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Applying the SIR model: can students advise the mayor of a small community?

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Abstract: This is an account of a modelling scenario that uses the SIR epidemic model. It was used in a third year applied mathematics subject. All students were enrolled in a mathematics degree of some type. Students are presented with the results of a test carried out on 100 individuals in a community containing 3000 people. From this they determined the number of infectious and recovered individuals in the population. Given the per capita recovery rate and making a suitable assumption about the number of infectious individuals at the start of the epidemic, they then estimate the infectious contact rate and from this the basic reproduction number. The mayor has asked the students to determine what will happen if no action is taken and to evaluate four policy options. They are asked to recommend the best course of action.

This scenario provides students with a problem where parameter values must be inferred from the information provided (one cannot be determined). They use the SIR model to provide public health recommendations, reinforcing their appreciation for the usefulness of mathematical modelling.

Our paper gives details of student presentations, and errors on the final exam, along with feedback to and from the instructor and the two student coauthors.

1 Introduction

A disease, the dynamics of which are governed by an SIR (Susceptible-Infectious-Recovered) model, is spreading through a small town (population 3000). The state government recently supplied the mayor with 100 test kits. The chief medical officer tested 100 individuals at random, and discovered that two individuals were infectious whilst a third of the individuals showed signs that they had

> *recovered from the infection. It is known that on average, it takes an individual two weeks to recover from the disease...*

The SIR model is a standard compartmental model for the spread of an infectious disease. The spread of the disease is modelled by two processes. First, the interaction between susceptible and infectious individuals may lead to the former becoming infectious. The second process is recovery from the disease, all infectious individuals eventually recover — entering the recovered class.

This paper provides a modelling problem for students familiar with the SIR model. In this scenario, the students reside in a small town which has been hit by a disease. The mayor has identified that they are the only residents with modelling expertise and has provided them with what is known about the disease. The mayor has outlined four policy options that are under consideration, the students are asked to evaluate these and recommend two of them. There is insufficient information to determine one of the model parameters. The students must therefore either decide upon the best way to estimate this parameter or how to incorporate this uncertainty into their recommendations.

The structure of this paper is as follows. The problem setting is fleshed out in Section 2. Two version of the modelling scenario were used. In the first, two groups of students gave competing presentations to the mayor. The second version appeared on the exam paper. Solutions to the common part of both formulations are provided in Section 3. In Section 4 one of the two presenting groups (CG & MW) outlines how they approached this task. In Section 5 the student responses to the exam question, problems B 2.1 & BE 2.3, are discussed. The exam question had two components. In the first, students estimate the basic reproduction number of the disease. This was intended as a stepping-stone to evaluating the policy options. Unfortunately, almost all students failed to correctly determine the basic reproduction number. This suggests a need to modify the content of the subject. In Section 5.3.2 the policy advice that the students provided is discussed. This reveals a range of misconceptions and mistakes. It also reveals that some students have excellent insights into the *political* practicalities of implementing policy. Two students came to opposing opinions on whether a two month lockdown would be acceptable to the population. It sometimes reveals an imprecision in the use of language: how long is a lockdown of "just under two months"? In Section 6 a variety of extensions to the modelling scenario are outlined. Finally, conclusions are drawn in Section 7.

Appendix A contains reflections upon the use of this problem. These are provided by the student group and the lecturer in Appendices A.1 & A.2 respectively. The students reflect on on how they designed their presentation and on their peers. In Section A.2.2 MW reflects on the need to critically identify important content when giving a presentation with limited time — not a skill addressed in our undergraduate (or postgraduate) curriculum. The lecturer provides a reflection on the presentation, and on what he learnt from some of the mistakes that students made on the exam paper.

The remainder of this introduction is structured as follows. Section 1.1 provides a short recap of the SIR model. Section 1.2 provides background information about the subject in which the modelling scenario was used. Section 1.3 discusses some other modelling resources involving the SIR and related models.

1.1 The SIR model

The SIR model is covered in many textbooks, including those on mathematical biology [6, 17] as well as mathematical epidemiology [5, 15]. A reader unfamiliar with this model is encouraged to consult Ketcheson [12], who provides an accessible, brief, and non-technical overview of the model including simple ways to model intervention strategies. So armed, they can consult one of the aforementioned books for more detail.

In an *epidemic model*, it is assumed that the propagation of the disease is being modelled over a sufficiently short time span that changes in the population size due to births and non-disease related deaths can be ignored. The SIR epidemic model has the property that the population size (N) is constant: individuals are neither created nor destroyed, at most a susceptible (S) becomes infectious (I) before recovering from the disease (R). In symbols,

$$N = S + I + R. \tag{1.1}$$

The SIR epidemic model is given by equations (1.2a)–(1.2c).

The rate of change of the number of susceptible individuals is

$$\frac{dS}{dt} = -\beta IS, \qquad S(0) = S_i. \tag{1.2a}$$

The rate of change of the number of infective individuals is

$$\frac{dI}{dt} = \beta IS - bI, \qquad I(0) = I_i. \tag{1.2b}$$

The rate of change of the number of recovered individuals is

$$\frac{dR}{dt} = bI, \qquad R(0) = R_i. \tag{1.2c}$$

This model contains two parameters: the infectious contact rate, β, and the per-capita recovery rate, b. The latter has a very nice interpretation. It is the reciprocal of the mean time that it takes for an individual to move between the infectious and recovered compartments. In principle, the per-capita recovery time can be estimated from data, whereas the infectious contact rate is estimated by fitting the model to data.

When a newly emerging disease is introduced into a community, then no individuals have recovered from the disease because no-one has ever been exposed to it. Hence, the initial condition for the number of recovered individuals is $R_i = 0$.

1.2 Subject framework

The modelling scenario, detailed in Section 2, was used in the subject MATH329 (Medical Mathematics) which "provides an introduction to mathematical modelling as applied to several areas in medicine and biology". This subject is taken by students in the third year of their degree. (In Australia, many undergraduate degrees are three years in duration.

Students with higher marks may stay on for a fourth year of study, graduating with an honours degree.)

The Australian academic system uses several words that may not be understood by American readers. For example, we use "session" where Americans use "semester". This section is called "Subject framework", an American would call it "Class framework".

MATH 329 is divided into two equally weighted parts: the first is taught in the first seven weeks of session, and the second taught in the remaining six weeks. During each of the thirteen weeks of session MATH329 has one two-hour lecture and a single one-hour tutorial. In 2022, there were 17 enrolled students, 16 undergraduates and one postgraduate. For tutorials in the second half of session, students work on tutorial sheets in two groups of four and three groups of three. Each group gave two tutorial presentations. (The questions for the presentations were released two weeks ahead of the presentation). One group gave their presentations in week 9, two groups gave presentations in weeks 11 and two in week 13. Every non-presenting group marked each presenting group's presentations.

The lecturer in the second half of session was MIN. The remaining authors formed one of the two groups that gave a presentation in week 13 on the problem presented in Section 2. The student coauthors were enrolled on the Bachelor of Engineering (Honours)–Bachelor of Mathematics degree (CG) and the Bachelor of Mathematics Education degree (MW). (This group's third member withdrew from the subject prior to the presentation).

1.3 Literature review

A variety of questions on the SIR model can be found in textbooks, such as [5, 6, 15, 17]. We start our literature review with two articles about the teaching of mathematical epidemiology. We then discuss six peer-reviewed resources that are available through the SIMIODE web site (Systems Initiative for Modelling Investigations & Opportunities With Differential Equations). We finish by describing three resources which fit outside the aforementioned sections.

1.3.1 The teaching of mathematical epidemiology

Bernardi and Arminian [4] outline a subject in mathematical epidemiology based around the use of historical documents and primary sources. Their subject starts with the Indian plague epidemic of the 1900s, moves onto Kermack's and McKendrick's classic paper [11], and finishes with the West African Ebola epidemic (2014–2016). An important component of their subject is that students fit simple models to real data.

During COVID-19 universities across the world transitioned from face-to-face teaching to online teaching. Barg [3] discusses how a reading and writing project in an introductory differential equations subject was transitioned into a professor-student research group collaborative project investigating the spread of COVID-19. This involved students collecting data and using a variety of models to estimate parameters. Further, the research group were faced with issues such as "when does the early time period end?" and whether the proposed definition was valid for outbreaks in different parts of the globe.

1.3.2 SIMIODE resources

1.3.2.1 Non-SIR models.

Two resources use the exponential and logistic growth models to investigate the spread of a disease [8, 24]. Winkel [24] shows how the spread of a disease can be simulated using graph paper and M&M candies. Driskell [8] asks students to fit data for the 2014 Ebola outbreak in West Africa to both models. Throughout the worksheet, students are posed a mixture of quantitative and qualitative questions.

1.3.2.2 SIR models

Winkel [25] uses the 1666 plague data from the village of Eyam. Students build the SIR model from scratch and estimate the values of the two parameters. This is achieved by bounding the parameter values, gridding the range of values, and running through all the grid points to find the one which gives the lowest least-squares error. A useful addition to this paper would be a discussion on how to estimate the initial bounds: the Mathematica code contains the comment that the search space is "much refined from trial and error".

Three resources use data from Murray [17] for the spread of influenza through an English boarding school [16, 19, 26] Miller [16] provides scaffolding enabling students to estimate parameter values from the data. This project was written so that it could be used by students with a variety of mathematical backgrounds. Pepper [19] provides an alternative resource for this data set. The best-fit parameter values are provided and students asked to answer three questions about the spread of the disease.

Winkel [26] uses a modified SIR model to analyse the boarding school data. He replaces the standard formulation for the incidence of the disease, i.e. the term βSI in equations (1.2a) & (1.2b), by the saturating term, $\beta I \cdot S/(K_m + S)$. Two methods are used to estimate the three model parameters from the data. The first uses a gridding approach, as in the paper [25]. The second uses the method of steepest descent. The model with the saturating term has the lower sum of square errors. What (MIN) likes about this resource is the evaluation of the two models using the Akaike Information Criterion. This shows that, for this data set, the traditional way of modelling the incidence term is better.

Greer [10] uses the SIR model to investigate student knowledge of the relationship between the solutions of the system, and the derivatives of the solutions. Given six graphs, they can determine which correspond to the state variables and which correspond to their first derivatives. This exercise is then repeated for a scenario in which the infectious contact rate (β) is piecewise-constant. This reflects a situation in which the introduction of counter-measures reduces its value.

Wang [23] provides a tutorial, motivated by COVID-19, for students with no prior experience of the SIR model. Students integrate system (1.2) using GeoGebra to gain an appreciation of the possible outcomes from simulations. They are then asked a sequence of questions which takes them through analytical insights into the model's solution. Finally, the model is extended to include deaths from the group of infective individuals.

1.3.3 Other resources

Ledder and Homp [13] discuss an extension of the SEIR model for COVID-19. The first extension splits the infectious group into symptomatic and asymptomatic infectious individuals. The second extension allows for symptomatic individuals to be hospitalised. The final extension allows hospitalised individuals to either recover or die. A strength of this paper is its discussion on how to estimate model parameters.

The authors discuss the use of the SEIR model for students with wide-ranging mathematical backgrounds. At one extreme it was used as an advanced project, at the other it was used in a general education subject for liberal arts students. Between these extremes, it was used in an introductory differential equations subject and in a subject for practicing secondary school mathematics teachers. Depending upon the background of the students, they are given either a spreadsheet or a computer implementation of the model. In the latter case, a graphical user interface hides the code, students provide values for a subset of the parameters. In either case, students use the model to assess the effects of public health policies and community behavior. The emphasis is on interpretation of results.

