people like to call math anxiety, y'know? The, uhm, juice -- it calms me down."

This pronouncement of the situation - was it the resolution or resignation with which verdict and treatment were offered? - echoed for some time in my mind. The whole thing seemed so incredible. Still, dysfunctional stress disorders of any shape or form can be quite debilitating and often require serious attention. Obsessive-compulsive rituals are commonly maladaptive endpoints to sensory stress overload. Why, then, should a phobic reaction to mathematics, a learned behavior like any other, be different?

While the above true tale depicts (maybe) an extreme-value, if you will, of radical math trauma, it nonetheless motivates an important question. To what extent does such learned behavior influence and impair the process of learning the subject matter? Often we ascribe poor conceptual skills and low aptitude scores on exams to weakly rehearsed fundamentals and inferior mechanical training at the primary and secondary school level. To no uncertain degree this is a contituent part of the problem. But somewhere into this equation how do we factor the more intangible, not easily quantified, components of cognitive stress? Or of social norms, which inhibit (or at least do not actively recruit) positive expectations of success (eg., for women) in the quantitative sciences, mathematics in particular. Early on, a rehearsed pattern of failure becomes not only well established and familiar, but an acceptable, and ultimately relied upon, event.

The opportunity to examine this is greater detail surfaced unexpectedly within the traditional format of a first-year Calculus course I was teaching at Pitzer College. The majority of beginning Calc customers at Pitzer are primarily social science students with a fairly sparse backlog of mathematical sophistication and technique.

By the time we reached the Fundamental Thm. (circa Thanksgiving this happens) several students complained of crisis proportion difficulties and felt, in general, too "intimidated" to fathom what was going on. We threw around different ways of framing hard-to-get topics (selling Riemann Sums was like forcefeeding strychnine!), still nothing seemed to penetrate the psychic barrier they had created.

I had an idea: in an attempt to identify this psychological impasse or at least to gain some measureable insight into the freak-out factors that were exponentialy gaining ground, I hustled the group down to the campus bookstore. We proceeded to the "self-help" psych aisle and I issued the following instructions. Grab the title most applicable to the sense of dread presently experienced. These were the four editions selected:

1. Feel the Fear (And Do It Anyway) - Susan Jeffers, Ph.D.
2. Stress, Sanity and Survival - Robert Woolfolk, Ph.D., Frank Richardson, Ph.D.
3. The Relaxation Response - Herbert Benson, M.D.
4. On Death and Dying - Elisabeth Kubler-Ross

A common denominator? Interesting that the primary target or focus of each of the above centered on fear and fear syndromes, physiological stress responses and "anxiety feedback loops." Anxiety feedback loops?! Speak of this to your garden-variety, off-the-street mathematician and see what response this evokes! According to the above-listed lit, a simple 3-fold model describes the "fight-or-flight" response. An exogenous fear stimulus (environmental cue) **triggers** endogenous (visceral) stress. Stress hormones, inturn, instigate a sometimes alarming array of somatic symptoms (rapid pulse, acclerated cardio-pulmonary activity, etc.); a sense of panic and pervasive fear consumes the conscious mind in its attempt to label and interpret the state of physical dis-ease. This cognitive labeling or negative ideation of the somatic symptoms then becomes the necessary cue to promote rpetition of the cycle. And in the most chronic cases the above may become an infinite loop (unbounded?)!

The salient point here is that fear-anticipation and fear-reaction are essentially equivalent events as the cycle replicates. But this fact of life seems particularly germaine to the math-phobic's reality: the student subscribes to an inherent lack of ability, or lack of success solving problems. The acceptance of this failure becomes a rehearsed and expected event. Together, these perceptions become a self-propelling force, creating a familiar feed-back dynamic with which the student becomes alienated or disaffected and/or both.

Learned behavior can be unlearned; to disrupt this cycle of negativity one must orient and efffect an appropriate labeling of fear-cues and re-orient these signals towards positive expectation. Cognitive distortions must be recognized and disallowed. In the context of mathematical problem solving, this prescription points to a structured reinforcement of successful outcomes - seed positive results so that this, in effect, becomes the norm. After all, this certainly is the case with the most accelerated students. They are driven by the conifdence of repeated sucess many times over; and this formats the necessary creative mobility to tackle the more difficult, abstract situations.

One way of creating positive expectation and diminishing withdrawel and isolation, I have found, is by utilizing extra-curricular groups. Meeting outside of class, and optimally in as informal an environment as possible (coffee house, baseball field, Mt. Baldy!!), the trick is to encourage interactive problem solving and out-loud (to eachother) communication of the concepts and idea. The best model for this (that I know of) is the workshop format agendized so successfully by Uri Treisman at Berkeley (initially set up for minority students taking Calculus, recently this has served as the blueprint many other institutions have adopted for math and other quantitative-science disciplines). Loosely simulating the structure I employed proctoring Treisman-planted workshops at Cal Poly Pomona, groups of students were assembled to tackle (at first) easy and (later on) more difficult problematic topics relating to concepts offered in lecture. Verbal discussion and exchange of ideas and solutions were the only components of operation. This give-and-take free-for-all of problem-solving technique was Marxist oriented: communal exchange and <u>shared</u> perceptions were the ingredients demanded. Individual initiative was fostered and channeled as feedback to the rest of the group.

This symbiotic nature of idea-sharing was quickly infectious and virtually synergistic in scope. Insofar as promoting paths of solution-productive thought, the collective insight seemed to out-distance the sum of the constituent parts!

Eventually the best results were recruited with more challenging problems which permitted a greater degree of discussion and verbal description of the underlying ideas involved. Here's an example: Sketch $y = x^n + \sin^m x$ (m=n=1 is usually a text-assigned homework exercise; so try m=n=2, what info does 1st and 2nd derivative relay? Now, who can generalize for different m and n, eg. m,n both even, both odd, etc). Another example: Compute the derivative of x raised to itself n times (if your idea of fun is logarithmic differentiation! -see below*).

Pushing through fear is less frightening than living with the underlying fear that comes from a feeling of helplessness," notes <u>Feel The Fear</u>'s Dr. Jeffers. Functionally this is the moral of the story. And it's a message that plays well with helpless-feeling students.

*: One person came up with a recursion formula,

$$d/dx\,(x*n) = (x*n)\,d/dx[x*(n-1)]\ln x + (x*n)[x*(n-1)]x^{-1}$$

where "x*n" indicates x raised to itself n times.

However the onus of trumping up silly problems incurs the responsibility of providing complete solutions:

$$d/dx\,(x*n) = \Big[\prod_{i=1}^{n}(x*i)\Big](\ln x)^{n} + \sum_{j=0}^{n-2}\Big[\prod_{i=j}^{n}(x*i)(\ln x)^{n-(j+1)}x^{-1}\Big] + (x*n)(x*(n-1))x^{-1}$$

<u>Then</u> came the question about $d^n/dx^n\,(x*n)$!!!!

An innovative curriculum idea

for the first two years

of college mathematics

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of Mathematical Association of America

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Introduction

> It was a late Friday afternoon. Two of our department's faculty members were lounging in the mathematics study/tutor room, talking with a bright mathematics student who was trying to make an argument. "Just grant me this," Sandy was saying. "Let x be a rational number, and let y be the next larger rational number." We were not willing to do such letting. Our questions about the nature of the number $(x+y)/2$ were greeted with puzzlement. Yes, it is a rational number. Yes, it is bigger than x but less than y. But so what?

Sandy was "filtering out" the contradiction that was so apparent to us as mathematicians. We were both reminded again of how "discrete" college freshmen's ideas of numbers are. Over the years in school they learn about integers first, then scatter in some fractions, and eventually throw in a few irrationals such as pi and square roots. But the notion of a continuum is beyond most of them when they enter college.

The disturbing realization on that Friday afternoon was that Sandy was not a bright freshman mathematics student. Sandy was a bright junior mathematics major, enrolled in our advanced calculus course. Sandy's previous 2-1/2 years of college mathematics had done little to develop a sense of the continuum. Calculus texts universally ignore this conceptual problem, and our calculus courses are virtually defined by the chosen text. So the Sandy phenomenon should not surprise us so much as it distresses us.

Sandy's limitations are broader than a superficial concept of the continuum, however. In reality, very few junior mathematics majors are well prepared to do well in the junior level courses we wish to teach. Our complaints include 1) lack of skill at the formal reasoning needed to understand and create proofs and other arguments; 2) lack of visualization ability; 3) inability to read mathematical writing, even expository writing; 4) poor skills at communicating mathematically; and 5) a lack of awareness of what we, and our colleagues in science, think is important about mathematics.

Perhaps our overriding frustration, however, is that our sophomores do not have a good basis on which to make a decision about majoring in mathematics or the sciences. Those who decide to enroll in junior level mathematics courses have been successful at the mechanical calculations learned in calculus and linear algebra - the same mechanics that have driven away many other students, especially women and minority students. Not only have they encountered precious little of the logical reasoning that makes up so much of the discipline; most of them have not experienced the elegance of beautiful mathematical relationships, the tension between intuition and logic, the breadth of exciting mathematical ideas beyond analysis, or the thrill of <u>doing mathematics</u>: asking questions, investigating special cases, making conjectures, trying to prove them, revising conjectures, and successful proofs.

Although we don´t pretend that college freshmen and sophomores can engage in research on the forefront of mathematics, we have seen students in our non-majors courses acquire a better feel for mathematical processes than our own majors do after two years of our courses. Students in our "Mathematics for the Liberal Arts" course understand the important role that mathematics has played in the development of civilization better than those who intend to do or use mathematics in their careers.

Our collective frustrations have led my department faculty to do some intensive thinking together (and with our other colleagues in the science division) about these problems since the fall of 1986. What I say here is my own organization of those thoughts. At least one other member of our department is here today to give other perspectives.

Our situation is very typical. Augsburg College is a liberal arts college of about 1500 students. As in most colleges, the first two years for students majoring in mathematics or the sciences include three semesters of calculus (two for biology majors), and - for mathematics, physics, and computer science majors - a semester of linear algebra. A few students outside the science division take Calculus I to satisfy a distribution requirement.

Occasionally I hear from mathematical colleagues at other schools the opinion that "If they can´t get it through these courses then they should major in something else." At Augsburg, however, we are asking if "these courses" really show students a true picture of mathematics, if they prepare students to become mathematicians or users of mathematics as well as other courses might. To get a handle on these questions, we made a list of goals that we want all of our mathematics and science majors to accomplish, at least eventually.

Our goals

Our goals can be stated rather simply. They are

The student should be able to

A. learn mathematics independently;

B. do mathematics; and

C. communicate mathematically.

D. The student should understand the importance of mathematics.

Let´s look at these in turn.

Learning mathematics, as we all have been trained to say, is not a spectator sport. It requires practice. Usually when we say this we

mean working lots of mechanical exercises. We also might include "problem solving," in at least one of its many interpretations.

