Differential Equations for a Changing World: How to Engage Students in Learning and Applying Differential Equations

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Differential Equations for a Changing World: How to Engage Students in Learning and Applying Differential Equations

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Abstract: In this article, I share my decade-long experience teaching an intensive five-week summer Differential Equation course covering complex topics and tips for creating an interactive and supportive learning environment to optimize student engagement. This article provides my detailed approach to planning and teaching an asynchronous course with rigor and flexibility for each student. An interactive teaching approach and variety of learning activities will augment students’ mathematical fluency and appreciation of the importance of differential equations in modeling a wide variety of real-world situations with special attention to ways differential equations can be relevant to creating public policy.

1 Introduction

Differential Equations has been a difficult undergraduate mathematics course [1] for both instructors and students alike for a long time. Building upon foundations taught in calculus I and II, the Differential Equations course potentially has tremendous impact on students’ field of study and future careers. The University of Massachusetts Dartmouth (UMass Dartmouth) is a public university dedicated to engaging pedagogy and innovative research. Our Mathematics Department has been committed to student success through the implementation of a five-week summer online Differential Equations course to augment student learning.

This five-week summer session Differential Equations course was taught in-person until

Notes: UMassD previously had three 3-hour lectures a week and daily assignments for the summer Differential Equations course before the COVID pandemic. We started the online summer course in 2020 with three 3-hour live online sessions a week in Summer 2020 and 2021. Since 2022, the course has been asynchronous remote. The class began on Monday of the first week with class ending on the last Thursday then final exam was held on the last Friday of the 5-week summer session. Students were required to complete daily course work except the holidays.
the outbreak of COVID-19 in spring 2020. We all had concerns regarding the effectiveness and challenge of a condensed 5-week summer asynchronous (fully online) class. Our course objectives were expansive: to understand the fundamental methods of ordinary differential equations (ODEs), to master qualitative analysis and numerical methods in solving nonlinear differential equations, and to gain substantive understanding of mathematical modeling with ODEs or other advanced methods. This article will highlight strategies to optimize students’ engagement and show how we can empower students’ learning through a dynamic rigorous curriculum including the use of multimedia, team exercises, incorporation of real-world applications of ODEs and an interactive discussion forum.

2 Course Topics and Modules

Given the asynchronous nature of the course, many adjustments were needed to develop more concise course notes, engaging lecture videos, and detailed assessments. The following Table 1 to Table 4 are listed as reference for an abbreviated course syllabus in a condensed Differential Equations course which follows closely the textbook [3]. For example, Unit 1 necessitated a different sequence of course topics in [2, 3] initially covering the classification and initial value problems of ODEs, solution curves with phase line and direction fields, separable equations, first order linear ODEs and their applications, exact equations, integrating factors, and solutions by substitutions.

The final exam was scheduled on Day 32, it covered Unit 1-4 and practice homework questions. Please note that our asynchronous online class began on Monday of the first week and ended on the last Thursday with a final exam on the last Friday of the 5-week summer session. Students were required to complete course work daily except on holidays.

| Unit 1 – First-Order Linear Ordinary Differential Equations (ODEs) (9 Days) | 1.1- Classification of ODEs (Day 1) 1.2- Initial Value Problems (Day 2) 1.3- Solution Curves with Phase Line, Direction Fields (Day 2) 2.1- Separable Equations (Day 3) 2.2- First-Order Linear ODEs (Day 4) 2.3- Continued (Day 5) 2.4- Exact Equations (Day 6) 2.5- Continued (Day 7) 2.6- Solutions by Substitutions (Day 8) | Daily Course Work:  
Read 1.1-1.2 & submit Quiz  
Read 2.1 & submit Quiz  
Read 2.2 & submit Quiz  
Read 2.3 & submit Quiz  
Read 2.4 & submit Quiz  
Read 2.5 & submit Quiz  
Activities: 
Practice homework problems 
Discussion board assignments for Unit 1. 
Exam Unit 1 on Day 9 |

| Table 1: Course schedule for Unit 1 |  |  |
### Table 2: Course schedule for Unit 2

<table>
<thead>
<tr>
<th>Unit 2 – Higher-Order Linear ODEs with Constant Coefficients (8 Days)</th>
<th>Daily Course Work:</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1- Theorems for Linear Ordinary Differential Equations (Day 10)</td>
<td>Read 4.1-4.3 &amp; submit Quiz (16 questions)</td>
</tr>
<tr>
<td>4.3- Homogeneous Equation with Constant Coefficients (Day 10-12)</td>
<td>Read 4.4 &amp; submit 4.4 Quiz (14 questions)</td>
</tr>
<tr>
<td>Part 1- Distinct Real Roots</td>
<td>Read 2.6 &amp; submit Quiz</td>
</tr>
<tr>
<td>Part 2- Repeated Roots</td>
<td>Activities:</td>
</tr>
<tr>
<td>Part 3- Complex Roots</td>
<td>Practice homework problems</td>
</tr>
<tr>
<td>4.4- Undetermined Coefficients, Superposition Approach (Day 13-15)</td>
<td>Discussion board assignments for Unit 2.</td>
</tr>
<tr>
<td>2.6- Numerical Solutions (Day 16)</td>
<td>Exam Unit 2 on Day 17</td>
</tr>
</tbody>
</table>

