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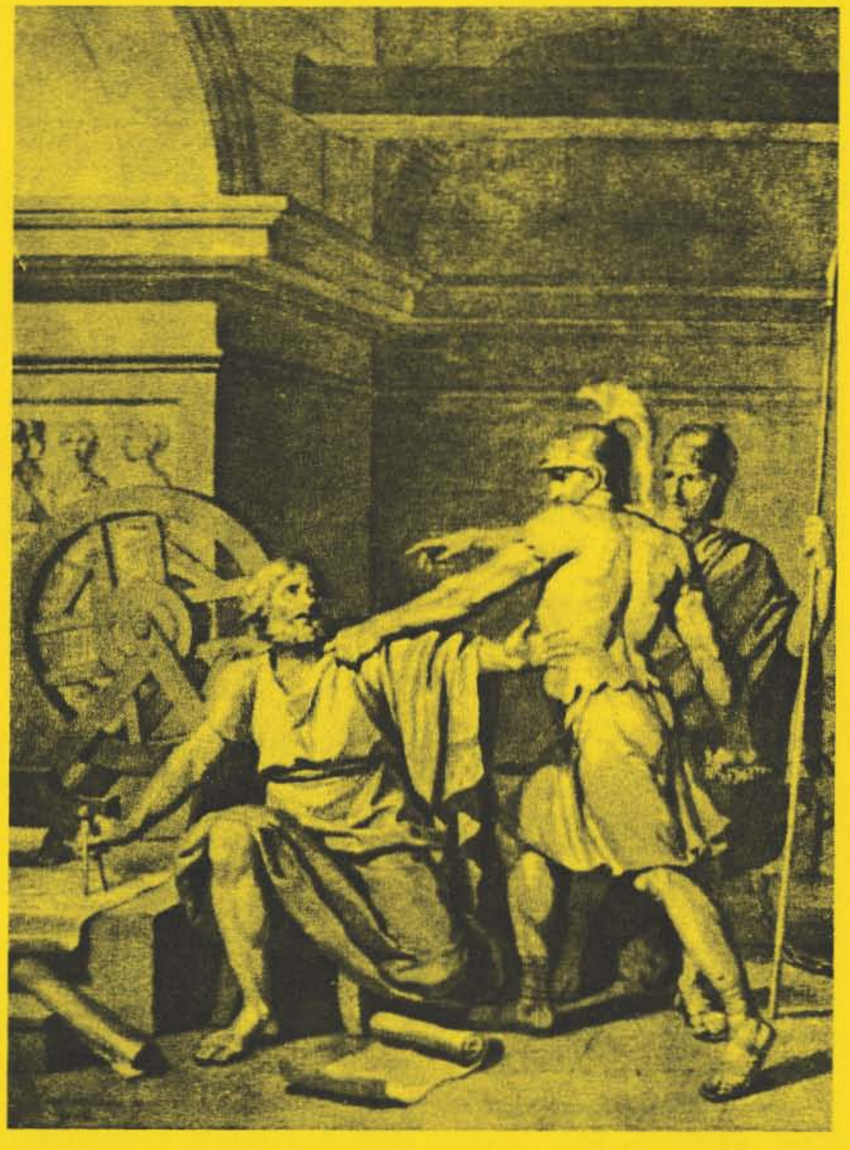

INVITATION TO AUTHORS

Essays, book reviews, syllabi and letters are welcome. Two copies, double spaced should be sent to Alvin White, HUM. MATH. NET., Harvey Mudd College, Claremont, CA 91711. If possible, avoid footnotes and put references and bibliography at the end using a consistent style. If you use a word processor please send a diskette in addition to the typed paper. The Newsletter is assembled using Microsoft Word 4.0 on a Macintosh. It is possible, however, to convert from other word processing systems. Clean typed copy can be scanned (but not dot matrix). Your essay should have a title, your name and address, and a brief summary. Your telephone number (not for publication) would be helpful.

COVER

Archimedes being killed by a Roman soldier while trying to complete a geometric proof.
(by Le Barbier, 18th Century)
From photo of painting in Cornell University Library.
Thanks to Judith Broadwin, Jericho Senior High School, New York.

Supported by a grant from the EXXON EDUCATION FOUNDATION

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From Newsletter #1

August 3, 1987

Dear Colleague,

This newsletter follows a three-day **Conference to Examine Mathematics as a Humanistic Discipline** in Claremont 1986 supported by the Exxon Education Foundation, and a special session at the AMS-MAA meeting in San Antonio January 1987. A common response of the thirty-six mathematicians at the conference was, "I was startled to see so many who shared my feelings."

Two related themes that emerged from the conference were 1) teaching mathematics humanistically, and 2) teaching humanistic mathematics. The first theme sought to place the student more centrally in the position of inquirer than is generally the case, while at the same time acknowledging the emotional climate of the activity of learning mathematics. What students could learn from each other, and how they might better come to understand mathematics as a meaningful rather than an arbitrary discipline were among the ideas of the first theme.

The second theme was focused less upon the nature of the teaching and learning environment and more upon the need to reconstruct the curriculum and the discipline of mathematics itself. The reconstruction would relate mathematical discoveries to personal courage, relate discovery to verification, mathematics to science, truth to utility, and in general, to relate mathematics to the culture in which it is embedded.

Humanistic dimensions of mathematics discussed at the conference included:

a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished versions to be "merely technical."

b) An appreciation for the human dimensions that motivate discovery — competition, cooperation, the urge for holistic pictures.

c) An understanding of the value judgments implied in the growth of any discipline. Logic alone never completely accounts for *what* is investigated, *how* it is investigated, and *why* it is investigated.

d) There is a need for new teaching, learning formats that will help wean our students from a view of knowledge as certain, to-be-received.

e) The opportunity for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems and to participate in controversy over mathematical issues.

f) Opportunities for faculty to do research on issues relating to teaching, and to be respected for that area of research.

This newsletter, also supported by Exxon, is part of an effort to fulfill the hopes of the participants. Others who have heard about the conferences have enthusiastically joined the effort. The newsletter will help create a network of mathematicians and others who are interested in sharing their ideas and experiences related to the conference themes. The network will be a community of support extending over many campuses that will end the isolation that individuals may feel. There are lots of good ideas, lots of experimentation, and lots of frustration because of isolation and lack of support. In addition to informally sharing bibliographic references, syllabi, accounts of successes and failures,..., the network might formally support writing, team-teaching, exchanges, conferences,... .

Please send references, essays, half-baked ideas, proposals, suggestions, and whatever you think appropriate for this quarterly newsletter. Also send names of colleagues who should be added to the mailing list. All mail should be addressed to

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This issue contains some papers and excerpts of papers that were presented at the conferences.

EDITORIAL

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The panel discussion on Humanistic Mathematics in Louisville January 7, 1990 was a lively session where many of the 200 in the audience actively participated. The remarks by Philip Davis are included in this newsletter. The other speakers were Ubiratan D'Ambrosio, Jerry Martin, Deputy Director of NEH, and I moderated. The Network is sponsoring Sessions on Contributed Papers at the January 1991 meetings in San Francisco. See Focus (MAA) for details.

There was some discussion in Louisville about the Network becoming a Special Interest Group (SIG) of the MAA. There is also the possibility that the Network could be represented on some committees of the MAA. What do you think? Volunteers or nominations? The Newsletter needs additional associate editors to solicit and edit essays and book reviews, report on local and regional events, and perhaps represent the Network on MAA committees.

A conference — MATHEMATICS: A HUMANISTIC DISCIPLINE was organized by Mary Sapienza of Newton North High School and her colleagues April 26, 1990. The conference was held in cooperation with the Mathematics faculties of Emmanuel College and Simmons College, Boston with financial support from the Newton Schools Foundation.

A similar conference was convened several months ago by the CUNY Discussion Group, New York.

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Financial support from the Exxon Education Foundation will be phased out soon. Alternative funding is being sought. Other sources of money are members and readers. Subcriptions of $25 will cover the cost of four issues. Smaller amounts will be set for those who are retired, students or unemployed. Tax deductible donations for uncovered costs are also requested. A subscription form is on the back page.

THE HUMANISTIC ASPECTS OF MATHEMATICS AND THEIR IMPORTANCE

(A talk given at Conference on Humanistic Mathematics, Louisville, Ky., January 17, 1990.)

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We are now living in a period in which at least four revolutions are working themselves out — not independently. There are revolutions of ethnic, familial, and gender related values. There are the political and economic revolutions of Eastern Europe, which, in a way, represent the failure of one great idea put forward by rational thinkers to model human realities. There is the media revolution which is already changing the basic patterns of communication, socialization, and learning. There is the computer revolution, bringing with it and enormously facilitating mathematizations of every aspect of our lives.

"Any serious fundamental change in the intellectual outlook of human society must necessarily be followed by an educational revolution. It may be delayed for a generation by vested interests or by the passionate attachment of leaders of thought to the cycle of ideas within which they received their own mental stimulus at an impressionable age. But the law is inexorable that education, to be living and effective, must be directed to informing pupils with those ideas, and to creating for them those capacities which will enable them to appreciate the current thought of their epoch." (A.N. Whitehead, The Aims of Education. The Mathematical Curriculum.)

Mathematics lives in both the technological and the humanistic cultures. It exhibits features that are science-like and features that one normally associates with the humanities.

What are some of the features of mathematics that are humanistic? I seek parallels with literature, which I take as a paradigm of humanistic expression. (And I follow here Jacques Barzun in his 1972 Bollingen Lectures.)

Mathematics, like literature, has metaphor. (Models.)

Mathematics, like poetry, has ambiguity. (Will the **true** geometry please stand up?)

Mathematics possesses an aesthetic component which is strong and which is immediately apparent to the practitioner at the higher levels of the subject.

Like poetry (which, according to T.S. Eliot, cannot be totally written down,) I would assert that mathematics cannot be totally formalized.

Mathematics has paradox.

Mathematics has mystery and can convey awe.

Mathematics has a sense of outcome, a feeling of rightness, and a sense of catharsis.

Mathematics is allied to and has contributed mightily to philosophy.

Mathematics has contributed to theology. It grasps for the transcendental and, in so doing, can be a surrogate for traditional forms of religious expression.

Like literature, mathematics can be an avenue of mental escape from this world.

Like contemporary literature, mathematics is done in a vacuum of belief, if one interprets this vacuum to refer to a formalist philosophy which most (pure) mathematicians adopt when queried about the essential nature of the materials they work with.

Like literature, mathematics exhibits both redemptive and destructive features.

Mathematics has a history. I emphasize this point, not because it is something unique to mathematics, as human ideas of whatever sort presumably have histories, but because it is so often asserted that the truths of mathematics are atemporal, and hence, stand outside history.

Like anthropology and literature, mathematics embodies mythologies. I use the term "myth" not to mean that which is false, but that which is accepted as normative. My friend and coauthor, Reuben Hersh, has written about the four myths of mathematics: its unity, objectivity, universality, and certainty.

