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Although I have always had an interest in this topic, my interest increased greatly when I was developing a Survey of Mathematics course for liberal arts students. The textbooks available for such a course stress remedial math and mundane (often contrived) applications. The approaches used in these books, in my opinion, do not give the students a true picture of what mathematics is or the role mathematics plays in our culture. Liberal arts students and educated people outside of mathematics (and some inside) do not have an understanding of mathematics as a vast and important body of knowledge that is an integral part of our culture and a significant force in the development of this culture.

In an article in a recent news magazine, a student stated that "We in the humanities deal with ideas while those in the sciences deal merely with skills". What is disturbing is that this likely reflects the attitude of his professors. What is not realized is that mathematics deals primarily with ideas and that these ideas are some of the most profound and important in our culture.

Recently the University of Wisconsin Centers revised their general education and associate degree requirements. They have adopted Introductory College Algebra (i.e., high school advanced algebra) as a competency requirement for the associate degree. I tried to convince people on the curriculum committee that this was not desirable (It should be an entrance requirement) and not appropriate mathematics for a liberal education in college. The committee could only focus on mathematical skills; they seemed to have no concept of the culturological contributions of mathematics.

In communication between Wisconsin Governor Thompson and University of Wisconsin President Shaw regarding assessment in the University of Wisconsin System, one of them referred to "quantitative skills" and the other to "mathematics". What is disturbing is that likely they thought they were talking about the same thing.

We need to communicate to our students and the public in general that mathematics is not a set of skills (bag of tricks, if you will) to solve mundane problems, but a vast body of knowledge that is an integral part of our culture.

In Mathematics in Western Culture (1953), Morris Kline states, "After an unbroken tradition of many centuries, mathematics has ceased to be generally considered as an integral part of culture in our era of mass education." He further states, "Almost everyone knows that mathematics serves the very practical purpose of dictating engineering design. Fewer people seem to be aware that mathematics carries the main burden of scientific reasoning and is the core of the major theories of physical science. It is even less widely known that mathematics has determined the direction and content of much philosophical thought, has destroyed and rebuilt religious doctrine, has supplied substance to economics and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe. Despite these by no means modest contributions to our life and thought, educated people almost universally reject mathematics as an intellectual interest." In his book, Kline discusses the relationship of mathematics to music, art, philosophy, religion, and literature as well as science.

Raymond Wilder has written a couple of books of significance in this area. They are Evolution of Mathematical Concepts: An Elementary Study and Mathematics as a Cultural System. One important concept Wilder discusses is that mathematical development can be categorized as the result of (i) environmental stress - motivation imposed from outside mathematics (e.g., commerce and simply keeping track of livestock motivating the development of the natural numbers, physics and astronomy motivating the development of calculus, economics motivating developments in game theory) and (ii) hereditary stress - motivation from within mathematics itself (e.g., development of negative (false) numbers, imaginary numbers, Non-Euclidean geometry, quaternions). I prefer to refer to the categories as cultural stress and intellectual curiosity. I stress this dichotomy in my Survey of Math course (and make mention of it in my other courses) and try to convince

my students that utilitarianism is not the reason to study mathematics - much important mathematics was developed out of intellectual curiosity (hereditary stress) without any idea that it would become useful, and later, sometimes centuries later, it proved to be of utilitarian value.

Another excellent source of information is the "Ascent of Man" television series done by Jacob Bronowski. Two of the video tapes in the series, "Music of the Spheres" and "The Majestic Clockwork" have mathematical themes. In the first, Bronowski traces the development of mathematics and culturally related aspects from the Pythagoreans, where Bronowski says mathematics began through the beginnings of the change in mathematics from a static to a dynamic description of nature. The relationship between music and mathematics was discovered by the Pythagoreans (6th century, B.C.). I understand that in some ancient universities, music was considered a branch of mathematics. Someone once said, "Mathematics is the music of the mind; music is the mathematics of the soul." I know I said it, but I believe I read it somewhere else first. Bronowski traces the flow of culture to the Islamic domination where he says the mathematician and the artist were one (and says he means that literally). The Islamic period art was very mathematical and made extensive use of symmetry in its patterns. Another concept that relates mathematicians to art is perspective (Albrecht Dürer, 15th century). This was the foundation for the branch of mathematics known as projective geometry. Related to this is the development of graph paper which was invented by the artists, not the mathematicians. Bronowski then traces the development through Spain, into Europe and the development of calculus as a response to the needs of physics and astronomy. We see here the changing role of mathematics — commerce, astronomy (& astrology), engineering, music, art, physics, and more recently economics (Morgenstern and von Neumann), and biology. Part of the maturity of an intellectual pursuit is the mathematization of the subject.

