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Using Topology to Explore Mathematics Education Reform

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Abstract

Mathematics education is a constant topic of conversation in the United States. Many attempts have been made over the last fifty years to reform teaching methods and improve student results. Recent ideas have emphasized problem-solving to make math feel more applicable and enjoyable. Additionally, the widespread problem of "math anxiety" has been tackled by creating lessons that are more discussion-based than drill-based to shift focus from speed and accuracy. In this thesis, we explored past reform goals and some added goals concerning students' perceptions of mathematics. We developed and tested a pilot workshop in topology, a creative and intuitive field of mathematics, for use in 4th–6th grade classrooms. Preliminary results suggest some success in altering student views on mathematics.

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Chapter 1

Introduction

This thesis explores elementary mathematics education, and the many past attempts at reforming mathematics curricula and teaching methods. Several proposed reform ideas were reviewed and their main goals were analyzed. Through this analysis, we identified some valuable goals, as well as a few that could be altered slightly for improvement. A few additional goals were then added. This thesis aims to explore how these goals can be combined to present mathematics most effectively to young students in order for them to develop a strong mathematical base, get excited about mathematics, and better understand the field as a whole.

To gain more insight into the potential of these reform ideas, we incorporated them into a set of lessons that adapt the study of topology for use in a 4th–6th grade classroom. Thus, we created an elementary topology workshop and tested it in a local 5th grade classroom. To the average mathematician, teaching topology to elementary students might seem like a crazy, unrealistic feat. Topology is considered an upper-level subject in mathematics and many students don't encounter it until graduate school. The intent, however, was not to establish a rigorous topology curriculum for young students. Instead, topology was used to introduce some rich mathematical ideas that are accessible to a 4th–6th grade mind. An early exposure to such valuable concepts can help to build a foundation for the students in their future endeavors in mathematics. Bringing topology lessons into elementary classrooms not only provides an enriching experience for the students, but is a chance to reflect on the efficacy of several reformative ideas for mathematics education as a whole.

The introduction of a topic like topology into early education holds strong potential to alter the way in which young students view mathematics. It is the perfect topic from which to get a taste of upper-level material. The visual nature of topology makes it intuitive, so it lends itself to a young audience. While topology is typically taught to college and graduate mathematics students in a rigorous manner, it's possible to gain a strong understanding of the fundamental concepts through purely visual demonstrations. Additionally, topology relies on some core mathematical ideas that are very valuable for students to understand at an elementary level. By bringing topology into the classroom, students are familiarized with these foundational concepts in a non-obvious way to keep the material from seeming dry and compulsory. The students' positive reactions to the subject and their ability to grasp information presented through a topological lens during the workshop reinforced my belief that topology could be used to create powerful elementary mathematics lessons.

Chapter 2

Review of Mathematics Education Reform

The conversation about improving math education, especially in the United States, is ongoing and highly controversial. Schools in the United States consistently perform at a mediocre level when compared with schools across the rest of the world. In 2007, for example, "15-year-olds in the United States ranked 25th among their peers in 30 developed nations in math literacy and problem solving" (Lewin, 2008). And as performance fails to improve, students in the United States have shown a "steadily declining interest in math ... as [they] progress through the grades" (Drew, 2011). The failures of mathematics education in the United States pose a great threat to the growth of the country. In fact, "policy analysts and pundits have expressed anxiety, even fear, about whether the United States will continue to be a world leader economically" (Drew, 2011). With this and other concerns in mind, many proposals have arisen for the reform of mathematics education in the United States.

2.1 Past Ideas for Reform

By far one of the most common goals in mathematics education reform is a desire to focus on problem-solving to highlight the applications of mathematics and make the subject feel more useful in everyday life. It is believed that problem-solving contributes to the practical use of mathematics and teaches students to be adaptable (Resnick, 1987). Problem-solving is also advocated as a way to develop mathematical thinking as a tool for daily life, since it can be applied to many unfamiliar situations

(Cockcroft, 1982). A particular example of the goal to focus on problem-solving is discussed by Conrad Wolfram, the Strategic and International Director of Wolfram Research. Wolfram is a major proponent of "Computer-Based Math," a movement to "redefine math education away from historical hand-calculating techniques and toward real-life problem-solving situations that drive high-concept math understanding and experience" (Wolfram, 2014). Wolfram's idea for reform takes advantage of the power of computation and suggests using computers in mathematics lessons and exams to allow students to solve more realistic problems, like choosing "the best life insurance policy" (Wolfram, 2010).

While problem-solving should be the focus of mathematics education, it is not clear that the main benefit of this reform is to prove to students that math is applicable in their everyday lives. The process of solving large, application-based problems lets the students figure out on their own how certain math skills might be of use in finding a solution. By giving them a problem description, realistic or not, students are given the opportunity to brainstorm the ways in which their mathematical skill-set can contribute to a method for solution. Discovery of application seems to be a much stronger way to emphasize the importance of mathematics than simply incorporating an application into a lesson clearly framed around a certain mathematical skill.

In addition to applications of mathematics, several reform leaders have mentioned that we must also show students the creative nature of mathematics. It is a common perception, not just among young students, that math is a rigid field in which there is a strict method with which to find the one correct answer. Seymour Papert discussed how this phenomenon is rooted in the fact that the "difference between between the work of a child in an elementary mathematics class and that of a mathematician is not in the subject matter ... but in the fact that the mathematician is creatively engaged in the pursuit of a personally meaningful project" (Papert, 1972). Students see mathematics as rigid because they are not often given projects that free them of the rigidity of fixed lessons. Math, however, is not a fixed set of facts but a wide open subject that is, and will always be, improving and expanding. Even mathematicians do not understand everything about math, so everyone has the potential to contribute to our collective knowledge of the field. Papert digs into how we might be able to "put children in a better position to *do* mathematics rather than merely to learn *about it*" (Papert, 1972).

Another common goal among proposals for mathematics education reform is to make math more "fun" for students, especially through the use of games. Because of "continuing advances in technology" and "the widespread popularity of entertaining computer games," educational video games have become a hot topic of conversation and "an increasing number of educators are using instructional games in formal school settings" (Kebritchi, Hirumi, and Bai, 2010). Keith Devlin, Stanford mathematician and Executive Director of the University's Human-Sciences and Technologies Advanced Research Institute, is a major advocate of the use of role-playing video games to teach math in a fun way and keep students engaged (Devlin, 2010). While it is critical that we try to get kids more excited during math lessons, it may not be necessary to try to hide the math behind a video game. Instead, we need to work toward changing the perception that math is not fun or cool. If a teacher is truly excited about the material and thinks it is cool, it is likely that his or her enthusiasm will rub off on students. Also, introducing math topics that are not as rigid or dry - such as topology - to reinforce ideas in basic mathematics can help engage and motivate students. Furthermore, creating a video game that "meaningfully enhances learning is a difficult task" (Elliott, Adams, and Bruckman, 2002). In a study of the efficacy of AquaMOOSE, a "3D graphical environment designed to allow students to explore 3D math" (Elliott and Bruckman, 2002), it was found that the game was very unsuccessful. AquaMOOSE had "little impact of what the students actually learned in the classroom" and it was challenging to meet students' high expectations for video game software (Elliott, Adams, and Bruckman, 2002).