Ledder and Homp [14] discuss two models for the spread of an epidemic that are appropriate for students with a strong background in algebra and functions but not calculus. The first model is an individual based model, the second is a continuous time SEIR model. The focus is on extending these basic models to include strategies to reduce the spread of the disease. Exercises are provided for both models, with an indication of what level of skills in calculus and programming are required for each exercise.

The SEIR model is carefully built up, with all the assumptions underpinning it clearly explained. The model contains three processes and the functionality by which each of these is modelled is carefully explained. The authors then explain how the model parameters can be determined. In particular, they solve the model in the early stages of the spread of a disease, during which the number of exposed and infectious individuals both increase exponentially. They show how measurements of the exponential growth can be used to estimate the basic reproduction number. The exposition ends by obtaining an equation for the fraction of the population that remains susceptible when the disease has run its course. Supplementing the many exercises provided throughout the exposition, the paper finishes with five research projects.

Nelson [18] discusses student responses when they were asked to write a short report for the mayor of a (hypothetical) small town in response to the mayor's plan to eliminate a contagious disease by locking the town down for three weeks. The approaches that students took in constructing their reports are summarised. Many students saw the parallel between the modelling scenario and concurrent discussions in the communities that they came from with regard to the spread of COVID-19. They accepted the idea that mathematicians need to be able communicate ideas in the form of written report.

2 The problem

In this section we provide the problem formulation. Two slightly differing formulations were used: one for the presentation in the week 13 tutorial and one in the final exam. As part of their preparation for the final examination, all groups were advised that to develop solutions for all presentation questions. This message was reinforced in a review class in the recess week between the end of teaching and the examination period. In the base problem, B 2.1 we indicate the common part of the two formulations. We then indicate the slight difference between the formulations for the presentation and exam, problems BP 2.2 and BE 2.3 respectively.

Problem 2.1 (Base problem). A disease, the dynamics of which are governed by a SIR model, is spreading through a small town (population 3000). The state government recently supplied the mayor with 100 test kits. The chief medical officer tested 100 individuals at random and discovered that two individuals were infectious whilst a third of the individuals showed signs that they had recovered from the infection. It is known that on average it takes an individual two weeks to recover from the disease.

1. Estimate the pairwise infectious contact rate. Hence estimate the value for the basic reproduction number.
2. How many people will be infected if the mayor takes no action?
3. The mayor's options include the following.
   - Introduce a strict quarantine by locking down all households, reducing the the infectious contact rate to zero. The streets will be patrolled by the police and citizens will only be allowed to leave their homes for medical emergencies.
   - Introducing social distancing reduces the pairwise infectious rate by 20%.
   - Introducing proper hand hygiene reduces the pairwise infectious rate by 30%.
   - Mandating the wearing of cotton masks reduces the pairwise infectious rate by 40%.

The second part of the problem formulation for the tutorial read as follows.

Problem 2.2 (Extension for presentation question). There are only two groups of citizens with advanced mathematics training in this community. Each group has been asked to give a presentation of no more than *ten minutes* outlining your recommended course of action. You should carefully consider what exactly it is you want to tell the mayor during your presentation.

The formulation for the final exam read as follows.

Problem 2.3 (Extension for examination question). You are the only individual in the community with advanced mathematics training. You are asked to provide the mayor with written advice outlining your recommended course of action.

You are asked to provide the mayor with the best course of action to minimise the total number of infections in the community. You are also asked to provide a second recommendation. In both cases you're are asked to provide sufficient details of any calculations so they can be checked.

In both the presentation and exam question, it was left to students to determine how to deliver the advice and the extent to which they should explain the mathematics underlying it. An important difference between the questions is that in the examination problem BE 2.3, students are told that the mayor wants to minimise the number of infections in the community. The lecturer (MIN) thought that it was desirable to provide this guidance in an exam. For the presentation question, he thought it better to leave an ambiguity as to how evaluate the options; forcing the students to make their own judgement.

3 Solution to the base problem B 2.1

In this section the mathematics of the model, as expected to be determined by the students in answering Problem B 2.1, is described. The aim in Section 3.1 is to estimate the number of individuals that are infected if the mayor allows the disease to run through the population. Prior to this calculation, the infectious contact rate and the basic reproduction number are estimated. The number of individuals initially infected with the disease is unknown. In Section 3.2 the sensitivity in the values of basic reproduction number and the total number of infections as a consequence of this uncertainty are investigated.

3.1 Determining the total number of infections through the course of the epidemic

The problem statement does not specify the number of individuals within the community in each of the compartments in the SIR model. Neither does it specify the parameter values. We start by estimating the number of individuals that are susceptible, infectious, and recovered within the town by scaling the information obtained by the chief medical officer. From this we estimate that there are currently sixty individuals that are infectious and thirty that are recovered. Hence, the number of susceptible individuals is 2910.

The parameters in the model are the per-capita recovery rate, b, and the infectious contact rate, β. The former is the reciprocal of the mean time that it takes for an individual to recover from the disease. This is two weeks. Taking the unit's of time to be days then

$$b = \frac{1}{14}\,\text{days}^{-1}. \tag{3.1}$$

We estimate the infectious contact rate using a *quotient* differential equation. Dividing equation (1.2a) by equation (1.2c) then we obtain the quotient differential equation

$$\frac{\mathrm{d}S}{\mathrm{d}R} = -\frac{\beta}{b}\cdot S.$$

(*Formally*, in obtaining this equation we are assuming that at time $t = 0$ there was at least one infectious individual in the population). The solution of this differential equation is

$$S\left(t\right) = S_i \exp\left[-\frac{\beta}{b}\cdot R\left(t\right)\right]. \tag{3.2}$$

Hence, from this solution, we obtain the following equation for the infectious contact rate

$$\beta = \frac{b}{R(t)} \ln\left[\frac{S_i}{S(t)}\right]. \tag{3.3}$$

We estimated the number of susceptibles and recovered individuals in the community using the chief medical officer's data. We determined the per-capita recovery rate from the information provided. If we knew the initial number of susceptibles, then we could estimate the pairwise infectious contact rate using equation (3.3). The required information is not specified in the question, students are required to make an educated guess. If they think that the disease was introduced into the community by a single infectious individual then $I_i = 1$ and the initial number of susceptibles was $S_i = 2999$. If they think that the disease was introduced by two individuals then $I_i = 2$ and $S_i = 2998$. Assume that the disease was introduced by a single infectious individual. Then

$$\beta = \frac{1}{14} \cdot \frac{1}{30} \cdot \ln\left[\frac{2999}{2910}\right] \approx 7.173 \times 10^{-5} \,\text{individuals}^{-1}\text{day}^{-1}. \tag{3.4}$$

For a SIR disease the basic reproduction number (R_0) is given by

$$R_0 = \frac{\beta K}{b}. \tag{3.5}$$

From the problem formulation we know that $K = 3000$ individuals. Thus

$$R_0 = 7.173 \times 10^{-5} \,\left(\text{individuals}^{-1}\text{day}^{-1}\right) \times 3000\,(\text{individuals}) \times 14\,\,(\text{days}) = 3.01. \tag{3.6}$$

How do we estimate the number of individuals that are infected if the mayor takes no action? To do this we use equation (3.2) in conjunction with the equation

$$S(t) + I(t) + R(t) = K.$$

It can be shown that [5, 6, 15, 17]

$$\lim_{t \to \infty} I(t) = 0.$$

Consequently when the epidemic has run its course, i.e. in the limit that $t \to \infty$, we have

$$S_\infty + R_\infty = K,$$

where R_∞ and S_∞ are the number of recovered and susceptible individuals at the end of the epidemic respectively. Combining this last equation with equation 3.2 gives

$$K - R_\infty = S_i \exp\left[-\frac{\beta}{b} R_\infty\right]. \tag{3.7}$$

As the only way to enter the recovered class is to be infected, the quantity R_∞ is the number of individuals that are infected if the mayor takes no action. Solving the equation

$$3000 - R_\infty = 2999 \exp\left[-7.173 \times 10^{-5} \times 14 R_\infty\right]$$

we find that

$$R_\infty = 2824.06 = 2824 \, \text{individuals}.$$

Should the normal rules of rounding be used to round up/down the number of individuals in each compartment? The epidemic ends when there is less than one infectious individual in the community. This implies that we should always round the number of infectious individuals down. MIN would always round the number of susceptibles up. Consequently, at the end of the epidemic, when there are no infectious individuals, MIN would *always* round the number of recovered individuals down.

3.2 How sensitive are our results to the initial number of infectious individuals?

In the previous section, the number of individuals infected during the course of the epidemic was estimated by assuming that initially there was one infected individual in the community. This is an assumption, perhaps a reasonable assumption but an assumption nevertheless. How sensitive are the values for the basic reproduction number and the number of individuals infected during the course of the epidemic to this assumption?"

The infectious contact rate is an *increasing* function of the initial number of susceptibles,

$$\frac{d\beta}{dS_i} = \frac{b}{R(t)} \cdot \frac{1}{S_i} > 0.$$

However, increasing the initial number of infectious individuals *decreases* the initial number of susceptibles. Therefore, increasing the initial number of infectious individuals decreases the estimated value for the infectious contact rate and consequently decreases the estimated value for the basic reproduction number.

It seems intuitive that reducing the basic reproduction number should reduce the number of individuals that are infected during the epidemic. It is easiest to investigate this numerically. Table 1 show that as the initial number of infectious individuals increases both the basic reproduction number and the number of recovered individuals decrease.

The usefulness of producing this type of sensitivity analysis was mentioned in lectures. The lecturer's solution to the examination question therefore contained this analysis. However, only two of the 16 students who took the final exam considered uncertainty in the initial number of infectious individuals. The other students picked a single initial condition; usually whether providing justification. As the question did not ask students to consider the uncertainty, they were not penalised for not doing so.

I_i	R_0	R_∞
1	3.01	2824
2	2.98	2817
3	2.95	2810
4	2.91	2802
5	2.88	2795

Table 1: The dependence of the basic reproduction number (R_0) and the number of individuals infected during the epidemic (R_∞) upon the initial number of infected individuals (I_i).

4 The presentation question: problem 2.2

The authors CG & MW formed one of the two groups that presented this problem. In this section they discuss the process by which they approached the problem, Section 4.1, and obtained their results, Section 4.2. The remainder of this section is written by CG & MW

4.1 How we worked on the presentation question

We received the week 13 tutorial sheet at the end of the tutorial in week 11. In the two weeks leading up to the presentation, we utilised various online communication to collaborate, such as Facebook Messenger, OneDrive and Zoom. We had four meetings to discuss the presentation: the first and the last in person and two Zoom meetings.

At the initial meeting, we discussed potential approaches to the question. We decided to attempt the question separately, discussing our approaches and comparing our findings at the next meeting.

In the second meeting, we compared each of our answers, mainly focusing on sections where they differed; specifically with the calculation of the infectious contact rate (β). We worked through some of our calculations together and decided upon a solution that we both supported. We then delegated tasks: Watanabe was in charge of the organisation of information and the layout of the presentation, and Goosen was in charge of simulating the SIR model to produce the figures for each management method.