Christopher Ormell, distinguished British philosopher of mathematical education, has written about the need for the **demythologizing** of mathematics.

Perhaps what one wants is not so much a

demythification but a de-hocus-pocus-ization to counteract a prevalent feeling among 99% of humans that mathematics is nothing but another form of magic.

It would be an undertaking of the first importance to work out in detail the parallels I have just suggested.

If mathematics exhibits humanistic features, then we may reasonably expect it to promote humanistic values.

Humanistic values are those which foster the consciousness of full human responsibility. To me, the phrase "mathematics as one of the humanities" means nothing if not that.

Mathematics is partly a language. Certain things are communicated by it. Things can be described, predicted, prescribed by it. **The ability of mathematics to provide frameworks of reality and of action, and its ability to change our perception of what is, is very great.**

Our world is rapidly becoming mathematized. Some of our best talent is spent putting mathematizations in place, creating and moving around abstractions. Cosmologists do it. Inside traders do it.

David Berlinski, a mathematician, philosopher, and polemicist, has written that once a mathematization has been put in place, it is all but impossible to remove: "Mathematical descriptions...tend to drive out all the others. Mathematics is often a matter of bondage with <u>things</u> in thrall to <u>theories</u>. Strong theories make for weak objects."

Philosophically, it would be better if we ceased regarding mathematics as that great, objective, people-free, supra-moral, atemporal, reservoir of eternal truths, whose procedures we chant mindlessly and before whose inevitabilities we must bend the knee.

With mathematizations proliferating (whether we install them ourselves or import them), the principal role of the teacher should **not** be to expound formal aspects in a way that may be better accomplished by other means, or to drill on topics that have been automated out even at the market place.

If mathematics is taught simply as the learning of procedures, then none of the humanistic elements can be grasped. It is rare that the teaching of mathematics, elementary or advanced, conveys or fosters the humanistic aspects of the subject.

If there has been a shortfall of self-examination, it is not in the examination of its own inner material by its own methodology, but it is in a steady refusal to examine how the characteristic features of mathematical thought operate on us and affect us.

The principal role of the human teacher should be to humanize the subject.

To teach mathematics as one of the humanities means nothing less than to teach that it possesses the awesome power to influence and change our lives, and to teach that we who use it and foster it must subject it to constant study and scrutiny.

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MATHEMATICS - A SIGNIFICANT FORCE IN OUR CULTURE

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Although I have always had an interest in this topic, my interest increased greatly when I was developing a Survey of Mathematics course for liberal arts students. The textbooks available for such a course stress remedial math and mundane (often contrived) applications. The approaches used in these books, in my opinion, do not give the students a true picture of what mathematics is or the role mathematics plays in our culture. Liberal arts students and educated people outside of mathematics (and some inside) do not have an understanding of mathematics as a vast and important body of knowledge that is an integral part of our culture and a significant force in the development of this culture.

In an article in a recent news magazine, a student stated that "We in the humanities deal with ideas while those in the sciences deal merely with skills". What is disturbing is that this likely reflects the attitude of his professors. What is not realized is that mathematics deals primarily with ideas and that these ideas are some of the most profound and important in our culture.

Recently the University of Wisconsin Centers revised their general education and associate degree requirements. They have adopted Introductory College Algebra (i.e., high school advanced algebra) as a competency requirement for the associate degree. I tried to convince people on the curriculum committee that this was not desirable (It should be an entrance requirement) and not appropriate mathematics for a liberal education in college. The committee could only focus on mathematical skills; they seemed to have no concept of the culturological contributions of mathematics.

In communication between Wisconsin Governor Thompson and University of Wisconsin President Shaw regarding assessment in the University of Wisconsin System, one of them referred to "quantitative skills" and the other to "mathematics". What is disturbing is that likely they thought they were talking about the same thing.

We need to communicate to our students and the public in general that mathematics is <u>not</u> a set of skills (bag of tricks, if you will) to solve mundane problems, but a vast body of knowledge that is an integral part of our culture.

In <u>Mathematics in Western Culture</u> (1953), Morris Kline states, "After an unbroken tradition of many centuries, mathematics has ceased to be generally considered as an integral part of culture in our era of mass education." He further states, "Almost everyone knows that mathematics serves the very practical purpose of dictating engineering design. Fewer people seem to be aware that mathematics carries the main burden of scientific reasoning and is the core of the major theories of physical science. It is even less widely known that mathematics has determined the direction and content of much philosophical thought, has destroyed and rebuilt religious doctrine, has supplied substance to economics and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe. Despite these by no means modest contributions to our life and thought, educated people almost universally reject mathematics as an intellectual interest." In his book, Kline discusses the relationship of mathematics to music, art, philosophy, religion, and literature as well as science.

Raymond Wilder has written a couple of books of significance in this area. They are <u>Evolution of Mathematical Concepts: An Elementary Study</u> and <u>Mathematics as a Cultural System</u>. One important concept Wilder discusses is that mathematical development can be categorized as the result of (i) environmental stress - motivation imposed from outside mathematics (e.g., commerce and simply keeping track of livestock motivating the development of the natural numbers, physics and astronomy motivating the development of calculus, economics motivating developments in game theory) and (ii) hereditary stress - motivation from within mathematics itself (e.g., development of negative (false) numbers, imaginary numbers, Non-Euclidean geometry, quaternions). I prefer to refer to the categories as cultural stress and intellectual curiosity. I stress this dichotomy in my Survey of Math course (and make mention of it in my other courses) and try to convince

my students that utilitarianism is not the reason to study mathematics - much important mathematics was developed out of intellectual curiosity (hereditary stress) without any idea that it would become useful, and later, sometimes centuries later, it proved to be of utilitarian value.

Another excellent source of information is the "Ascent of Man" television series done by Jacob Bronowski. Two of the video tapes in the series, "Music of the Spheres" and "The Majestic Clockwork" have mathematical themes. In the first, Bronowski traces the development of mathematics and culturally related aspects from the Pythagoreans, where Bronowski says mathematics began through the beginnings of the change in mathematics from a static to a dynamic description of nature. The relationship between music and mathematics was discovered by the Pythagoreans (6th century, B.C.). I understand that in some ancient universities, music was considered a branch of mathematics. Someone once said, "Mathematics is the music of the mind; music is the mathematics of the soul." I know I said it, but I believe I read it somewhere else first. Bronowski traces the flow of culture to the Islamic domination where he says the mathematician and the artist were one (and says he means that literally). The Islamic period art was very mathematical and made extensive use of symmetry in its patterns. Another concept that relates mathematicians to art is perspective (Albrecht Dürer, 15th century). This was the foundation for the branch of mathematics known as projective geometry. Related to this is the development of graph paper which was invented by the artists, not the mathematicians. Bronowski then traces the development through Spain, into Europe and the development of calculus as a response to the needs of physics and astronomy. We see here the changing role of mathematics — commerce, astronomy (& astrology), engineering, music, art, physics, and more recently economics (Morgenstern and von Neumann), and biology. Part of the maturity of an intellectual pursuit is the mathematization of the subject.

We get some perspective on impatience with resistance to change to the metric system when we realize it took Europe 400 years to accept the Hindu-Arabic numeration system over the Roman numeration system despite its obvious superiority in many ways.

The other mathematically oriented film, "The Majestic Clockwork", discusses Newton and Einstein and their contributions as well as their relationship to their peers and the public. One interesting story is a discussion between Edmond Halley and Newton when Halley asked Newton what would be the path of a body if the force acting on it varied inversely as the square of the distance. Newton responded immediately that it would be an ellipse. Halley asked how he knew, and Newton replied that he had calculated it. Halley asked to see the calculation. It took Newton three years to develop the proof and the document was hundreds of pages long. Another view that appeared in Scientific American suggests that Newton got the idea that it was an ellipse from Robert Hooke, but that Hooke was not a powerful enough mathematician to develop it deductively.

What, then, is the appropriate mathematics for the liberal arts college student. In my opinion, it is the study of mathematics as an integral part of our culture, not only the contributions of mathematics to our culture, but how the culture has influenced the development of mathematics. I have been attempting to convey this in my Survey of Mathematics course. Sometimes it bombs, and sometimes, as last semester, the students do very well and are very generous in showing their appreciation for the class.

I decided that in this course, I would try to instill in the students an appreciation of mathematics as an integral part of our culture, and at the same time develop, in a somewhat deductive manner, a mathematically significant relationship. I chose the Euler relation, $e^{i\theta} = \cos\theta + i\sin\theta$ because of its great unifying nature. It relates the exponential function, imaginary numbers, and trigonometry; seemingly diverse concepts. Of course, other topics come up along the way; some are needed for the development and some are interesting, I hope, digressions. Here is a brief outline of my lectures.

I. Number Systems - development of the "number tree" from the positive integers to quaternions including motivation for each extension and geometric considerations.

II. Mathematical Systems
   1. Groups
   2. Fields
   3. Ordered Fields
   4. Complete Ordered Fields

III. Exponents, the Exponential Function, and the Logarithmic Function - including application in growth and decay.

IV. Sequences and Series - informal discussion including convergence, divergence and intuitive concept of limits.

V. Probability, Binomial Distribution & Binomial Theorem
   1. Empirical and theoretical probabilities

2. Counting procedures (elementary combinatorics)
3. Proof of Binomial Theorem by use of combinations
4. Development of

$$e^x = \lim_{n\to\infty}\left(1 + \frac{x}{n}\right)^n$$

by use of Binomial Theorem

VI. Elementary graphing concepts (Cartesian coordinate system).

VII. Wrapping Function and Trigonometry
1. Basic concepts
2. Sum and difference formulas
3. Trigonometric form of complex numbers
4. DeMoivre's theorem and other theorems

VIII. Euler Relation
1. Proof of Euler Relation
2. Development of Maclaurin Series representation of sine and cosine.

Students in my survey class often complain that they don't "understand" some of the mathematics that we discuss. I am also asked by my colleagues if students in this type of class understand all the mathematics. I like to tell them a story about John von Neumann that I found in a book by Gary Zukow, The Dancing Wu Li Masters (this is a book about the new physics). When von Neumann was working at Los Alamos on the hydrogen bomb project, a young physicist asked von Neumann for help on a difficult problem. von Neumann responded, "simple, this can be solved using the method of characteristics." After an explanation, the physicist indicated that he did not understand the method of characteristics. "Young man", von Neumann said, "in mathematics you don't understand things, you just get used to them." This tells me that it is possible for people to get an understanding of the place of mathematics in our culture and an appreciation for mathematics without necessarily understanding all the mathematics. If it's good enough for von Neumann, it is good enough for me.

Another aspect of this course is independent study on the part of the students. Throughout the semester, students read selections chosen from a bibliography (the current state of this evolving bibliography is included in this paper), make periodic oral reports, and three written reports. The material they are reading may or may not have anything to do with my lectures, but is geared toward culturological aspects of mathematics, and class discussions of the oral reports are steered toward this.