### Table 3: Course schedule for Unit 3

<table>
<thead>
<tr>
<th>Unit 3 - ODEs with Variable Coefficients, Systems of Linear Differential Equations (6 Days)</th>
<th>Daily Course Work:</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2- Reduction of Order (Day 18)</td>
<td>Read 4.2 &amp; submit Quiz</td>
</tr>
<tr>
<td>4.6- Variation of Parameters, Cauchy-Euler Equation (Day 19)</td>
<td>Read 4.6 &amp; submit Quiz</td>
</tr>
<tr>
<td>8.1- Preliminary Theorem for Linear Systems (Day 20) including 2 questions relate to Cauchy-Euler Equation</td>
<td>Read 8.1-8.2 &amp; submit 8.1-8.2 Quiz 1</td>
</tr>
<tr>
<td>8.2- Homogeneous Systems (Day 20-22)</td>
<td>Read 8.2 &amp; submit 8.2 Quiz 2</td>
</tr>
<tr>
<td>Part 1- Distinct Real Eigenvalues</td>
<td>Read 8.2 &amp; submit 8.2 Quiz 3</td>
</tr>
<tr>
<td>Part 2- Complex Eigenvalues</td>
<td>Activities:</td>
</tr>
<tr>
<td>Part 3- Repeated Eigenvalues</td>
<td>Practice homework problems</td>
</tr>
<tr>
<td></td>
<td>Discussion board assignments for Unit 3.</td>
</tr>
</tbody>
</table>

Exam Unit 3 on Day 23
3 Pedagogical Philosophy

In this asynchronous summer class, students are provided with daily assignments to ensure everyone is moving at the same pace. Expectations were clearly delineated in a welcome video including the course requirements, the academic policy, and introducing myCourses site, where students participate in class activities and complete their coursework and exam. To encourage class participation and engagement, completion of activities comprised 10% final grade. Examples of participation included:

- Watching videos, reading textbook/notes/slides and completing selected practice homework solutions.

- Participating in the discussion forum either by asking or replying to a peer’s questions related to the course and daily assigned practice homework.

- Working on practice exams and mathematical modeling problems in a small group, presenting the group work during instructor’s three office hours for each unit, and then posting solutions on the discussion board.

Engaging students beyond watching lectures and completing assignments was crucial. Students were heavily encouraged to post questions or reply to a peer’s question(s) daily about learned concepts and practice homework problems in online discussion forum. At the end of each unit, they were expected to work on written practice exam problems and ODE modeling problems in a small group. Each group consisted of three or four members with defined roles including a facilitator or leader to coordinate each member’s role and completion of tasks. Each member was expected to contribute and help each
other complete the group work and one member was designated as the group monitor or recorder to post the group work. The role designation rotated amongst group members so each student was able to take on the leadership role and learn actively from each other. The experience of practicing presenting their solutions during office hours in a low-stakes environment was key to help students gain confidence and mastery in their mathematics skills and emulate a real-world environment where they will need to communicate complex ideas in front of a group.

For each section, I developed concise notes with clearly explained concepts and meticulous selected examples as well as developed videos for each unit. We utilized the philosophy of spaced repetition – retrieving and applying the learned knowledge in daily quizzes and discussion posts to cement concepts. At the beginning of each unit, students were reminded of objectives and solved practice homework problems (ungraded) with the help of summary notes and videos. Students then needed to complete a sub-unit quiz within 24 hours, with a maximum of two attempts to obtain a minimum of 60% as the prerequisite passing score to move on to the next sub-unit.

The unit exams covered topics and concepts that were also tested upon in sub-unit quizzes. To minimize online cheating, the screen only shows one question each time, students have only one attempt to complete time-limited exams and could not go back to a previous question to change answers. Again, students were required to upload their handwritten solutions to explain the problem-solving process as this allowed me to give the partial credits for their line of thinking. I also announced that when some students provide incomplete work with missed steps to solve problems, then I will interview them and adjust their scores if necessary. In order to be able to take the final exam, students need to get at least 60% on the last exam unit.