At the third meeting we shared our progress. Watanabe asked if it would be useful to put all of the management methods onto a single figure for a better comparison. Following this, we discussed how to calculate the infectious contact rate for all the non-lockdown methods. We debated whether it was better to add or multiply the fractional reductions in the infectious contact rate to obtain the combined value. We decided on multiplication due to the interactive nature of the preventative measures. For instance, suppose that mask wearing and social distancing are both enforced. We thought of this as social distancing being enforced on top of mask wearing, which has already reduced the value of the infectious contact rate.

We had our fourth meeting in person, on the morning of the presentation. At this, we compiled our material and had a practice run of the presentation. We worried about the presentation being boring, so Goosen added a few memes to make it more entertaining. When going through the slides, Watanabe asked if it was possible to obtain the percentage of people infected throughout the course of the disease if no action was taken; this would present a clearer view of the impact of the disease on the community. Goosen suggested that another point of comparison would be to show the percentage of people infected for each management method. Since the SIR model did not differentiate between individuals who fully recovered from the disease, those who partially recovered, and those who died, the team decided that the best management method would be one in which the percentage of infected people was minimised, therefore minimising the possible deceased population.

4.2 How we obtained our results

In this section we predominantly discuss how we obtained our results. Interlaced into this are observations about our presentation. For example, at the start of the tutorial a student from the class was assigned to be "the mayor" of the town. We addressed this student as "Mr Mayor" throughout the presentation.

4.2.1 Assumptions

We started the presentation by stating the population size and outlining how many people were initially infectious, recovered and susceptible. The compartment sizes were calculated as fractionally proportional to the sample of 100. We stated these and that we assumed the sample was a perfect representation of the community. The details of the calculation were not included, as they are not necessary for the comprehension of the presentation. For example, we stated that as 2 out of 100 people were infectious in the sample we expect 60 out of 3000 people to be infectious in the town.

The average time to recover from this disease was specified in the question to be 14 days. However, the death rate of the disease was not specified. Thus, we needed to look at options to avoid additional people being infected.

Within the scope of this project, the reduction in the 'quality of life' for the citizens of the disease ridden town is the level of deviation that they experience when compared to their lives prior to the implementation of any safety regulations. We made the assumption that the quality of life decreases as more regulations are implemented and their lives are subjected to larger deviations from their norm. (The other group did not consider 'quality of life' nor 'deviation from normal life'.)

4.2.2 Contact rate and the basic reproduction number

We then explained that there are two significant numbers in our analysis: the infectious contact rate, β, and the basic reproduction number, R_0. We did not discuss the SIR model at this stage as we wanted to highlight the fact that the disease is spreading rapidly among the community. We explained the significance of these two values as follows:

- the pairwise infectious rate measures the contact rate between susceptible and infectious individuals. Its value does not give an idea as to how the epidemic is progressing through the community, it indicates how 'quickly' the disease travels from one individual to another. Its value changes as countermeasures are implemented.

- The basic reproduction number was explained as being the number of people infected by one infectious individual. For example, if the basic reproduction number is two, then each infectious person infects two additional people. We want this value to be less than one, thereby each infectious individual infects less than one person. In this case there are fewer people being infected than those infectious — the disease is under control.

4.2.3 The mayor takes no action

It was assumed that the disease is eradicated on the day in which the number of infectious individuals drops below one, as there cannot be less than one infectious person. Figure 1 shows that if no preventative methods are implemented the disease peaks with 907 infectious individuals on day 33. Dealing with a large volume of cases this early will place significant strain on the hospital system, potentially impacting the lives of citizens and leading to panic. The disease is completely eradicated after 163 days.

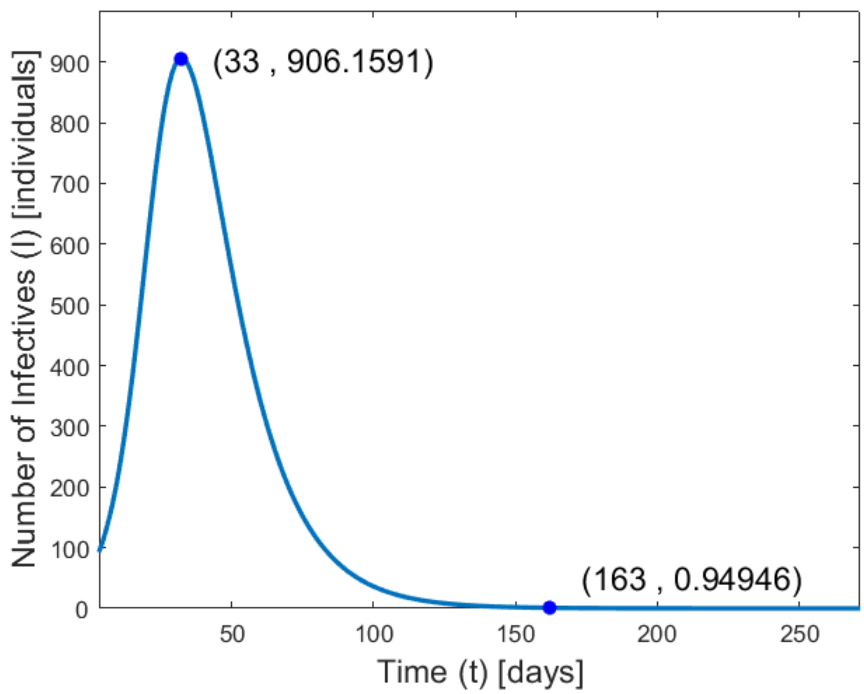

Figure 1: The number of infectious individuals in the community as a function of time if the mayor takes no action. The times at which the maximum number of infectious individuals is reached and the disease is eradicated are indicated.

When no action is taken to prevent the spread of the disease, 94.13% of the population are infected throughout the course of the disease. In the worst case, in which all infectious individuals die, only 6% of the town's population survive if the mayor does nothing. Preventative measures are required to reduce this percentage. Any method yielding a percentage lower than 94% is more effective than taking no action.

4.2.4 The options open to the mayor

The question identified the possibility of implementing four safety measures.

1. Lockdown (100% effective).
2. Social Distancing (20% effective).
3. Hand Hygiene (30% effective).
4. Masks (40% effective).

The effectiveness parameter refers to the reduction in the infectious contact rate if that method is implemented. For example, as social distancing is 20% effective it reduces the value of β by 20%. In addition we considered two combinations of options.

5. Combined Method (66.4% effective) — combining social distancing, hand hygiene and masks.
6. The Hybrid Method — 31 days of lockdown and then the combined method.

Based upon the effectiveness percentages alone we identified that the best option would be a Lockdown. We analysed each method to test this hypothesis and to provide a recommendation to the mayor. As the extent of implementation for each measure was not specified, the team assumed the effectiveness percentages are only valid under the strictest circumstances. For example lockdown is not only implemented between households, but also within them. Similarly, masks must be worn both in public spaces and in one's own home. This strict assumption was made to ensure that the team could give a fair recommendation based not only on the statistics obtained from their calculations, but also on the decreased quality of life that each method has on the town's citizens.

In discussing each scenario we presented a figure showing how the number of infectious individuals in the community varies as a function of time if the counter-measure is immediately mandated. Each figure also indicated the time at which the maximum number of infectious individuals is reached and the eradication time of the disease. Rather than presenting a separate figure for each scenario, as in the presentation, the simulations are combined in Figure 2. Similarly, the data points are provided in Table 2.

4.2.5 Scenario one: the mayor introduces lockdown

Prior to discussing this option we summarised the main characteristics of the SIR model. This included the three main groups of people and the parameters used in the calculations.

Table 2 shows that if a lockdown is introduced then the number of infectious individuals peaks at its initial value: there are no additional infectious people, and the percentage of the population infected is 3%. This is significantly lower than the 94% infected if no action is taken. It only take 58 days to eradicate the disease, compared to 163 days if no action is taken. As the possibility of disease-related fatalities is minimised by ensuring no additional people are infected, lockdown is the best method to eradicate the disease.

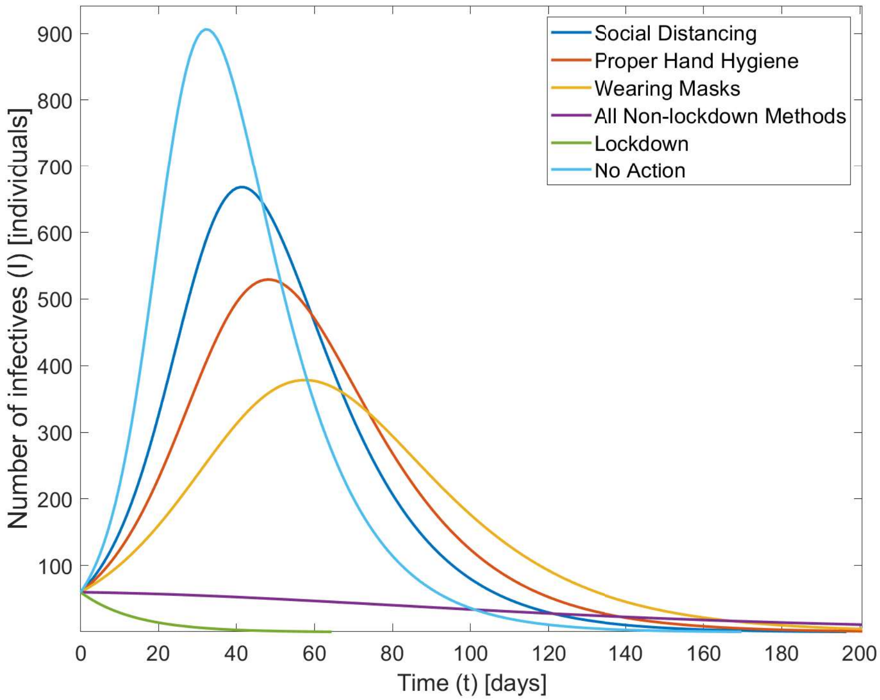

Figure 2: The number of infectious individuals in the community as a function of time and of the method that the mayor mandates.

Scenario	Peak Infectives	Eradication time	Additional infections
No action	(33, 906.16)	(163, 0.95)	2734
(1) Lockdown	(0, 60)	(58, 0.95)	0
(2) Social distancing	(42, 668.08)	(186, 0.98)	2554
(3) Hand Hygiene	(49, 529.30)	(210, 0.97)	2390
(4) Masks	(59, 378.32)	(243, 0.99)	2134
(5) (2–4)	(0, 60)	(389, 0.99)	502

Table 2: The times at which the maximum number of infectious individuals is reached, the eradication time and the number of *additional infections* following the introduction of the specified counter-measure(s). The corresponding number of infectious individuals is also provided. In the last scenario the mayor mandates options two through four.

The introduction of a lockdown reduces the infectious contact rate to zero. This assumes all citizens comply with the lockdown protocol, which is unlikely. It also assumes that the lockdown is implemented immediately. This may cause difficulties, as citizens may not take the lockdown seriously and the enforcers may not have a protocol to manage a full scale lockdown. There may also be psychological problems following a sudden implementation of a lockdown, especially if the town has never experienced a lockdown.

4.2.6 Scenario two: the mayor introduces social distancing

Social distancing is the least effective measure under consideration: reducing the infectious contact rate by only 20%: $\beta_{\text{new}} = 0.8\beta_{\text{original}}$. Table 2 shows that introducing social distancing eradicates the disease in 188 days, slightly longer than if no action is taken (163). The time to peak infectious individuals is delayed by 9 days, providing both the government and the hospital system with a little more time to prepare for a large influx of infectious people.