I offer this approach as an alternative to the ever increasing emphasis in mathematics education on remediation, basic skills, and mundane applications of mathematics.

I would appreciate any comments you have and corrections and additions to the bibliography.

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HEURISTIC THINKING AND MATHEMATICS

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Thesis

Mathematics is created by human beings; it is a monumental artifact of human reasoning. Those who would study this field seriously need to know not only mathematical results, but also the reasoning processes by which these results are achieved. Human reasoning processes are the cement that holds key ideas together as well as the conduit through which information is generated. Among these reasoning processes are heuristics, or methods for inventing problem solutions. One who understands and uses heuristics sees better the cohesiveness of the subject and is more capable of discovering new information about it. For example, we can chunk whole courses down into a handful of key related ideas. Calculus is function, limit, derivative, integral and fundamental theorem. Linear algebra is vector space, basis, matrix and linear transformation. We can also take a single idea such as rate of change in calculus or vector in linear algebra and produce many different instances that embody the same fundamental idea. So, in the process of doing mathematics, we move information around; we regroup old information, we generate new information, and we manage it and mold it like a sculptor to suit our needs and to satisfy our aesthetic sense. In our models, we emphasize form instead of content, yet we expect our students to intuitively understand what that means. This uncanny ability humans possess to manage information should be a focal point of mathematical instruction. It should not be regarded as incidental subject matter which might be picked up by perceptive students. Some say the only way to become a good problem solver is to solve lots of problems. Hopefully, they mean that solving lots of problems is a necessary but not sufficient condition for becoming a good problem solver. Unfortunately, too many mathematics professors hold the opinion that their students will deduce or induce the important strategies of problem solving just by observation — the "immersion/osmosis" approach. This may be humanizing mathematics by requiring considerable human effort, but it is unrealistic in that it fails to stress those cognitive behaviors that uniquely characterize the human aspect of the development of mathematics.

Before our students can achieve real learning of mathematics, they need to develop a proper perspective on the nature of the subject, itself. There is a folklore about mathematics, and in this folklore are certain myths that bias their view on how to perceive and learn the subject.

Myths

Here are five common myths that tend to block mathematical learning:

Myth #1

Mathematics is a collection of isolated facts and tricky techniques that must be memorized.

This perception of mathematics overwhelms people. It is fostered by texts and teachers that present the subject in a fragmented way — by classifying problems and categorizing techniques for handling specific kinds of problems. A better perception is to visualize mathematics as a network of related ideas with means of moving to and from these ideas. It helps to understand that no problem exists in a vacuum — there are other problems related in some way to any given problem. Some memorization is undoubtedly necessary, but searching for ways to minimize memorization has the effect of maximizing the understanding of relationships between ideas.

Myth #2

Mathematical truth is absolute.

Those who would believe this have great difficulty being flexible. They also ascribe to mathematics a superhuman quality. Belief in a principle is good, but unwillingness to change our beliefs in the face of compelling evidence is foolish. We must always strive to answer the question "Why?" and dare to ask the question "What if...?"

Myth #3

Mathematics is an exact science.

This is a popular layman's belief. Mathematics deals with numbers; numbers are exact; therefore, mathematics is exact. "You either get the right answer or you don't." But we who work with mathematics know differently. Even in the theoretical foundations of mathematics, there are unresolved questions about the axioms. And when the axioms change, the mathematics changes. Moreover, whenever mathematics is used to model reality, approximation and estimation become the norm rather than the exception.

Myth #4

Mathematics deals primarily with symbolic representation and manipulation, so ordinary writing and speaking skills are not necessary for communicating mathematics.

Anyone who has graded written problem solutions or proofs or who has tried to evaluate oral presentations in mathematical topics has felt firsthand the effect of this attitude. While mathematics, like music or art, can be correctly regarded as a universal language, its clear communication in any culture depends on carefully constructed written and spoken language. As more mathematics develops, technical writing skill and precise verbal communication become increasingly essential to the user and doer of mathematics. One of the largest problems facing industry today is the inability of technical people to communicate.

Myth #5

Mathematics is something one does alone.

This is only a partial truth. True, some mathematics is perhaps most efficiently handled by individuals (e.g. intensely focused problem solving). But more often than not, complex problems requiring mathematical solutions are being resolved by teams and think tanks. In many instances, sharing ideas and teamwork enhances individual creativity rather than stifling it. Often it is simply too inefficient to try to produce an effective solution by relying only on individual inputs.

Some or all of these myths characterize the beliefs of our students about the nature of mathematics. As a condition of mathematical learning, we need to dispel such myths. Our students need to view mathematics as a cohesive body of knowledge. They need to challenge their own beliefs, to place a value on approximation, to communicate clearly and precisely, to learn to work together, and to incorporate mathematical heuristics into their own thinking.

Heuristics

George Polya defined "heuristic" as a rule of thumb or method by which a person invents a solution to a problem. The method of solution itself highlights the principal difference between an exercise and a problem. An exercise requires little, if any, creative thinking, except to classify it, identify a suitable algorithm, and execute the algorithm. A problem, on the other hand, requires that a method be invented, and its solution depends heavily on certain thinking strategies of the solver. There is a parallel distinction between algorithm and heuristic. An algorithm is a stepwise procedure that is class or problem-specific and which guarantees success if the steps are taken carefully. Heuristics, by contrast, transcend classes of problems, but they are tentative processes with no guarantee of success. For example, there is no guarantee that drawing a diagram will really help us solve a given problem. But there is some strength to the belief that application of heuristics over the long run increases our probability of success. Some heuristics identified by Polya which seem to characterize the mathematical thinking of experts are the following: devising a model, using analogies to discover related problems, searching for patterns, working backwards, applying successive approximation, reducing a problem to a simpler case, extending a problem by generalization, exploiting symmetry, checking, solving a problem in several different ways, and studying a correctly solved problem. Following Polya's method of teaching by inquiry, heuristics are best illustrated by asking lead-

ing questions that guide the problem solver to think in a certain way. Here are some examples:

(1) We know from geometry the relationship between a tangent and radial line intersecting on a circle is that of perpendicularity. Are there any similarities with respect to ellipses and hyperbolas ?

(2) Solving a quadratic equation with real coefficients is easy. What if we change the coefficients to complex numbers?

(3) We are to prove: Given an $e > 0$, we can find $\partial > 0$, such that $|f(x) - L| < e$ whenever $0 < |x - a| < \partial$. Where do we start? How do we chain our reasoning?

(4) Our problem is to devise a formula for counting the number of elements in k sets. What makes this problem difficult? Can we make it into an easier problem?

(5) Suppose you know the number of space compartments formed by k planes in general position. What is the effect of adding one more plane? How many new compartments are formed? Is there a related situation in lower dimension that may help?

(6) We want a fresh perspective on voting patterns in an election. Suppose we think of political issues as dimensions and voters as points in a geometric "issue space." Does this help us to understand or predict voting patterns?

(7) By suitable integration, we find that the area of an ellipse with semidiameters a and b, is πab. Is this result reasonable? Can we check this answer in different ways?

(8) We are to find an integer a such that the polynomial $x^2 - x + a$ divides the polynomial $x^{13} + x + 90$. Obviously, one method is to try various values for a and use long or synthetic division. But can we find a way to reduce the possibilities for a first?

(9) If linear transformations can be represented by matrices, is there a matrix operation that corresponds to composition of linear transformations?

Questions such as these are designed to elicit heuristic thinking from students. As with heuristics themselves, they do not guarantee success. But they are characteristic of the way in which mathematicians think, and students of mathematics need to get in touch with human reasoning in mathematics as well as the mathematics itself.

Classroom Suggestions

Formulated as questions, two principal educational issues have been raised in this paper, namely: (1) How do we dispel myths that tend to block mathematical learning? (2) How do we get our students to incorporate heuristics into their own thinking? These are tough problems because there are no unique solutions, and optimal solutions are yet to be found, if they exist at all. What works best for one professor may not work well at all for another. Based on my own experience over a number of years in the mathematics classroom, here are some suggestions I would make:

(1) Encourage students to learn to carry on a conversation with themselves, using sequences of internalized leading "self-questions" that elicit heuristic thinking. To help accomplish this, ask lots of heuristic-oriented questions in class, making students active participants rather than passive spectators. Have them step back and examine their own thinking, comparing it with the thinking of good problem solvers, and then use heuristic-oriented questions and answers in their communication with others.

(2) Hold problem solution critiques occasionally at the blackboard, where a student presents his/her solution to the group (including comments on reasoning), and the group discusses the solution and the reasoning. This is like a painting critique in an art class or analysis of a musical composition, except here the masterpiece is a problem solution.

(3) Have students work in teams on a project (e.g. mathematical modeling, mini-research project, computer-supported solution). This helps them to actively recruit and observe the ideas of others, using their expertise when necessary. By sharing ideas, students practice communication.

(4) Require some technical writing, preferably early in the student's mathematical careers. Often students have difficulty with mathematics because they have not taken the time to formulate precise questions. Have them write down questions or write out a complete problem solution or proof. This forces them to organize their thinking and it also improves their ability to communicate.

(5) Assign fewer exercises and more problems. Higher expectations of students, kept reasonable, usually lead to better performance.

Mathematical instruction needs to change. As mathematics professors, we need to dispel myths that block mathematical learning; we also need to get familiar with and communicate explicitly those heuristic processes that differentiate experts from novices in our field, so that our students will incorporate these thinking strategies into their own perception of what mathematics is really about. Through mutual discovery and invention, students and teachers together can trace the path of mathematical thought as a truly human endeavor.

Preparing Teachers to Teach Mathematics within a Humanistic Perspective

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Introduction

As a newly hired faculty member of the Mathematics Department of the University of Campinas Brazil, I was assigned the course Finite Mathematics, for the 1988 academic year. It was a one year long course in the program for future secondary mathematics teachers[1]. This course was embedded in a new program that was implemented as of March 1988. It was the very first night-time program to be offered by that University.

About the program

The new program was intended to be different from that offered during the day, in an attempt to make change occur in secondary teacher education. It was designed to prepare secondary mathematics teachers within an approach that had several "humanistic" characteristics. For example, throughout the program mathematics courses and education courses would be integrated. The different mathematics courses would exemplify different methodologies proposed in education courses.

It was expected that the students choosing a night program would be quite distinct from their daytime colleagues, with differing personal characteristics such as age, maturity, responsibilities and experiences, as well as having quite different career goals and professional aims. The program was designed to cater to the needs of this different population.