But learning mathematics also involves a number of other skills. For one thing, to learn mathematics effectively a student must be able to read and listen to mathematics with understanding. Without getting into a discussion of understanding, I'll say that understanding what is read and heard seems to have several prerequisites:

a knowledge of the shorthand used in mathematical communication;

an ability to follow logical arguments;

understanding of a representative set of mathematical concepts (so that the distance from a new idea to an old one is fairly small); and

familiarity with the processes the author or speaker may have used to arrive at the results being described.

These processes are what I mean by **doing mathematics**, our second goal. They include

modeling problems in mathematical terms, whether the problems arise out of "real world" situations or from questions about extending other mathematics;

generating data to search for patterns, including data from special cases;

making conjectures based on seeing patterns in data;

trying to prove or disprove those conjectures, reformulating them, and looking at specific cases, pitting intuition against logic, with each being informed by the other; and

asking exploratory questions that will lead to new problems.

All of these activities involve, of course, understanding a representative set of mathematical concepts.

Our third goal, **communicating mathematically** in an effective way, can be summarized as "writing and speaking well." But what's really involved here? Writing and speech teachers can tell you in much more detail, of course, but I list a few ideas. Besides what must be rather standard for all fields - the necessity for an awareness of audience level and the ability to organize ideas - I can think of three requirements especially important to good mathematical writing:

knowledge of the shorthand of mathematical communication - the subtleties of quantifiers, and the careful use of English connectives to communicate the direction of an argument;

a sensitivity to the power and limitations of mathematical symbols and a feeling for how to achieve an effective balance between symbols and words; and, of course,

an understanding of a representative set of mathematical concepts.

By the **importance** of mathematics I certainly include the successes of mathematical models for solving practical problems - and their limitations, too. But I also refer to the effects of mathematics on the development of civilization, about which our non-majors learn something but mathematicians learn about only by accident if at all. Especially at a liberal arts college like Augsburg, with an awareness that the most creative contributors to a field are those who can look at problems from many different perspectives, it´s important for mathematics graduates to have seen the many faces of mathematics that history has exposed.

Experiences to achieve these goals

Given that these are our major goals, what kinds of experiences are most likely to help students achieve them? Just restating what I have said will give us some major categories for these experiences; after doing so I shall expand upon them briefly.

We should provide students with <u>coached practice</u> at

reading mathematics,
listening to mathematics,
following logical arguments,
mathematical modeling,
generating mathematical data,
making conjectures,
creating mathematical proofs,
posing mathematical problems,
writing mathematics, and
speaking about mathematics;

our students should study the effects of mathematics on civilization;

and our students should encounter a representative set of mathematical concepts.

Let me expand on these three categories.

The term **coached practice** reminds me of football training or perhaps music lessons. Coaching involves demonstrating or describing how to do something, and then (even in team sports) giving a good bit of advice to individuals, based on their performance. It goes well beyond telling a group of people how to do something, asking them to do it on their own, and then testing them to see if they can do it. But isn´t that what most of us do when we try to teach mathematics? Our approach works

fairly well for teaching skills at working exercises; but teaching more complex skills and concepts requires giving a good bit more individual attention. For whatever reason, instead of giving that attention we abandon the teaching of higher level skills, and Sandy arrives at the junior year without having obtained them.

In discussing the **influence of mathematics on civilization** we have meant the ways mathematics has influenced - and, naturally enough, been affected by - areas of life that, only in the last few hundred years, have come to be seen as different from mathematics. Alphabetically, some of these areas are

architecture,
literature,
music,
mysticism,
painting,
philosophy,
science, and
sculpture.

And here we are again at **understanding a representative set of mathematical concepts.** In one sense, the exact nature of these concepts diminishes in importance if our other goals are met. Does it matter whether or not we´ve spent a class period integrating by partial fractions if students can learn such techniques and even the ideas behind them on their own? On the other hand, the sample of ideas the student has learned must be representative. With no understanding of the concept of integration, learning about integration by partial fractions would be left to the few Ramanujans of the world.

But what must the set of ideas represent? Mathematics, of course. And what all does mathematics include? Think, now: geometry. Number theory. Algebra. Topology. Probability. Statistics. Graph theory. Combinatorics. Yes, even analysis. Can we get a representative sample of mathematical ideas by teaching calculus and linear algebra? No, not even if we throw in a little "discrete mathematics." We need a much broader sample to have a collection of ideas that are more-or-less dense in the continuum of ideas that make up contemporary mathematics.

We can learn something, we think, from other disciplines. In the first year of a physics major, the student takes "General Physics." In the first year of a sociology major, students study "Principles of Sociology." These courses introduce students to the big ideas and techniques of the field. To design a real "principles of mathematics" course we must do more than redesign Sandy´s calculus course for a new century.

Implementation ideas

So how do we accomplish all this? We at Augsburg have been experimenting with a number of teaching ideas in our current courses, and some of them show promise for accomplishing some of the goals we have set above. For example:

Some of us assign the students to read a section of the text in advance, and we spend the class time answering their questions about it. We try to avoid just repeating the book in our own words. When the students have no more questions, we talk about an application or work some problems involving the ideas they have read about. They leave with an assignment that includes not only working more problems but also reading another part of the text.

Instead of introducing a new topic with "It´s very important to know this," we sometimes start with a problem. It may be a practical problem, but it might also be a problem that intrigues students through its mystery or even nonsense. In analyzing the problem together we develop some mathematics ideas, and, eventually - perhaps after several days or even weeks - solve the problem.

Encouraging discussion among students works well for creating an atmosphere of doing mathematics. As we implement a program that focuses on learning and process goals, we should take the opportunity to substitute quality for quantity.

If we want students to learn to learn by listening, they must not be deprived of lectures. They need to be coached, however, in how to listen effectively to mathematical talks.

Several of us require our students to write. I, for example, insist on a careful presentation, using complete, grammatical sentences, of each solution. I critique the presentation as well as the content, and if it´s not done well I ask for revisions. Others use journal writing or summary term papers. We ask students to explain concepts in their own words - a humbling experience for us all, but one that allows us to coach more individually.

Some of us ask students to prepare and deliver short oral presentations in class, even in the freshman year. Again, these are critiqued for style as well as content.

In our "Mathematics for the Liberal Arts course" we have found that a powerful way to demonstrate the influence of mathematics on civilization is to arrange the topics chronologically. We frequently use a "pseudo-historical" approach, choosing mathematical topics most appropriate to the historical period in question. This organization scheme has the additional advantage of gradually increasing the level of abstraction and thus the level of difficulty.

The notion of encountering ideas repeatedly, at increasing levels of difficulty, is known as a "spiral approach." We find that method very

effective within our courses as well as through the curriculum as a whole. Our thoughts about curriculum design are that during the first year we might move chronologically through a representative set of mathematical ideas, and then spiral through the main ideas another time during the second year, with less emphasis on cultural influences and with more detail and a higher level of rigor. In our standard junior and senior courses the students would encounter the same ideas a third time (at least), with even more details and an even higher level of rigor.

Some side benefits

We anticipate two important positive side effects of an approach like this. First, we hope that this approach will "hook" those students who in the past have been discouraged by the tedium and apparent irrelevance of the current calculus sequence. Research indicates that, for example, many women and minority students have been "turned off" by an apparently non-humanistic discipline. If the students can experience some of the ecstasy of doing mathematics, they will come to regard the discipline as less cold and inflexible.

Second, those students who are taking a mathematics course at this level purely to satisfy a liberal arts requirement will achieve a much better understanding of the breadth of mathematics and its importance to the development of society than they do in their current calculus course.

Augsburg's agenda

We see the five steps in our curriculum development process:

In the first step, we need to add to our collection of stimulating problems whose exploration will help us (with the students) "create" many ideas of mathematics. At the same time, we need to be addressing some of the logistical questions, such as making sure that our program meshes with others well enough so that transfer students are not handicapped too greatly.

Then we need to decide which problems are most likely to lead to discussions of particular ideas, and in which order they might be posed.

Some teaching methods are more appropriate for some problems than for others. At times we might want use lectures, at other times emphasize reading, and for other topics "invent" the mathematical ideas together through class discussion.

Since reading plays such an important role in any college course, students need to have written materials concerning a wide variety of mathematical topics at an appropriate level of difficulty. Of all the books we know of that address a variety of topics, none has both the

scope we desire and a level appropriate for students who are currently taking the calculus sequence. So we need to create our own text, using already-written materials as much as possible and writing our own to fill in gaps.

This brings me to where you come in. We need help. What stimulating problems have you used? What excellent reading do you know of about these various topics? Would you be interested in joining us in this new venture? in helping us design courses? in class testing ideas?

I'd be glad to hear from anyone willing to help us address the Sandy phenomenon.

I've found recently that many of my technical classes have at times become completely overwhelming in content. Some concepts which are no longer intuitively obvious and increasingly complex occupy a state of indeterminancy. They tease me by appearing at one moment to be within my scope of understanding, only at the next moment to be flying back out into vagueness with lazy, complacent ignorance flowing in to fill the void. In order to solve this problem, I have learned that when one of these ideas arrives for a brief stay in comprehension I must fasten it down. This is accomplished by harpooning it with blind acceptance of certain background information that is too tedious for me to appreciate. Much of the beauty of the concept is destroyed in the resulting mutilation and the cables holding it in place are not strong. However, they are easy to erect. Eventually, if other concepts that are founded on the one tied down grow too abstract and complex themselves, the cables will snap and the whole assembly will disintegrate and blow away.

In this intense, academic environment there is also no time for self-congratulations for a concept recently mastered. There's no time to enjoy the view from the new ledge you have just ascended to on the climb for better understanding. There's no time because at that very moment parts of the ledge are collapsing while a landslide of new information is bombarding you from above. Self-confidence is a very dangerous, unpredictable instrument in your bag of climbing tools. It should be implemented with a great deal of care. It is a balloon of delicate material tied around your waist that can give you some buoyancy to make the climb a little easier. However, a small

shard of misfortune can eaily leave a gaping hole in it. New problems and tasks then collect in its empty husk weighing you down and complicating your task of learning. If you have an overabundance of self-confidence, though, your balloon provides too much vertical lift. Involved in the ceremonies of narcissism, you ignore or accidently miss details and vital features of the obstacle you are trying to surmount. A heavy fog of conceited ignorance begins to settle over the environment, diminishing your range of sight until finally a hidden obstruction breaks your bag of hot air sending you plummeting in a freefall from which it is difficult to recover. In either case, a blessing has suddenly become a cursed burden.

So what is a student to do when surrounded by a vast body of destitute ideas all crying out for attention, and time and enthusiam are scarce resources? Pondering the task itself for too long is enough to drive a self-wielded dagger through the toughest armors of assuredness.

5/1/88
Chris Jewell
Class of '91
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FOR WHOM NOBEL TOLLS

By

William Dunham

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It is well-known that Nobel Prizes
Come in many shapes and sizes.
But one is missing from the list -
The Nobel Math Prize does not exist.

There is a widely-held suspicion
That might account for this omission:
Perhaps it's <u>jealousy</u> that's to blame
For mathematicians' absence from the Nobel game.