Many ideas for improvements in mathematics education also revolve around the goal of decreasing math anxiety in students. Math anxiety, or "feelings of tension or worry that interfere with mathematical performance in daily life and school settings," is a highly researched phenomenon that has been seen in students as early as first grade (Vukovic, Kieffer, Bailey, and Harari, 2013). Students with math anxiety tend to avoid math, which "ultimately undercuts their math competence" (Ashcraft, 2002). Math anxiety is a major problem not only because it affects how students perform, but also because it affects how much math they retain during transitions between grades (Vukovic, Kieffer, Bailey, and Harari, 2013). Math anxiety often stems from our treatment of math as a performance subject (Boaler, 2012). There have been many approaches suggested to tackle the problem of math anxiety. One promising approach involves a focus on "guesstimating" in math classrooms (Park and Brannon, 2013). The process of "guesstimating" the answer to a math problem takes the emphasis away from using a formula to find the exact, correct answer and moves the focus toward the thought process involved in intuiting what the answer might be. This idea helps students to gain a full understanding of the problem, rather than popping out an answer without thinking deeply about its implications.

A final interesting suggestion for reform is an increased use of manipulatives in mathematics lessons. Manipulatives, or hands-on instructional tools, can have a moderate to large impact on student retention of material (Carbonneau, Marley, and Selig, 2013). Many students struggle with purely symbolic mathematics lessons, and manipulatives help them to visualize the subject matter. Furthermore, "many problems [in student learning] could be avoided altogether by allowing students to work with concrete models prior to dealing with them on a more abstract level" (Moch, 2008). This is especially true of younger students, as they are "assumed to have a greater dependency on physically interacting with their environment to construct meaning" (Carbonneau, Marley, and Selig, 2013).

2.2 Additional Ideas for Reform

Often, reform of education is focused primarily on producing results and proficiency. Even the goal of making math lessons more fun is motivated by getting students to pay attention and learn the material. Reform goals should also focus on altering the overall perception of mathematics among young students. We need to see a greater general appreciation of mathematics and a better understanding of the subject and everything it encompasses.

Presenting math as a creative field can be a means of altering student understanding of what it means to be a mathematician. As discussed by Seymour Papert, we can close the gap between the rigid way in which mathematics is taught and the creative way in which mathematicians do math (Papert, 1972). It is also critical, however, to be explicit when talking to students about what mathematicians do, as they are likely not aware that mathematicians are creators of mathematics, rather than simply people who understand a lot of math. The introduction of more creatively engaging lessons might help to catch students' attention in the classroom, but is not enough to present mathematics as a viable career option.

Another change we need to see in young students' perceptions of mathematics is a recognition that failure is okay. The majority of students are not comfortable with getting things wrong, but mathematicians *often* fail many times before they are right. The process of understanding why a certain method of solving a problem is not working is very valuable in figuring out a more effective method and also adds to a general base of understanding in the field of mathematics. Thus, students should be encouraged to make logical guesses at how to solve problems, even if these guesses are completely wrong; analyzing mistakes is a real-world skill to which all young students should be exposed. It also helps students understand the amount of creative thinking that goes into problem-solving. Finding the best solution to a problem is not so much about using the right formula; it is about combining your mathematical skills in innovative ways and comparing different methods to decide upon the best one.

Finally, it is important for students to be given a sense of what types of math lie beyond elementary and even secondary education. Only a very small portion of mathematics is seen before the college level, and this is reduced to computation that is simply a pre-requisite for any of the more complex and fascinating mathematical topics. It is true that many advanced topics are not accessible to younger students because of the necessary mathematical skills to solve interesting problems. However, the conceptual ideas involved in these topics can often be presented as a motivation for learning lower-level math; students are not likely to be satisfied doing tedious calculations in grade school unless they get a glimpse of how it might later pay off. Exposure to the concepts involved in many areas of math can also help to broaden students' definitions of mathematics.

Chapter 3

Project Goals

To address the ideas for reform presented in the previous chapter, we created a short pilot workshop for a 4th–6th grade audience as a medium through which to explore the various mathematics education reform goals. The workshop was tested over two days in the 5th grade classroom of Erik Dahl at Chaparral Elementary School in Claremont, California during the Spring 2014 semester.

The workshop was focused on topology for several reasons. Firstly, it is a subject capable of grasping young people, as it differs greatly from other topics introduced at the elementary level. More importantly, though, introducing topology into 4th–6th grade classrooms had the potential to test the education reform goals discussed in the previous chapter. Since topology is such an active field in mathematics, it provides a chance to show young students that math is not a dead subject with rules set in stone. Rather, it is a creative subject that is constantly changing and to which they have an opportunity to contribute. Discussing the unknowns in topology presents math as a subject in which we do not have all the answers, a viewpoint which is not commonly seen in early mathematics education.

Furthermore, topology is a subject that is very rarely seen before college and is often not even seen until graduate school. It is, however, a fascinating subject that feels much less dry than many of the topics seen in elementary and secondary education. By giving students a glimpse of such an intuitive topic in mathematics, they are able to get a sense of what they might see if they continue in mathematics past high school. Knowing that topology is out there will additionally help to expand their understanding of the field of mathematics and what it entails.

One may be concerned about how much of the topology the students would be able to conceptualize, so a goal of the project was simply to gauge what kinds of upper-level ideas can be picked up by a 4th–6th grade audience. Even if the students were unable to retain the lessons, they would still get a taste of what kinds of math they might encounter at a college level. By giving them a sense of what math looks like after primary education, they could possibly gain some enthusiasm for the subject and broaden their definitions of mathematics and mathematicians.

Finally, it is a challenging exercise to craft lessons containing upper-level material—that is rarely seen outside a college setting—into language accessible to 4th–6th grade students. A goal of the project was to successfully present some topological ideas to the students at an appropriate level for them to understand.

After piloting the lessons, they were edited based on their results in the classroom in hopes of creating polished lesson plans for future use by others. This thesis includes those lesson plans, so they can be distributed to any interested 4th–6th grade teachers. To gather results for revision, both pre-workshop and post-workshop evaluations were administered to the participating students. These evaluations provide a variety of data, including the students' perceptions of mathematics and their grasp of the fundamental concepts presented in the workshop. Each question was tailored to one of the education reform goals addressed in the workshop, which provided some hard data along with my personal observations to determine the efficacy of the workshop. The questions from the evaluations and the students' answers to these questions are presented in Appendices D and E.

The information in the following two sections details the lessons covered in the two days of the workshop. The lessons are presented in the form of instructions to a teacher. The goal was to make it possible for the lessons to be easily recreated by anyone with access to my thesis. In doing this, the workshop will remain an open-source project that can be edited and added to by others. Additional materials, such as activity sheets and worksheets are included in the appendix.

3.1 Day 1: Let's Learn Topology!

This lesson will introduce students to the field of topology and its differences from classical geometry. The students will gain an understanding of the terms "rubber sheet geometry" and "continuous deformation" through hands-on exploration. By the end of the lesson, students will feel comfortable classifying topological objects into groups based on equivalence and will understand the definition of genus. Also, the students will recognize equivalence as something that can be defined in more than one way.