Another characteristic of the program was that it was designed specifically for future secondary mathematics teachers. Traditionally the pre-service program was very similar to the pure mathematics programs, except that the students were required to take a few less mathematics courses that were substituted by courses offered by the School of Education. All mathematics courses were those designed for any field of mathematics, with no relationship at all to the students future professional goals. The criticism voiced by Morris Kline (1977), that many American universities do not prepare future mathematicians for their role as teachers can equally be made of the mathematics programs of most Brazilian institutions of higher education. Therefore, the intention of this new program was to deal with this problem raised on many occasions, especially by students of previous programs who pursued teaching careers.

Other characteristics of the program that allow us to categorize it as a program educating future teachers within a humanistic perspective are the following:

a) Each instructor deals with the students as future teachers. All students in the classes are pursuing the same degree.

b) From the beginning of the students' involvement in the program they are pursuing their professional development. All work done in the courses is related to their future professional endeavors. Whatever the discipline being taught, some reference is made to the teaching and learning of mathematics.

c) Whenever possible, relationships are drawn between the theoretical aspects dealt with in the education core of the curriculum and the practical aspects of mathematics learning, dealt with in the content core of the curriculum.

d) A special emphasis is given to practical experiences, students' contact with children occurs much earlier than in traditional programs. This rests on the belief that these students construct their understanding of mathematics learning and teaching based on their experiences in the teaching-learning process.

About the students

The population of students enrolled in the program is unique to the institution. The individuals in this group were working full-time with very little time out of the classroom to invest in studying. With daily classes from 7 pm to 11 pm and an 8 am to 5 pm workday, one could not expect much work during the scarce hours that are left in their days.

There was something special about the group with respect to their feelings towards mathematics. Unlike other groups of students with career goals that are not the teaching of mathematics, these students did not appear, at first, to feel anxious about

their mathematical ability. This proved to be a false assessment when they began preparing for their calculus examinations. Somehow, due to attitudes of the calculus instructor, they never felt comfortable with their expertise in problem solving to deal with calculus problems. Due to the nature of their calculus examinations, it was clear to them that they were safer by memorizing as many formal solution processes as possible. Unfortunately, within the perspective adopted in the course, this did not generate legitimate mathematical thinking, instigating reproductive thinking rather than productive thinking by the students.

About the Finite mathematics course

The intent of this report is to discuss the teaching perspectives assumed throughout the 1988 academic year in the Finite mathematics course and relate the successes and difficulties of educating teachers for work within a humanistic approach.

The basic principles of the course, in line with the program, were the following:

a) These students were being prepared to be future mathematics teachers for grades 5 through 12.

b) They had had a school lifetime's experience with mathematics and consequently had developed their own beliefs and attitudes about mathematics, its learning and its teaching.

Several studies have documented attitudes and beliefs of individuals with respect to mathematics teaching and learning (see Schoenfeld, 1985; Thompson, 1985; Robinet, 1989). Many of these results were confirmed with the pre-service group being discussed.

c) The dichotomy between mathematical content and mathematical teaching methods had to be broken. This could only happen through a joint effort of the content core and the educational core. We know that the education core alone has traditionally had very little success in breaking this dichotomy. Thompson (1985) states that:

> There is research evidence that teachers' conceptions and practices, particularly those of beginning teachers, are largely influenced by their schooling experiences prior to entering methods of teaching courses. ... We need to explore ways in which to articulate mathematical content courses that teachers are required to take with methods courses, so that the learning experiences are consistent with those advocated in the latter. (pg. 292)

With these three aspects as guiding forces for the work of the year, the curriculum was designed for the course.

Basically the approach would be a problem-solving approach involving small group work and occasional large group discussions.

Instruction was designed within a constructivist paradigm. Experiences are believed to be essential for the construction of ones' concepts, hence the learning experiences about the teaching of mathematics and legitimate involvement with mathematical thinking should be an integral part of the experience of future teachers. According to Kline (1977) mathematics courses fail for never involving students in the creative act of doing mathematics which would include "the fumbling, the guessing, the blundering, the mental struggles, the testing of hypotheses, the frustrations, the false proofs, the insights, and other acts of the creative process" (pg. 129). These aspects of mathematical thinking are rarely present in the students' mathematics experiences, unless of course they pursue graduate work in this field, when they will, all of a sudden, be expected to understand this as being legitimate mathematics (experiences which are reserved exclusively for research mathematicians).

With these concerns in mind, looking towards providing experiences that would involve students in legitimate mathematical thinking, a problem-solving approach was the main mode of instruction selected for the course. Every class period began with a list of problems to be solved. This list occasionally took more than a class period to be completed, but each group was given as much time as they required for the solution. Groups finishing earlier were given additional problems, which never generated any complaints. During the solution process the instructor went around the classroom observing students at work and groups interacting and often modeling metacognitive behavior, serving as external monitors to students during problem-solving activities. Students were constantly reminded that the group work did not consist of finding a solution, but that each and every member of the group reach a solution upon which the group agreed. Furthermore, every member of the group should be sufficiently convinced of the solution to be able to explain it in the end.

Group work consisted of the normal interactions of a group of individuals pursuing a similar goal. However, when a member was satisfied with his/her

solution they should assume the role of "teacher" in the sense of trying to help colleagues with their own solutions. As was to be expected solution processes differed greatly and it was the role of the acting "teacher" to try to understand the colleagues' solution, and ask appropriate questions that would focus on misinterpretations in the understanding of the problem or in execution errors.

At the end of a problem set a large group discussion took place. The intent was to formalize any concepts that had arisen, to clear any doubts about any of the problems through intra-group discussions. Basically, this was the moment to discuss both mathematical concepts and dynamics of group interaction. The discussion of students' beliefs and preconceived notions about the nature of mathematics, the nature of learning and teaching mathematics were often part of the large group debate. In general these were issues raised as a consequence of conflicts between the activities in which students were involved and their previous experiences in learning mathematics.

Tensions and Conflicts

There were several difficulties in the implementation of a problem-solving methodology with this group. Some of these were felt as tensions between the students and the instructor. At other times conflicts were observed in the students, between their previous experiences and consequent expectations and the new experiences they were having. Examples of these tensions and conflicts are described here.

a) Students were quite resistant to change and resentful that they were not told exactly how to solve the problems before being given the problem. They considered the sequence of activities to "inhibit" their ability to solve the problems given. Comments were often heard of the type: "I think this pattern is probably a geometric sequence, or an arithmetic sequence..." Revealing that students would first attempt to categorize problems as being of a type for which they had a pre-established solution sequence. In doing this students soon became aware of their lack of thorough understanding of several concepts which they were throwing around for consideration.

b) Students commented that the instructor "did nothing during classes". They were unhappy by the fact that they had such few notes from classes.

c) Students did not know how to work in small groups. They would work independently and then often reveal their beliefs about effective mathematics teaching by telling colleagues who were struggling with a problem exactly how to solve it. Colleagues would readily say "Oh, now I understand," revealing their beliefs about the process of learning mathematics. The instructor would respond to the situation by asking questions about the solution process used or giving a modified version of the problem or an isomorphic version of the problem. This often led to a discussion with the students about the meaning of "understanding".

d) In the beginning of the semester students would constantly ask: is this right? is the answer "x"? To which there would be a response with a question about the solution process or another problem. Very often the students would say: "Oh, I guess its wrong, or you would just have said yes or smiled." They were usually expecting the positive reinforcement that had always cued correct answers throughout their learning experience.

e) Students were not accustomed to having an active role in mathematics classes.

f) Evaluation: assessment of this type of instruction has many difficulties. First it is not possible to use a traditional form of individual work to evaluate student progress. Second, evaluation is not a static process that can be pinned down to a certain moment, it is a continuous process, possible by the fact that the instructor is in continuous observation of the class members and their activities. Students resist any type of evaluation procedure with which they are unfamiliar and consequently do not trust. Since evaluation is a delicate issue and often put aside in pre-service education programs, in fact one might venture to say that it is avoided in most cases, this became a major discussion point in the large group work. The evaluation procedures used during the course were proposed by the students. Immediately after which we discussed the pros and cons of each procedure, both from the students perspective and from the instructor's perspective. An example of such a procedure was the following: a three step evaluation process in which students would solve, individually, a problem: this would be handed in, then they would discuss the problem in a group and rewrite the solution, incorporating the new aspects gained from group discussion. This process generated much discussion and many pros and cons were raised, but probably the most important consequence was a definite change of attitude toward the group work. Students decided that the period of individual work was essential for the success of group interactions. During the individual work period students thought on their own and were able to each contribute to the group discussions.

Observations

Of course, final results will only be noted on a long term basis, by analyzing students' mathematical thinking skills in other classes or their teaching skills when they actually assume mathematics classes. It is the observation of these individuals in their teaching practice that will permit an analysis of the validity of the procedures with respect to their teaching practice, the program's ultimate goal.

However towards the end of the year observations were made of changes in students' behavior and attitudes about teaching and learning mathematics. The observations discussed here are primarily interpretations of students' comments in small and large group discussions as well as observations of small group interactions.

Students attempted non-routine solutions with more ease even in other mathematics classes, apparently they were no longer stuck to one solution procedure.

Through the group work it became clear that knowing how to solve a problem was not enough to teach an individual the solution, in fact students claimed that it was quite easy to find a solution to a problem without really knowing what they were doing.

Students claimed that "showing someone how to solve a problem defeats the purpose of education, you spoil the positive feelings and attitudes that are consequent of the efforts of solution and being able to come to a result and being convinced that you have found a solution".

Students, at first, were quite surprised at the number of different solutions possible to a problem. Furthermore, they were also surprised, when put in the "teacher" role in the group activities how difficult it is to ignore your own solution and attempt to understand your colleague's solution ("student" role), trying to look for possible errors in the understanding of the problem or in the solution process itself. The "teacher" role assumed on occasion by every student helped develop their questioning skills. It was "difficult to hold your tongue" when all you really wanted to do was say: do this. This was a statement made by a few of the students and was confirmed by the reduction in the number of occasions in which students were observed simply telling colleagues how to find a solution.

There is a form of intrinsic motivation in a problem-solving environment, this conclusion can be drawn from the following fact: classes were held from 9 pm to 10:30 pm. Unlike previous teaching experiences, rarely did students involuntarily signal the end of class (by closing notebooks, or becoming fidgety and looking at their watches). On almost all nature: "already?! It feels like we had just begun. I was just starting to warm up." Students were busy and active to the very end of classes. On rare occasions did the instructor have to draw students' attention back to the problems at hand.

In trying to assess the nature of this motivation students commented on the challenge of each problem situation. And that group work made them feel comfortable in taking risks at solutions. Normally, in the privacy of their own work the negative feelings of failure would soon overwhelm their desire to continue working on the problem.

Conclusion

In order for teachers to feel confident that there are alternative ways of teaching mathematics that are more effective than the traditional methods, they must experience different learning situations themselves. The knowledge they have acquired through rote must be challenged. In other words there must be some conflict created in their beliefs about effective learning and teaching of mathematics.