We get some perspective on impatience with resistance to change to the metric system when we realize it took Europe 400 years to accept the Hindu-Arabic numeration system over the Roman numeration system despite its obvious superiority in many ways.

The other mathematically oriented film, "The Majestic Clockwork", discusses Newton and Einstein and their contributions as well as their relationship to their peers and the public. One interesting story is a discussion between Edmond Halley and Newton when Halley asked Newton what would be the path of a body if the force acting on it varied inversely as the square of the distance. Newton responded immediately that it would be an ellipse. Halley asked how he knew, and Newton replied that he had calculated it. Halley asked to see the calculation. It took Newton three years to develop the proof and the document was hundreds of pages long. Another view that appeared in Scientific American suggests that Newton got the idea that it was an ellipse from Robert Hooke, but that Hooke was not a powerful enough mathematician to develop it deductively.

What, then, is the appropriate mathematics for the liberal arts college student. In my opinion, it is the study of mathematics as an integral part of our culture, not only the contributions of mathematics to our culture, but how the culture has influenced the development of mathematics. I have been attempting to convey this in my Survey of Mathematics course. Sometimes it bombs, and sometimes, as last semester, the students do very well and are very generous in showing their appreciation for the class.

I decided that in this course, I would try to instill in the students an appreciation of mathematics as an integral part of our culture, and at the same time develop, in a somewhat deductive manner, a mathematically significant relationship. I chose the Euler relation, $e^{i\theta} = \cos\theta + i\sin\theta$ because of its great unifying nature. It relates the exponential function, imaginary numbers, and trigonometry; seemingly diverse concepts. Of course, other topics come up along the way; some are needed for the development and some are interesting, I hope, digressions. Here is a brief outline of my lectures.

I. Number Systems - development of the "number tree" from the positive integers to quaternions including motivation for each extension and geometric considerations.

II. Mathematical Systems
   1. Groups
   2. Fields
   3. Ordered Fields
   4. Complete Ordered Fields

III. Exponents, the Exponential Function, and the Logarithmic Function - including application in growth and decay.

IV. Sequences and Series - informal discussion including convergence, divergence and intuitive concept of limits.

V. Probability, Binomial Distribution & Binomial Theorem
   1. Empirical and theoretical probabilities

2. Counting procedures (elementary combinatorics)
3. Proof of Binomial Theorem by use of combinations
4. Development of

$$e^x = \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$$

by use of Binomial Theorem

VI. Elementary graphing concepts (Cartesian coordinate system).

VII. Wrapping Function and Trigonometry
1. Basic concepts
2. Sum and difference formulas
3. Trigonometric form of complex numbers
4. DeMoivre's theorem and other theorems

VIII. Euler Relation
1. Proof of Euler Relation
2. Development of Maclaurin Series representation of sine and cosine.

Students in my survey class often complain that they don't "understand" some of the mathematics that we discuss. I am also asked by my colleagues if students in this type of class understand all the mathematics. I like to tell them a story about John von Neumann that I found in a book by Gary Zukow, The Dancing Wu Li Masters (this is a book about the new physics). When von Neumann was working at Los Alamos on the hydrogen bomb project, a young physicist asked von Neumann for help on a difficult problem. von Neumann responded, "simple, this can be solved using the method of characteristics." After an explanation, the physicist indicated that he did not understand the method of characteristics. "Young man", von Neumann said, "in mathematics you don't understand things, you just get used to them." This tells me that it is possible for people to get an understanding of the place of mathematics in our culture and an appreciation for mathematics without necessarily understanding all the mathematics. If it's good enough for von Neumann, it is good enough for me.

Another aspect of this course is independent study on the part of the students. Throughout the semester, students read selections chosen from a bibliography (the current state of this evolving bibliography is included in this paper), make periodic oral reports, and three written reports. The material they are reading may or may not have anything to do with my lectures, but is geared toward culturological aspects of mathematics, and class discussions of the oral reports are steered toward this.

I offer this approach as an alternative to the ever increasing emphasis in mathematics education on remediation, basic skills, and mundane applications of mathematics.

I would appreciate any comments you have and corrections and additions to the bibliography.