Instructions:

1. Before introducing topology, lead the students in a discussion about classical geometry. Make a list on the board of words they know from geometry and talk about what properties of shapes they think are important in the subject. Specifically, ask them to identify what makes two objects equal in classical geometry. Hopefully, the students will mention words like angle, length, volume, etc. If not, plant them into the discussion and ask the students if they know what these words mean.

2. Once the students are refreshed on what they know about geometry, ask them whether a square and a circle are equal in classical geometry. Then, when they say no, tell them that there's another type of geometry—called topology—in which they *are* equal, but wait to explain why. Emphasize that topology is a much newer field of math than classical geometry and that it is still being created by current mathematicians.

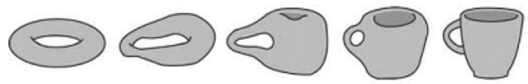
3. Now, explain that topology is also known as "rubber sheet geometry," because it is a kind of math in which bending and stretching an object—as if it's made of rubber— does not change it. These kind of alterations are called "continuous deformations."

4. Show the students an example on the board by drawing two circles of different sizes. Explain that the smaller circle can be stretched to create the larger one, so the two are considered equal in topology.

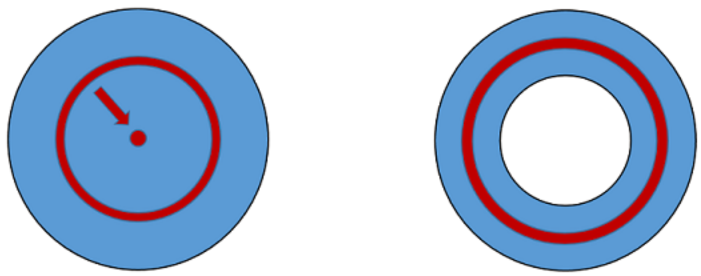
5. Lead the students through the **Play-doh Geometry Activity** (Appendix A).

6. After the activity, the students will understand that the number of holes in an object is an important quality in topology. Explain that this number of holes is called the "genus" of an object.

7. Reiterate that objects with the same genus can morph into one another, like with the play-doh transformations from the activity. Use the following visual of a donut morphing into a coffee cup as an example:



8. Reiterate that an item with one hole cannot be morphed into an object with zero holes. This is because, on a genus 0 surface, we can place a circle anywhere and shrink it down to a point. On a genus 1 surface, however, there is somewhere that we can place a circle so that it cannot shrink down to a point. Ask the students to identify where to place the circle (around the surface's hole) and make sure they understand why we can't contract the circle to a point.



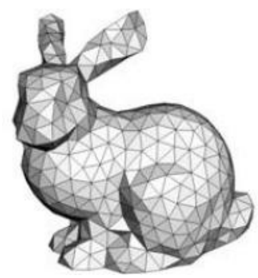
3.2 Day 2: Using Geometry to Understand Topology

In this lesson, students will learn that older, more developed fields in mathematics are used as a base for understanding newer topics. Specifically, students will explore how classical geometry lies at the foundation of topology. The students will be introduced to triangulations and will use them to calculate Euler characteristic. Finally, students will look at an application of triangulations and subdivisions. This application will show the students just how diverse the field of mathematics can be and will expand their definition of mathematics.

Instructions:

1. Begin the lesson by explaining that, since topology is such a new topic in mathematics, we often use older topics like classical geometry to better understand it. For example, we can learn interesting information about topological surfaces by breaking them down into simpler geometric shapes. In topology, it is most common to break down a

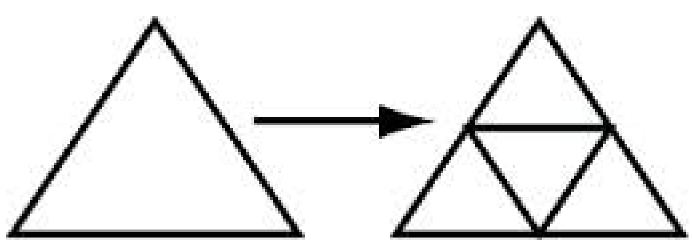
surface into triangles, which is called "triangulating" the surface. Be sure to show some visual examples of triangulated objects, like the image below, so that the students understand this concept.



2. Explain that once we have triangulated an object, we can count up geometric qualities like vertices, edges, and faces to make discoveries about the object. Make sure to review what each of these terms means on the board.

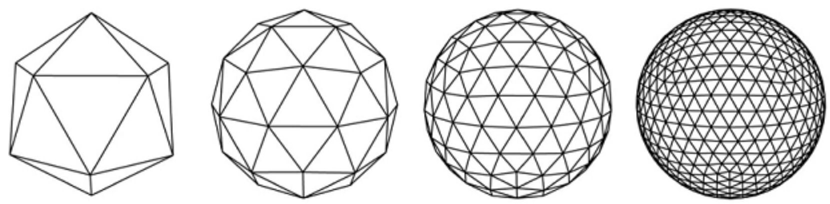
3. Tell students that they will now explore some triangulated surfaces in an activity by counting up their vertices, edges, and faces and using those quantities to calculate a special number. Then, lead the students through the **Euler Characteristic Activity** (Appendix C).

4. Now, the students should understand that Euler Characteristic is invariant under "play-doh" transformations, or continuous deformations. With this fact in mind, ask the students whether the Euler Characteristic of an triangulated object will change if we break each triangle into smaller triangles, as in the image below. Explain that this process is called "subdividing" the triangles.



5. Students should recognize that by breaking the triangles down into smaller ones, we are only doing "play-doh" transformations to the triangulated object, so the Euler Characteristic does not change. If

they have trouble, guide them using the below image of a triangulated object being subdivided several times.



6. Tell the students that you can prove that the Euler Characteristic by only looking at one of the triangles and it's subdivision. Draw a triangle and its first subdivision on the board, and ask the students if they can guess how to prove that these both have the same Euler characteristic. Most likely, the students will try to calculate the Euler characteristic of each to see that they are the same. If not, guide them to do this with you by counting up the vertices, edges, and faces for each. Then, by looking at the changes in vertices, edges, and faces from the triangle to its subdivision, show the students that the net gain in the subdivision is zero. Thus, the Euler characteristic will not change.

7. Discuss with the students that, since subdividing a single triangle does not change its Euler characteristic, then subdividing lots of triangles on a triangulated object will similarly not change the Euler characteristic of that object.

8. Ask the students what happens to a triangulated object as you subdivide it further and further. They should notice that the object becomes smoother. Emphasize the idea that if we subdivide a triangulated object far enough the triangles become too small for the eye to see. Even though the object is still made of triangles, it is a very close approximation to a perfectly smooth object, not made of triangles.

9. Tell the students that this idea of creating approximations of smooth surfaces out of small triangles is how computers make all of their favorite animated movies. Show them some example videos of how animators use technology to subdivide triangulated images far enough to reach smooth limit surfaces and create flawless images. Ask the student why the computers might like working with surfaces made up of geometric shapes like triangles rather than smooth surfaces. See if they can guess that computers can more quickly process straight lines, so triangles are simpler for the computers to interpret.