It is not through simply discussing the importance of a humanistic approach to the teaching of mathematics that we will effectively create conflicting situations that challenge their beliefs. It is from these challenges and their resolutions that learning will occur, and in fact learning about the teaching and learning of mathematics can be the focus of these challenges and should be the focus for future teachers.

The experience revealed the urgent need for reform in teacher education programs especially with respect to the content courses and the dissolution of the dichotomy between the content and education cores of teacher preparation programs. It became clear that beliefs about effective mathematics teaching overpower any learning that may occur in methods classes, and maybe explain how the traditional teaching of mathematics has perpetuated throughout the years in spite of all attempts to reform and change mathematics instruction in schools.

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[1] In Brazil the University pre-service mathematics program certifies teachers for grades 5 - 12. In some cases university graduates pursue teaching careers in small private colleges.

Advanced Displacement Exam

compiled by
Robert Messer
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Match the correct term with each definition.

1. That which Noah built.
2. The parrot left.
3. What a bloodhound does in tracking a woman.
4. Title for a knight named Koll.
5. A sunburned man.
6. Undergarment made of sea moss.
7. Two grandmothers from Llelo.
8. Aromatic measure.
9. The nominal in love.
10. Occupied lavatory on an airplane.
11. A writer for a hotel.
12. Avoid dull males.
13. Sneaky slobbering.
14. Animal hair induces tingle.
15. What one does to unwanted trees.
16. What the captain said when the boat was bombed.
17. What the acorn said when it grew up.
18. What people do in a canoe.
19. What you do with yarn and needles.
20. Jewish festival greeted the lady.
21. Frost covers Indian cymbals.
22. Do this when it rains.
23. A dog sitting in an ice chest.
24. Can George Washington run for reelection?

Algebra	Decagon	Perimeter
Arc	Dimension	Perpendicular
Axiom	Geometry	Polygon
Center	Horizontal	Slide rule
Centimeter	Hypotenuse	Tangent
Circle	Inscribe	Unit
Coincide	Parallelogram	Vertical
Cosecant	Parsec	Zero

Real Needs of School Children

"Paper distributed at the final plenary meeting of the New York City Chancellor's Committee for the Improvement of Mathematics Education"
December 16, 1987

Hassler Whitney

Here is a child, on the way to school in a large city. Where are his or her thoughts? "What's the use? Big brother is in the remedial class, and knows he will always be there. Teacher asked me a silly little question yesterday and praised me for answering. Teacher explains all the obvious things, but I have no idea what she is talking about. I just want to get out."

We are in a time of unrest, pressures, and sham. Budgets are balanced by manipulating figures and people are elected for uttering fine sounding phrases. "Help the minorities take part in the coming economy of the city," and the first demand is to throw out the principal if the school doesn't measure up (the test scores remain low). In the urge to show "accountability," the teachers are told to drill and monitor the children more. Isolated facts and processes are poured into the empty heads; connections among these splinters are missing and the whole has no relevance to the children's lives. The more the pressure, the worse the results.

With help and support, the teachers could aid the children towards true growth for a decent future life. By letting the children bring in prices of all sorts of things, and discuss related questions among themselves and with the teacher, with much reading and writing, they will soon have a solid and extended sense of "place value," at present badly missing in the classroom. And , in groups, finding secure ways of subtracting one amount from another, subtraction becomes one topic, controlled by the child. (There is now way to "remediate" a morass of isolated notions, all mixed up with each other.) In such ways, true progress is made, tests are passed through control of the topic, and the next topic can be taken up similarly; the class is alive and flourishing.

How can one go after better ways with different groups talking in different directions, with very different objectives in mind? Under true leadership, a study group can be set up which will go deeply into the actual functioning of the schools, staying on basic goals and needs, and keeping constant contact with all groups concerned to resolve the basic issues so separating us at present.

Mathematics and Ethics

Reuben Hersh
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I want to start off by correcting any possible false impression that I'm going to tell you what is ethical, or that I've solved any big problem regarding mathematics and ethics, because I certainly haven't, and make no such claim. Of course, the next question you ask is, why am I standing up here anyhow? It's only because I have thought about the question, and in the process of thinking about it have had some ideas which I'd like to offer you.

The observation that got me started on this was that in many professional fields, there has been for a while a well established concern with ethics. What that means varies from field to field. But the idea that a professional association of engineers or statisticians might concern itself with ethical behavior in that field is not radical at all. It's a very standard thing. Often it's done officially by the establishment. Often there are active concerns on the part of special organizations, editorials in journals, and so on.

One of the first organizations of this type that I had contact with, long before I was a mathematician, was the Society for Social Responsibility in Science. I'm not sure it still exists. In its day, the 50's and the 60's, it was primarily concerned with nuclear arms, nuclear warfare, nuclear destruction of the human race. It consisted largely of physicists, many of them Quakers or Quaker sympathizers. They took the position that there was a question of social responsibility, for the physicist particularly, whether he should be working on nuclear weapons. Some people refused to work on nuclear weapons, or quit military jobs. Whether you agree with that or not, this was a legitimate issue in the physics community.

Another example arose with the environmentalist movement. Barry Commoner, of St. Louis, was an outstanding spokesman. This movement involved biologists and also chemists, because chemists do a lot of polluting. Not chemists themselves, the things that chemists create. There again, was a question of social responsibility, which is one aspect of ethics.

I have in my band two actual codes of ethics. One was adopted by the statisticians' society, and the other by the professional engineers' society. These are not so political. They have more to do with proper behavior toward one's client, ethical issues of that sort. No doubt you could find other examples.

For a mathematician, it's natural to ask, how come we don't seem to be concerned about ethical issues or discuss them? It is true, as many of you know, that recently there was a referendum in the American Mathematics Society. There was a long-drawn-out political hassle, and in the end seven motions were passed by the membership. The one that is probably most controversial says that the Society should not involve itself in helping the Star Wars SDI activity to recruit among A. M. S. members. That issue certainly has ethical implications. But it was a one-time, ad hoc thing, not an indication of continuing concern or involvement with ethical issues by mathematicians. In my opinion, the reason it became a big issue in the AMS was that there had already developed strong opposition to the SDI among physicists and computer scientists, both in individual departments and in national organizations. I think that was why some mathematicians felt that we should also get involved. In the end, after a lot of back and forth haggling, the membership approved the anti-SDI motion. So there is an example of an ethical issue that did come before and actually passed the American Math Society. That's not the main thing I want to talk about. I just mention it because some of you might have it on your mind and might remember it.

The thing that is striking, you see, is that in all the other examples I've given-the biologists' involvement in environmental issues, and the chemists as well, and the physicists in nuclear war, and the statisticians requiring that if you are a good statistician you won't give away your client's data-these are all different, but they have one thing in common. They are all in some way intrinsic to the actual practice of the particular profession. The physicists are the ones who make the bombs, the chemists are the ones who pollute, and so on. When I thought about the situation of mathematicians, I found I was oscillating between two different viewpoints. On the one hand, a mathematician is somebody who solves a problem or proves a theorem and, of course, publishes it. And it's hard to see significant ethical content in improving the value of a constant in some formula or calculating something new, say cohomology of some group. You might say it's beautiful or you might say it's difficult, but it's hard to see any good or evil there,

in the way physicists and biologists, and so on, do have ethical problems. On the other hand, if you step back from that particular way of looking at the role of mathematicians and just think about your own activity, or mine, think of what we actually do, daily and yearly, there are constant decisions and conflicts involving right and wrong.

The ethical demands of all the scientific groups seem to fall into three categories: What you owe the client, what you owe your profession and what you owe the public. Now, if you are a math professor, the word "client" may be unfamiliar. Who is the client, anyhow? But there is always a client, in the sense of the one who's paying your salary. The ethics of the statisticians and engineers are almost entirely concerned with duties to the client. And then there is the profession. What do you owe your partner, colleague, or fellow professional? In some ethical codes, that's up at the top. I think that's the way it is with lawyers and doctors. Doing something unethical means treating some other lawyer or doctor unfairly. Duty to the public is an afterthought.

Now to the mathematicians. I can list five different categories of people to whom we have duties: staff, students, colleagues, administrators, and ourselves. First are the staff, the people who do the work that we don't want to do. It would be interesting to think about the situation or treatment of the non- faculty employees of your department. Do you regard it as equitable? If you don't, does anybody ever try to do anything about it?

Then there are the students. For instance, there is the problem of mathematical illiteracy. I don't mean to suggest that we owe students mathematical illiteracy. Rather, the existence of mathematical illiteracy poses an ethical issue. Is the prevalence of mathematical illiteracy among students in part a responsibility of us, their teachers? If so, what can we do about it? This issue needs to be mentioned because so many of us deny our responsibility and blame the high schools. Next example: grading. Again, we don't usually think of this as an ethical issue. We try to make it a mechanical matter, a rule, and let a machine do it. But despite our machinery, there are always hassles and disagreements about grades. I think that the grade you finally give, whether it's a number or a letter, is not just an objective application of some rule, but also to some extent an ethical choice. What do you think is more important, more valuable than something else? I would say grading should be included in the ethical life of the mathematician. I've had a student from some place in the Near East tell me that if I didn't change his grade, he'd have to go home and go in the army and get killed, and it would be my fault. For all I knew it might have been true, except I didn't change his grade and he was still there a year later. That's an extreme example of an ethical issue: murder associated with grades.

Finally, gender and ethnicity. This has been subject of a good deal of talk in recent years. There are, to some extent, special programs to help women and to help Blacks, Hispanics and Indians to attain a higher level in mathematics. Not many people here, I would guess, are involved in that activity. And there are certainly differences of opinion about it. But it's a clear case of an ethical issue.

Colleagues. This, I think, is the big one, the one that most of us are most involved in. Hiring, tenure, promotion-these are the issues that department meetings hassle about. Sometimes, I suppose, decisions are made entirely on an objective basis of what's best for the department. And then again, sometimes people try to help their friends. But before you get down to who gets hired, and who gets tenure, I think there has to be some value, some assumptions about what's important, what's legitimate, what you want to do. It's usually assumed that this is already given. Everybody is supposed to know already what the department needs to do to improve itself. But actually, that's not tenable. The standards for hiring, promotions, etc.. we subject to differences of opinion, depending on what you believe in, what you think is the right thing for the department to be doing. In other words, your ethical stance.

Here is a story about an ethical problem in relations between colleagues. It's a little out of date, but interesting. You probably know that back in the '30's many mathematicians were leaving Germany in order not to be killed. Emil Artin was one of the great algebraists of his time. Artin wasn't Jewish, but his wife was, and they had two kids. Artin was approached by Helmut Hasse, who was another outstanding algebraist. Hasse was almost a pure Aryan, though he did have a Jewish ancestor someplace back in his family tree. He had become the head of the Institute at Gottingen after Courant and Weyl and Neugebauer had been kicked out. Artin was planning to leave because of his wife being Jewish, their children half-Jewish. Hasse said he could give Artin a deal. The kids could be made Aryan!