Social distancing measures, compared to taking no action, reduce the number of additional infected individuals through the epidemic, from 2734 to 2554, and decreases the peak number of infectious individuals, from 907 to 669. Overall, 88.10% of the town's population are infectious at some stage throughout the course of the disease, as opposed to 94% when no counter-measures are introduced.

The calculations assume all citizens abide by the social distancing protocols, even within households. It is unlikely that citizens are completely compliant. However, it is more likely that people follow social distancing protocols than lockdown protocols, as social distancing does not reduce their quality of life quite as much.

4.2.7 Scenario three: the mayor introduces hand hygiene

Implementing hand hygiene regulations is 30% effective: $\beta_{\text{new}} = 0.7\beta_{\text{original}}$. Table 2 shows that immediately introducing hand hygiene eradicates the disease after 210 days. The peak number of infectious individuals is 530 on day 49. The number of additional infected people is 2390, decreasing the percentage of infected people to 82.65%.

4.2.8 Scenario four: the mayor introduces masks

With a 40% effectiveness, $\beta_{\text{new}} = 0.6\beta_{\text{original}}$, mask regulations are the second best single implementation method. This yields an infected population percentage of 74.11%. Table 2 shows that the disease is eradicated after 243 days and that the peak number of infectious individuals is 379, occurring on day 59.

If only modelling results are considered, implementing mask regulations is the best single non-lockdown method. However the quality of life that each method has on the citizens of the town should also be considered. The implementation of any single non-lockdown method extends the duration of the disease further than if no action is taken: citizens are required to abide by the appropriate regulations for up to 8 months. Despite these efforts, at least 74% of the population are infected over the course of the epidemic which, in the worst case scenario, corresponds to the fatality of around 2134 people.

4.2.9 Comparing results from scenarios two to four

Comparing scenarios 2–4, i.e. social distancing, hand hygiene regulations, and masks, against what happens when no action is taken it is seen that as the effectiveness of the method increases, the peak number of infectious individuals decreases and shifts to the right.

The reduction in the peak number of infectious individuals places less strain on the medical system. The increase in the time to peak infections gives both the government and the hospital system additional time to prepare for a large influx of patients. The percentage of the population infected through the course of the disease decreases as the effectiveness increases. Hence increasing the effectiveness of the implemented measures decreases both the strain on the hospital system and the number of disease-related fatalities.

However, increasing the effectiveness of the safety measures extends the duration of the disease. This increase has significant impacts on the quality of life for the citizens. Although implementing hand hygiene or social distancing regulations reduce the overall number of infected people, this is at the expense of requiring citizens to follow the implemented rules for a longer period of time than that required if no action is taken.

4.2.10 Scenario five: The mayor combines all non-lockdown methods

Implementing masks is the best non-lockdown method. However, we were apprehensive about the possibility of 74.11% of the population dying. We therefore investigated combining non-lockdown methods to decrease the total number of infections. There are multiple ways of combining these methods, for example one could implement both mask and social distancing regulations or combine social distancing with hand hygiene regulations etc.

The most effective method is to combine all three non-lockdown methods. With the 20% effectiveness of implementing social distancing, 30% from introducing hand hygiene regulations, and 40% from the use of masks, the new value of the infectious contact rate is

$$\beta_{\text{new}} = 0.8 \times 0.7 \times 0.6 \times \beta_{\text{original}} = 0.336\beta_{\text{original}}.$$

Implementing all three non-lockdown methods yields an infected population percentage of 19.71%, with only 502 additional infections before eradication. Table 2 shows that peak infectious individuals occurs on day zero: no additional strain is placed on the medical system as the influx of patients never exceeds the original number of infected people. However, although 19.71% is significantly lower than the 74.11% obtained from introducing the best possible single non-lockdown method, the implementation of all three non-lockdown methods takes 388 days to eradicate the disease. This is significantly longer than the eight months required when masks are worn, the seven months of hand hygiene regulations, the six months of social distancing, and the two months required to eradicate the disease when a lockdown is imposed. Our calculation assumes that the population follows all three regulations. This is unlikely. This combined method reduces the quality of living more than the implementation of any single method.

4.2.11 Comparison of methods

At this stage in our presentation we used Figure 2 to compare all the methods. In so doing we divided the curves into three groups. The first group contains ‘Lockdown’ and ‘All non-lockdown methods’. The third group is ‘No Action’. The second group contains the remaining three methods. This grouping allowed us to better communicate the differences in the effectiveness of each method. We observed that these groupings represent the decrease in quality of life: group one methods impact the quality of life the most while group three methods do not impact the quality of life. We were determined to factor into our recommendation both the modelling results and the corresponding change in the quality of life. We found ourselves in a quandary as the most effective methods have the most significant effect on the quality of life.

At this stage we decided to re-examine our conception of 'quality of life'. Originally, we assumed that is the degree of deviation that the citizens experience in their life due to the imposition of rules regulating how they behave. Combining the intrusiveness of the countermeasures with their duration measures the change in the quality of life. The former was partly evaluated through our experiences of counter-measured used in Australia. Upon re-evaluation we decided to define the base-line to be how the citizen lived before the introduction of the disease to their town.

Using our original definition, imposing no regulations does not change quality-of-life. Under our new definition, it does as over 94% of the population are infected by the disease. Potentially 2734 people die in the worst case scenario. Hence, taking no action has the most significant impact on the quality of life for the citizens. On these grounds it was eliminated as a recommendation. Similarly, region two methods were eliminated as they have a minimum infected population of 74% — in the worst case scenario, around three quarters of the population die. Therefore, only region one methods are left as recommendations.

4.2.12 The hybrid method (combining all four single methods)

The lockdown method ensures that no additional individuals are infected, minimising the percentage of infected individuals. We were therefore partial to recommending this method over all other non-lockdown options. However, a lockdown can affect an individual's mental health and emotional well being.

In view of these considerations we explored a new option with the aim of reducing the time that the town spends in lockdown. This starts with an immediate lockdown for 31 days. When this is lifted all three non-lockdown methods are imposed. This two-prong approach halves the lockdown time and minimises the number of infected individuals.

The team's hypothesis was confirmed by our simulation, shown in Figure 3. There are only 132 additional people infected throughout the course of the disease. This is significantly lower than methods excluding lockdown, and only slightly higher than obtained from implementing lockdown. However, the trend of increased effectiveness causing an increase in disease duration remains. The hybrid method takes 593 days to eradicate the disease.

Whilst the hybrid method reduces the duration of the lockdown, it has the longest disease duration. Its lockdown period is only one month shorter than that required for a full lockdown. The team decided that spending over a year abiding to three safety regulations would have more impact on the quality of life than requiring an additional a month in lockdown. Consequently, the hybrid method was not recommended.

4.2.13 Recommendation to the mayor

Having considered all methods only two remained: implementing a lockdown or implementing everything but lockdown. With these methods 3% and 19% of the population are

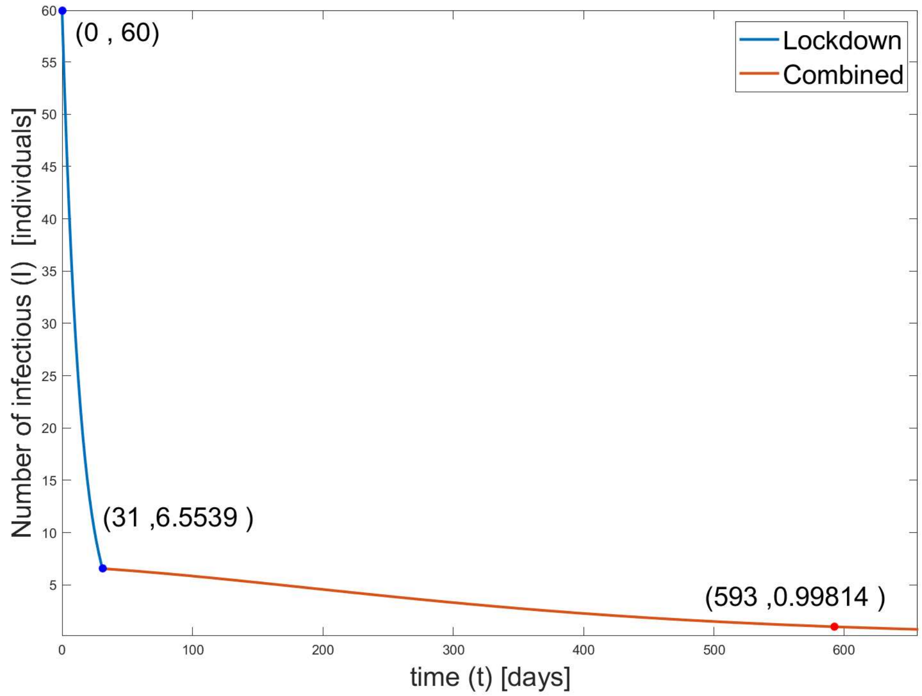

Figure 3: Hybrid method. The number of infectious individuals in the community as a function of time if the mayor immediately mandates 31 days of lockdown followed by implementing the combined method. The times at which the maximum number of infectious individuals is reached and the disease is eradicated are indicated.

infected respectively. The lockdown method is more suitable as it minimises the number of infected people and consequently the number of possible fatalities.

The team also wanted to consider the quality of life for the citizens of the town. It was decided that the duration of the implemented measures would impact the quality of life: abiding by one method for two months may be much easier then enduring another method for a whole year, even if the first method imposes more restrictions. Since lockdown only requires 58 days to eliminate the disease while the combined method requires 391 days, the team decided that the former has a impact on the quality of life.

Hence we recommended that the mayor implement a lockdown. During the presentation we mentioned that the spread of the disease may deviate from the model predictions. Therefore, although implementing lockdown for two months should eradicate the disease, the team also recommended implementing the combined method after the lockdown period to minimise the impact effect of any unexpected events. No time frame was specified for the precautionary use of countermeasures after the lockdown is raised. This was left to the discretion of the mayor. The team considered this recommendation to differ from the hybrid method discussed in Section 4.2.12 as it is imposed as a safety precaution after the two month lockdown period has theoretically eradicated the disease.

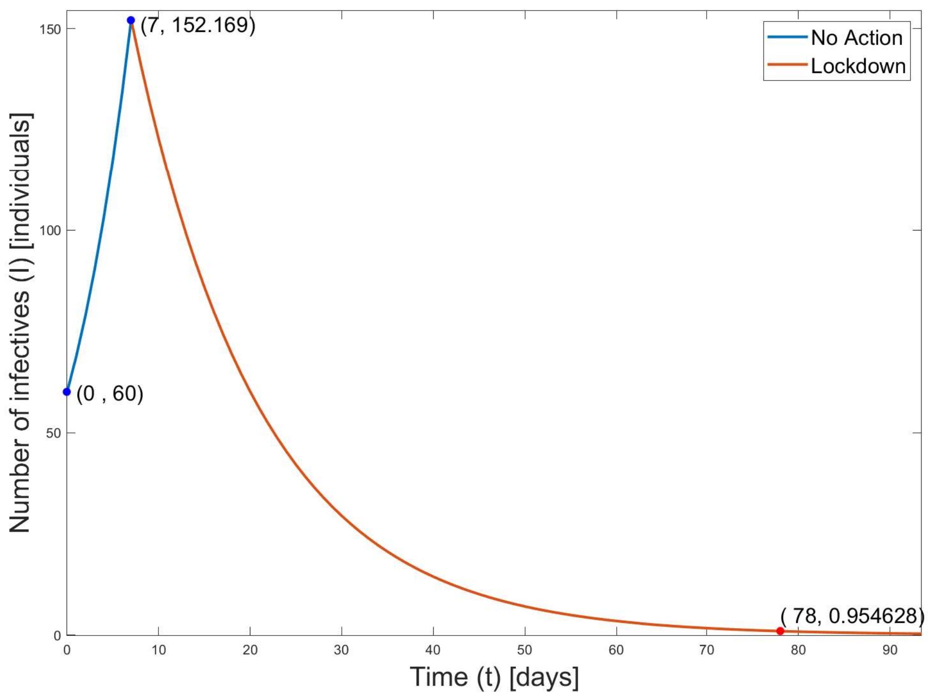

Figure 4: The number of infectious individuals in the community as a function of time if there is a seven-day delay before the mayor introduces a *lockdown*. The times at which the maximum number of infectious individuals is reached and the disease is eradicated are indicated.