Chapter 4

Workshop Results

4.1 Data and Analysis

Before and after the pilot workshop in Erik Dahl's 5th grade classroom at Chaparral Elementary School, the students were asked to complete evaluations that were designed to test the efficacy of the workshop in meeting the set goals. The full lists of student answers can be seen in Appendices D and E.

1. **How much do you like math?**
   (Scale from 1 to 5, "Not at all" to "Very much")

   Pre-Survey Average Score: $3.49 \pm .21$
   Post-Survey Average Score: $3.61 \pm .22$

   The difference between the two averages is not statistically significant, so we cannot conclude anything about how the workshop affected how much the students like math.

2. **What do you like about math?**

   One interesting change in the students' answers from the pre-survey to the post-survey was that, in the pre-survey, 11% of students said that they enjoyed math because of the challenge it provides, while in the post-survey, 28% mentioned this as something they enjoy. This change is fairly large, and may be a result of the students learning that mathematicians work on problems that take years and years to solve.

It was mentioned in the workshop that there are fields of math, like topology, with unsolved and very challenging problems, and that this was a really cool aspect of mathematics because it means that any of them are capable of contributing to the field. The students may have picked up on my excitement about the potential rewards of pursuing a challenge. Additionally, in the post-surveys, 26% of the students mentioned topology as something they like about math, which was a really exciting thing to see. It is clear that the students were intrigued by the topic enough to mention it specifically.

3. **What do you dislike about math?**

   The percentage of students who wrote that something they disliked about math was that it was hard or that they were bad at it decreased from 37% to 23%, showing that the difficulty of mathematics was not as much of a turn-off after the workshop.

4. **Do you think you are good at math?**
   (Scale from 1 to 5, "Not at all" to "Very much")

   Pre-Survey Average Score: 3.97 $\pm$ .16
   Pre-Survey Average Score: 3.98 $\pm$ .15

   The difference between the two averages is not statistically significant, so we cannot conclude anything about how the workshop affected students' perceptions of their math abilities.

5. **Why do you think so?**

   The answers to this question were interesting because, before the workshop, 49% of students mentioned their grades in math as an indicator of why they chose their rating, while after the workshop, only 32% of responses mentioned grades. It appears that the students picked up on the message that getting the correct answer to a problem on the first try isn't the most important indicator of your understanding of that problem or of your ability to eventually find the answer. Furthermore, some of the post-survey responses showed an understanding that mathematics is sometimes a slow and difficult process, but that doesn't indicate inadequacy. One student said, "I pull through, although I sometimes struggle here and there." Another said, "my friends and teachers help me a lot and I'm improving," which also shows a recognition that getting help is not a sign of weakness.

6. **Do you think math can be creative?**

   Pre-Survey

   Yes: 46%
   I'm Not Sure: 43%
   No: 11%

   Post-Survey

   Yes: 74%
   I'm Not Sure: 23%
   No: 3%

   This dramatic change reflects the students' overall realization of the creativity of topology and mathematics, ideas which were strongly emphasized during the workshop.

7. **Describe math:**

   The answers to this prompt did not show much variety in answers between the pre- and post-surveys. After the workshop, however, there were definitely more mentions of topology and shapes in the students descriptions. It cannot be said, however, that the students' definitions of mathematics were significantly altered as a result of the workshop.

8. **What do mathematicians do?**

   Students answers to this question clearly reflected what they were shown in the workshop and answers generally became much more specific. Before the workshop, 17% of students had answers that simply stated that mathematicians are good at math. This percentage dropped to 6% in the post-surveys. Also, the notion that mathematicians answer math questions very quickly, which appeared in 6% of pre-survey responses, disappeared completely.

9. **What is topology?**

   The answers to this question were very impressive. In fact, 23% of students mentioned the term "continuous deformation" within their answers and many more described the concept of continuous deformation. Almost all of the answers showed a strong understanding of the ideas from topology presented during the workshop.

10. **Why is topology useful?**

    The students continued to show how much they learned in their answers to this question. 23% of students mentioned that topology can be used to make animations in movies or video games and 10% mentioned that we use topology to categorize objects in different ways. Students also used vocabulary picked up during the workshop in their answers, with mentions of approximations and genus. A few students also recognized that topology could help them with geometry.

11. **What else did you learn during the topology workshop?**

    Answers continued to include vocabulary from the workshop, including mentions of continuous deformation, genus, and Euler characteristic. Some students mentioned that they learned about geometric properties, like vertices, faces, and edges. These answers show that bringing topology into the classroom was effective in reviewing standard elementary math. Lastly, there were several more mentions of the fact that topology is used in animation.

Overall, the student responses on the surveys suggest that the workshop was successful. The students showed a lot of growth within a very short workshop, which helps to demonstrate the potential of the proposed ideas in altering the perception of mathematics in young students.

4.2 Personal Reflection

From my perspective, the workshop was very successful. The students seemed to be engaged throughout and definitely enjoyed themselves. Additionally, the students followed along with the material, so I think that I presented the information at an appropriate pace. Working in a 5th grade classroom was a really exciting experience, and I learned a lot about what catches the students' attention and how to control the room. It would have been nice, however, to have had a second time to test the workshop. Another run-through would give me a chance to utilize the skills I gained from the first one. Also, I now have a better sense of how long each section and activity takes, so I'd be able to better time the workshop so that no section feels rushed.

The students received the workshop very positively. I was shocked at their enthusiasm right from the start of the first lesson. When the students entered the room and saw "Let's Learn Topology!" on the screen at the front of the classroom, there was a buzz of chatter, despite the fact that none knew anything about topology. This warm welcome definitely gave me a lot of confidence, and I was able to feed off of the energy in the room to keep the workshop moving along. The students were extremely responsive. When I told them that they would be doing college-level math and that they would also get to use play-doh during the lesson, they seemed very excited. This initial excitement started things out on the right foot and held throughout the rest of the lesson. The students remained engaged, answering questions and repeating key terms without having to be asked twice. Overall, I was very pleased with the amount of student participation and the general reception of the workshop. I had a ton of fun throughout both of my two visits.

The first day, a general introduction to topology lesson based mainly around the pipe cleaner and play-doh activity, ran very smoothly. From the start of the lesson, the students picked up on the concepts very quickly. They seemed comfortable talking about and utilizing continuous transformations during the pipe cleaner and play-doh activity. The students especially enjoyed the minute I gave them to build whatever they wanted with the play-doh before we began the play-doh section of the activity worksheet. This free time increased their concentration for the rest of the activity, as they were able to get the excitement of playing with play-doh out of their systems. During the pipe cleaner and play-doh activity, the students were kept busy with the worksheet and got really into each of the questions. I particularly liked one student's insistence that a sphere of play-doh could indeed be continuously deformed into a donut if it was a jelly donut.