Do you see any ethical questions there? Hasse was a great mathematician. After the war he was quoted as being annoyed that some of the de-Nazification programs instituted by the American army were too severe. And he wasn't the worst. There were people like Teichmuller and Bieberbach,

brilliant mathematicians who were whole-hearted, all-out Nazis. Their ideology affected their professional work too, driving people like Landau off the lecture platform. Probably it could never happen here. But racism is a problem everywhere. It's not only a political problem, it's an ethical problem. We tend, many of us, to throw it under the rug, to think it's of no relevance to us. But maybe learning a little history will enlighten us about that. So much for ethical problems between colleagues.

Finally, what do we owe to the Dean, the Provost, the Chancellor? What they usually expect is, you should get grants, visibility, things like that. That demand from the administrators is based on certain values. It's based on a particular idea of what the university is, and what the department should become. If those values are accepted, then our present conditions follow absolutely. What the math department should be doing is to get out there and bring it in! But this kind of value system is also arguable. There are some of us who think otherwise. And recognizing that there is an ethical conflict here can only help to clarify our possibilities and our alternatives.

Now, to yourself! Does anybody here remember Polonius? It's in Hamlet, just before he got Hamlet's sword through his gut. "And this above all, to thine own self be true."

I've carefully avoided at any point giving you my own values. So there's no way anyone can disagree with me. I've just listed points of value judgment in our profession. Probably there are others that I have forgotten. But I'm sure you're ready to point out that this really has nothing to do with mathematics. It has to do with academic life. A French professor, even a mechanical engineering professor, would be involved in all these issues. I've been assuming that we're all academics. Of course, this isn't true. People here may be working in industry or other things. But being an instructor or professor involves you in all these interactions with people: students, faculty, staff, administration. And these all have an ethical component. However, this does not really deal with the issue I started with, which was, what about mathematicians as mathematicians?

Are there issues we have to face, just because we're mathematicians, the way engineers have ethical issues they have to face? Here I think we have to recognize the irritatingly vague line between pure and applied mathematics. To the extent that it is really involved in the so-called "real world," applied mathematics obviously brings in the same ethical issues as engineering, or any other applied science. For instance, nowadays people are using big computers to figure out secondary oil recovery. The people who do this are both geophysicists and applied mathematicians. The ethical issues for the applied mathematician are the same as for the geophysicist. What are the consequences of this activity for the environment, for the economy? To the extent that an applied mathematician gets involved with a real world activity, like geology or engineering, he has to deal with the ethical issues of that field, not because he's a mathematician, but because he's involved in that application.

Therefore let me acknowledge the separation, and ask, what about pure mathematics? Mathematicians who merely prove theorems. Is there any ethical component comparable to what you find in other fields of science? Of course, depending on what you include as ethics, you can say yes or no. "It's unethical to prove an ugly theorem." "It's unethical to republish under a different title a trivial paper that you have already published." As expressions of the taste or the standards of the field, these statements are correct. But still, one laughs at the word "ethical" here. It just doesn't make sense to use the same language for such issues of taste in pure mathematics as for air pollution or nuclear war. There are "ethical" issues in pure mathematical research. But they cannot withstand comparison with the major issues of human survival arising in "real world" science.

In pure mathematics, when restricted just to research and not considering the rest of our professional life, the ethical component is very small. Not zero, but so small it's hard to take very seriously. In fact, this may be a characteristic, a defining characteristic of pure mathematics. I can't think of any other field of which you could say that. That's why people say pure mathematicians live in an ivory tower. One answer to this could be, "Well, this is fine! There's no need for mathematicians to have a code of ethics, because what we do matters so little that we can do whatever we like." And I might agree with that. I'm not going to start advocating a code of ethics in mathematics at this point. But when I think about this attitude, I find it rather scary. Because it means that if we become totally immersed in research on pure mathematics, we can enter a mental state which is rather inhuman, rather totally cut off from humanity. That's a thing we could worry about a little bit.

Therefore I come to a conclusion for most of us, those who are not doing pure research a hundred percent of the time, who are not in the institutes for advanced studies, but have students and colleagues and staff and administrators. We mathematicians, I think, have a special need to take all these other responsibilities very seriously. Because unlike people

in other fields, our research work does not automatically involve human concern. My conclusion: If our research work is almost devoid of ethical content, then it becomes all the more essential to heed our general ethical obligation as citizens, teachers and colleagues, lest the temptation of the ivory tower rob us of our human nature.

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Teaching Global Issues Through Mathematics

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One significant way to show that mathematics is a humanistic discipline is to give a course that helps provide understanding of the many critical problems that face the world today.

Teaching global issues through mathematics has several important advantages:

1) Students' eternal questions about mathematics: "Why do I have to learn this?" and "What relevance does it have?" are answered. Students' motivation toward mathematics is greatly increased, as they see how mathematics can provide knowledge and understanding of current critical issues.

2) Coherence is provided to a mathematics course by focusing all the mathematics on global issues.

3) Students are not lulled by vague generalities; they are able to integrate hard data. For example, students do not get only a general idea that population has been increasing rapidly; by plotting a graph of world population growth versus time and solving related mathematical problems, students obtain a firmer understanding of the nature of population growth and related terms such as doubling time and exponential growth. By solving a problem related to the fact that in 1986, the world spent $900 billion on the military, an amount equal to the income of the poorer half of the world's people, students get valuable insight into the huge sums spent for the military, as well as the tremendous poverty in much of the world.

4) Students become aware that their studies can provide the valuable background information necessary for them to play an active role in helping to solve world problems.

5) The relationships between variables such as population, pollution, hunger, energy, waste, and the arms race are clearly shown. Stress can be placed on the important ecological principle that everything is connected to everything else and hence that you can't do one thing without affecting many other things.

A course relating mathematics to global issues can be valuable in several situations:

1) for liberal arts students, who need some mathematics to meet graduation requirements. Currently these students generally take a smattering of subjects such as set theory, logic, math history, number theory, and introduction to statistics, with no coherence and little relationship to current issues. A math course related to global problems can be related to material they're learning in their classes in such subjects as history, economics, and political science.

2) as an elective to provide useful background information for careers of students in such fields as ecology, business, technology, engineering, and education.

3) as an enrichment course for high school students who wish additional math, but don't need subjects like calculus or computers for their career goals.

During the past 14 years I have related mathematics and global issues in a course "Mathematics and the Environment" at the College of Staten Island. It is a three credit, elective course, designed primarily for liberal arts students. After several years of using a wide variety of background materials, many of which are discussed later, I wrote a text, <u>Mathematics and Global Survival</u>. (See annotated references and sources.)

Some examples of mathematics problems that have been used in the course are:

1) It has been estimated that the average American has 50 times the impact on the environment (in terms of resource consumption and pollution) as does a person in an underdeveloped country. How many people in these countries have the same impact as 246 million Americans (1988 population)?

2) From 1946 to 1968, consumption of plastics per person increased by 1024 percent. For every pound of plastics used in 1946, how many were used in 1968? Repeat for synthetic fibers (1792% increase),nitrogen fertilizer (534% increase), synthetic organic chemicals (495% increase), and aluminum (317% increase).

3) In 1988, Nicaragua's population was doubling every 20 years. At that rate, how many people would there be in Nicaragua in 100 years, for every one there today?

4) Draw a circle diagram showing the population for the major regions of the world. (This utilizes data from the Population Reference Bureau's 1988 World Population Data Sheet).

Other kinds of mathematics problems which can be presented using information on the population Data Sheet involve computing histograms for such things as birth rates, physical quality of life indices, etc., and investigation correlations between such variables as birth rates and per capita GNPs.

5) Plot a graph of total production of electrical energy in the U.S. versus time, using the data given below.

See Table at end

This graph shows the great increase in electrical energy production (it doubled about every 10 years, until recently) and helps students understand the many recent problems related to energy.

6) The average diet in the U.S. requires about 3.5 acres per person. The diet of an average person in the underdeveloped world requires about 1/5th of an acre. What is the ratio of acres required by a person in an underdeveloped country? (Note: In 1978, there was about one arable acre per person in the world, and population is growing rapidly).

If it is not possible or desirable to give a complete course relating mathematics and global issues, a mini-course could be given. For example, there could be a variety of math problems related to one issue such as energy, the arms race, population, or hunger. Or, if it were necessary to teach one math topic such as computing percentages, drawing graphs, or working with sequences, a variety of global issues and problems could be covered related to the appropriate mathematics topic.

To add to class interest, news and magazine articles related to the course can be discussed. There are several articles daily related to some aspect of the course. This helps show the relevance of the mathematics covered to everyday events.

The course can also be related to events such as Sun day, food day, and U.N. conferences related to population, hunger, environment, water resources, habitat, decertification, and disarmament. From time to time, films and slide presentations can be shown to improve student awareness and understanding.

Instead of a final examination, students can report to the class on some global issue they have researched, using mathematical concepts covered in the course. These reports enable the class to get an introduction to a wide variety of important issues, not otherwise covered, or to gain more knowledge and understanding of topics that were covered. Student reports have been on topics such as solar energy, noise pollution, the greenhouse effect, ozone depletion, destruction of tropical rain forests, pesticides, world hunger, and surveys of student opinion on various issues.

Through the course, students learn of the relevance of mathematics to global problems. They become very aware of global issues and the need for fundamental changes to avoid future crises. Many students have stated that their personal habits were changed through participation in the course, and they often spoke to friends and relatives about the need to solve global problems.

In closing, I feel strongly that this type of course can make major contributions to mathematics education and global awareness. I hope that similar courses will soon be offered at many other colleges, high schools, and perhaps even elementary schools.

A discussion of sources for further mathematical problems follows:

1. The population Reference Bureau, Inc., (777 14th St. NW, Suite 800, Washington, D.C. 20005) is an excellent source for information related to population. We have already referred to their annual World Population Data Sheet which gives a wealth of information on the world's regions and countries. They also have a "Population Handbook" which has a comprehensive summary of demographic techniques with many sample problems related to the World Population data sheets. Other valuable mate-

rial includes population sheets, teaching modules, and bulletins. Two bulletins with especially useful information (statistics, graphs and charts) on population are "Man's Population Predicament," Vol. 27, No. 2, 1971, and "Our Population Predicament: A New Look," Vol. 34, No. 5, Dec., 1979.

Special data sheets and other background material with many graphs, charts, and data were produced related to the International Year of the Child, in 1979, and International Women's Year, in 1980.

2. Mathematics and Global Survival, by Richard H. Schwartz (Ginn Press, 160 Gould Street, Needham Heights, MA 02194-2310; 1-800-428-GINN).

A 290 page text book with a wide variety of mathematics problems related to pollution, hunger, resource scarcity, energy, the arms race, and rapid population growth.