4.2.14 What happens if there is a delay before the mayor makes their decision?

Although not mentioned in the question, the team thought it was important to investigate an additional scenario in which the countermeasures are not immediately implemented. This was considered more realistic, as it often takes time for decisions to be made and implemented. To investigate this we re-calculated the effectiveness of locking the town down if there is a one-week delay before its implementation.

After a one-week delay, the number of infected individuals increases to 153. Once lockdown is implemented the number of infectious individuals decreases exponentially, as shown in Figure 4. The disease is eradicated after 78 days, as opposed to 57 days without the delay, with only 142 additional infections. This delayed method only extends the lockdown period by one month which is reasonable considering the other options range from 163 days (no action) to 592 days (hybrid method). Despite a seven-day implementation delay, the infected percentage is similar to the hybrid method (7.39%). As the main objective is to minimise the percentage of infected individuals, the team recommends the lockdown method, even if a seven-day consideration period was taken.

4.2.15 Limitations to our advice

Throughout the presentation we described the limitations of the SIR model and about other modelling assumptions made. At this point we summarised some of our assumptions.

- The sample of 100 people correctly captures the demographics of the town.
- The majority of the calculations assume that the countermeasures are implemented immediately.
- Citizens abide by any regulations.
- All model parameters are fixed. If the parameters of the disease change, our recommendations may become outdated. Examples of unpredictable changes includes a treatment being found which decreases the severity of the disease, or a new variant of the disease is introduced that is more contagious.

For our second point, a delay in implementation, we recommended a re-calculation close to the implementation date to obtain a more accurate assessment of the options. For our third point we recommended calculations be performed to investigate what happens if this assumption breaks down. Goosen explained this in the presentation with the observation that "there is always one person who believes in their right to infect others".

We concluded the presentation by conveying the limitations of the SIR model to the mayor.

- It fails to differentiate between individuals that have recovered from the disease and those that died from it. In this situation the use of the SIRD model would be better.
- It assumes that all infected individuals are infectious, and therefore the terms 'infected' and 'infectious' are interchangeable. The SEIR model should be employed.
- There are also models that take the strain on the medical system into account.

5 The exam question: problems 2.1 and 2.3

On the exam paper 13 marks allocated to this question. There were five marks for estimating the basic reproduction number, two marks for determining the number of individuals infected if the mayor takes no action, and six marks for a discussion of the recommendations. Due to changes in exam processes as a consequence of COVID-19 the exam was open book and uninvigilated.

5.1 Estimating the basic reproduction number

This part of the question was poorly done, with only one student receiving full marks and one student receiving almost full marks. In addition to losing marks for making errors, many students lost marks because they failed to adequately explain what they were doing and why they were doing it. In the following the lecturer (MIN) discusses how students went wrong estimating the basic reproduction number. It is hoped that this discussion will prove useful for instructors who use this modelling scenario.

Four students incorrectly scaled up the results from the sampling carried out by the chief medical officer.

- One student failed to scale the data, assuming there was one recovered and two infectious individuals in the town.
- One student correctly deduced that there were thirty recovered individuals. However, they incorrectly deduced that there were thirty infectious individuals.
- Two students misinterpreted the question, believing that one-third of the test sample had recovered from the disease. They took this to imply that there were 1000 recovered individuals in the town. A different deduction is possible from this incorrect premise. If one-third of the individuals in the sample size of 100 individuals showed signs of recovery how many individuals showed signs of recovery? We can not say that it is 33.333 individuals, should we take the number to be 33? Scaling up, this gives the number of recovered individuals as 990.

Two students showed a misunderstanding of the basic reproduction number, arguing that the recovered individual was responsible for the presence of the two infectious individuals within the sampling set. This meant that the basic reproduction number is two. From this incorrect value they calculated the corresponding infectious contact rate.

Five students correctly scaled the test data to obtain the number of recovered and infectious individuals in the community. However, they did not realise that the initial number of infectious individuals is an unknown quantity and thought that its value could be determined from the supplied information.

- One student argued that the presence of one recovered and two infectious individuals in the test data implied that the disease was introduced into the community by three individuals.

  Although an incorrect deduction, the fortuitous nature of this error meant that their estimated value for the basic reproduction number was in the right ball park, corresponding to the case $I_i = 3$ in Table 1.

- One student noted that there was one recovered individual in the sample set. They then noted that the mean recovery time from the disease is fourteen days. From this, they deduced that the disease must have been introduced into the community fourteen days ago — to give sufficient time for the infectious individual to recover. As the test had revealed that there were thirty recovered individuals, they deduced that the disease was introduced into the community by 30 infectious individuals.

  This student forgot that the mean recovery time is the *mean* time taken for individuals to recover from the disease. The recovery time for any individual could be shorter or greater than this.

- In using equation (3.3) three students used initial values $S_i = 3000$ and $R_i = 0$. This implies that the disease was introduced by no infectious individuals. Although equation (3.3) is derived under the assumption that the initial number of infectious individuals is *positive*, it still makes sense for an initial number of *zero* infectives. This gives a higher value for the basic reproduction number than those given in Table 1 ($R_0 = 3.05$). It might be argued that this is a sensible way to side-step the issue of not knowing the initial number of infectious individuals in the community as it provides a worst-case estimate for the basic reproduction number. However, if that was the reason for this assumption it needed to be spelled out in the report.

Two students assumed that the value for the basic reproduction number was 2.6, a value mentioned in the lecture notes. They then calculated the required value for the infectious contact rate. One student did not answer the question: although they correctly estimated the infectious contact rate, they did not calculate the basic reproduction number. One student had no idea how to estimate the basic reproduction number.

5.2 How many people are infected if the mayor does nothing?

Students who had an incorrect answer to the first part of the question could score 80% on this part if their answer was 'correctly incorrect', i.e. their estimate of the number of infected individuals was correct given their incorrect assumptions. Thirteen students received a mark of at least 80% on this question.

No student used equation (3.7) to find the number of recovered individuals at the end of the epidemic. They found this by integrating system (1.2). This gives the correct answer, provided that the integration period is sufficiently long. As the system had to be integrated to determine the eradication time, it was more efficient to determine the number of recovered individuals by integrating system (1.2). In the following the mistakes made by the three students who scored less than 80% on this question are identified.

Instead of inspecting the final line of the solution matrix to find the number of recovered individuals one student instead summed the entries in the column corresponding to the number of infectious individuals as a function of time. Their answer indicated that the number of individuals infected in the community was greater than the number of individuals in the community. This should have raised a red flag.

Although a second student used the wrong initial conditions ($I_i = 2$, $R_i = 1$) and the wrong value for the basic reproduction number ($R_0 = 4.2857$) they correctly determined the number of susceptibles remaining at the end of the epidemic. However, they incorrectly determined the number of people infected during the epidemic; writing

$$R_\infty = 3000 - 44 - 3 = 2953.$$

Deducting the initial number of infected individuals from the total number of infectious individuals is inconsistent with the way that equation (3.7) is used. The student could have received additional partial credit if they had remembered that they are providing advice to the mayor and provided some context for their answer. For example, they might have written something along the following lines.

"In calculating the number of individuals infected during the epidemic I have removed the three individuals that were initially either infectious or recovered. These three individuals will always be in these categories, regardless of what counter measures are introduced. Removing them from the number of individuals infected during the course of the epidemic provides a better baseline to evaluate the effectiveness of your options."

One could equally argue that the correct number is given by

$$R_\infty = 3000 - 44 - 90 = 2866,$$

as whatever policy the mayor implements can not change the fact that ninety individuals have caught the disease at the time the countermeasures are started.

The third student did not provide any reasoning behind their calculation. As they failed to upload their MATLAB code to the course management system, it was impossible to determine their error. Consequently, they scored zero on this question.

5.3 Providing advice to the mayor

In the final part of the exam question, students had to evaluate the options available to the mayor and provide advice on what course of action to take. There were three marks for each of these components. Students who had an incorrect starting point to evaluate the options could score a maximum of 80% for the first part of this question. However, they could score 100% on the remaining part — provided that their advice was consistent with their evaluations of the options.

The exam question BE 2.3 provides one metric to evaluate the policy options. However, students were free to include additional metrics. The evaluation of the options is discussed in the next section.

5.3.1 Evaluating the policy options

The exam question B 2.3, specifies that the policy options are evaluated using the number of individuals infected during the epidemic. Another important measure, mentioned throughout the subject, is the extinction time. Therefore, for each course of action, the students were expected to find the total number of infected individuals and the extinction time. Other characterisations are possible, for example the maximum number of infectious individuals and/or the time to peak infectious individuals — as discussed in Section 4.

Of the options available to the mayor, the eradication time can only be determined analytically for the lockdown option. Assuming that the introduction of a strict quarantine reduces the value of the infectious contact rate to zero, system (1.2) essentially reduces to

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -bI, \quad I_i = I_0.$$

The solution of this equation is,

$$I = I_0 \exp\left[-bt\right].$$

The extinction time, t_{erad}, is

$$t_{\text{erad}} = \frac{1}{b} \log I_0. \tag{5.1}$$

Using the values $b = 1/14\,\text{days}^{-1}$ and $I_0 = 60$ individuals we find that

$$t_{\text{erad}} = 57.3\,\text{days}.$$

The disease is eradicated if the quarantine runs for 58 days. (The normal rules of rounding should *not* be used; there is a second wave of infections if the lockdown is lifted after 57 days). The number of individuals infected by the disease is 90: the sum of the infectious and recovered individuals when the lockdown starts. No student used equation (5.1) to calculate the eradication time, it was found by integrating system (1.2).

The other options being considered by the mayor must be investigated by integrating the model. Students had previously calculated the number of individuals infected if the mayor does nothing. They were expected to realise that they should also find the eradication time for the case when the mayor implements no countermeasures.

Table 3 shows the eradication time and the number of individuals infected for each option. In order to obtain their characterisations, students had to determine how each option changes the infectious contact rate. For example, social distancing reduces the pairwise infectious rate by 20%: $\beta_{\text{social distancing}} = 0.8\beta_{\text{old}}$. Two students wrote $\beta_{\text{social distancing}} = 0.2\beta_{\text{old}}$ — an 80% reduction. The effect of the other options upon the infectious contact rate was similarly incorrectly determined.