The enthusiasm from the first day immediately reappeared when I came back the following week for the second half of the workshop, which was focused on using geometry to understand topology. The students seemed very happy to have me back and greeted me with excitement. We began the lesson with a refresher from the last lesson, which appeared to be very helpful and definitely reinforced the idea of continuous deformation in the students' minds. Before introducing triangulations to the class, we reviewed what they knew about triangles. We defined vertices, edges, and faces, since they would need this vocabulary for the Euler characteristic activity. The students knew these terms, but it took them a minute to remember them, so I'm really glad that I set aside some time to refresh their memories. Once we began the Euler characteristic activity, the students were very comfortable working with the ideas and counting up the vertices, edges, and faces of their objects despite the difficulty of the activity. This task also held their attention well and they were not discouraged by the challenge. Also, placing the students in groups of four for the activity worked really nicely, as each student was able to have a particular job to focus on in calculating their object's Euler characteristic.

Though the workshop testing was overwhelmingly positive, there are several improvements that I would like to make to ensure a smoother run-through. Firstly, in the pipe cleaner and play-doh activity, there were a few very common errors. Several students claimed to be able to create an 'S' by continuously deforming a pipe cleaner square. Similarly, a few students thought that they could continuously deform a sphere of play-doh into a donut by rolling it into a snake and joining the two ends. To avoid both of these misconceptions, I need to be more explicit about the rules of continuous deformations. Specifically, I should have mentioned that continuous deformations must be reversible. Thus, any gluing that creates or removes a hole from an object is disallowed. You cannot create an 'S' from a pipe cleaner square, since you would need to glue the pipe cleaner to remove the hole and you cannot create a donut from a play-doh snake, since you would need to glue the ends to one another and create a hole. Furthermore, I would like to better enforce working in pairs for the pipe cleaner and play-doh activity. I think that this would foster discussion about the worksheet and eliminate some of the confusion about the rules of continuous deformation.

I would also like to change the Euler characteristic activity to make the triangulations of the objects simpler. The triangulations I used were fairly complex and caused the activity to run well over the time I had tentatively allotted for the activity. Also, I did not have enough of each color sticker for the students to be able to assign separated colors for labeling vertices, edges, and faces. I think this would have made counting slightly easier. The stickers I brought also could have been larger, which would have allowed the students to number them, rather than having to tally on a separate piece of paper each sticker they placed on the object. Finally, since the Euler activity went longer than expected, the conclusion of the lesson involving approximations and animation were very rushed. Given more time, I think I could have led the students to a stronger understanding of the ideas of triangulation, subdivision, and approximation. Some of the things I may have done with more time include letting the students try to subdivide a triangle on the board, proving that subdivision does not change Euler characteristic, discussing in more depth why computers use geometric approximations, and showing videos about Pixar's OpenSubdiv technology.

Chapter 5

Conclusion

This senior thesis project in mathematics enters the highly debated conversation about the possibilities for reform in math education. The project began with research into the conversation thus far, reading the proposed reform ideas of some of the leaders in the field of mathematics education. Research showed a strong overlap between many of the proposed ideas. In particular, many take on the problem of math anxiety by attempting to de-emphasize that mathematics is a performance subject, which is a very common perception. Past reform ideas also work to put more focus on problem-solving to highlight the applications of the field. Lastly, there was a common goal of making mathematics lessons more fun, but the common approach of trying to hide math within video games or similar mediums did not seem effective.

After reviewing past ideas for reform, we extended these with some important additional goals. One additional goal involved working to change general perceptions of mathematics to impact whether students viewed math as a fun subject. Another added goal was presenting mathematics as a creative subject, since it is more often viewed as rigid and dry. If students recognize how much room there is for creativity in problem-solving, and also how much math there is still to be discovered, they might be more excited about contributing their unique thoughts during class. It is not fun to feel like your ideas are bound to be wrong because they do not fit a strict set of criteria. It is also important for students to recognize that failure is okay, especially in mathematics. Trying an incorrect or inefficient method of solving a problem only helps us to understand how to improve and design a better method. The performance-based presentation of mathematics in elementary education does not reflect this idea, though it is one seen every day by mathematicians. Finally, providing students with a further glimpse of what lies beyond high school in the field of mathematics can serve to inspire and motivate them to persist with the fundamentals. Introducing basic concepts from higher-level mathematics can motivate students to continue on in the field past grade school. It is important for students to gain a broad understanding of the breadth of mathematics, and this can be achieved by giving them a sneak peek into upper-level mathematics topics.

With these ideas in mind, a two-day pilot workshop on topology was designed for 4th–6th grade students as a means to explore possible mathematics education reform. This workshop was tested in March 2014 in Erik Dahl's 5th grade classroom at Chaparral Elementary School in Claremont, California. The students were presented with two lessons on topology. First, they were introduced to topology with a lesson about continuous deformations and genus, which used play-doh to demonstrate topological ideas. On the second day, the uses of geometry in the study of topology were discussed through an exploration of triangulations and Euler characteristic. The students were also taught about subdivisions of triangulated objects and how they are used as approximations when creating animations.

Before and after the workshop, the students were asked to complete evaluations in order to analyze the efficacy of the workshop in meeting the desired goals. The student evaluations showed that they learned a large amount and the results of their surveys showed the explored reform goals to be very promising. The students clearly picked up many of the topological ideas presented in the workshop, as they used a lot of the workshop vocabulary in their post-survey answers. Furthermore, there were interesting changes in survey responses that showed improvement in the students' perception of mathematics. Notably, the number of students who viewed math as a creative subject increased dramatically, as did the number of students who enjoy the challenge of mathematics. The students also appeared to learn that getting good grades is not the most valuable indicator of success in mathematics. As a whole, this pilot workshop demonstrated the potential of the reform goals that were chosen to be explored. Introducing topology to young students is a strong example of how student perceptions of math can be broadened to set the stage for increased participation in, and excitement about, the field of mathematics.

Appendix A

Activity: Play-doh Geometry

In this activity, students will use pipe cleaners and play-doh to explore introductory concepts in the field of topology. Topology, which is often called "rubber sheet geometry," is an area of mathematics that studies properties that remain unchanged under continuous deformations. Informally, if an object can be bent and stretched to form another, those two objects are topologically equivalent. The students will utilize the malleability of pipe cleaners and play-doh to determine whether or not certain shapes are equal in a topological sense. Specifically, the aim is for students to recognize that the "genus," or number of holes in an object, is an important quantity in topology.

Materials:

- 1 **Play-doh Geometry Worksheet** per pair of students (Appendix B)
- 1 pipe cleaner per student, pre-formed into a square
- 1 ball of play-doh per student, pre-formed into a sphere

Instructions:

1. Break students into pairs.
2. Give each pair of students a worksheet and each student a pipe cleaner square and play-doh sphere.

3. Direct students to look at **Exercise 1** on the worksheet. Explain that they must try to transform their pipe cleaner square into the objects on the list of shapes, but they must keep the square flat on the table and cannot break the pipe cleaner.

4. Once students finish **Exercise 1**, they can move on to **Exercise 2**, which is a 3-dimensional version of the pipe cleaner activity. In this activity, they are allowed to bend and stretch their play-doh to try to form the various shapes, but they cannot break or poke holes in the play-doh.

5. Allow students enough time to attempt both activities. As each pair works, check in to make sure they understand the rules.