For Alfred Nobel had become aware
Of his wife's impassioned love affair
With a mathematician, who held her tight
And thought that she was DYNAMITE.

Then Nobel, reacting as expected,
Vowed, "Mathematicians shall be neglected!
And if it's Sweden they want to see,
Let them take a tour and pay the fee!"

O, the lack of a Math Prize is indeed a curse,
Yet I have to admit it could have been worse -
What if Madame Nobel's infamous tryst
Had been instead with a pacifist ?

MATHEMATICAL METAPHORS FROM ADVANCED PLACEMENT STUDENTS

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The images and perceptions of mathematics held by many math avoidant people often surprise me. I encourage these people to develop their metaphors and stories for me. (See Buerk 1982, Buerk 1985, and Potter 1978.) The more I listen to the math avoidant, the more I wonder what the images of those more successful with mathematics might be. I am beginning to ask and want to share some responses with you.

In May I asked two different classes of Advanced Placement high school students to develop their images of math. After an introduction by the classroom teacher, I asked students individually:

1) to list words they would use to describe math (5-7 minutes),
2) to imagine themselves in a situation of doing math and to list all their feelings while doing math (5-7 minutes),
3) to list all of the objects (nouns, things) that math is like for them (5-7 minutes),
4) and finally to read over their three lists and write a paragraph beginning, "For me math is like a..."

The paragraphs of two-thirds of these students follow.

FOR ME MATH IS LIKE A
For me math is kind of like an incredible book that you have to read through an infinite number of times. The first time you get the general idea, but until you reach the end you really have no idea what's going on in relation to anything else. Each successive reading brings out more meaning and little unimportant asides suddenly relate to everything else in the most amazing way. It has to be read an infinite number of times because there's always more meaning to be found. But the book would have to be infinitely long as well, because math has no beginning or end and there's no place to stop. Like the book, questions posed at the end may be answered by some tiny thing at the beginning. Sometimes it seems hopeless and completely irrelevant, but if you force yourself to continue it will suddenly clear up and make more sense than you ever thought possible. You have to have faith that it's worth plowing through it.

FOR ME MATH IS LIKE A
Math is often like a track race, because sometimes it works and sometimes it doesn't. My experiences with math have sometimes been frustrating in practice although I might work hard at it and see the overall picture well. Running can be like this because although you can work hard for a long time, so-called success comes down to one race, which either could be good or not, depending on the day. In the same way, math sometimes "clicks" and becomes great fun and beautiful, but other times seems just annoying, and although perhaps exciting, out of reach.

FOR ME MATH IS LIKE A

For me math is like an egg. It is simple and one can hold it in one's hand and roll it about. But then it has a surface that is smooth in some places, rough in others, but always pure white. It can be delicate and breakable, but remarkably tough when, for instance, you try to crush it from the ends. It may seem like an oval or a simple circle, but it is a slight contortion. And, best of all, deep inside rests the baby of the world, a wonder for all to speculate upon, for we only see it after the egg breaks. We know it is in there, living; if only we could get to it without breaking the shell.

FOR ME MATH IS LIKE A

For me math is like a maze or rather a huge maze made up of many smaller ones which you have to finish first in order to get to the end. Except it really has no end. There are rewards, though, on different levels as you finish various submazes, and excitement when you discover a pattern or a new path. It's fun to explore and to succeed and it's worth the frustration and sense of being lost that comes when you have taken 20 wrong turns in the last hour. Math and mazes are tedious in the areas you already know but exciting when you discover something new. They are very rewarding; they bring great senses of accomplishment.

FOR ME MATH IS LIKE A

For me math is like a strange object in a glass case where the object intrigues you but you can only guess. Sometimes you can spend hours standing looking at it and still can't figure out what it is. Sometimes you can pin it down and easily walk away. It is when the object is something you can't figure out that stimulates, fascinates, and makes you eager to go to other sources to come to an idea of what it is. But what you decide the object is depends on your background and education. There are many approaches to take you to a conclusion.

FOR ME MATH IS LIKE A

For me math is like a puzzle that is all blue sky with a couple clouds in it so that most of the time it is just luck whether you get the pieces together, sometimes it's easier because of the clouds. Actually, math is really easy for me when I don't really have to figure anything out. That's when it's satisfying, when you have a bunch of formulas and you can plod through a bunch of problems with them - not really easy, obvious problems but hard ones where you have to figure out which method to use first to solve it. So math has been alright so far, I'm just scared when I think about trying to apply it to a problem in real life, or when I think about discovering something new. It just seems so impossible, like the puzzle. That's why I won't enter math contests. Maybe I'm just scared I'll fail. Math has always been really competitive for me. It's like a race, but there always has to be an end because if there wasn't I think I wouldn't feel compelled to do it.

FOR ME MATH IS LIKE A

Math is like a puzzle because it contains a million pieces that fit together perfectly only if you understand the relations of the pieces to each other and each piece helps put the others into perspective.

FOR ME MATH IS LIKE A
For me math is like trying to eat a jawbreaker that is too big to swallow and too hard to chew. It dissolves away a little at a time, very slowly, but it tastes good along the way - and there's the promise of the sweet, soft center.

FOR ME MATH IS LIKE A
For me math is like a strong, secret hiding place that I can escape to, think in, and be comforted by. It's in another world away from interruptions. It's filled with rare objects, unknown objects that I am drawn to and mesmerized by. I stare at each one of these objects for long periods of time - I have all the time in the world in this place - trying to unravel its mysteries, trying to find out what they really consist of. Some are easy to discover, others are very difficult, almost impossible. When I`ve had no luck with these more difficult objects, I wander back into the cave, the center of this secret place. In this cave are many different crystals and many beautiful, but well known rocks and gems. Here lie all the basic `truths.' The foundations upon which everything is based. Then I go back out again, renewed with strength.

FOR ME MATH IS LIKE A
For me math is like a Bach fugue. In a fugue there is an opening phrase, the very foundation of the fugue. It is used and spun out. But as soon as you've stated this idea another voice enters in. Horizontally there are a lot of single voices going on, all developed from the same basic assumption, but alone wouldn't make it music. It is important that the voices go together well, the vertical harmonies of the different lines. They're all separate but together and only together they make harmonic and musical sense. The contrapuntal voice leading.... One line alone in a fugue might be beautiful but the depth comes into existence because of the complicated interweaving patterns. There is no way to find all the structures and connections but it is a nice challenge to look for them.

The difference though is, that a fugue ends and the search ends at a certain point. A fugue was set up in its complexity by a human being and finished, while math, though created by humans, never ends.

The different "branches" of math are the voices of the fugue that are built(?) on the same assumption, developing different things from there and not contradicting each other.

FOR ME MATH IS LIKE A
For me math is like a toolbox. The tools in the toolbox represent the concepts, formulas, and techniques needed to solve problems. However, I could always use the wrong tool, or maybe my toolbox doesn't have the tool I need. The tools can be used to construct something, or they can be used to strip down a complicated machine so that all the parts can be analyzed. Some tools can become obsolete if I acquire new ones. When working with the tools of mathematics, I could just as easily use them to fulfill my needs by solving the problems.

FOR ME MATH IS LIKE A
For me math is like a non-existent rubbery wall of silly putty that, while you can imprint cartoons on it, shoves you back time after time. A shapeless blob that has to be molded into something obvious and simple.

FOR ME MATH IS LIKE A

For me math is like a human being (one not always easy for everyone to deal with). My first exposure to math was Winroth's "How many can you do in three minutes?" (no,it was learning to draw eights in nursery school - also frustrating) and memorizing times tables. I hated both - the pressure and the rote learning was boring, and I was a daydreaming type who didn't like the pressure of time limits, I guess. Then came a mysterious thing called "pie" (sic) that my parents and brother talked about, over my head, at dinner, and the physics my parents discussed. I was a girl and I didn't like boys and math, which was boy's stuff. Gradually I had to get to know both - and I love math now because it's interesting. Math is temperamental and not easy for everyone to get along with. An eccentric personality that, though varied, still is oriented with one goal in mind, and, even when you know and like him/her/it, can be intimidating because it sounds pretentious, stuffy, over-intelligent, eccentric-like your typical stereotype physicist or mathematician. But sometime he/she/it has flashes of brilliance where "it" says something incredible and you say - wow. What a neat person.

FOR ME MATH IS LIKE A

For me math is like a hunt or an adventure. You have a seemingly impossible problem in front of you, and all these formulas and methods in your head. You have to keep pulling out methods and formulas until you begin to weaken the beast, and keep working until you've turned the lion into a pussycat. Once you've accomplished the feat, you can again say you're King of the Jungle and go out in search of the next lion to be tamed. You find the next one - and it's huge! You begin to try your different weapons, but it thwarts them all off. But finally, exhausted, you remember L'Hopital's Rule, and finally the battle is in your favor. From then on it's an easy fight, and you win again.

FOR ME MATH IS LIKE A

Math is like a name, you are given names to ideas that don't even exist, it's so abstract, you call the sky "sky" but if you wrote "sky" on a piece of paper it wouldn't be the sky so that's what numbers, real and imaginary are. But then we made laws for the numbers, and with the first defining statements all the rest followed. When someone took a step in one direction we had to keep walking that way. Math could have developed in a completely different manner, but it would still be just as correct. It's important to remember that math isn't really a reality , it's a philosophy, a way of relating tangible things. The fact that it does such a good job is amazing to me. The thing that excites me the most about math is the connections, for example, the number, e, shows up mysteriously in so many places, as do the golden ratio and pi. But also the connections between logic and proofs in math and debating arguing a point. Math is in many ways a magic, a supernatural entity that we can attain, this makes it very special. Yeah, yeah and I really think I know what I'm talking about. Math can be amazing if you can appreciate the many things it has made us capable of. I said that math was simply a philosophy, which is true, but look at what this philosophy, law, that at one point did not exist, has blossomed into. Math is a creation of man, or is it?

FOR ME MATH IS LIKE A

For me, math is like a jigsaw puzzle with hundreds of pieces. It can be frustrating when I can't grasp a concept or can't find a mistake in a problem I did wrong. On the other hand, when a concept does make sense, it's like putting many pieces of the puzzle together at once. It's also puzzle-like in that it's intriguing how so many seemingly unrelated pieces can fit into a coherent whole which is visible in nature as well as in textbooks. It's rewarding like a puzzle when a concept is fully understood - hence, the puzzle is finished.

FOR ME MATH IS LIKE A

For me math is like a trial - a test of my intelligence. I feel a lot of pressure to do well. This pressure comes from several different places. First, at home: my father is a mathematician and my older brother has always been a math whiz. I am expected to be the same way, but my interests lie in other areas (music, poetry...). For me math holds no interest. Thus the only point (to me) of learning it is to succeed, to excel, to please others. I like to think slowly and figure things out. When my Dad helps me with math I feel pressured - like if I don't know/understand something he will think I'm stupid.