MATHEMATICS AND OUR CULTURE BIBLIOGRAPHY

Abbot, Edwin A., Flatland, 1987, Penguin (3.95) ISBN: 0-14-0076158

Aleksandroy, A.D. et.al. Mathematics: The Contents, Methods and Meanings (3 Vols.), 1969, MIT Pr. ($30.00 set) ISBN: 0-262-51014-6

Altwerger, Samuel I., Modern Mathematics, MacMillan, 1960

Bakst, Aaron, Mathematics: It's Magic and Mastery, Van Nostrand (1952)

Barker, Stephen Francis, Philosophy of Mathematics, Prentice-Hall, 1964

Bates, Grace E. and Kiokenmeister, Fred L., The Real Number System, Allyn & Bacon, 1960

Baum, John D., Structure of the Real Number System, Prentice-Hall, 1967

Bell, Eric T. The Development of Mathematics , McGraw-Hill, 1945

Bell, Eric T., The Last Problem, Simon & Schuster, 1961

Bell, Eric T., Men of Mathematics, 1937, Sts (10.75) ISBN: 0-671-46401-9, Fireside

Bergamini, David, Mathematics, Time, 1963

Berkeley, Edmund C., A Guide to Mathematics for the Intelligent Non-mathematician, Simon & Schuster, 1966

Berlinghoff, A Very Brief History of Mathematics

Berlinghoff, William P., Mathematics: The Art of Reason, Heath, 1968

Boehm, George, ed; The Mathematical Sciences; MIT Press, 1969

Boehm, George; The New World of Mathematics; Dial Press, 1958

Boyer, Carl B., A History of Mathematics, Wiley, 1968

Britton, Jack R., University Mathematics, W. H. Freeman, 1965

Bush, George C. and Obreanu, Phillip E., Basic Concepts of Mathematics , Holt, Rinehart & Winston, 1965

Bush, Grace A., and Young, John E., Foundations of Mathematics, McGraw, 1968

Cadwell, James H., Topics in Mathematical

Recreations, Cambridge, 1966
Asimov, Isaac, Realm of Algebra, Fawcett, 1967
Coleman, John A., Relativity for the Layman, Macmillan, 1959
Cooley, Hollis, Introduction to Mathematics, Houghton-Mifflin, 1949
Copeland, Richard W., How Children Learn Mathematics, Macmillan, 1979
Altshiller-Court, Nathan, Mathematics in Fun and in Ernest, Dial Press, 1958
Courant, Richard, and Robbins, Herbert, What Is Mathematics, Oxford University Press, 1941
Craig, Robert T., Modern Principles of Mathematics, Prentice-Hall, 1969
Crossley, John N. What Is Mathematical, Oxford University Press, 1972
Cundy, H. Martyn, and Rollett, A.P., Mathematical Models, Clarendon Press, 1961
Cutler, Anne, and McShane, Rudolph, The Trachtenberg Speed System of Basic Mathematics, Doubleday, 1960
Dantzig, Tobias, Number, the Language of Science, Macmillan, 1939
Dantzig, Tobias, The Bequest of the Greeks, Scribner, 1955
Davis, Philip J., 3.1416 and All That, Birkhauser, 1985
Davis, Philip J. The Love of Large Numbers, Random House, 1961
Davis, Philip J., Descartes's Dream, Harcourt Brace-Jovanovich, 1986
Davis, Philip J. & Hersh, Reuben, The Mathematical Experience, Birkhauser, 1981
Douglis, Avron, Ideas in Mathematics, Saunders, 1970
Dowdy, S.M., Mathematics: Arts and Science, Wiley, 1971
Duncan, Dewey C., Arithmetic in a Liberal Education, McGraw-Hill, 1969
Durbin, John R., Mathematics: Its Spirit and Evolution, Allyn & Bacon, 1973
Evans, Trevor, Fundamentals of Mathematics, Prentice-Hall, 1959
Eves, Howard W., Introduction to the History of Mathematics, Holt, Rinehart and Winston, 1969
Eves, Howard W., An Introduction to the Foundations and Fundamental Concepts of Mathematics, Holt, Rinehart & Winston, 1965
Feferman, Solomon, The Number Systems, Addison-Wesley, 1964
Feldzamen, A.N., Numbers and Such, Prentice-Hall, 1968
Felix, Lucienne, The Modern Aspect of Mathematics, Basic Books, 1960
Friedricks, Kurt O., From Pythagorus to Einstein, Random House, 1965
Fuchs, Walter Robert, Mathematics For the Modern Mind, Macmillan, 1967
Gamow, George, One, Two Three, ...Infinity, Viking Press, 1961
Gardner, K. L., Discovering Modern Algebra, Oxford University Press, 1966
Gemignani, Michael C., Basic Concepts of Mathematics and Logic, Addison-Wesley, 1968
Grossman, Israel, and Magnus, Wilhelm, Groups and their Graphs, Mathematical Assoc. of America, 1964
Groza, Vivian Shaw, A Survey of Mathematics, Elementary Concepts and Their Historical Development, Holt, Rinehart & Winston, 1968
Haag, Vincent H., and Western, Donald W., Introduction to College Mathematics, Holt, Rinehart, & Winston, 1968
Hall, Todd, Carl Friedrick Gauss, MIT Press, 1970
Halt, M. J. and McIntosh, Art, The Scope of Mathematics, Oxford University Press, 1966
Hamilton, and Landin, The Structure of Arithmetic
Hardy, Godfrey, H., A Mathematician's Apology, Rev. Ed., The University Press, (Cambridge), 1969
Hogben, Lancelot T., Mathematics for the Million, W. W. Norton & Co., 1951
Hogben, Lancelot T., Mathematics in the Making, Doubleday, 1960
Huff, Darrell, How to Lie With Statistics, Norton, 1954
Jacobs, Harold R., Mathematics, A Human Endeavor, W. H. Freeman, 1970
Jacobs, Judith, ed; Perspectives On Women and Mathematics, ERIC, 1978
Johnson, Donovan A. and Pahty, Robert, The New Mathematics in Our School, Macmillan, 1966
Jones, Burton, W., Elementary Concepts of Mathematics, Macmillan, 1963
Kasner, Edward, Mathematics and the Imagination, Simon & Schuster, 1940
Kemeny, John B., Random Essays on Mathematics, Education and Computers, Prentice Hall, 1964
Kemeny, John B., The New Direction in Mathematics
Kempf, The New Math Made Simple, Doubleday, 1966