Option	t_{erad}	R_∞	β	R_0
Do Nothing	162	2824	β_{original}	3.01
Lockdown	58	90	0	0
Social distancing	187	2643	$0.8\beta_{\text{original}}$	2.408
Hand hygiene	209	2480	$0.7\beta_{\text{original}}$	2.107
Masks	242	2224	$0.6\beta_{\text{original}}$	1.806
Last 3 countermeasures	388	595	$0.336\beta_{\text{original}}$	1.01136

Table 3: The time for the disease to be eradicated (days) and the number of recovered individuals when the disease has run its course as a function of the options available to the mayor (determined by MIN). The table also includes the cases when nothing is done, and when all non-lockdown counter-measures are implemented. The eradication times have been rounded up whilst the number of recovered individuals has been rounded down. The last two columns indicate how the proposed course of actions modifies the values for the infectious contact rate and the basic reproduction number.

The numbers in the third columns of Tables 2 & 3 differ. There are two reasons for this. The first is that in the former the students calculated the number of individuals infected *after* the mayor has received their advice. The lecturer calculated the *total* number of individuals infected during the course of the epidemic. The difference between these quantities should be 90: the number of individuals infected when the mayor receives the advice. A second reason for the difference is in the rounding of infectious individuals.

The students (CG& MW) noted that decreasing the infectious contact rate increases the time to eradication. Table 3 shows the infectious contact rate for each course of action and the corresponding value for the basic reproduction number. As the basic reproduction number is reduced towards one, the time to eradication increases. The lecturer has not seen this phenomenon discussed in the literature. Possibly, it was not known in the pre-COVID world. Perhaps it was known only to a few experts. For values of time larger than the time to peak infectious individuals, the number of infectious individuals decreases. However, susceptibles are still being infected. Consequently, there is a decaying source of new infectives. It seems plausible that the decay time for this source term increases as the infectious contact rate decreases, provided that the corresponding value of the basic reproduction number is larger than one. This pushes out the eradication time.

Ten students received a mark of at least 80% for their characterisation of the effectiveness of the four options. What mistakes were made by the other students? Two students only characterised the options that they recommended: this was quarantining and the wearing of face masks. Two students provided advice without characterising the options, both recommending quarantining followed by combining the three remaining counter-measures. One student only characterised quarantining. The final student wrongly interpreted their MATLAB output.

5.3.2 Analysis of advice

In this section MIN summarises student recommendations. In addition to characterising the options available to the mayor, Table 3 also considers combining all countermeasures. Although not mentioned on the exam question it was investigated by six students. This raises the issue as to how to determine the effect of combining options. A plausible assumption is that the effects are multiplicative. This gives $\beta_{\text{combined}} = 0.366\beta_{\text{old}}$. Two students incorrectly assumed that the effects are additive, assuming $\beta_{\text{combined}} = 0.10\beta_{\text{old}}$.

Eleven students recommended a lockdown of sufficient duration to drive the number of infectious individuals to zero. Three students recommended a shorter lockdown followed by the implementation of some form of counter-measures. Two students made a recommendation that did not involve a lockdown. Not all of the 11 students that recommended a lockdown correctly calculated the lockdown period, either due to errors made earlier in the question or due to incorrectly rounding of the eradication time.

One student recommended a lock-down of 79 days. (This student included a seven-day delay between receipt of the recommendation by the mayor and the implementation of the lockdown. This delay increases the number of infectious individuals in the community at the start of the lockdown). They noted that "such a long lockdown may be undesirable".

One student recommended a lockdown "of just under two months". Depending upon the time of the year this is a period of between 59 and 62 days. What does "just under" mean? (It would be interesting to ask a different cohort of students how long a period "of just under two months" is). Another calculated that a lockdown of 59 days was required and recommended introducing a two-month lockdown. They commented that

There is an understanding that a lockdown decreases the quality of life for citizens, however as it will only last for two months, this is not a significant issue. It is also recommended that the mayor should introduce all non-lockdown measures after the two-month lockdown period, just to be safe.

One student determined that after a lockdown of 60 days, "only one infected will remain and so it shouldn't spread from that". (An incorrect statement.) Two students recommended that the town be locked down for 58 days, two for 57 days and one for 48 days. One student made a mistake that was also made by several Australian political pundits during the early stages of COVID-19. They told the mayor that, as the recovery time is 14 days, "it would be safe to reopen the town after a lockdown of two weeks". Finally, one student recommended a lockdown without providing a duration.

Three students proposed that the mayor impose a two-prong strategy. Firstly, lockdown the town for a period of time less than that required to eradicate the disease. Secondly, impose specified countermeasures upon the lifting of the lockdown.

The first student found that a lockdown eradicates the disease after 63 days. (This incorrect estimate was due to assuming that there were 90 infectious individuals in the community at the start of the lockdown.) They thought that such a lengthy lockdown might be politically undesirable, c.f. the views of the student who recommended a two month lockdown period. In view of the potential adverse consequences of a long lockdown, they recommended a shorter lockdown of 21 days followed by the imposition of masks for two weeks. Although they could have used MATLAB code to determine the state-of-play at the end of the five week period, they did not do so. This student thought it was useful to remind the mayor "please note that 14 days of infection is an average. Some individuals will take more or less time to recover".

The second student correctly determined that a lockdown of three weeks leaves 14 infectious individuals in the population. Following the lockdown, they proposed combining all counter-measures. They provided numerical simulations for the cases $R_0 = 2.88$ and $R_0 = 3.01$. From these they deduced that the eradication time varies from 396 to 530 days and that the number of individuals catching the disease would vary from 236 to 809. (It is neither evident from their MATLAB code nor their discussion that they changed the infectious contact rate to reflect the imposition of counter-measures).

The third student recommended "an initial lockdown of around 30 days to prevent a large spike in infections and then release the town while implementing all safety options to allow the economy to recover". (In Australia, some political commentators argued strongly against the imposition of lockdowns due to their economic consequences).

Two students recommended courses of action that did not use a lockdown. One suggested either combining social distancing with hand hygiene or imposing the use of masks. However, they wrongly assumed that combining social distancing with masks would decrease the infectious contact rate by 50%. The second student commented that "introducing a strict quarantine by locking down all households immediately will be the best option…this plan is a bit too harsh and not practicable in real life". (Their calculations showed that a lockdown of 98 days was required). Rather than imposing such a long lockdown they recommended combining all three countermeasures.

A number of students commented upon the dichotomy that, excepting the case of a lockdown, as the eradication time decreases the number of infected individuals increases. Consequently, courses of action that lower the number of infections must be implemented for longer than those with a higher infection count. The decision to prioritise eradication time over total infections or vice-versa is a political decision, not a modelling decision. How long will the community be willing to comply with restrictions?

6 Extensions of the modelling scenario

The modelling scenario, problem B 2.1 and either problem BP 2.2 or problem BE 2.3, can be extended in many directions. Student answers suggest new avenues to explore. For example, two students considered the possibility that individuals may die from the disease. This raises the issue as to whether changes in population size, due to disease-related death, should be incorporated into the model (1.2). This issue is discussed in lectures. For a disease with a low mortality rate it is acceptable to leave the model as it is. One of the students commented that “as the SIR model does not differentiate between a dead person, and a recovered person, it is best to minimize the number of overall infectives” — the course of action being pursued by the mayor (in the exam question).

All-but-one student assumed, implicit in the question, that their policy recommendation is made on the day that they receive the mayor’s request, that their decision is communicated to the mayor on the same day, that the mayor decides on their course of action on the same day, and that the policy is implemented the following day. One student included a “7-day delay” between the mayor contacting them and the policy being implemented. Having identified the appropriate initial conditions and the value of the infectious contact rate they ran system (1.2) over a period of seven days to discover the values of the state variables when the policy is implemented. They used the values of the state variables at this point as the starting point for characterising the policy options. Incorporation of a delay of this type is an interesting scenario for exploration.

Another avenue to explore is for students to take into account uncertainty in the number of infectious individuals whom introduced the disease into the community. Uncertainty could also be introduced into the per-capita recovery rate (b). An extension along these lines is to give students a set of recovery times, asking them to identify both the mean value and the standard deviation. With these options, one can also investigate how the variability in the infectious contact rate due to the uncertainty in how the disease was introduced into the community effects predictions.

The modelling scenario uses an estimate of the spread of the disease in the population from the use of 100 test kits. A second set of test kits might be received at a latter date. These can be used to improve the estimate of the infectious contact rate. This refinement might be beyond what is reasonable to expect students to do in the period between receiving a tutorial sheet and giving their presentation.

This modelling exercise uses the premise that the point-estimates obtained from testing one hundred individuals can be scaled up to obtain the number of infectious and recovered individuals in the population. Given the diverse background of students in the class, this seemed reasonable. (All students had taken at least one statistics subject. Some students had taken no more, whereas others were completing a major in Applied Statistics.)

A 95% confidence interval around the point estimate can be estimated. When the sample size is not small and the proportion is not close to 0 or 1, this is usually based on the normal approximation to the binomial. This gives symmetric confidence intervals. When the number of cases in the sample is small (usually taken as less than 5) 'exact' and asymmetric confidence intervals can be constructed based on the binomial distribution. There are several options available. The traditional one is the Clopper-Pearson method. The web-based calculator [1] calculates confidence intervals using a variety of methods. Confidence intervals for the proportion of infected individuals are reported in Table 4.

	Number positive	Sample Size	Proportion	Lower 95% CL	Higher 95% CL
Normal approx.	2	100	0.02	-0.0074	0.0474
Clopper-Pearson exact	2	100	0.02	0.0024	0.0704
Wilson	2	100	0.02	0.0055	0.0700
Jeffreys	2	100	0.02	0.0042	0.0626
Agresti-Coull	2	100	0.02	0.0011	0.0744

Table 4: Estimate of confidence interval for the fraction of infectious individuals in the population using a variety of approaches. Calculations obtained using [1].

The proportion of infected individuals in the population can not be negative. Consequently, the normal approximation should not be used to estimate the confidence interval

7 Conclusions

We have discussed an open-ended modelling question for the spread of an infectious disease in a closed community. Based on information provided, students had to calculate quantities such as the number of susceptible, infectious, and recovered individuals in the town and from this determine the infectious contact rate. They used this information to predict what will happen if the mayor takes no action and to evaluate four policy options From this, they made two policy recommendations.

The students were divided into five groups. The modelling question comprising B 2.1 & 2.2 was used as a presentation question in the tutorial of week 13. Two of the five groups gave a ten-minute presentation to the mayor. They had to determine what information to include as part of the presentation, i.e. should they gloss over or 'hide' the details of mathematical calculations? Should they present graphs or present information in tables? In Section 4 one of the groups describes how they went about their presentation. They have also provided a reflections on their experience working on this problem, in Section A.1, and on the feedback that they received from their peers, in Section A.2.2. Both presentations were of a high quality and enjoyed by the class, leading to numerous questions both from the floor and exchanges between representatives of both groups.

The final exam contained a question comprising B 2.1 & BE 2.3. Unfortunately, only one student scored full marks on problem B 2.1; a second student scored almost full marks. The exam scripts revealed students making a variety of mistakes. In view of the types of mistakes made, in future the question will be split over two tutorial sheets with its first appearance being restricted to problem B 2.1.

Analysing the exam answers has identified a variety of errors and misconceptions. These can be used in questions with students asked to identify if a statement is incorrect or correct, justifying their answer. In doing so, some correct statements will be used to prevent students from assuming that the statements are always incorrect.