But the pressure also comes from the whole system of the advanced math sequence. People are expected to keep up with the speed of these courses. If I didn't understand something (a common experience) there was no time to ask questions. We had too much material to cover. Basically the system is inflexible. It is geared towards ONE kind of student: fast thinking, smart... and if you are not these things you are left behind.

Another thing: there is a big "competition" (unspoken) between my level math class and the level just below - a real snob attitude of those more advanced towards those less advanced. This competitive attitude is not one I work well around. I get angry and vent that anger toward the subject <u>(math)</u> not towards the people.

DEVELOP YOUR OWN METAPHOR FOR MATHEMATICS! PLEASE WRITE IT UP AND SEND IT TO ME. I WILL SEND SOME/ALL(?) OF THESE TO AL WHITE FOR THE NEXT NEWSLETTER!

Have you seen Lynn Steen's metaphor in <u>Science</u> (1988)? Where are others written down?

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AJB/VM

26th April 1988

Professor A. White
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Dear Professor White

Thank you for sending me a copy of Newsletters 1 and 2. I enclose with this letter a copy of the report of work that I undertook at Brunel University, London, whilst on sabbatical leave from the City University, London.

I think that the report deserves a wider audience than it has thus far had, and I would like you to consider it for publication within the auspices of the Humanistic Mathematics Network.

I look forward to any further material from you,

Yours sincerely

Dr Anthony Briginshaw

1. Some observations on teaching undergraduate mathematics.

Introduction.

This Report is an essay on current perspectives in mathematics education. It is largely concerned with undergraduate mathematics education, and focuses particularly on how mathematics is taught to first and second year engineering undergraduates. It is clear that, as always, such a narrow focus cannot fail to have ramifications in neighbouring areas, and I shall at least mention the following:

(i) the history and philosophy of mathematics,

(ii) the treatment of mathematics in schools,

(iii) the great success of mathematics as the servant of physics and engineering,

(iv) misconceptions of what mathematics is and what it seeks to achieve,

(v) behavioural phenomena in the lecture meeting, and

(vi) methods of assessment of lecturer performance.

The essay is a distillation from a period of intensive enquiry undertaken throughout the 1986/7 session, a session which was spent as study leave in the Department of Mathematics at Brunel University.

The net result of the enquiry is that many more questions were raised than I could answer, so that in this Report I re-iterate those questions and outline whatever I have achieved in the way of an answer.

2. Teaching mathematics: the mathematical language.

Teachers of mathematics, at all levels, are concerned with the transmission of learning. They use, as the medium of communication, a hybrid language, part English, part Mathematics . In one sense the mathematical language is not a natural language, in that it has not grown as a readily accessible medium for communication and for general discourse. In another sense the language is a natural one, for it has grown with and alongside physics and engineering and has come to describe scientific and engineering usage and procedure with unrivalled success. Indeed, we might say that it is a medium for discourse within certain communities, the various communities of science and engineering. However, the mathematical language is succinct and economical, and, indeed, it has been honed to economical perfection through centuries of selection (Cajori, 1928) . Thus, unlike English, or any similar natural language of general written or spoken discourse it has very low "redundancy". Yet high redundancy seems to be a quality which indicates that the language is a good vehicle for the transmission of information. Thus the mathematical language would seem a priori to be a poor vehicle for communication to learners! In spite of that, I suggest that, with care, this difficulty is subject to automatic alleviation in the currency of a course of mathematical instruction. Briefly, there is a continual need for conceptual scaffolding throughout the course, and this scaffolding is the means by which a quite small number of major concepts and techniques is acquired. When facility and confidence with these concepts and techniques is achieved, by accretion, and with a large degree of overkill and redundancy, the objectives of the course are attained. In a University mathematics course, some familiarity with the axiomatic method and with logical implication is also acquired, but not as much as we think; to most students, especially those for which mathematics is not a major subject, facility and confidence are quite enough .

3. Language and communication.

Claud Shannon is regarded as the founder of the modern theory of communication. Shannon's original work (1948) gave rise to the collaboration of Shannon and Weaver (1949) and that was followed by the publication of Bell's book (1953). Since then, the field has expanded vigorously and that expansion has been matched by prolific publication. We need only touch here on the very basic idea of the theory, and, in particular, introduce the concept of entropy, a measure of the quantity of information which is transmitted per symbol of language.

Colloquially, we envisage a distinction between "language" and "code". "Language" (literally "tongue") is perhaps a naturally occurring spoken or written system of discourse. On the other hand, "code" is a restricted, curtailed, economical or displaced version of such a language. More technically, the word "code" has also come to indicate strings of symbols with which we address a machine. However, if we look upon a code as a device used for protecting security, then a good code is one which transmits a message to insiders and which is impenetrable to outsiders. On the other hand, a poor code is one which transmits a message to all and is not impenetrable. Thus English is a good language but a poor code (for English speakers) and Mathematics is a poor language but a good code (for English speakers!).

4. Self information of an event; entropy of a set of events.

Shannon defined the self information, $I(E)$, of an event E as a function of the probability p of its occurrence.

It is $I(E) = \log\left(\frac{1}{p}\right)$ which defines the self information of E, or, in other words, the quantity of information which is transmitted when E

occurs. The convention is that base 2 logarithms are used for this measure and that I(E) is measured in "bits". Notice that the quantity of information transmitted by E increases with its "surprise" value.

For a set of events $\{E_i\}$ with respective probabilities of occurrence $\{p_i\}$ we define the <u>entropy</u>, $H\{E_i\}$ by

$$H\{E_i\} = \sum p_i \log\left(\frac{1}{p_i}\right) .$$

We can look upon entropy as the average information transmitted per event for a sequence of events.

5. <u>Redundancy of a code or language</u>.

If a code has n symbols with actual probabilities of occurrence $\{p_i\}$, then we may calculate two numbers, the actual entropy and the equiprobable entropy (that which would occur if all symbols were of equiprobable occurrence).

The quantity $R = 1 - \dfrac{\text{actual entropy}}{\text{equiprobable entropy}}$ (see Usher, 1984)

is then referred to as the <u>redundancy</u> of the code. Notice that the equiprobable entropy is the one which corresponds to the case of <u>maximal</u> information transfer per symbol. Thus, if the actual entropy is low, then the code is a relatively inefficient information transfer mechanism. It is, correspondingly, a good vehicle for communication, for, even if the receiver misreads or ignores some of the symbols, the receiver may still get the import of the message.

Both Shannon and Weaver (1949) and Bell (1953) estimated the redundancy of written English at around 80%, basing their calculations on a <u>conditional entropy</u>, assuming intersymbol influence.Thus, a conditional entropy

$$H(j/i) = -\sum_i \sum_j p(i,j) \log p(j/i)$$

takes the place of the cruder measure H of paragraph 4 above. Here, in particular, H(j/i) assumes influence only in adjacent symbols, and p(i,j), p(j/i) are, respectively, joint and conditional probabilities; there may be more complicated intersymbol influence. With this means of estimation, I suspect a very low value for redundancy of the mathematical language, which implies that it is a good code, but a poor vehicle for communication (except to those who are already "fluent").

6. Tactical surprise in undergraduate mathematics courses.

If we accept that the mathematical language has low redundancy then that fact must affect teaching style. Thus in an undergraduate mathematics course, especially one where mathematics is not the major study, so that motivation may be low, the lecturer must seek to exploit whatever tricks and strategies he can command to achieve two ends. Firstly, to optimize receiver tuning (student attention) for a given information flow, a matter which is not wholly affected by the nature of the subject matter. Secondly, to maximize the information flow. Thus, in order to catch and hold student attention those tricks may encompass changes of pace and vocal tone and appropriate use of humour and anecdote and may stretch to limited histrionics. To maximize the information flow, on the other hand, tactical surprise might be used in one of two ways, either at the motivational stage of a new section or within the development of a set of theorems by a judicious selection of pattern and proof. It might occur by appeal to any facility for pattern recognition that students may already have acquired whether it be a recognition of analogue in structure or in usage. It might occur in a particularly neat or succinct set of implications which justify a technique or procedure.

Whatever attempts are made at tactical surprise, however, it is difficult to achieve, and, worse still, it is often post facto. The limits of tactical surprise, indeed, must be set alongside the realization that mathematics is mostly formal, methodical and economical, in other words, it is intrinsically not surprising.

This is allied, consciously or unconsciously, to the attention profile which is associated with a target student group, which roughly indicates a variable attention span, with a lack of receptivity, both at 17-20 minutes and at 34-37 minutes. These occur willynilly in a fifty minute lecture presentation. There are, as I have said, two types of surprise. Motivational surprise, by which we hope to capture attention by pointing out the utility of a prospective technique before outlining the details, or by pointing out the structural beauty of the mathematics in its own right; that is, respectively, motivation either at the modelling level or at the aesthetic level of cognition. It is facile to assume that the former should be reserved for undergraduate engineers and the latter for students of pure mathematics, I have not found such restrictions to be an effective aid to good communication for either group. The second type of surprise is dramatic surprise and it is a phenomenon, or collection of phenomena that is very difficult to describe adequately; let us say that it has something to do with the lecturer as performer, and his or her ability to engage in limited histrionics or to inject appropriate humour or expertly to pace the flow of information to suit the target group.

7. Strategic redundancy in undergraduate mathematics courses.

If the mathematical language is such a fundamentally poor medium for communication, yet the whole point of it is to communicate that which cannot be communicated in English, how is it that generations of scientists and engineers come through unscathed? In my view, because of

the strategic redundancy inherent in the undergraduate mathematics course. Much of the time and effort spent in, for example, a first year methods course for engineering students is scaffolding. It is essential for building the edifice of mathematical knowledge with which a student proceeds to year two, but it is redundant at the end of year one. On my estimation, perhaps 90% of the work and effort expended in a first year course is scaffolding. It is that part by which the course is motivated topic by topic and by which each topic development is given plausibility and each resulting technique is given facility. It is essential for the successful execution of the course, but it is redundant when the course is over.

Concerning the bedrock knowledge of mathematics that a student actually needs, that is, those basic ideas of definition, notation and technique which occur over and over again in physics and engineering, we might conclude that if they could be implanted in memory banks and logic circuits direct, it could be done in one tenth of the normal course length.

It is in the provision of scaffolding that the intrinsic information transfer redundancy occurs for the mathematics lecture course.

8. Mastery of the mathematical language.

In speaking of English as a language we recognise such terms as essay, article, novel, poem, description, reporting, etc., and we distinguish between creative writing and criticism; equally we may refer to written material as being classic, modern, mainstream or avant garde. To a quite marked degree these concepts have their analogues in mathematics insofar as we treat mathematics as a language in its own right. The analogies go some way toward explaining some of the snobberies that arise between

teachers of undergraduates must come from the ranks of the "research mathematicians". Unfortunately, this is as misleading as saying that every poet can be a reporter, essayist or playwright, in other words, it is an empty assertion, usually made without any attempt to analyse what is going on, whether in research or in teaching. Another snobbery attempts to devalue "teaching" vis-a-vis "research", this is rather naive, too, for in order to validate research the researcher must communicate it, and the process of communication of new ideas even to peers is still "teaching". Naturally "new writing" is for aficianados i.e. academic mathematicians, who often are themselves, "writers". There are certainly "critics" of "new writing", necessarily themselves peers, and often playing the role of assessors, before publication. Too often there is the danger that, in the exciting world of avant garde "mathematical writing", critics and aficianados alike will too readily place new writing before mathematically immature minds. That danger is perennial in mathematics, as in music, or art, or literature.