Kennedy, Hubert, C., Selected Works of Guiseppe Peano, University of Toronto Press, 1973
Kline, Morris, Mathematical Thought From Ancient to Modern Times, Oxford Univ. Press, 1972
Kline, Morris, Mathematics and the Physical World, Doubleday, 1963
Kline, Morris, Mathematics and the Search for Knowledge, Oxford Univ. Press, 1985
Kline, Morris, Mathematics in the Modern World, Readings From Scientific American, W. H. Freeman, 1968
Kline, Morris, Mathematics in Western Culture, Oxford University Press, 1974
Kline, Morris, Mathematics: The Loss of Certainty, Oxford Univ. Press, 1980
Kogelman, Stanley & Warren, Joseph, Mind Over Matter, Dial Press, 1978
Kramer, Edna E., The Nature and Growth of Mathematics, Hawthorne Books, 1970
Kramer, Edna E., The Main Stream of Mathematics, Oxford Univ. Press, 1967
Lieber, Lillian, The Education of T.C. Mits, Norton, 1972
Lieber, Lillian, Human Values and Science, Art, and Mathematics, Norton, 1961
Lieber, Lillian, Infinity, Holt, Rinehart & Winston, 1953
Lieber, Mits, Wits and Logic, Norton, 1947
Lieber, Relativity
Linn, Charles F., The Golden Mean: Mathematics and the Fine Arts, Doubleday, 1974
Mahoney, Michael Sean, The Mathematical Career of Pierre de Fermat, Princeton Univ. Press, 1973
Mancill, Julian Dossy, Contemporary Mathematics, Allyn & Bacon, 1966
May Lola J. and Moss, Ruth, New Math for Adults Only, Harcourt, Brace & World, 1966
Meschkowski, Herbert, Evolution of Mathematical Thought, Holden-Day, 1965
Meserve, Bruce Elwyn, Mathematics For Secondary School Teachers, Prentice-Hall, 1972
Meserve, Bruce Elwyn, and Sobel, Max A., Elements of Mathematics, Prentice-Hall, 1968
Meserve, Bruce and Sobel, Max A., Introduction to Mathematics, Prentice-Hall, 1978
Midoniek, Henrietta 0., The Treasury of Mathematics, Philosophical Library, 1965
Moritz, Robert Edouard, On Mathematics and Mathematicians, Dover Publications, 1968, 1942
Moses, Richardson, and (M. Richardson), Fundamentals of Mathematics, MacMillan Co., 1941
Mueller, Arithmetic, Its Structure and Content, Prentice Hall, 1956, 1964
Muir, Jane; Of Men and Numbers: The Story of the Great Mathematicians, Dodd, 1961
Munroe, Marshall Evans, The Language of Mathematicians, Michigan Press, 1963
National Council of Teachers of Mathematics, Enrichment Mathematics For the Grades, 27 Serials, Books Demand UMI
National Council of Teachers of Mathematics, Enrichment Mathematics For the High School, 28th Serials, Books Demand, UMI
National Council of Teachers of Mathematics, The Growth of Mathematical Ideas For Grades K-12, The Council, 1959, 24th yearbook
National Council of Teachers of Mathematics, Topics in Mathematics For Elementary Teachers, 29th yearbook, 1964
Newman, James Roy, The World of Mathematics, Simon and Schuster
Newson, Carroll Vincent, Mathematical Discourses, Prentice-Hall, 1964
Niven, Ivan Morton, Mathematics of Choice, Random House, 1965
Niven, Ivan, Numbers: Rational and Irrational, Random House, 1961
Ohmer, Merlin Maurice, Mathematics for a Liberal Education, Addison-Wesley, 1971
Ore, Oystein, Graphs and their Uses, Math Assoc. of America, 1963
Ore, Oystein, Number Theory and Its History, McGraw-Hill Book Co., 1948
Osen, Lynn M., Women in Mathematics, MIT Press, 1974
Podriza, Mathematics: An Introduction
Polya, George, How To Solve It
Rademacher, Hans, The Enjoyment of Mathematics, Princeton University Press, 1957
Reichmann, William John, Use and Abuse of Statistics, Oxford Univ. Press, 1961, 1962, 1971, 1975
Reid, Constance, Hilbert, Springer-Verlag, 1970
Reiel, Constance, Zero to Infinity, Cromwell, 1960
Richardson, Moses, Fundamentals of Mathematics, The Macmillan Company, 1941
Runyon, Richard, P. How Numbers Lie, Greene Press, 1981
Sanford, Vera, A Short History of Mathematics, Houghton-Mifflin Co., 1930, 1958
Sawyer, Walter W., Mathematician's Delight, Penguin Books, 1959
Sawyer, Walter Warwick, What Is Calculus About?, Random House, 1961
Schaff, William Leonard, Basic Concepts of