3. The Limits to Growth, by D. Meadows, et al, New York, Signet, 1972.

While somewhat dated and controversial this book uses several concepts related to mathematics, including exponential growth, mathematical models, feedback loops, quantification of variables, and many charts and graphs. It gives students valuable practice at interpreting material with both mathematical and global applications, while providing a warning about the results of continued unrestrained growth.

4. The Complete Ecology Factbook, J. Deedy and P. Nobile, editors, Garden City, N.Y., Doubleday, 1972.

Much ecological data, graphs, and charts. Excellent background materials for the construction of mathematical problems.

5. World Military and Social Expenditures, 1987-88, by Ruth L. Sivard, World Priorities Publications, Box 25140, Washington, D.C. 20007

An Excellent annual publication which has much information on arms expenditures and the impact on social issues. Many charts, tables, and graphs are provided.

6. The United States and World Development Agenda, 1985-86, by John W. Sewell, et al and the Staff of the Overseas Development Council, New Brunswick, N.J.: Transaction Books, 1985.

Much information is presented in the form of tables, charts, and graphs, related to developmental issues such as trade, the arms race, population growth, education, health, and many more.

7. The Mathematics of the Energy Crises, R. Gagliardi, Westmont, New Jersey, Intergalactic Publishing Co., 1977.

Over 130 mathematical problems related to energy for secondary students.

8. Mathematics in Energy, National Science Teachers Association (John M. Fowler, Project Director), Nov. 1978 (can be obtained from the U.S. Department of Energy Technical Information Center, P.O. Box 62, Oak Ridge, Tennessee, 37830)

Energy problems involving fractions, decimals, percents, and graphs. Also has energy activities involving mathematics problems and a summary of energy facts.

9. Energy-Environment Source Book, John M. Fowler, National Science Teachers Association, 1742 Connecticut Avenue, N.W., Washington, D.C. 20009, 1975.

A very comprehensive discussion of all aspects of energy extraction, conversion, and use, and related environmental problems. Many graphs, tables, and charts provide useful information for mathematics problems.

10. Mathematics for Ecologists, I. Chaston, London, Butterworth & Co., 1971.

An introduction to basic principles of advanced mathematics (calculus and linear algebra) applied to ecological problems.

YEAR	ELECTRICAL ENERGY (trillion Kilowatt hours)
1902	6
1912	25
1920	57
1930	114
1940	180
1950	389
1960	844
1970	1640
1975	2003
1980	2286
1986	2489

A Social View of Mathematics
Implications for Mathematics Education

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In recent literature one increasingly finds the proposal that we take a more social view of mathematics, but the intention can vary considerably. On the one hand, it can mean a recognition of the social nature of teacher/pupil interaction, and the significance of the social context for mathematics education, perhaps the last school subject to concern itself with anything other than content and the manner of its presentation. On the other hand it can be a recognition of the invasion of the mathematics classroom by controversial issues. In Britain recently, the Prime Minister complained that children in our schools are learning anti-racist mathematics instead of arithmetic. In another instance the conservative press complained about a public examination question that contained several parts asking pupils to read from a graph of arms expenditure by Nato and the Warsaw Pact. It ended with a question concerning the number of weeks of arms expenditure that would be required to feed the starving peoples of the world. It is obvious which part of the question was considered objectionable.

The intention of this paper is to propose that there are distinct consequences of a social view of mathematical knowledge, and to briefly present two examples. In order to do this I will first indicate my use of the notion of a social view of mathematics. I take this to apply to mathematical knowledge itself, in that the history of mathematics is not one of the gradual revelation of absolute truths, but, as with all knowledge, the consequence of people's ideas, interest, conflicts and patronage, and is culturally and temporally relative. Mathematical knowledge is a social construction, the meaning of a concept such as 'polyhedron' for example, following Lakatos, is negotiated and adapted according to convention and agreement, through proofs as explanations, leading to basic refutable statements. It is not the case that there exists, in some universal sense, a concept called 'polyhedron', which merely needs discovery and explication. This equally applies to notions of proof, truth and rigor, by which we justify particular areas of mathematics. Consequently, there is no natural or logical necessity to the state of mathematical knowledge at present. Undoubtedly we have a body of mathematical knowledge, that generally works, but it is in the nature of a library of accumulated experience, rather than universal truths. In any case, the 'it' refers to that collection of books on the shelves of the particular libraries that we frequent. Bishop, D'Ambrosio, Gerdes and others highlight our culturally restricted view of mathematical concepts. I suggest that the consequences of a social view of mathematical knowledge itself are far-reaching, including:

1) that there are alternative mathematical concepts, the direction of mathematical development is not a necessary one;

2) that, since mathematical truths have always been taken as the paradigm of true propositions in philosophy in general, taking this last bastion of certainty as itself relative is quite fundamental to our whole notion of knowledge;

3) that there is a full sociology of knowledge, dealing symmetrically and impartially with 'true' mathematics as well as 'false';

4) that the world 'out there', including the mathematical, is unknowable in any universal a priori sense.

This notion was proposed by the radical constructivists at the last PME conference in Montreal, and it is important to recognize that this is a central problem for philosophy today, as well as for mathematics education.

Taking this alternative view of mathematics, there are many possible consequences, for teaching styles, curriculum development etc., and I have described these elsewhere (Lerman 1983, 1986, 1987). I will develop here just two illustrations of implications for mathematics education, namely political and social education through mathematics, and the notion of ability.

Firstly, I suggest that teachers of mathematics can no longer sit in the school staffroom, believing that values enter every classroom except the mathematics one, and this not simply and solely because of arguments such as that education is everyone's responsibility. Since mathematics is as much a social construction as any other form of knowledge, it is culture-bound and value-laden. A

strong case can be made for characterizing mathematical values as sexist, for instance. Further, sociological analyses such as those of Freire, Apple and others propose that knowledge is power, i.e. that different conceptions of knowledge reflect different forms of social relationships and control. Freire, for instance, describes the 'banking concept' of education, whereby students activities are restricted to storing, filing and retrieving, as against the 'problem-posing' concept, whereby people see themselves as owning the mathematics, and empowered to both pose questions and propose solutions. This latter notion resonates strongly with the ideas of Stephen Brown and others in mathematics education.

Mathematics indeed serves a central function as a tool of government and power groups, since it is used to justify all sorts of policies and decisions, including closing coal mines, fighting inflation rather that poverty, under-funding social services and health, and in the Britain disbanding the Inner London Education Authority. Education for a critical mathematics places power in the hands of people to have some control over their own lives, and in particular to have such control. Perhaps we have, in the end, more responsibility that any other school subject, not less, for political and social education.

Secondly, notions of ability in mathematics are dependent on theoretical interpretations of learning and understanding, and are not in themselves fixed, certain and value-free. It is a form of platonism, that 'understanding' is a description of a particular completed mental state, much like the recall of forms known by the immortal soul. However, if concepts are themselves social constructions, determined by their use and consequently negotiable, the notion 'understanding' has a quite different meaning. Generally, we tend to see mathematics as, to quote Hart et al (Hart 1981), a "very difficult" subject, that some people seem able to do, and others not, despite many years of learning mathematics. If mathematics is about certain kinds of interactions with the world around, the application of certain ways of thought, or a particular language game, there is no reason why it should be very difficult. We have all encountered instances of children and adults performing sometimes complex mathematical tasks successfully, and more important comfortably, in everyday life, but failing to repeat those same tasks, achieve the same success, or indeed feel comfortable, in the mathematics classroom. This brings into question in a quite fundamental manner our notions of ability, and demands discussion, rethinking and new directions of research. Clearly, new ways of learning call on new ways of assessment, and interpretation of 'ability'. Such very different directions as those described by, e.g. Cobb (1986), focusing on the child's constructions, which in general examine children's grasp of things taught by the teacher. This latter notion of children's understanding is the focus of much on-going discussion and research, but the point being made here is that the concept 'ability' is related to the concept of 'understanding' with which one is working, and not some absolute concept. If we encourage children's understanding of mathematics through independent work in investigations, and problem-posing, and on computers, through Logo for example, we need different ways to assess their progress. 'Understanding' by these approaches, does not mean the successful application of a learned algorithm, and thus cannot be identified by a traditional pencil and paper test, developed within a Piagetian framework of hierarchies of concepts. Yet we adhere to this mode of assessment of children's mathematical ability.

In conclusion, as long ago as 1972, Rene Thom suggested that "all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics". This paper is a contribution to an examination of the ways in which alternative philosophies bear fruit in mathematics education.

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WHAT HAS MATHEMATICS GOT TO DO WITH VALUES?

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The popular view of mathematics, and mathematics education in particular, can be described as follows:

(a) Whilst mathematics education can be used to bring politics into the classroom, or to teach children particular values, and in fact mathematics itself can be used for moral or political ends in society, this is all really about uses or perhaps abuses of mathematics.

(b) In essence, mathematical knowledge is about pure concepts, relationships, pattern and structure. It is concerned with proof, and its truths are timeless, certain and absolute.

(c) We may argue about the causes of the Second World War, or whether Lowry was a great artist, we may discuss the role of religion today, but "5 + 6 = 11" is a truth now and forever.

(d) Any other mathematics is inconceivable, and so it is quite independent of time, of place, of culture and even of the people who invented or discovered it.

It is generally recognized that education is not just about the passing on of certain bodies of knowledge, but is also a preparation for life in society. This is enough justification in itself for anti-racist and anti-sexist mathematics. Look at the kinds of examples we draw on in the teaching of mathematics at the moment: percentage increases in pay; simple and compound interest; profit and loss; hire purchase; exchange rates and angles of missile projection to hit a target. Why shouldn't we use examples to reveal prejudice and injustice, and raise children's awareness of social issues? Reactions to the occurrence of these kinds of questions in mathematics lessons, or examinations, as seen in the Daily Telegraph, the Sunday Times and other newspapers, simply reveal how strong are the hidden messages of British values. We have come to accept questions on those topics above, and it is only when we see something about an unusual issue for mathematics, such as SMILE 'O' Level question about expenditure on arms, and the cost of feeding the starving peoples of the world, that people begin to worry.

So, we can point out to the critics of this kind of work in mathematics, that questions we have been using for years convey certain messages, and it rather hypocritical, to say the least, to single out some messages from others, particularly when they are about charity to the Third World, or opposing Apartheid, both issues that British Governments claim to support.

However, the reaction could be that we should make mathematics completely free of all messages. Return it to its pure state, where it is about numbers, patterns, skills, and procedures. This reaction, I believe, is one that most mathematicians and in particular mathematics teachers would give, and consequently it is essential to examine the very heart of the issue, the nature of mathematics itself. A strong case can be made, I am proposing, from within mathematics, for confronting social issues in the mathematics classroom, and in relation to this volume in particular, the issue of racism.