As I have already noted in paragraph 5, the mathematical language is likely to have a low redundancy. Does this imply that it is inherently a poor vehicle for SOCIAL (i.e. educational or classroom) communication? That, of course, does not deny the fact that once the language is mastered, and fluency is gained, the language can be used with confidence as the outstandingly good vehicle for SCIENTIFIC communication that it is.

9. Taxonomies.

A taxonomy is a categorisation of a discipline by way of various traits and qualities, those of Bloom and Piaget, for example, are decided by "depth of cognisance". Following Jolliffe and Ponsford (1986), I propose the following taxonomy as being the most sharply focused as a means of cognitive analysis regarding mathematics learning at post 16 ages. In

particular, it is valid for the first and second years of an undergraduate mathematics or mathematics-related degree course. One would expect that incursions would be made into levels 0,1,2, in varying measures at post 16 ages and the implication of the classification is self evidently that the sequence 0,1,2,3 indicates an increasing depth of cognisance.

LEVEL 0	NUMBER	SPATIAL AWARENESS	FUNCTION
LEVEL 1 (Skill acquisition)	LANGUAGE	NOTATION	TECHNIQUE
LEVEL 2a (Abstraction)	ABSTRACTION	FORMALITY	STRUCTURE
LEVEL 2b (Logical implication)	PROOF	RIGOUR	AXIOMATICS
LEVEL 2c (Mathematical utility)	MODELLING	MATHEMATICS/PHYSICS INTERFACE	
LEVEL 2d (Analogy)	SPECIALIZATION	GENERALIZATION	ANALOGY
LEVEL 3 (Invention)	ADVANCED INTUITION	INVENTION	

We might ask which of these qualities is to be regarded as being of primary importance in the communication of mathematics as a service discipline, and particularly in the communication of mathematics to engineers, whether they are already qualified or in training. My use of the word "training" of course provokes an immediate objection from any self respecting educator. An honours degree programme in engineering is not just a training course, it should stretch the intellect and enable the aspiring engineer both to understand current practice and to venture beyond it. We need to encourage people to THINK and provide them with the intellectual equipment firstly to be able to respond to extreme and anomolous behaviour, and finally to be capable of engineering invention, however modest, on their own behalf.

Of course mathematics lecturers should, ideally, themselves be well aware of the interplay between mathematics, physics and engineering, not only as it

stands now, but as it has developed over the centuries. That is asking a great deal of the mathematics lecturer, with the implication that he or she should have some knowledge of both the history and philosophy of mathematics, as well as an overview of many areas of current mathematical practice, including some insights into how mathematics fulfils its modelling role. For an overview of what mathematics is, see Temple (1981), Howson (1972), and Roman (1975). For a discussion of the extent to which such polymath qualities are feasible in the modern world, at the same time being compatible with the demands put on the research mathematician, see Kline (1977, 1980).

10. Pedagogical perspectives.

Clearly the traditional approach to the teaching of undergraduate mathematics will increasingly become subject to modification as the result of the availability of computer aids. Structured learning is already available as a means of self-paced instruction under the general title of "Keller Plan". There are several modifications of this plan in current use, and the scheme has proved to be an effective means of communicating service mathematics material when it is allied to back-up facilities in the form of video taped lectures and typed hand-outs. It has certain drawbacks in the proliferation of new problem sheets which are required to ensure that real progress takes place in successive years. It also suffers, in my view, from the fact that it seems not to take into account a general behavioural trait, namely that learning is not, at all levels of the taxonomy (or even at any of them) an instantaneous process. At what level of skills is the Keller Plan working, for example? Are other aspects of knowledge and understanding accumulating with various time delays with the Keller Plan as with the traditional lecture method?

It has become standard practice in many institutions to give lectures in mathematics to quite large groups and provide tutorial back-up in smaller groups. That tutorial back-up usually consists of working of problem sheets together with individual attention. It would seem that micro-computer networks have an increasingly large role to play in this area, for there seems to be no reason why they should not be useful in providing illustrations especially using the graphics facilities that they possess, or in the demonstration of model solutions. What they cannot do, in my view, is to teach concept, nor can they motivate, nor be capable of the surprise, humour or timing that is the hallmark of the expert human communicator.

We are reluctant to accept that knowledge transfer is a multiply fuzzy process, that is that the knowledge is inevitably fuzzy in conception prior to transmission, then it is fuzzily transmitted and finally fuzzily received. In the course of a lecture programme we witness several learning phenomena at work. Depending on the cognitive level there are several indices of delay; there are also cross-disciplinary effects of great sublety by which just the flavour of one lecturer's approach in one discipline will pay dividends in some unforseen way elsewhere.

In addition, there is enormous redundancy and overkill built into the traditional lecture programme method of tuition, and that possibly accounts for what seems to be its continuing success, or at least acceptability.

11. History of reforms in mathematics teaching.

The year 1871 saw one of the great causes célèbres of mathematical education in full flow. The dispute arose between the ranks of the teachers of school mathematics on the one hand and the scions of the University of Cambridge on the other. In particular, on the school side, were members of the College of

Preceptors, and, particularly, a formation called the Association for the Improvement of Geometrical Teaching; on the University side such famous names as Todhunter, Kelland and Dodgson.

What, then, was the argument about? It was about the balance between the formality and rigour that University mathematicians demand in their version of the mathematical disciplines, and the hands-on experience and plausibility that school teachers judge to be essential in their role.

It is a dispute that recurs constantly at all levels of mathematics teaching, from primary school course to undergraduate course, and it is one whose intensity was to peak again in the era of the "modern mathematics" controversy.

In 1871, as in 1969, University mathematicians somewhat arrogantly assumed that they, and only they, knew what mathematics was really about, and they tried to use their position as masters of the mathematical high ground to deny others a say in the educational process. The Universities did not have the argument all their own way, however, opposition was strong, not least at the various thriving technical schools, both in London, and elsewhere. That opposition would almost certainly have been reinforced by opinion in the nation's premier military academies, for outstanding mathematics teachers and text book writers were to be found at the Royal Military Academy (Woolwich), the Royal Naval College (Greenwich), and the Royal Military College (Sandhurst). These institutions were at the leading edge when it came to instruction in the techniques and procedures of applicable mathematics. So, too, in London, were the Cowper Street School, off City Road, and the Borough Road school, and, outside the capital, the various northern Mechanical Institutes, including the Manchester Institute.

In 1969, the titles of "reformer" and "traditionalist" were interchanged; the proposers of reform were in the Universities, its opponents, such as they were, in the schools. The issue, however, was the same, how to reconcile the conflicting needs of rigorous mathematics and vocational mathematics.

"Mathematics is formal, logical.and wonderful" would say the reformers; unable to deny this, prospective critics of the reform were muted in their opposition. Indeed, the educational difficulty that "modern mathematics" provokes is only manifest when the reformers bring forth their next implication: "therefore we must teach it formally and logically in our schools and universities."

To say that "mathematics is formal, logical and wonderful" is far less than half the story. It is also intuitive, inventive and pragmatic. Its acquisition is cumulative, but that accumulation is selective . Though it is successful, its success is not capable of being complete . Though it might have been thought once to be independent of experience, it seems now more likely to be quasi-empirical. That prospect will have the greatest possible effect on how it is taught, at all levels.

12. Quasi-empiricism: what is it?

Within the last twenty years, especially since the work of Lakatos, (1967, 1976, 1978) attention has been focused on mathematics AS IT IS not as axiomatists have conceived that IT OUGHT TO BE. In Tymoczko (1986), there is an extended critique of the platonist, logicist, formalist and intuitionist positions, and the philosophical basis of mathematics is re-examined. The extreme convolutions to which Hilbert, Russell and others were reduced in their search for the perfect axiom system, which were shown to be vain by

Godel, WOULD HAVE MADE mathematics dry, automatic and computer generatable. Fortunately, it is not dry, automatic and computer generatable. But what is it, then?

Mathematics is as it is practised. It develops now as mathematicians have always experienced its development, by generalization, specialization and analogy, by cross-fertilization with physics, by conjecture and refutation, by abstraction, and by intuition tempered with rigour.

These facts about mathematical reality are not observable without some appreciation of how mathematicians worked in the past, and this is nowhere better illustrated than by George Polya in his article in Tymoczko (1986) concerning $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Euler guessed at a result, but not wildly; he made just the right guess, the one IN THE IDIOM OF MATHEMATICS. But the guess, the conjecture, was not motivated from set theoretic foundations, nor with formal logic. It was the way that all mathematicians, at their most creative, work.

Tymoczko (1986) describes the field in which mathematicians work as "mathematics without foundations". That is to say, the search for firm foundations at a very deep level is vain. That did not stop Euler "doing mathematics". Should it stop us? Of course not.

My own conception of mathematical activity is as work on a mosaic of knowledge, never to be completed, and resting on somewhat spongey "foundations", but <u>fascinating</u>; and not only that, <u>useful</u>. Thus mathematics cannot be divorced from experience, whether foundationist philosophers like it or not. Lakatos defines "quasi-empiricism" as follows:

> the axioms or basic principles of the theory are the results of bold speculation that have survived the test of severe criticism, (Tymoczko, 1986).

Thus a theory stands UNTIL IT IS FALSIFIED.

Perhaps it is a little vague to talk of having "survived the test of severe criticism", but that seems to be the best that we can do.

We may perhaps look upon "severe criticism" as the operation of "rigour" in the following way. Bold speculation leads, and if it misleads into error by way of a proferred contradiction or counter example, then one of a succession of logical filters will indicate how and why. If the speculation is still unexplained, and if it escapes all existing filters then we have to rethink the position and design a new logical filter. The collection of logical filters is called "rigour".

13. High ground v high peaks.

What is the best preparation for a university lecturer in mathematics? It is tempting to assert that clearly, those with a good overview of what mathematics is will be the best teachers. It is then a short step to asserting that there are certainly such persons around, and that they are clearly research mathematicians, those who have the deepest knowledge of certain aspects of mathematical theory and practice. I dispute this last assertion. In two ways. Communication of some fund of knowledge requires two, at least, ingredients, a fund of knowledge to communicate and the ability to communicate it. The fund of knowledge is available to two categories of academics, those who occupy the high peaks in the mathematical landscape - the researchers, and those who occupy the high ground, what I shall call scholars. Constituents of neither group are guaranteed an innate ability to communicate well the material at their command, but the scholars are more likely to want to do it.