Elementary Mathematics. Weber-Schmidt, 1971, 1982

SMSG, Studies in Mathematics. Vol. IX

Singh, Jag., Great Ideas of Modern Mathematics: Their Nature and Use. Hutchinson, London, 1962

Smith, David Eugene, Number Stories of Long Ago. Nat. Council of Teachers of Math, 1965

Smith, David Eugene, History of Mathematics. Dover Publications, 1958

Spector, Lawrence, Liberal Arts Mathematics. Addison-Wesley Pub. Co., 1971

Spitznagel, Edward L., Selected Topics in Mathematics. Holt, Rinehart & Winston, 1971

Stein, Sherman K., Mathematics: The Man-Made Universe. W. H. Freeman, 1963

Steinhaus, Hugo, Mathematical Snapshots. Oxford Univ. Press, 1960

Struik, Dirk Jan, A Concise History of Mathematics. Dover Publications, 1948

Struik, Dirk Jan, A Source Book in Mathematics. Harvard Univ. Press, 1969

Suppes, Patrick and Hill, Shirley, First Course in Mathematical Logic. Blaisdell Publishing Company, 1964

Suppes, Patrick, Introduction to Logic. Van Nostrand Company, 1957

Swain, Robert Lomond and Nichols, Eugene D., Understanding Arithmetic. Holt, Rinehart & Winston, 1965

Tanur, Judith M., Statistics: A Guide to the Unknown. Wadsworth & Brooks/Cole, Advanced Books & Software, 1985

Taviss, Irene, The Computer Impact. Prentice-Hall, 1970

Thomas, Computers

Tietze, Heinrich, Famous Problems of Mathematics. Graylock Press, 1965

Ulam, Stranislaw M., Adventures of a Mathematician. Scribner, 1976

Vergara, William Charles, Mathematics In Everyday Things. Harper, 1959

Vilenken, Naum Iakovlevich, Stories About Sets. Academic Press

Newsom, Carol Vincent, Introduction to College Mathematics. Prentice-Hall, 1954

Weinberg, George H., Statistics, An Intuitive Approach. Cole Publishing Company, 1969

White, Stephen, Students, Scholars and Parents. Doubleday, 1966

Whitehead, Alfred North, Introduction to Mathematics. University Press, 1967

Wiener, Norbert, Ex Prodigy. M.I.T. Press (1964 c1953)

Wilder, Raymond, Evolution of Mathematical Concepts; J. Wiley & Sons, 1968

Wilder, Raymond, Introduction to the Foundation of Mathematics. Wiley, 1965

Wilder, Raymond Louis, Mathematics As a Cultural System. J. Wiley, 1965

Willerding, Margaret, Elementary Mathematics. Wiley, 1970

Wren, Frank Lynwood, Basic Mathematical Concepts. McGraw-Hill, 1965