6. When students are done attempting both activities, lead a discussion about what they discovered. Go through the items in the checklists for each activity and ask whether the students were or were not able to create them. Then, use the following questions to help the students identify key ideas:

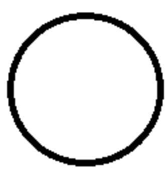
   Key Questions:

   - In the pipe cleaner exercise, what was different about the shapes you could create and the shapes you could not?
   - For the shapes you could not create, what rules would you need to break to be able to create them?
   - In the play-doh exercise, what properties of a shape make it impossible to form (under the rules of the activity) from a ball of play-doh?
   - (Note: Ask once the students recognize holes in the play-doh as an important characteristic.) Does the number of holes matter? If you started with the donut, could you create the letter B? Why or why not?

Appendix B

Worksheet: Play-doh Geometry

Exercise 1: Place your pipe cleaner square flat on your desk. Try to bend your pipe cleaner into each of the following shapes without lifting it off the desk or breaking the pipe cleaner. Circle YES if you can bend the pipe cleaner into the shape and NO if you cannot.



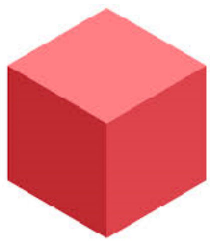
a. YES OR NO

S

b. YES OR NO

c. YES OR NO

Exercise 2: Now try a similar exercise using a ball of play-doh. Try to mold your play-doh into each of the following shapes without tearing through the play-doh. Circle YES if you can mold the play-doh into the shape and NO if you cannot.



a. YES OR NO

b. YES OR NO

c. YES OR NO

d. YES OR NO

e. YES OR NO

f. YES OR NO

Appendix C

Activity: Euler Characteristic

In this activity, students will work in groups to calculate the Euler Characteristic of an object. They will use a triangulation of that object to count up the vertices, edges, and faces of the triangulation and calculate the Euler Characteristic using the formula

$$\chi = V - E + F.$$

The students will then make connections between the Euler Characteristics of certain objects to understand that it is an invariant under the continuous deformation of an object, or "play-doh deformations" from the previous activity. This will lead students to a more complete understanding of topological equivalence through more quantitative proof.

Materials:

- Several objects pre-"triangulated" with a Sharpie (e.g. ball, rod, cup, mug, donut)

- Small, colored circular stickers to label vertices, edges, and faces, pre-numbered (different colors for vertices, edges, faces)

Instructions:

1. Break students into small groups.
2. Give each group of students a triangulated object and three sets of colored stickers for counting.
3. First direct students to count the vertices of their object. Designate a colored sticker set for vertices and have each group place use their numbered stickers starting at 1 to label every vertex on their object. Have each team choose a member to record their number of vertices.
4. Designate a second colored sticker set for edges and have the students repeat the labeling process for the edges of their object. Make sure they again record their number of edges.
5. Finally, have the students use their third colored sticker set to label all the faces of their object. Make sure they record this number.
6. Write the formula
$$V - E + F$$

on the board and direct each group to use the formula to combine the number of vertices, edges, and faces of their object to get a number. Have each group write the entire formula and answer on the board. (Ex: $4 - 6 + 4 = 2$)

7. Have a representative of each group stand at the board next to their equation, holding their group's object. Lead a class discussion about the results of the activity using the following questions:

   Key Questions:

   - Which objects produced the same number? What do these objects have in common? Hint: What if these objects were made out of play-doh?
   - Which produced different numbers? What is different about these objects?

8. Explain that the number the students calculated is called the Euler Characteristic of an object and that it is a number that stays the same under play-doh transformations (i.e., no matter how much we bend and stretch an object, it's Euler Characteristic will not change!) Make sure students understand that if two objects are equal in topology, they must have the same Euler Characteristic. This is because for two objects to be equal, we must be able to bend and stretch them to form the other.

Appendix D

Student Pre-Survey Responses

1. **How much do you like math?**
   (Scale from 1 to 5, "Not at all" to "Very much")

   Average Score: 3.49 $\pm$ .21

2. **What do you like about math?** I like doing things like multiplication;The easy problems; I like the fractions; I like math because the challenge and the success of getting it right; I like when we do fun things with math; Some things Mr. Dahl makes easier and fun; I like to do the problem solving and looking at all aspects of the problem; It makes me smarter; I like the challenge; That it is easy; I like multiplication; That you can learn how to do things that I will need in the real world; All the numbers and strategy; I like that it is used for almost everything; Learning new things each day; It has many ways to solving it; I like adding; Multiplying; Algebra, multiplying and dividing fractions; Multiplication (of fractions, too); Addition and subtraction; Well I like math because I think it's fun and I want to get better at math; It is just like interesting and it is kind of like music because you need to count; I like math because sometimes it can be challenging and fun; I like that you can do things in many different ways and you can use it to learn about science; I like fractions and subtracting, adding, dividing, and multiplying them; That sometimes if I get the hang of it, it can become fun; I like doing division problems; I like division and multiplication; I like that math can be fun sometimes and also sort of challenging; Math is fun and is pretty easy for me. It is very interesting and sometimes a little hard; It stays in your mind and at the right moment it helps you; It is always fun! The math al-

ways feeds my brain; I like that there is a bunch of ways to do only 1 problem; How you can figure out almost everything with it.

3. **What do you dislike about math?** I don't like math problems that have a lot of steps; The hard problems; I don't like the division; I don't like not understanding something; Most things; Some things are challenging and I don't get the way Mr. Dahl describes them; I can't think of anything that I don't like about math; How much time it takes and how complicated it is; Nothing; Sometimes it can be difficult; I'm not too good at division of fractions so division of fractions; That there are too many ways to do things and then we jump around and it's just confusing; Nothing really; I dislike that you can't count all the numbers in the world; That I got a lot of homework sometimes; It can be very complicated and frustrating; Division and dividing fractions; Everything else [besides multiplying]; Nothing except showing work; Long division; Everything with fractions; Um sometimes I don't like math because some of it is really hard for me, and it's boring; I dislike how adding and subtracting numbers work; Sometimes it's a little too hard; I don't like that sometimes you have to do a lot of work to solve a problem; Nothing; Sometimes I don't get the method or it gets too hard; Decimals; Decimals, percents, and fractions; Sometimes it can be too easy and become boring; I don't dislike anything about math and it is very fun; It really confuses me; Nothing; I don't like it because a lot of times there are too many steps which confuses me; Division!!!!!!!!

4. **Do you think you are good at math?**
   (Scale from 1 to 5, "Not at all" to "Very much")

   Average Score: $3.97 \pm .16$

5. **Why do you think so?** (5) I think so because I understand math; (2) Because it is challenging; (2) Because I have a hard time doing it; (5) Because I never get frustrated and can figure it out by myself; (4) Because I get good grades in math; (3) Because sometimes I don't think I apply myself as I could. But I also get average grades in math; (5) I get high scores on my math tests and I understand how to do the problems; (4) Because I do my work in my head; (4) I always get good grades; (5) I get high scores on tests; (3) It is harder because you have to get improper fractions then divide; (4) Because I handle it pretty well. Few areas I have some difficulties; (4) Because I pass

most of my tests; (5) Whenever I take a test I get 90% and up; (4) Because I get so high grades and low grades; (4) I am pretty good at understanding instructions; (3) I think I am a three because some things I get and some things I do not get; (4) Because; (5) I placed first in all fifth graders for math field day; (5) Because I love it a lot and I get high scores on my tests; (3) Because I always get bad grades. I get confused and forget the ways to do it; (3) Well cause I'm really struggling in math and it's hard to me; (4) Because I get what my teacher teaches us; (4) Because I get good grades in math; (5) I usually get good grades in math; (5) Probably because I get good grades and I always know what to do for each problem; (3) I always need the teacher to explain it for me again; (5) I get really good scores on tests; (3) It's confusing to me when it gets into decimals, percentage, and fractions; (5) Because I have taken special advanced math classes; (5) I think I am good at math because I always understand math concepts; (3) It just makes me need help and have a headache; (4) I think so because I get most of the hard work done but I mess up on like the easiest problem; (4) Well I'm OK at math. I'm not the best at math but I'm fairly good. A four; (3) Because some things I get and some things I don't get.