14. Assessment of teaching skills.

Following the foregoing discussions of what exactly is the practice of mathematics and what are the various theoretical problems of communicating it to others, there remains the problem of judging what is good communication practice, and who achieves it and how? That matter is the subject of Briginshaw and Newby (1987), which is submitted for publication elsewhere. Briefly, that paper observes that mathematics is particularly difficult to teach. The reasons for this are two-fold, and I have attempted to explain them more fully in this Report. Firstly, the mathematical language has low redundancy, and is not an easy vehicle for good communication; secondly, mathematics operates at so many levels of cognition that the multiply fuzzy ways in which mathematical knowledge is focused, transmitted and received are an order of magnitude more complex than those for a non-scientific discipline. Briginshaw and Newby (1987) conclude that a major input to the assessment of the teaching of mathematics to undergraduates must be by way of anonymous student questionnaire. They have therefore attempted to design a model questionnaire which is both skill specific (i.e. it judges ability to communicate) and subject specific (it focuses specifically on mathematics).

15. Acknowledgements

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GOALS FOR MATHEMATICS EDUCATION

1. To give the educated person a fund of mathematical analogies and the ability to use them.

2. To enable the educated person to further create mathematical analogies for his own use when needed.

3. To enable the educated person to understand how and to what effect mathematical analogies have been used significantly for our civilization and our culture.

4. To enable the educated person to intelligently anticipate directions, areas, ways in which mathematical analogies, new or old, might have further significant effect for our civilization and our culture.

The latter two are rexpression of Peter Hilton's dictum, that we should "strive to awaken in as many people as possible, irrespective of their chosen vocation, an awareness of the nature of our science, and its significance for our civilization, material and spiritual."

The 'analogies' mentioned in 1 through 4 above need not be applications in the traditional sense; they may often have more to do with the relationships between ideas than with sensed reality; they may in fact be pure mathematics.

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MATHEMATICS AND ITS APPLICATION

Jack V. Wales, Jr.

MATHEMATICS AND ITS APPLICATION

When mathematics is taught using lots of applications and the applicability of mathematics is emphasized (perhaps to the point of becoming the guide and justification of mathematics education), students are likely to come to believe that the mathematics is "in" the worldly circumstances which are the focus of the application at hand. Such a belief does not encourage the investigation of the limitations of the application of mathematics to the situation. Further, it is ground for coming to see worldly reality as a mathematical edifice which must be conquered, and oneself as feeble before it. It would be better to teach that mathematics is the study of an independent, extant reality; and that any application of mathematics is the postulating of an analogy between a set of worldly circumstances and a set of mathematical circumstances. Then the coincidence of pattern is a cause for delight, and the limitations are expected, even if they are not seen.

I wish here to argue that we mathematics teachers should not feel that it is necessary to justify whatever mathematics we teach by its applicability, and further that even the most applicable mathematics should be presented as being worthy of study independent of its applicability, not least because its application will be done best when its independence is accepted. By 'application' I mean not only the use of mathematical patterns to increase control or predictive power in practical situations, but also the use of mathematical patterns to increase understanding in practical or impractical or even fantastical situations.

When I was learning to fly, my instructors told me, "The throttle controls your altitude, the elevator controls your speed, not the other way around." I now believe that is a distortion, but it was a very useful one. When one begins flying an airplane, he has a natural tendency to believe that the throttle controls the airplane's speed and the elevator makes the plane go up or down. In fact, the situation is much more complicated than that; and my instructors' dictum was instrumental in my learning to use the two controls effectively. I would use it today if I were to take the controls of an airplane again.

I don't think the phenomenon of an idea being of good effect even if it is a distortion is a particularly unusual one. Another example might be the dictum, "There is no important difference between men and women." Or, "When acting in a play by Shakespeare, one should never give emphasis to a personal

pronoun." If those sentences are not true, acting as if they were might nevertheless have good effects because they counteract some deeply ingrained erroneous biases.

The idea I want to offer here is, I suspect, correct. Even if it is not, however, acting as if it were would have good effects. And so I want to present the idea here, examine some of its consequences, and defend that complex consisting of the idea and its consequences.

The idea is this: first, that mathematics is an account of an extant reality that is independent of physical reality or social reality; and second, that it is therefore appropriate to understand applications of mathematics in various arenas as analogies, analogies which often appear as metaphors.

I don't know, nor do I care, just what philosophical position I am taking when I say that mathematics is an account of an extant reality. The important point is that mathematical truth is independent of human judgment. 29 is prime. 27 is not prime. Those sentences are true not because I or any expert says so, but because 29 <u>is</u> prime and 27 is <u>not</u> prime. That is just the way they are. I don't know offhand whether forty-three times sixty-seven is equal to fifty-one times sixty-one, but there is no doubt that it either is or it isn't, independent of anything anyone does or says. Mathematical truth is about the least contingent truth around. Mathematical reality is there to be discovered or observed.

At the same time, mathematics gives an account of logical inevitability, not physical inevitability. "There is one thing...of which a geometry is <u>not</u> a picture, and...that is the so-called real world," says the famous mathematician, G. H. Hardy. (1) No one can mathematically prove that the sun will rise tomorrow or that it won't, that the stone will fall when released, that closing the circuit will cause the light bulb to come on, or whatever; physical reality may more or less coincide with mathematical pattern, but it is not constrained by it. This fact gives substance to the word 'unreasonable' in the title of Eugene Wigner's essay, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." (2)

What, then, is the relationship between mathematics and, say, physics? Is mathematics a language in which physics is expressed, as is suggested by the title of Tobias Dantzig's book, <u>Number: The Language of Science</u>? (3) If I am right that mathematics is an account of an extant reality, then it is certainly more than a language. Perhaps mathematical symbolism is a language, but the mathematical ideas, the mathematical facts, expressed by the symbols are not mere language. Rather, physics is a science which, among other things, makes analogies

between mathematical circumstances and physical circumstances. If extent of spatial separation is like number, and this particular spatial separation has an extent which we take to be like the number 5, then that one has an extent which is like the number 7, and furthermore, the two of them together constitute a spatial separation the extent of which is like the number 12. This is not a particularly unusual idea; mathematicians and scientists often refer to mathematical "models" of worldly circumstances, by which they don't mean anything too different from what I mean when I use the word 'analogy.'

Why, then, do I choose to use that somewhat unusual word? Because the word 'analogy' tends to remind us of certain things that the word 'model' does not. The important points are these: any analogy has two parts or sides, and the extent of validity of an analogy is always in question. (Note Wendell Berry's words, quoted later in this paper.)

There are, roughly speaking, two kinds of analogy: explicit and implicit, simile and metaphor. The literary critic Northrop Frye observes,

> In descriptive writing you have to be careful of associative language. You'll find that [simile], or likeness to something else, is very tricky to handle in description, because the differences are as important as the resemblances. As for metaphor, where you're really saying "this _is_ that," you're turning your back on logic and reason completely, because logically two things can never be the same thing and still remain two things....The motive for metaphor, according to Wallace Stevens, is a desire to associate, and finally to identify, the human mind with what goes on outside it, because the only genuine joy you can have is in those rare moments when you feel that although we may know in part, as Paul says, we are also a part of what we know. (4)

If application of mathematics is the making of analogies, then in much of human knowledge, mathematics is (ironically, for if mathematics is not logical, then nothing is) not a simile but a metaphor, in fact an unconcious metaphor, a culturally subconcious metaphor, for other sorts of reality.

Digression

The words left out by the ellipsis in the quote by Northrop Frye are,

> The poet, however, uses these two crude, primitive, archaic forms of thought in the most unhibited way, because his job is not to describe nature, but to show

> you a world completely absorbed and possessed by the human mind. So he produces what Baudelaire called a "suggestive magic including at the same time object and subject, the world outside the artist and the artist himself." (5)

And when Frye mentions Wallace Stevens, he is referring to the following poem by Stevens:

The Motive for Metaphor

You like it under the trees in autumn,
Because everything is half dead.
The wind moves like a cripple among the leaves
And repeats words without meaning.

In the same way, you were happy in spring,
With the half colors of quarter-things,
The slightly brighter sky, the melting clouds,
The single bird, the obscure moon--

The obscure moon lighting an obscure world
Of things that would never be quite expressed,
Where you yourself were never quite yourself
And did not want nor have to be,

Desiring the exhilarations of changes:
The motive for metaphor, shrinking from
The weight of primary noon,
The A B C of being,

The ruddy temper, the hammer
Of red and blue, the hard sound--
Steel against intimation--the sharp flash,
The vital, arrogant, fatal, dominant X.

Frye's contrast between "describing nature" and "showing a world possessed by the human mind," and Stevens's use of technical symbols (A B C, X, weight, primary noon, temper, steel) for what is shrunk from when one is motivated to metaphor, suggest that I am off base, that the whole point of what they are talking about is the contrast between poetry on the one hand and the "objective" worlds described by math and science on the other. But remember that Stevens's title is "The Motive for Metaphor." Is not science an (in some ways quite successful) attempt by the human mind to absorb and possess a world?

It is one of the ironies of Man's present condition that a motive of identification should have been a driving force behind what seems to so many to be the source of so much alienation, i.e., the application of mathematics to worldly circumstances.

As I put the finishing touches on this essay, I am starting to read a new book by the philosopher William Barrett, in which I found this:

> We note the extraordinary power and constructivity of the human mind in producing the great edifice of modern science. And yet, precisely here occurs one of the supreme ironies of modern history: The structure that most emphatically exhibits the power of mind nevertheless leads to the denigration of the human mind. The success of the physical sciences leads to the attitude of scientific materialism, according to which the mind becomes, in one way or another, merely the passive plaything of material forces. The offspring turns against its parent. We forget what we should have learned from Kant: that the imprint of mind is everywhere on the body of this science, and without the founding power of mind it would not exist.
>
> The irony here is not one that we can merely sit back and enjoy aesthetically. This doubt of the mind, in its actual consequences, in the lives of individuals and societies, provides one of the ordeals that modern civilization will have to go through. (6)

Return

Frye suggests earlier in his essay that as science moves from data towards laws, it "moves toward imagination," it tends to invoke mathematics, which, along with literature and music, is (he says) a language of the imagination. "A highly developed science and a highly developed art are very close together, psychologically and otherwise." (7) Pursuit of mathematics is the pursuit of understanding. (Henry Pollack has said that the essence of science is the right to repeat an experiment, while the essence of mathematics is the right to understand.(8)) Surely understanding is, in Frye's terms, the identification of the human mind with something outside it, the attribution of meaning to the coincidence of pattern. In the case of applications of mathematics, meaning is often attributed to the coincidence of mathematical patterns with patterns of worldly circumstances. Carried along by the logical inevitability of mathematics, undaunted by obscurities in the coincidence of the patterns, users of mathematical applications can come to believe that the mathematics is "in" the worldly circumstances. (9) Understanding becomes "knowledge,"

Digression

Scott Buchanan gives a more thorough account of this in his book Poetry and Mathematics. Here are some relevant excerpts:

"Belief is the natural attitude of a thwarted mind. It

arises from fatigue and confusion....For the most part confusion is of two sorts, one involving symbols, and the other metaphysical nostalgia, the tendency of thought toward the absolute." (10)

For Buchanan, symbols are things (aesthetic objects) which point to ideas (intellectual objects). He explains the confusion involving symbols as follows:

> The aesthetic properties of ceremony, formula, natural processes are intimations of complex and profound intellectual objects, but the difficulties of intellectual clarification and discrimination leave the mind in various attitudes of belief. For every intellectual object, half-comprehended, there is an aesthetic object before which we bow in more or less deep reverence. Pure aesthetic contemplation and complete intellectual clarity are seldom found in human beings, and any middle ground is touched with credulity and idolatry. (11)

As to the metaphysical nostalgia,

> [a stage of mathematical discovery] results when we can see the <u>relations</u> holding between <u>qualities</u>.... Mathematical functions find elementary values in qualities. Qualities find their relations in the functions of mathematics. Whenever this happens, a system is recognized, and it takes on a quasi-independence and reality. Often the effect on the thinker is a conviction. Belief attaches itself only to such systems. The further expansions and the wider assumptions are ignored and there is a resting point for thought in a mathematico-poetic allegory. (12)

Of course, we are dealing here not with a disorder or aberration in human thought processes, but with the very nature of thought itself. Buchanan's account has some similarities with this description by Ernst Cassirer:

> [For the religious genius,] the power of his belief first proves itself in being made public. He must communicate his belief to others, he must fill them with his own religious passion and fervor, in order to be certain of his belief. This is possible only by means of religious constructs--constructs which begin as symbols and end as dogmas. Thus, even here, every initial expression of feeling is already the beginning of an alienation. It is the destiny and, in a sense, the immanent tragedy of every spiritual form that it can never overcome this inner tension; to extinguish it is to extinguish the life of the spirit. For the life

> of the spirit consists is this very act of severing what is whole in order that what has been severed may be even more securely united. (13)

And, in Cassirer's view, this pattern runs deep. Later, but talking about the same example, he says,

> And so it is that here, too, we find the same oscillation which sets in within all forms of culture as they begin to take shape[,]...the ceaseless and irresistible rhythm of life itself. (14)

<u>Return</u>

and that knowledge is passed from one human being to another, often as subject matter in academic courses. And so we can see how mathematical circumstances, which are intrinsically logical, become illogically (but perhaps more or less appropriately) identified with things they are not, namely worldly circumstances.

Mathematics is often a tremendously effective metaphor, "unreasonably effective," in Eugene Wigner's words. But there is danger here. Essayist Wendell Berry, in his volume on culture and agriculture in the United States, refers in passing to

> ...the model of the scientists and planners:... an exclusive, narrowly defined ideal which affects destructively whatever it does not include. (15)

Joseph Weizenbaum, a computer scientist and teacher of computer science, speaks this message to his fellow computer science teachers:

> I...affirm that the computer is a powerful new metaphor for helping us to understand many aspects of the world, but that it enslaves the mind that has no other metaphors and few other resources to call on. The world is many things, and no single framework is large enough to contain them all, neither that of man's science nor that of his poetry, neither that of calculating reason nor that of pure intuition. And just as love of music does not suffice to enable one to play the violin--one must also master the craft of the instrument and of music itself--so is it not enough to love humanity in order to help it survive. The teacher's calling to teach his craft is therefore an honorable one. But he must do more than that: he must teach more than one metaphor, and he must teach more by the example of his conduct than by what he writes on the blackboard. He must teach the limitations of his tools as well as their power. (16)

Mathematics is, like the computer, a powerful metaphor. Unlike the computer, it is not a new metaphor. Any metaphor can take on excessive significance in a human imagination. Perhaps because mathematics is not a new metaphor, we are inured to it a bit; but that may just mean that we have lost the nervous uncertainty that comes with venturing into new territory, but not seen enough failure of the metaphor to undermine our credulity. Metaphor is a wonderful thing, not at all to be scorned. Even if it were not wonderful, it would probably be inevitable. (17) The motive for metaphor is a glorious aspect of human nature. G. Spencer-Brown observes, "That mathematics, in common with other art forms, can lead us beyond ordinary existence, and can show us something of the structure in which all creation hangs together, is no new idea." (18) But we need not take away from the value of the metaphor, or from the metaphorical experience, indeed we will enhance them, if we keep in mind the question, "What are the limits of this metaphor?"

If the application of mathematics is the construction of analogy, whether simile or metaphor; and if we are to effectively understand the limits of the analogy; then we must have some understanding of each of the two things being compared, and that understanding must be wider than what is immediately relevant to the analogy. Hearing Macbeth say of Duncan,

> After life's fitful fever he sleeps well, (19)

we bring to bear all our experience of life, fever, sleep, and death; and if we did not know enough of sleep and death, of life and fever, to know many ways in which they are not alike, as well as ways in which they are, then the metaphors would not be as rich and effective, the sentence not as beautiful, as they are. Of course, at the same time, analogies broaden our experience, deepen our understanding, give us new insights into the things being compared; indeed, that is their very function.

What does all this mean for the teaching and learning of mathematics? It means we should teach and learn mathematics beyond that which is "relevant," that which appears explicitly in applications of practical importance. The perennial students' question, "When will we ever use this?" is a misguided question, one to which we should not succumb. That is not to say that we should refuse to answer it; but we should deny that the question is determinative of what is important in a person's study of mathematics. Mathematics is one side of a myriad of important analogies; if we are to understand that side, then we must understand, we must teach and learn, mathematics itself. (And, of course, the study of mathematics itself has its own rewards, quite apart from applicability. Alfred North Whitehead describes the pursuit of mathematics as "a divine madness of the human spirit, a refuge from the goading urgency of contingent happenings." (20) Hardy says, "Real mathematics...must be

justified as art if it can be justified at all." (21) Scott Buchanan observes, "The structures with which mathematics deals are more like lace, the leaves of trees, and the play of light and shadow on a meadow or a human face, than they are like buildings and machines, the least of their representatives." (22)) Knowledge of calculus has vocational value for engineers and others, but its value is increased by deep understanding of the mathematical theory of calculus; knowledge of statistics is perhaps necessary for informed citizenship; knowledge of arithmetic has survival value; knowledge of number theory may have none of these, but it strengthens one's understanding of mathematics as an independent reality which is in some of its facets analogous to some facets of other kinds of reality. (23)

It also means that we should resist, and teach our students to resist, any tendency to neglect those aspects of other sorts of reality that do not fit into those analogies with mathematical reality that we call applications. We should in fact look for them. We should seek to understand the limitations of our analogies, and we will understand them better if we know what is beyond them, on both sides. We will be the richer for knowing what of mathematical reality does not fit the physical circumstances, and what of physical reality does not fit the mathematical circumstances, of whatever mathematical application we are dealing with.

We should, in short, let mathematics be, just as other disciplines are, the pursuit of ways of seeing, the pursuit of visions. We should teach our students to look for mathematical analogies, to delight in them when they find them, to stretch them and test them and savor them, but not to be consumed by them lest they, we, suffer the fate of all tragic heroes. (24) We will lay the proper foundation if we teach them mathematics itself, as independent, extant reality, whose "applications" are in fact analogies which often appear as metaphors.

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Notes

(1) G. H. Hardy, "What is Geometry?," Presidential Address to the Mathematical Association, 1925.

(2) Eugene P. Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," Communications on Pure and Applied Mathematics, Vol. XIII, 001-14 (1960).

(3) Tobias Dantzig, Number: The Language of Science (Garden City, NY: Doubleday & Company, Inc., 1954)

(4) Northrop Frye, The Educated Imagination (Bloomington: Indiana University Press, 1971), pp. 32-3.

(5) Ibid, pp. 32-3.

(6) William Barrett, Death of the Soul (Garden City, NY: Anchor Press/Doubleday, 1986), p. 75.

(7) Frye, The Educated Imagination, pp. 23-4. Perhaps one of the more important differences has to do with belief. See the second digression.

(8) This was in a talk that Dr. Pollack gave on "The History of an Application" at the 1986 Woodrow Wilson Summer Institute on High School Mathematics in Princeton, New Jersey.

(9) As part of its promotion of "Math Education Month" (April, 1987), the National Council of Teachers of Mathematics is offering for sale a bumper sticker which reads, "Math Keeps the World in Motion." Of course, as is proper for a bumper sticker, the sentence is ambiguous, suggestive, a play on words. It points to the central role of mathematics in modern technology, government, economics, etc. But it also suggests that math causes the earth to rotate on its axis and revolve around the sun, the automobiles and telephones to work, and so on. It can be read as ironic (in a couple of subtly different ways), but for one who cannot see the irony, it could be an intimidating and depressing, an unfortunate, message.

(10) Scott Buchanan, Poetry and Mathematics (Chicago: The University of Chicago Press, 1975), p. 135.

(11) Ibid, p. 140.

(12) Ibid, pp. 146-7.

(13) Ernst Cassirer, The Logic of the Humanities (New Haven: Yale University Press, 1974), pp. 115-6.

(14) Ibid, p. 214.

(15) Wendell Berry, <u>The Unsettling of America: Culture and Agriculture</u> (San Francisco: Sierra Club Books, 1986), p. 112.

(16) Joseph Weizenbaum, <u>Computer Power and Human Reason</u> (San Francisco: W. H. Freeman and Company, 1976), p. 277.

(17) When looking up the sentence about belief and the thwarted mind, I discovered how much my thinking in this essay had been affected by Buchanan's book. For example, the last sentence in one of his chapters is, "Any history of thought might begin and end with the statement that man is an analogical animal." Buchanan, <u>Poetry and Mathematics</u>, p. 141.

(18) G. Spencer-Brown, <u>Laws of Form</u> (New York: E. P. Dutton, 1979), p. xxix.

(19) This example will be recognized as borrowed from Hardy.

(20) Alfred North Whitehead, <u>Science and the Modern World</u> (New York: The New American Library of World Literature, 1954), p. 22.

(21) G. H. Hardy, <u>A Mathematician's Apology</u> (New York: Cambridge University Press, 1977), p. 139.

(22) Buchanan, <u>Poetry and Mathematics</u>, p. 36.

(23) Buchanan makes several comments relevant to mathematical pedagogy. Here is one that relates to the use of mathematical applications in the classroom:

> Mathematics is not a compendium of memorizable formula and magically manipulated figures. Sometimes it uses formulae and manipulates figures, but it does this because it is concerned with ideas already familiar to the ordinary mind, but needing special sets of words or symbols for the sake of precise expression and efficient communication. Further, the abstraction thus signalized, which most people from bad emotional habits fear, is actually much more familiar to the untrained mind than any observed facts could possibly be. Abstract ideas are of the very tissue of the human mind. For this reason and for many others, illustration of mathematics by concrete event, fact, or object is never as effective as illustration by equally abstract analogous ideas.

Ibid, pp. 35-6. Of course, in this passage Buchanan is speaking of the use of the concrete to illustrate mathematics, rather than the use of mathematics to explain the concrete.