To summarize, the following is the claim that I will be examining here:

Mathematics seems to have a character all of its own, and a position of unique significance in discussions about knowledge, because of its hold on certainty and truth, and because of its purely abstract nature. The mathematical knowledge that we have is fixed, timeless and absolute, as are the logical methods that are used to deduce or calculate. If this is essentially the nature of mathematics, then it has nothing to do with social content, or values. Mathematics teaching should be kept free of all such material.

The evidence against this claim is beginning to mount up. First, I will give some examples from the mathematics classroom that appear to show some fundamental changes taking place. Then some examples will be drawn from philosophy of mathematics.

Examples from mathematics education:

1) Child methods of working, e.g.

$$\frac{1}{2} \div \frac{3}{5} = \frac{5}{10} \div \frac{6}{10} \text{ (common denominators?)}$$

$$= \frac{5 \div 6}{10 \div 10} \quad \text{(this only works for x!)}$$

$$= \frac{5 \div 6}{1}$$

$$= \frac{5}{6} \quad \text{correct!}$$

Now it is not a new idea to suggest that pupils often solve problems in ways that are completely different from the method we taught. But what does this do for our understanding of pupils' replies to our questions? One's first reaction to choosing common denominators for a division of fractions would be that we only do that for addition and subtraction. We would have destroyed that pupil's confidence, and identification with her/his own mathematical thinking. Or else the pupil would have gone on ignoring the teacher! It suggests, perhaps, that the teacher has to listen to every answer given by pupils, and treat them as potentially correct. They require testing and discussion by the class, before rejection or adoption as a good method. This is a very different function for the mathematics teacher from the traditional one of the conveyor of knowledge and algorithms, and arbiter of right and wrong answers

2) The view of the CSMS group [Hart et al 1981], supported by Cockcroft, is that mathematics is a very difficult subject. This is a very worrying statement. After all, what is mathematics about, if not certain kinds of interaction, that we all experience, with the world around us? If this is the case, why should it be so difficult? That is, does our view of mathematics perhaps act as a kind of self-fulling prophecy?

If our notion of mathematics is that it is hierarchical, and that one must learn it in order, from basic concepts to more abstract and difficult ones, and that the mathematical progression is mirrored by our psychological development, and indeed depends on it, then our curriculum, teaching styles, expectations, testing and much else are structured in a most rigid fashion. Yet there is growing evidence to show that young un-taught children (in the traditional sense of the word) often show understanding of concepts that in our hierarchy should only be accessible at 'formal operational' stage. The point is, that I am suggesting that our work is structured by our theories, and so is our testing. Is it then any surprise that our tests confirm our theories?

3) An investigation from a well-known source [SMILE 1981]:

"Consider triangles with integer sides.

There are 3 triangles with perimeter 12 units. Investigate."

No methods, skills or procedures. Not even a question! I have had PGCE and also BEd students ask me what they are supposed to do. Others have chosen to work on rectangles, areas, angles, triangles with other perimeters, etc. What is more, as the teacher, I don't have the answers. Even if I worked on the question for several hours the night before, I would still only have answered the questions that I asked.

With this kind of mathematical work, which is becoming more common in schools now, since GCSE criteria include course work, extended pieces of work, and investigations, the teacher/pupil relationship is changing. We are no longer the possessors of all the knowledge, passing it on in snippets as and when we feel the pupils are ready. All the people in the classroom are participating together in doing mathematics, whether we are aware of the change or not!

4) Here are three quotes concerning the excitement of mathematical creativity, in an adult and then in two children, aged 10 and about 5:

"This fascinated and excited him, spurring him on to feverish activity... He relaxed, satisfied... After coffee I wanted to work but the tension was unbearable... He felt a strange mixture of disappointment at his failure and elation because he felt he knew why he had failed." [Tall 1980 p. 25-34]

"About ten minutes later it happened. Sandra jumped up, knocked over her chair and ālmost shouted "you can!". It was clearly time for a get-together. Sandra described her discovery... She was really thrilled and I believe the others were pleased for her as well." [Atkins 1984 p.3]

"Consider Kevin, who was presented with ten drinking straws of differing lengths. Before I said a word about the straws, he picked them up and said to me,

'I know what I'm going to do,' and proceeded, on his own, to seriate them by length... It wasn't easy for him. He needed a good deal of trial and error as he set about developing his system." [Duckworth 1972 p. 219]

Are these not descriptions of the same kind of activity, despite the differing levels of "sophistication" of the mathematics? One's first reaction to the latter two extracts, perhaps, is how wonderful to have that excitement of discovery, and creativity in the mathematics classroom. Further reflection perhaps leads to the idea that what characterizes mathematics is not sets, quadratic equations, or calculus, but the doing of the business of mathematics, at all levels.

<u>Examples from Philosophy of Mathematics</u>

It is vital but not quite enough, for mathematics teachers and educationalists to believe that our notions of mathematics and of mathematics education have changed. The problem is that we ourselves remain convinced that we must ultimately look to the 'real' mathematicians, in the universities, at the forefront of knowledge, and see what they have to say. All of us are products of the mould, either directly from university or polytechnic teaching, or indirectly at colleges of education, from tutors who were themselves from that tradition.

Here are some quotes from recent, and not so recent, writing of mathematicians reflecting on the nature of their activity, or from philosophers of mathematics who spend al the time reflecting!:

"...all mathematical pedagogy, even if scarcely coherent, rests on a Philosophy of Mathematics" [Thom 1973 p. 204]

"...when he (the professional mathematician) is doing mathematics, he is convinced that he is dealing with an objective reality whose properties he is attempting to determine. But when challenged to give a philosophical account of this reality, he finds it easier to pretend that he does not believe in it after all." [Hersh 1979 p. 32]

"Mathematics is able to deal successfully only with the simplest of situations, more precisely, with a complex situation only to the extent that rare good fortune makes this complex situation hinge upon a few dominant simple factors." [Kac et al 1986 p. 21-22]

"Logic may explain mathematics but cannot prove it. It leads to sophisticated speculation which is anything but trivially true. The domain of triviality is limited to the uninteresting decidable kernel of arithmetic and logic - but even this trivial kernel might sometime be overthrown..." [Lakatos 1978 p. 19]

"Insofar as the propositions of mathematics give an account of reality they are not certain; and insofar as they are certain they do not describe reality." [Einstein 1921]

Obviously these are only extracts, chosen to illustrate a particular point of view. Others can be selected that would demonstrate support for the more traditional view of mathematical knowledge.

Which is correct? The traditional absolutist view, that mathematics is about truth, proof, certainty, structure, and that we the teachers possess some of that, and must convey it to pupils? Or what is sometimes called the fallibilist, or relativist view that all knowledge is relative to time and place, and hence to culture and values? By this latter view, all the knowledge that we have is a library, a body of experience, much of which is well-corroborated and supported, and successful. After all, buildings stay up, most of the time, and space research has taken people to the moon and back. But that knowledge is always vulnerable to new ideas and discoveries, and revolutionary change, as history shows. And it is not the only way that mathematics could have developed.

These are rival perceptions of mathematics, and they have, as I have hinted at, consequent major and significant effects on the teaching of mathematics in schools.

The difficulty here is that we have no certain way of making a universally acceptable choice. The criteria we use for deciding which of two rival theories is better, are themselves open to choice! One way of preferring seems to be which theory is the richer in the sense of the ideas for investigation and research (and possible refutation) generated by it. From this criterion, the fallibilist view is very rich in consequences for study and action, not least of all in the area of concern of this volume.

Mathematics and Values

If one holds the view that mathematical knowledge is social in nature, then one cannot get away from involvement in values. The mathematical ideas that one is teaching would have originated in a certain time and place, as a response to some social needs, whether of the individual mathematician, or of the wider community, scientific or otherwise. History of mathematics becomes a primary source of information for finding out what is mathematics, how it develops, and how it functions in society. It is not simply finding out who discovered binary numbers, or when logarithm tables were first put together. Instead, just as we can examine why the Greeks preferred geometry to number, or why British mathematicians did not develop non-Euclidean geometries, we can also look at how the Chinese were using 'Pascal's Triangle' centuries before he was born, or why Frege developed the supremely abstract mathematical logic. We can be impartial with respect to 'correct' mathematics or 'wrong' mathematics. We can also visualize how mathematics could have developed quite differently.

The South American educationalist Paulo Freire [e.g. Freire and Shor 1987] describes what I have called the traditional or absolutist view as the 'banking concept'. The image he describes is of the teacher baking ideas in the minds of pupils, whose only activities are storing, filing, classifying and retrieving. The alternative view he calls the 'problem-posing concept', whereby people see themselves as having the power to engage in problems that dominate their lives, pose questions for themselves, and develop solutions. He sees the former as associated with oppression, and the latter with freedom.

This may seem somewhat extreme, especially for such a cold climate as Britain! But if one considers what happens when children examine, in the mathematics classroom, racist headlines in the Sun, unemployment patterns in Britain by region, gender and race, the economics of Apartheid, distributions of wealth in Britain and the world etc., the word 'freedom' is not at all inappropriate. By these kinds of study our students do gain freedom, the freedom to examine the assumptions of the society in which they live. These assumptions draw on mathematical ideas and techniques, be they decisions about what constitutes an 'unemployed' person, with a view to keeping the total down, or up, for party-political ends, or what percentage figure is a real increase in funding for the National Health Service.

These mathematical techniques are often developed just in order to solve these kinds of problems, set in the terms of reference of the required solution. This is yet a further illustration of the social nature of mathematical knowledge. Certainly that knowledge gains some objectivity, in that another person or group somewhere else can read about those techniques, in the learned journals, and use them or adapt them to their own particular situation or problem. It cannot be said to have gained objectivity in the sense of corresponding to the 'real world', however, since that reality has been established in certain socially determined ideas. In developing an equation to decide whether to close a coal mine, one has the choice to introduce an element to take account of the social impact on the community, or not. There are no absolute rules to be applied in such a situation.

Conclusion

Mathematics is treated as a special case, by parent, governors, industry, commerce, the DES, pupils and teachers. It can be said that teachers are perhaps suspicious of the reason for this, and pupils single out mathematics both for the importance of qualification that can result, and for the most negative reactions! But there is no doubting the importance with which it is endowed by society. Mathematical knowledge is similarly treated as a special case in the field of scientific, philosophical, sociological and historical thought.

I have attempted to show that one way of looking at mathematical knowledge is to see it to be as much socially determine as any other area of knowledge, and that it has as little claim to timeless and absolute truth as any other. Mathematics teachers cannot claim that issues of justice, morality, freedom, values, are for the discussions in English lessons, or History, or Personal and Social Education, or Geography, but not Mathematics. It could even be claimed that we have a special responsibility, since mathematical techniques and methods are often developed for, and used in, social decisions. They are always developed by people in particular places at particular times, they reflect the current ideas of the mathematical community, for instance in the policies to fund certain research and PhD students or not, in editorial decisions to publish papers in journals or not, and finally they cannot be said to correspond to reality in any definite and absolute sense.

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