The wording of the exam question should be tightened to remind students that they are providing advice to the mayor, so that they are discouraged from writing mathematical expressions without context. Further, they should provide more explanation of the procedures that they used to evaluate the options; including how they estimated the missing parameter values.

The lecturer found it fascinating that ideas which were introduced by the student group in their presentation on problem BP 2.2 were developed by other students in their exam answers. Evidently, some students were paying close attention to the presentation.

A number of suggestions have been made, some motivated by student comments/answers, in how this modelling scenario an be extended. Following a comment made by a referee, the lecture content of this subject will be revised to include a small section on estimating confidence intervals. (It is interesting to note that a 95% confidence interval can be obtained even when there are zero positives in the sample set.) An expert in sample survey would want to take one step back. How were the one hundred individuals that were tested selected? Do they truly represent a random sampling of the population? Sample design is outside the scope of MATH 329, although it is an area of expertise in our school.

As a final avenue for exploration, one might develop this scenario to include the introduction of a vaccine. As a final reflection, the lecturer observes that without COVID-19 it is inconceivable that he would have developed this modelling scenario.

Appendix A Reflection

This appendix contains reflections from the students, in Section A.1, and the lecturer, in Section A.2. Student reflection on their presentation feedback is provided in Section A.2.2

A.1 The students' perspective

A.1.1 Watanabe

Set during the COVID-19 pandemic, the presentation task provided a timely and socially relevant scenario to apply skills and knowledge acquired in MATH 329. The open-ended nature of the task allowed us to explore both the usefulness and limitations of the SIR model and a variety of hypothetical scenarios The skills required for this task had been practised throughout the semester in the weekly assessments and tutorial questions with the opportunity to receive feedback from the lecturer. This approach was effective in building confidence in applying the mathematical knowledge and skills to draw conclusions. Thus, in the presentation, I focused on the clear communication of mathematics.

Due to the simplistic and hypothetical nature of the scenario, Carrin and I made it transparent that our model had neglected several issues that would impact the spread of the disease in real life, such as disobedience to preventative measures, for example, not everyone wearing masks properly, and the introduction of new strains of the disease.

As I write my reflection, I remember my first-year educational psychology class where we learned about prominent learning theorists, such as Piaget and Vygotsky. Reflecting on the pedagogical approaches in MATH 329 highlighted the connection between these theories and the presentation task, which was particularly interesting to me as an early-career school teacher. Working with a peer with a different educational background — Carrin is studying engineering and is far more competent at coding allowed us to complement each other's skills. I learned immensely from Carrin's competence in MATLAB, allowing me to work in my zone of proximal development (ZPD) as supported by Vygotsky's social constructivist perspective towards learning [22]. The presentation task served as a collaborative project-based learning opportunity which allowed exchange of ideas and skills with peers [2]. Piaget's [20] idea of schemas was also in play in the task, as we drew on the skills and concepts learned each week in the tutorials to complete the presentation.

Meaningful incorporation of information communication technology (ICT) in learning is highly encouraged in Australian educational settings, and is identified within two standard descriptors in the Australian Professional Standards for Teachers (2.6 and 3.4) [9]. This task is an example of a learning activity where ICT, namely MATLAB, plays a meaningful and essential role. MATLAB allowed us to immediately test our hypotheses, supporting discovery learning [7]. For example, we initially hypothesised that the use of 'less effective' preventative measures (such as hand hygiene instead of lockdown) would lead to more individuals being infected, and that such methods would result in a longer eradication time. By simultaneously plotting all the results, we saw that the less effective preventative measures both reach the peak number of infectious individuals faster and result in a higher number of individuals being infected. However, the disease is eradicated more quickly. The immediate feedback we gained from using MATLAB is an example of an effective redefinition of a learning task according to the SAMR (Substitution, augmentation, modification, and redefinition) model of technology integration [21], which was also heavily discussed in my initial teacher education program.

As we assumed that the mayor did not have a mathematical background, I wanted to ensure that the presentation was communicated using non-mathematician-friendly language, which I also prefer as a school teacher. I particularly enjoyed this process as I gain satisfaction from working with students who do not like mathematics, and showing them that maths is not scary! I focused on: minimising the use of jargon, providing simple definitions, and concise explanations and examples to clarify ideas without oversimplifying the SIR model. For example, the basic reproduction number was presented as "an indication of how rapidly the disease is spreading" and "how many people one infectious individual infects, and that we want this to be less than 1". I attempted to ensure that we were selective about the most important aspects of the model.

The familiar context of the task's provided intrinsic motivation for many of us to understand the way in which a disease spreads in a community. However, I feel that the minimal quantitative significance of the assessment hindered students' extrinsic motivation to explore the topic deeper. The assignment only weighed 5% of the overall grade for the subject. Given the wide scope of skills and knowledge assessed in this task, which covered several of the subject learning outcomes, it would have been more appropriate for the task to be weighed a higher proportion of the overall grade. Furthermore, this task could be linked to other aspects of the course to efficiently assess student learning.

Throughout the presentation, we emphasised that our calculations are based on a very simple mathematical model, thus would not be an accurate reflection of the spread of the disease in real life. A suggested addition to the task could be a literature review and a requirement to use the findings from this to calculate a margin of error based on recent literature. This would benefit in assessing the accuracy of the mathematical model and improving students' research skills.

A.1.2 Goosen

The presentation question was an interesting method to showcase a real-world application of mathematics. It not only improved my understanding of the SIR model, but also informed me about the roles of mathematicians in industry and the significance of developing models to simulate the results of implementing competing options. The presentation question taught me the importance of creating a fair and reasonable comparison of results to aid the decision-making process based on unbiased mathematical analysis.

The lack of information given in the question encouraged me to develop my own assumptions regarding the model. This also contributed to my appreciation of the question as an example of a real-world scenario since in reality, required information may not be readily available and the majority of population based data are obtained from surveys or sample populations. Conjectures about the initial number of people in each compartment of the SIR model and the addition of a non-mathematical parameter that refers to the citizens' 'quality of life' were decided upon by both groups that presented. This resulted in both groups having different calculations. This openness to interpretation further encouraged teamwork which provided another layer of real-world application to the project as most industry projects require collaboration.

The presentation question explicitly required the use of the SIR model, the simplest model examined in class. This led me to believe that the purpose of the presentation question was not necessarily to challenge the students academically, but rather to stimulate critical thinking and inspire awareness of the limitations of using models to simulate real-world systems and situations. This was one of the reasons that Mahi and I decided to include a section on our assumptions and the limitations of the model in our presentation to ensure that the mayor understood the boundaries of our mathematical predictions and the uncertainty of its accuracy in representing the real world.

Collaborating with Mahi was very informative as her education background enabled her to view the scenario with a more humane perspective, as opposed to my more analytical view. She introduced the idea of considering the citizen's quality of life and organised our results for the presentation to make it easier for the audience to follow. She helped simplify my explanations so that the mayor, with a non-mathematical background, could understand our findings. For example, our presentation did not include calculations for determining parameters such as the basic reproduction number and the infectious contact rate since she thought the mayor would neither be able to follow the working nor would care how they were determined. Her direct approach when organising the presentation ensured that we did not waste time explaining things that were not relevant to our recommendation. She emphasised that since our calculations were primarily focused on the immediate implementation of safety measures time was a precious factor; the major would not want to know how we derived our calculations. This was something I would not have considered I appreciated her insight as an educator and a presenter.

The team encountered some time management and communication problems while working on the project. This was mainly due to the short time frame of 2 weeks in which we had to perform the calculations, analyse the results, decide on a recommendation, and present our findings to the mayor. This short time frame was not itself a problem, however as Mahi and I lived quite far from each other and our communication was restricted to emailing and online video calls. This obstacle hindered productivity as we were not always on the same page. We did manage to have two in-person meetings in which the majority of our project was completed.

A.2 The lecturer's perspective

This section predominantly consists of the lecturer's reflection on the use of the modelling scenario as a presentation question and an examination question. However, it also contains the perspective of the two students on the feedback that they received on their presentation.

A.2.1 The presentation

Each group gave two presentations. In addition to providing written feedback, each non-presenting group gave an overall mark for the two presentations. Each non-presenting group decided for themselves the process by which they determined the group mark and provided their feedback. At the end of the tutorial the feedback and marks were collected. The lecturer's feedback was combined with that of the groups and emailed to each group. In an attempt to disguise who had provided which feedback,the comments from the lecturer were not placed as a block at the start/end of the feedback file.

The tutorial sheet in week 9 only contained 'standard questions'. Standard questions are those that test skills covered in the lecture materials and which can be answered within the confines of a tutorial. In addition to 'standard questions' the tutorial sheet in both weeks 11 and 13 contained a single open-ended question. The open-ended question used in the week 13 tutorial sheet comprised problem B 2.1 and problem BP 2.2. The open-ended question used in week 11 used the scenario described in [18]. In both questions students had to present policy recommendations to a major.

The group presenting in week nine answered two standard questions. Each group presenting in week eleven answered a different standard question. (The open-ended question was identical for both groups.) The same arrangement was employed in week 13.

For the open ended questions in weeks 11 and 13 a student from one of the non-presenting groups was selected to be the mayor (a different student in week 13 to the one chosen in week 11). The remainder of the non-presenting groups took the role of concerned citizens. It was left to the presenting groups to determine how they would engage with the mayor during their presentation. The benefits of requiring each group to give a presentation on an open-ended question only occurred to me after the week nine presentations. In future I will ensure that each group has an open-ended question to present.

Although each group gave two presentations, it was not required that each member of the group be involved in both presentations. It was left to the groups to determine how to deploy their members across the two questions. One of the groups contained a student that had transferred into the Bachelor of Mathematics degree from the Bachelor of Mathematics Education degree. The latter qualifies students to work as a high school mathematics teacher in New South Wales. This student was the sole member of their group to answer the technical question. Their presentation style was very different from that used by all other groups who answered technical questions. Evidently, this student applied what they had learned about mathematics pedagogy in the education part of their former degree. This raises the question as to how in future years to best 'deploy' students from the Bachelor of Mathematics Education degree, so that the other students are exposed to a different approach to answering technical questions.

Although both groups in week 13 gave good presentations, one of the groups gave a presentation that was clearly better. (This group comprised CG and MW). The positive feedback that this group received is given below.

- Liked the way you started by looking at the mayor: up close and personal.
- XY was dramatic at times.
- Very engaging.
- Very confident.
- Very good teamwork.
- Good consideration of "decision period".
- Good definition of eradication.
- Thorough explanation of each scenario. Drawbacks and other relevant information.
- Good combined graph.
- Good graphs of lockdown timing.
- A very comprehensive set of advice.
- Good recommendations.

The negative feedback received is given below. Some of these comments are not relevant within the context of the brief, i.e. a presentation to a mayor.

- Your numbers do not agree with our numbers. I do not think they are correct.
- How did they calculate that (3 options)?
- Mayor doesn't care about β and R_0… waste of time giving numbers.