(24) I have heard attributed to Mark Twain the remark, "It ain't what people don't know that causes all the trouble, it's what they do know that ain't so."

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Faculty of Mathematics & Science

Department of Mathematics and
Applied Mathematics

Our ref Your ref Date 1988-03-03

Dear Professor

Over a number of years by now, I have had a growing feeling of unease about the ways of science in the West, especially in the case of academic hard science.

It happened that during the last year, I have on several occasions been asked to express my views in writing on certain issues concerning science management. These were reports of about half a dozen pages, which after a while, gave me the feeling of being hopelessly scattered across the wider and deeper ranges of issues affecting present day science.

As a consequence, during the Christmas and New Year holiday - which here is the long, summer vacation - I could no longer help it and had to get the issue out of my mind, as if once and for all ...

The enclosed 93 pages followed ...

I am circulating the essay within a smaller circle of academics and a few other persons whose interests may include the state and development of science.

Since the USA happens to concentrate by far the most of the present day Western hard science, the essay may have the Americans as the main public.

In case you may find it of sufficient interest to read the essay, I would be most grateful for your possible comments.

To the extent that it may be appropriate to try to publish the essay, I would most appreciate suggestions for possible publishers who would be best placed from the point of view of the American public.

With best regards and wishes

Yours sincerely

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TRANSPARADIGM MATHEMATICS RESEARCH INITIATIVE (TMRI)

Present Trends and Alternatives

In their scientific research activity, humans face the unknown, in an attempt to bring ever more of it into the known.
In many of its essential moments, that encounter between us humans and the unknown does rather take place on the terms of the unknown than on ours.

Concerning our terms in facing the unknown, two aspects are paramount : our presently available knowledge and our future shorter or longer time interests. Both of these can, and in fact do strongly influence the outcome of our encounter with the unknown, not seldom through the limitations they impose on our capabilities, or perspective and vision, as well as on our subsequent approaches.

In contrast with recent past times, when for several decades we witnessed a spectacular expansion and deepening of scientific research achievements, during the last decade or two one can note signs of a slowdown if not stagnation, which appear to be intrrelated with the lately emerging social, economic, political, etc., trends.
Remedial actions, undertaken occasionally and on a rather limited scale in view of the recent budgetary constraints, tend to be motivated by economic or defense interests of specific and shorter term nature.

It is in this way that in facing the unknown, we increasingly tend to impose upon that encounter our interest dictated terms. The main instrument to emerge for that purpose is the so called 'science management', which is supposed to run all scientific research and every researcher, with the aim of maximizing certain specific, shorter term economic or defense benefits, by using tools such as the selective distribution of scarce research funding.
Such an approach can often show a success when considered on its own terms alone, yet its longer time effects on our encounter

with the unknown could be dramatically negative. Indeed, that approach is rather concerned with drawing upon some of the existing resources of science and research scientists, while with respect to the rest - in particular, fundamental research, or scientific scholarship - it exhibits a significantly lower interest and attention.
But above all, the present ways of science management prove to be incapable of understanding that, in facing the unknown, we cannot beyond certain limits 'manage' the terms of that encounter, least of all determine or dictate them, without endangering the whole of the scientific research venture.

We should indeed remember that the unknown is after all unknown ...
Moreover, our ways to it will essentially remain unknown.
Therefore, these ways can only be in part dictated by any of our own terms.
And it is an essential necessary condition that we are - and remain - sufficiently open to the terms of the unknown, terms wich themselves are never fully known and have to be found out continuously.
This is precisely why freedom of research - in particular, academic freedom - is an essential necessary condition in facing the unknown, since it alone can give us a chance to avoid the full imposition of our terms upon that encounter.

The novelty of the present situation is that science management comes to add a new term - namely, that of our various intersts - to the earlier, traditional term which has always been defined by the limits of our existing knowledge.
The extent to which scientific research has suffered over the ages from the limitations of the existing knowledge, understanding and set ways of thinking is well documented in the history of science.
A rather uniquely important and detailed study of that phenomenon was presented by Thomas S Kuhn in his 1962 book on 'The Structure of Scientific Revolutions', hailed at the time as '... a landmark in intellectual history ...'. Based on an impressive historical evidence, Kuhn shows the shocking fact that science, during its usual development, tends to fall into various 'paradigmatic traps'. In other words, scientists - and most likely the leading ones, who have worked for long in a given field and can impose their point of view - will get into certain habits of thinking which will inevitably and strongly precondition their way of facing the unknown. And rather as a rule, once a field of science gets settled into such a 'paradigm', it has a strong and long lasting tendency to stay there. Moreover, the way out can usually occur only through a 'scientific revolution' in the given field, brought about by a new 'paradigm' which manages to emerge, and against the domination of the old one, happens to be taken up by a sufficient number of younger scientists.

That traditional term defined by our given 'paradigms', and imposed by us on our encounter with the unknown, has proved historically to be sufficient in order to cause quite a bumpy ride along the dynamics of science, which has been punctuated by occasional 'scientific revolutions' coming to upset the given establishment of various ruling 'paradigms'.
By adding to that traditional dynamics of science the novelty of present day science management, we risk the major danger of staying in our 'paradigmatic traps' indefinitely.
Indeed, present day science management imposes an early and narrow specialization on researchers, prevents the emergence of science scholars with a wider and deeper understanding of related fields, subjects researchers to the short term pressures of 'publish or perish' and attaches various strings to research funding. In this way, we risk to have 'pacified' science once and for all, by eliminating any chance for future 'scientific revolutions'.
Certainly, the sheer emergence of any new 'paradigm' requires the development of a whole new system of ideas, which can only be accomplished by a research scientist who does not suffer from narrow specialization and can afford to consecrate quite a number of his or her most creative years to deeper and wider thinking, wondering and experimentation. Such a research scientist has to be free from undue concern with the hectic pressures of 'publish or perish', the strings attached to research funding, or the worries of promotion, tenure, etc.

It is essential to remember that, one of the most important powers of science comes precisely from the insights offered by the truly new and fundamental systems of ideas.
Paradigm bound research on the other hand, rather resembles learning a given, complicated game, and then playing it successfully with similarly trained top class competitors. Such a research can therefore hardly lead to more than adding new, better ways of playing within some rather fixed rules. Certainly, it will hardly lead to more, under the conditions imposed by present day science management.

The alternative however should not be sought in a return to the traditional science dynamics with its rather bumpy ways, in which essentially new contributions can only come as if against the system, after succeeding in 'scientific revolutions'.
The deficiency of that traditional dynamics of science has been that in its development, the 'main line' has so often become the 'only line' for peer recognition and appreciation, therefore creating a kind of totalitarian pressure upon science researchers dependent on public scientific opinion. Fortunately, 'scientific revolutions' could nevertheless occur now and then within that traditional way, and the 'paradigmatic' limitations of science were rather inner, own affairs, and were run accordingly, even if they led to oscillations between established and revolutionary periods. Today however, when few research scientists are of

independent financial means, the mentioned, or any other kind of totalitarian pressures can only be felt stronger.

Since present day science management has been added to the traditional dynamics of science, it seems as if we risk the danger that the 'only line' which is decided upon by that management may as well become the 'main line' in scientific research. In other words, a completely outside decision process is being substituted for the traditional inner dynamics of science, leading thus to the possibility of a significant strengthening of earlier 'one track', totalitarian type pressures, with the consequent increase in the danger of staying indefinitely in 'paradigmatic traps'.
Indeed, present day science management has doubly handicapped our relationship with the unknown.
Fistly, it has imposed upon research and researchers the terms of economic or defense interests, and is doing so more and more, to the exclusion of other considerations.
Secondly, it completely fails to be aware of - least of all address - the traditional danger of 'paradigmatic stagnation', danger which is reaching particularly high levels as a consequence of the mentioned first handicap.

Since the 'main line' or 'only line' tendencies 'from the above' may to a good extent be inevitable and may stay with us in the future, one of the possible corrective actions seems to be the setting up of grass root 'parallel lines'.

Namely, it appears that our critically important priority is not only to avoid indefinite stagnation in 'paradigmatic traps', but to go beyond the traditional bumpy dynamics of science as well. For that, we should devise ways and means by which we can not only avoid the negative consequences of the conflict between 'established' and 'emerging' paradigms, but we can in fact promote a rather continuous flow of ideas, which aim to bring about viable candidates for new and emerging paradigms. And precisely to the extent that the present day 'main' or 'only' lines cannot accommodate such a venture, certain 'parallel' lines may prove to be useful.
It should be noted that such a venture, if possible at all, seems to be particularly easier to accomplish in the case of research in mathematics, which as is well known, requires the lowest funding among hard sciences, unless the massive use of main frame computers is involved.
It should also be noted, and strongly emphasized that, even if for a longer time, that venture is to be confined to mathematics research alone, it is nevertheless most likely that its positive effects may go much beyond and reach into various other sciences. Indeed, we should only remember for instance the celebrated 1960 paper of the Nobel Prize winner physicist Eugene P Wigner on 'The Unreasonable Effectiveness of Mathematics in Natural Sciences'...

An Offer

Let us establish a network which as a first step, offers every interested mathematician the following services free of charge :

- lists of authors in various fields of mathematics with transparadigm research ideas, developed in various stages, up to completed, published or unpublished papers,

- sufficiently detailed abstracts of such ideas,

- lists of research mathematicians who exprss their interest in specific such ideas,

- information on poster sessions presenting such ideas at conferences and symposia,

- information on newsletters consecrated in part or as a whole to such ideas.

A network of that type does not need a centralized organization or administration, therefore it does not need funding. In fact, such a network should not even become centralized. And fortunately, it could hardly attempt to become so, in view of the fact that it only offers individuals information which cannot be monopolized.
Moreover, such a network can be started, restarted, expanded, etc., from the grass roots, by any number of interested individual research mathematicians.
It only requires the individual's use of the presently widespread and outstanding mathematics word processors, as well as minimal expenses connected with photocopying and mail.
However, it can offer the individual research mathematician a multiplicity of possibilities and potentialities which were never before available :
It can encourage, motivate and sustain his or her truly free research thinking, which so far, one would seldom dare to indulge in even as an intellectual hoby.
It can propagate his or her respective ideas among many possibly interested people, who would otherwise may remain unknonw to him or her. The eventual reactions may be particularly beneficial, enlightening or both.
It can present a mathematician with a range of unusual and surprising ideas from different fields, and offer the connection with their authors. It can in this way help in going beyond excessive narrow interests and specialization.
And on the whole, it may usher in a new spirit in mathematics research, and subsequently, in other sciences as well.

Proposal for a Motto

... If you happen to have nothing better to do, choose a mathematical paradox, try to understand what is involved in it, and have a go along one of the possible ways out ...
... Good luck to you for you next paradox ...

A Personal Request

If and when you may decide to give the above a try, please, be so very kind and place me on your mailing list.
I do most strongly promise to do the same for you.

Many thanks for your kind attention,

With very best regards and wishes,

Yours most sincerely,

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