6. **Do you think math can be creative?**

   Yes: 46%
   I'm Not Sure: 43%
   No: 11%

7. **Describe math:** Math is a way to find answers; Hard problems, division, multiplication, subtraction, and addition; I like the data graphs and that is why I think math can be creative; Math is the tool to figure out problems; Not my type of subject; Numbers and equations and sometimes hard; Creativity, fun, there are many fun things to do in math; Lots of numbers mixing with other numbers; Something you use everyday. You can make your own shape; Adding, subtracting, and finding sums; F-U-N (ish); A bit tedious; Math is fun and hard at the same time; It is fun, sometimes easy, and sometimes hard; Math can be fun sometimes and not fun sometimes, but I like it; Math is a part of science made to make science easier; Long, numbers, symbols, and shapes; Boring; Problems that can be done using one or more kinds of math; Math is a subject in school and a way or finding out

answers; Hard, but a little good for you; Well it's fun, boring, hard, sometimes easy, and creative; Quick, thoughtful and a bit creative; Different numbers in different orders; Math is a way to solve things with numbers and operations; Math is mostly problems; Math can be hard but if you get the hang of it then in some stuff it can be easy; Everything has to do with math; Everything you do is practically math; Math is the configuration of numbers. It can solve everyday problems; Math is very easy for me but sometimes math is a little hard; Math is the dividing of fractions, addition, subtraction, and multiplication; It is very interesting; It can be boring but it also can be fun; Using numbers to figure out a mystery.

8. **What do mathematicians do?** Mathematicians do very complicated problems that they know will work; They are very good at math and do a lot of it so they are smart; They calculate data and stuff and find percents of things; They help do problems like dividing equally or how something falls or goes at a certain rate; They are really good at math; Study math (I don't know); Solving problems, having fun, and being creative when solving the problems; I don't know, solve math?; They do difficult math problems that most people don't know how to solve; Math problems; Umm... they do math; Mathematicians teach people math; I forgot. :(; They are always finding more numbers in pi and finding new patterns to get the sum of a question; I forgot. :) ; Mathematicians are people that can do these problems easily and quickly; Mathematicians solve math very fast and they are good at math; Math; They create names for numbers, they figure out tricky coincidences, and they help other people with math; Mathematicians either teach people some math or they learn new things about math; They make up new ways to do things; They help people with math like me; Mathematicians always do math; They try to make up new things in math; Mathematicians try to come up with new concepts in math; Mathematicians do math as a living; They are math magicians?; I don't know; I don't know; Mathematicians know math very well and teach it to other people; They do math and are something like math professors; Math?; They are really good at math; They find and make up math. They also learn other stuff too; Figure out different ways to do math and also use already figured out math.

Appendix E

Student Post-Survey Responses

1. **How much do you like math?**
   (Scale from 1 to 5, "Not at all" to "Very much")

   Average Score: $3.61 \pm .22$

2. **What do you like about math?** I like multiplication; Math can be tricky and fun; I like multiplication and addition; It's very spread out and there are lots of kinds of math. It has hard parts and challenges, which suits me; I like to do things that make me see math differently; Math is fun and it's creative and sometimes cool; It helps you when you most need it; It is fun and you need to have a strategy when you answer; That some things are easier and I get them faster; Nothing really. Play-doh; The division; I like when we used the different objects and put the stickers on; I like adding and times; Nothing, except dividing and topology (I think that's how you spell it); I like how you learn new things every day; I like playing with play-doh to bend it into shapes / topology; I really like anything to do with fractions and decimals; You need to study it and can't just be good at it; It can be really fun because math has to do with objects, which means that we can create new objects by using clay or play-doh; I like the challenge of math; I like some of the questions because they are challenging; That it is easy; I like that it can always be hard so I can challenge myself; That there is multiple ways to do things; The challenge; That it can be fun and you might have a partner to challenge or we could play with play-doh; There are so many different subjects in math, like topology; I like a lot of things, including topology; It can be very creative and there are lots of different things and concepts to do; That

you can solve problems that seem hard easily; It is very interesting.

3. **What do you dislike about math?** I dislike long division; I don't dislike anything about math; I dislike division and fractions; Nothing; I don't like to do boring problems; Math can be really boring for me cause I really don't like math; Fractions; Division; It's challenging sometimes; Un-fun stuff; Reducing fractions; Fractions; I don't like division and fractions; Everything else (besides dividing and topology); About all the work we have to do; Fractions and estimating are my least favorite things in math; Nothing; Negative and positive numbers. I do not like the rules; Mostly decimals because they are hard to work with for me; Any memorizing names, it's annoying; I don't like some of the problems because they can be too hard; Some parts can be difficult; I can't think of anything I dislike; How challenging that it can be sometimes; The challenge; That you have to think so much and there are always hard problems; I dislike how math can sometimes be boring; It can be very long and boring to solve problems; That they jump around so much and give you a lot of time to learn something; Nothing.

4. **Do you think you are good at math?**
   (Scale from 1 to 5, "Not at all" to "Very much")

   Average Score: $3.98 \pm .15$

5. **Why do you think so?** (4) Because I get high scores on my tests; (5) Because I always get good scores and understand concepts; (3) Because I can understand what I do; (5) I do algebra books at home, and I am one grade ahead in math; (4) I think so because I know most of the stuff we do; (3) Cause I'm struggling; (3) Memorization?; (4) Because I get good grades on math tests most of the time; (3) Because it is hard sometimes so I don't apply myself as much; (3) Because I get good grades; (3) I have trouble and sometimes I'm good at math; (3) Because my friends and teachers help me a lot and I'm improving; (3) I think I am a three because some math is very challenging and some I get; (4) Cause I've always got a 5 on my report card; (4) I think so because I learned some new math when you came in; (3 or 4) I have no idea I just sometimes get good grades and sometimes don't; (5) I get good grades on my tests and I normally get what I have to do after 1 example or less; (4) Because I do not always get the rules; (5) I get good grades on it and I understand it; (5) Because usually I don't

get it wrong and know lots of types and styles of math; (4) Because I get most of the problems right; (5) I get high scores on tests; (5) I usually know most of the questions' answers but not all the time; (4) I'm not great at math but I'm good at it; (4) I usually get scores in the 90s, 80s, or 100%; (2) Because in some areas I'm strong and in others weak; (5) I understand the problems, get good grades on math tests, and I enjoy it; (5) I know the answers to a lot of math questions; (5) I almost always can understand numbers, algorithms, patterns, and concepts like topology; (4) Because I pull through, although I sometimes struggle here and there; (4) Well I do all the hard math right and mess up on easy equations.