- Yes, the mayor might not understand mathematical models very well but explaining what parameters were reduced to can not hurt
- Said no one else gets infected once lockdown starts, not true
- Why aren't masks effective — maybe provide statistics as to why lockdowns are more effective than masks + distancing.
- How did you get graphs/what did you reduce?
- Very long…

A.2.2 Student reflection on presentation feedback

A.2.2.1 Watanabe

I appreciate our peers for providing constructive feedback on our presentation. A variety of perspectives was presented in the feedback and I acknowledge that they are all valid and valuable. Examining the feedback, I view that the key in our presentation was critically identifying what should be included/excluded given the limited time frame. I feel that we did a reasonable job of critically assessing and prioritising information to be included in the presentation. However, in addition to reducing the volume of content in the presentation, we would have benefited from utilising various forms of media to further improve the efficiency of our communication.

Briefly stating of information intentionally omitted from the presentation would increase a sense of trust through increased transparency and allows the audience to seek further information should they wish. This would set a clear expectation for the audience in terms of what is to be explored in the presentation and build a stronger sense of trust in our recommendation. Furthermore, the feedback highlighted the importance of communicating with the client to understand their expectations.

The other pieces of feedback linked to the simplicity of the model, and the lack of explanation for the statements we assumed to be true, such as assuming that the contact rate is zero once lockdown has begun and why masks were considered to be less effective. I feel that this issue can be addressed in conjunction with the feedback that the presentation was rather long. Both of the above can be addressed by improving the academic rigour and adding an element of entertainment in the presentation. For example, we could have included a short advertisement-style video explaining the effectiveness of each of the preventative measures and the limitations of the model.

Other pieces of feedback were linked to wanting to know how the figures and numbers were produced. We were hesitant to overwhelm the audience by presenting a series of calculations, as we wanted to focus on the actions available in the short amount of time available. Although this is beyond the scope of the assessment, this issue could be addressed by presenting a user-friendly interface where figures of the number of infective individuals can be produced. I am imagining a software similar to GeoGebra where values for the parameters can be entered and users can instantly see the change in the graph, such as the dynamic construction presented by Figure 3 in [23]. It would be useful to present something like this for one or two of the methods that we presented to show that we did not pull these numbers out of a hat.

A.2.2.2 Goosen

Having examined the feedback given to my team there are some things that I would change about the presentation. I appreciate that the feedback of my peers is constructive criticism, which I can use to further develop my presentation skills. However, some points raised were contradicted by other points and so while engaging with feedback can help me improve, it is also important to identify which pieces of advice to listen to.

I appreciate the positive comments regarding the presentation, particularly those focusing on its content such as 'good graphs' or 'thorough explanation of each scenario'. These are things that Mahi and I worked on the most, specifically the graphs and methods of explaining the situations in non-mathematical language. It was great to see that our efforts reached the audience. I also appreciate the feedback involving our mannerisms such as being 'very confident' or 'started by looking at the mayor'. The team did not discuss these kinds of mannerisms, nor were we aware of displaying them during our presentation, it was a pleasent surprise to see them positively viewed by our peers.

However good it is to receive positive comments, most of the time an improvement in one's abilities is developed by engaging with the more constructive feedback. Some points made were arbitrary and subjective, for example some students did not agree with our calculations. I was not surprised by such a comment since the presentation question seemed open to interpretation and the feasibility of solutions are dependent on the assumptions made by each individual team.

Another comment stated that we gave too many numbers to the mayor, specifically the values of the infectious contact rate and the basic reproduction number which were provided at the start of the presentation. Although we felt a full-length derivation would be too intimidating to the mayor, we felt that identifying the values was necessary as it created a sense of academic transparency. My interpretation was that if the presentation were to be recorded and archived, as most discussions are in government, someone that watched our presentation would be able to generate the same figures and numbers by inputting the identified values for these parameters into a MATLAB script for the SIR model. In short, disclosing the parameter values we used, ensures that others can replicate our calculations and verify our findings. From the perspective of a student giving a presentation in class, this also means that other students can reproduce our figures.

As previously stated the feedback we received contained a few contradictory comments, particularly regarding our explanations. Some considered them to be 'thorough' and 'very comprehensive' while others considered them to be a 'waste of time'. These personal opinions are all valid, however it is difficult to produce a presentation that everyone agrees with. Throughout the project, my team had various meetings to discuss what information to include or exclude in the presentation. We ensured that everything we presented had a justifiable reason for being included, whether it was a percentage of the overall infected that we could use to compare the different methods, or the identification of parameter values included for transparency. In short, the contents of the presentation and level of explanation used was a decision that the team made considering the mayor, not our peers, as our primary audience.

In our presentation, we identified that an immediate lockdown reduces the infectious contact rate to zero. Therefore, no additional people are infected and the percentage of infected people is based on the 60 initially infected individuals. One comment pointed out the falsehood of this statement as it is still possible for people to become infected. This is a good example demonstrating the limitations of the SIR model when representing the real world. The team stated in the presentation that we only considered the strictest implementation of each method as the extent of implementation was not specified. This ensured a fair comparison of methods as we were not comparing loosely enforced hand hygiene regulations with a very strict enforcement of social distancing. Our assumption also aided in comparing the impact each method had on the quality of life of the citizens. With this assumption of strict implementation and the limitations of the SIR model in mind, our statement regarding zero additional infected individuals after the implementation of a lockdown was mathematically correct — if not true in reality. This comment shows a misunderstanding between the real world and the model's representation of the real world. The team was very transparent about the issues regarding this distinction, which was the main reason why we included our assumptions and the limitations of the model in the presentation. The use of a more complex model or more detailed assumptions would produce results that more accurately align with behaviours observed in the real world. It is important to note that the SIR model is one of the simplest models used to represent the spread of a disease and is not primarily used by epidemiologists.

The last comment was that the presentation was too long. In retrospect, I agree. We could have gone tcommenthrough our content quicker. Perhaps for a presentation to the mayor, it was a reasonable length of time. However, considering that the majority of the audience were mathematics students who were already familiar with the SIR model we could have picked up the pace a little bit. As I have previously mentioned, the presentation question was based on the simplest model that we studied in class. The audience was very familiar with the mathematical underpinnings of the presentation and it is understandable some may have found it repetitive. Increasing engagement was one of the main reasons why I suggested adding memes and popular show references to the presentation. Since the question explicitly identified the mayor as our target audience, our presentation was aimed at a non-mathematical audience which may have been boring to some students. The team may have benefited from aiming our presentation a bit more to the peers in our audience. However we would still need to consider the major as our primary audience.

I appreciate the feedback we received and acknowledge its subjective nature. In retrospect, the presentation could have been shorter, and we could have better catered to an audience of our peers. However, all information in the presentation was included for a reason and other than omitting the occasional explanation of our figures and calculations, I would change little. It is important to note that we had two weeks to prepare and the weighting of the presentation was minimal. Given these conditions, my team put in an appropriate amount of effort. We could have explored more implementation methods or provided a more in-depth study if the time frame and assessment weighting was increased. The time frame and assessment weighting for the task as written was sufficient.

(The lecturer disagrees that the presentation should aim at the level of the other students. The question stated that the target audience of the presentation was the mayor and I would have marked you down if you had given a presentation that was aimed instead at your peers. However, your presentation exceeded ten minutes and therefore it was too long. I made a mistake in not alerting when you approached the time limit and then stopping your presentation when it reached ten minutes. Lesson learned for next year.)

A.2.3 Lecturer's reflection on the exam question

In setting the question it did not occur to me that students would incorrectly scale the data obtained from the sampling of 100 individuals, i.e. they would not deduce that there are sixty infectious and thirty recovered individuals in the population. I assumed that this would be a straightforward calculation for third year students. Similarly, I assumed that when using equation (3.3) it would be evident that there is no way to know how many infectious individuals introduced the disease into the community. (I anticipated that students would assume that the number was one.) Although the lecture materials do not contain a question of this type, they do contain equation (3.7). Furthermore, the lectures contain a discussion of the influence that uncertainty in the initial number of infectious individuals has on model predictions when system (1.2) is integrated.

Although problem 2.1 appeared on the week 13 tutorial sheet, it used content introduced much earlier in the session. The next time I teach this subject, this problem will be placed on the tutorial sheet for week 9 or 11. It will be used both as a tutorial question that all groups submit a written answer on *and* as a question for presentation. The feedback on the written answers will allow group to identify their errors. More particularly, I hope that using this problem 2.1 as a group question will allow the weaker students to gain confidence in how to answer this type of question. Backing up the potential for using this to increase student understanding, any mistakes made by the presenting group can be picked up and discussed in the tutorial. These two feedback mechanisms should ensure that there are fewer errors in the exam on problem 2.1 — and hopefully prevent a situation where a student does not submit an answer because they do not know how to start.

As outlined in Section 5, students made a variety of errors. These can be used in future versions of this subject. For example, incorrect statements can be used as the basis for questions in lectures, tutorial sheets and assignments. Rather than informing students that the statements are incorrect, I would ask them to identify if they are correct or incorrect and to justify their answer. A number of correct statements/deductions would be thrown into the mix, to ensure that there is no certainty that such quotes are always incorrect.

Some students made recommendations that they did evaluate. For example, a lockdown of three weeks followed by the wearing of masks for two weeks. Students can be asked to evaluate such recommendations.

One error has not yet been mentioned. The infectious contact rate is not a scaled parameter, it has units of individuals^{-1}day^{-1}. A calculated value for this parameter that does include the appropriate units is incorrect. The answer given in equation 3.4, $\beta \approx 7.173 \times 10^{-5}$ individuals^{-1}day^{-1} is correct. The answer $\beta \approx 7.173 \times 10^{-5}$ is incorrect. Twelve students gave an answer without providing units and two gave incorrect units (day^{-1} and individuals day^{-1}). This leaves only two students who provided correct units. In retrospect, the importance of providing units for physical quantities was not emphasised in class. The lecturer assumed that students entering MATH329 would already know this.

A useful skill for modellers to develop is to ask themselves whether a result 'makes sense'. A result that does not make sense may indicate that you do not understand the modelling problem, i.e. there is an error in the model formulation. Alternatively, it may indicate that a more mundane mistake has been made. Many students find examinations stressful. It is therefore unsurprising that they sometimes turn off their faculty for critical reasoning. Here are some answers that students should have questioned.

1. Is it reasonable that an infectious disease is introduced into a community by the arrival of thirty individuals?
2. Is it plausible that the basic reproduction number is 256?
3. Is it likely, in a community of three thousand individuals, that when an infectious disease has run its course 21,706 individuals have been infected?
4. As the previous point, but with 42,900 infections.

Finally, in answering the exam question too many students presented mathematical calculations without providing any scaffolding as to what they were doing or why they were doing it. The rubric for the exam includes the proviso that "APPROPRIATE WORKING or a discussion of the working must be shown to earn full marks" so students could have been penalised for not doing so. The wording of problem 2.3 could be reworded to require students to justify all assumptions made and provide sufficient information so that calculations can be checked by another individual with advanced mathematical training.

Appendix B MATLAB code used to generate figures

The MATLAB code used to generate all the figures used by the student group is provided as supplementary material. All figures can be obtained by running the file `prompt.m`. The purposes of the various codes is described below.

`graphFunctionCompare.m`
Plots the number of infected people over time for each method on the same graph.

`graphFunctionHybrid.m`
Plots the number of infected people over time for the hybrid and Delayed method.

`Model.m`
Defines parameters and runs SIR model by calling `sir0.m`.

`graphFunction.m`
Plots the number of infected people over time for a single method.

`prompt.m`
This is the only script that the user needs to run. It provides prompt questions that require user input, which then calls other functions.

`sir0.m`
This function contains code that runs the SIR model.

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