6. **Do you think math can be creative?**

   Yes: 74%
   I'm Not Sure: 23%
   No: 3%

7. **Describe math:** During math it makes your mind constantly think in math, you never stop your brain; Math seems boring at first, but then it seems more interesting; Using numbers, geometry, and operations to solve things; Math is the configuration of numbers and shapes; A fun subject where you can look at all the aspects of the problem; Math is fun (sometimes), hard, and easy (every once in a while); Something we do everyday, we can even make our own shape; I think that math is like a swirl of numbers that makes sense; Fun, challenging, and creative; Adding, dividing, subtracting, and multiplying; Numbers, letters, and angles in different orders to form a problem; Math is the tool to figure out difficult problems; Numbers, shapes, objects, things like that; Math can be creative and not creative and numbers go on forever; Problems that make your mind think; It's a hard subject to be good at in my opinion; I think that math is fun because you can use play-doh to make shapes; Evil; Problems, numbers, signs, and shapes; Math is numbers and basically everything you do, even sports; Adding, subtracting, multiplying, dividing; Boring, but I don't mind topology; Numbers; Fun and hard at some points. I always want to be the one that gets the right answer; The adding, subtracting, multiplying, or division of two or more numbers in a sentence; Fun, cool, sometimes boring; Math is shapes, numbers and a lot more; A mystery; Math is a learning way in FUN!; Math is pretty

tricky but if you understand it, it will seem very easy; Math is a subject, and it is also used for helpful things throughout your life.

8. **What do mathematicians do?** They find out many ways to get answers to math problems; I think mathematicians get a problem and solve it, like the people who build rovers; They come up with new concepts, ways to do problems, and find patterns in math and science; Mathematicians are very good at math. Mathematicians try to teach math to the world; They do problems and solutions, and they figure out why the solution is what it is; They help you solve problems; They solve math problems to help people. They do math for a living; Make up math and shapes; They always find more numbers in pi. They are also finding more ways to get answers from questions; Teach other people math; They do math for a living; They can figure out how things can turn out; They find ways to make problems work out; Mathematicians study math; They do math as a job and they practice it a lot; They learn new ways and easier ways to do math; I think what mathematicians do is help you learn new things just not the boring things; Math for a living; Mathematicians do math for a living; They describe ways to make math easier; Calculate surveys of stuff; They are good at math; They do math for a living; Teach math and do math for a living; Math, geometry, topology?; They help you learn about math and make people become smart; Mathematicians do very hard problems and create new branches of math; They think through theorems; Mathematicians do math; Mathematicians do math and use math to solve problems; Mathematicians teach math and learn math.

9. **What is topology?** Topology shows that you could take an object and change it to another object; It's basically a harder way of geometry; Topology is when you can continuously deform an object and make it equal something else; Topology is the continuous deformation of shapes; The study of continuous deformation; Something that can bend without breaking it or putting it together; Topology is where you can make almost anything into a different shape; A series of shapes that you can change to look smooth; Topology is changing a shape into another shape without breaking its continuous deformation; The form of an object changing into another object; Shapes or objects that can be molded into different shapes or objects; That shapes can stretch, so in topology a square is a circle. It can also bend;

The idea of continuous deformation, which means that one item can stretch to become a new item without making holes; Topology is objects that equal or can equal other objects; Topology are shapes that can be formed as if rubber, which means you can turn a square into a circle; It's how you can bend shapes without breaking the shape; Topology is using shapes to stretch into different shapes like a donut into a coffee mug; A form of geometry with bending and molding; Topology is measuring and changing shapes into another shape. That is called continuous deformation; It's when you can make one shape into another it's called continuous deformation; It is to prove a circle is a square and a donut can be a coffee cup; Math; Math and seeing if one thing can change into another shape; Shapes and sizes; Topology is a fun way to learn about geometric figures. It also means constant changing; Topology is basically shapes and stuff, it's really cool; Topology is a form of geometry where everything is bendable and changeable. For example, in topology a circle is a square; Finding out if shapes are made with clay without breaking it; Topology is like geometry (I think); Topology is morphing shapes into something else by making everything seem like rubber; Topology is when you can bend shapes into one another.

10. **Why is topology useful?** They use it for movies and games; Because you can group shapes using different methods; You can use it to make approximations of something like in animation; With topology, we can see the relation between different shapes; You learn new things and you can understand more about math; It helps in everyday life; To shrink or grow something when it is too big or small; For movies and animated things; Topology is useful for making last minute changes; People use topology for animation, video games, and movies; It's useful because movies can be made with topology; It can be used for animation and how things are the same; You can see what shapes can change into other things in geometry; Topology is useful because we can see what goes into what; To get to know your shapes well and know what can be made as another thing. This would be useful for cooking; It helps you understand what you can do with shapes; Topology is useful because you can use it everywhere like in movies; To figure out a shape's area or perimeter; Topology is useful because it helps us learn about shapes; So you can see if a donut is a coffee cup too; Yes it useful because you can't put holes in a sculpture and you can change shapes; I don't know; ?; To see what

shapes are the same as others; It helps you with geometry; It really helps you with geometry; Topology is useful for a different outlook about shapes and objects; If you spend 7 hours doing something that cannot be done, topology helps if it is possible; It is useful to figure out angles and shapes; Topology is useful because you can use an item's genus number to classify objects; It's useful because you can learn things from it, and do fun things.

11. **What else did you learn during the topology workshop?** That a donut can turn into a coffee mug; That in continuous deformation you cannot break it; I learned about genus and they are the amount of holes an object has; I learned how to make a donut into a coffee cup; I learned what topology is and I learned about an object's Euler number; That a donut can be made out of a mug; I learned about vertices, faces, and edges; About faces and a lot of other things; I learned that a square can be a circle, a donut can be a coffee cup and much more; That some objects can be changed into other objects; I learned that a circle doesn't have to be a circle, it can be any shape as long as it doesn't have a hole in it; That a square is a circle but a square can't be a donut because you can't make holes and you can't attach; That math can be creative and shapes are more alike than I thought; That movies use topology; It's not possible to turn a shape into another shape that has a hole if it doesn't start with a hole; I learned that a donut can be a cup, which I never knew; I learned that topology is shapes that can be turned into different shapes and it is used in movies; That a circle = square = triangle. :) ; I learned a donut can turn into a mug. I thought that was really cool; That you can't make a hole in a shape; I learned that you can't bend an 'S' in a [circular] pipe cleaner; That a coffee cup can be turned into a donut; That a diamond can't change into an 'S' with pipe cleaner; That a donut is pretty much a coffee cup; That a donut can turn into a coffee cup; That [topology] is kind of easy and cool; I learned about continuous deformation, which is the infinite changing of objects; Nothing; I learned that a square can be a circle; That objects with no holes had a different [Euler] number than one with holes; I learned that a donut cannot be made out of play-doh because you can't break holes through it.

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