The practice homework problems are always open and available for students to practice not only for sub-unit quizzes but also for the unit exam, students are given time and opportunities to test their knowledge of the subject by solving practice problems. Through the interactive discussion forum of practice homework problems and real-life application of ODEs, daily quizzes and team exercises, students are able to augment mathematical fluency and to decrease test anxiety as they communicate mathematical concepts and explained their problem-solving approach. This allowed the course to maintain high standards of academic integrity and scholarly practice as listening to students’ presentations allowed me to assess and determine their true level of mastery of concepts.

4 Examples of Practice Problems and Projects

Engaging the 21st-century learner in appreciating the importance of differential equations, which are crucial to multiple fields including science, technology, social science, and finance, is paramount to the teaching differential equations successfully. Built upon what students have learned in Calculus I and II, the Differential Equations course can benefit students from a variety of fields and at all levels as it may be the last math course for
many students or serve as a cornerstone for the higher-level mathematics courses (such as Advanced Engineering Mathematics and Partial Differential Equations). Teaching the sophomore Differential Equations course is not just about narrating concepts and showing packaged formulas. More importantly, it is incumbent on us to show students the necessity of mathematical reasoning and application in everyday situations, so they are compelled to acquire analytical and problem-solving skills through a thoughtful curricular design emphasizing the applications of differential equations in real-world situations such as natural phenomena and public policy. I have provided below some sample questions that will hopefully be helpful to other instructors.

4.1 Project of modeling complicated pollution (Unit 1)

A 1000 gallon holding tank that catches runoff from some chemical process initially has 800 gallons of water with 2 ounces of pollution dissolved in it. Polluted water flows into the tank at a rate of 3 gal/hour and contains 5 ounces/ gal of pollution in it. A well-mixed solution leaves the tank at 3 gal/hour as well. When the amount of pollution in the holding tank reaches 500 ounces, the inflow of polluted water is cut off and fresh water will enter the tank at a decreased rate of 2 gal /hour while the outflow is increased to 4 gal/hour. Determine the amount of pollution in the tank at any time \( t \).

Solution: Let \( Q_1(t) \) be the amount of pollution in the tank. \( t_m \) be the time when the pollution reach the maximum allowed amount of 500 ounces, \( Q_1(t) \) be the amount of pollution in the tank at time \( t \), \( 0 \leq t \leq t_m \), \( Q_2(t) \) be the amount of pollution in the tank at time \( t \), \( t_m \leq t \leq t_{end} \), where \( t_{end} \) denotes the time when the tank will be empty.

If the amount of pollution ever reaches the maximum allowed amount, there will be a change in the situation. This will necessitate a change in the differential equation describing the process as well. But we could do the same way as with in mixing problems with ordinary differential equations.

First,

\[
\frac{dQ_1(t)}{dt} = 3(5) - 3\frac{Q_1(t)}{800}, \quad Q_1(0) = 20, \quad 0 \leq t \leq t_m ,
\]

which has solution

\[
Q_1(t) = 4000 - 3998e^{-\frac{3t}{800}}.
\]

Let \( Q_1(t) = 4000 - 3998e^{-\frac{3t}{800}} = 500 \) and solve for \( t = t_m = 35.475 \).

Second, once the pollution reaches the maximum amount of 500 ounces at \( t_m = 35.475 \), the tank needs to be cleared. To reduce the concentration of pollution to avoid any further damage, the freshwater are pumped into the tank at a rate of 2 gal /hour. Since the concentration of pollution is 0 in the freshwater and the input rate \( R_{in} \) of pollution is the product of the inflow concentration of pollution and the inflow rate of the solution (water), so \( R_{in} = 0 \).
While the gallons of the mixed fluid in the tank is decreasing from 800 gallons at a rate of (4-2)gal/hour, the gallons of fluid in the tank is 
\[ 800 - 2(t - t_m), \]
where \( t - t_m \) represents the time elapsed from \( t_m \) to any time \( t > t_m \). The concentration of pollution in the tank as well as in the outflow is \( \frac{Q_2(t)}{800 - 2(t - t_m)} \), so the output rate \( R_{out} \) of the pollution is
\[ R_{out} = 4 \times \frac{Q_2(t)}{800 - 2(t - t_m)}, \]
the net rate is \( R_{in} - R_{out} = -4 \times \frac{Q_2(t)}{800 - 2(t - t_m)} = -4 \times \frac{2Q_2(t)}{435.475 - t}. \)
And the ODE is
\[ \frac{dQ_2(t)}{dt} = -\frac{2Q_2(t)}{435.475 - t}, \quad Q_2(35.475) = 500, \quad 35.475 \leq t \leq t_{end}. \]
Next by solving the above ODE, we find \( Q_2(t) = \frac{(435.475-t)^2}{320} \).
Notice that \( Q_2(t) = \frac{(435.475-t)^2}{320} = 0 \) at \( t = 435.475, t_{end} - t_m = 435.475 - 35.475 = 400 \) hours to empty the tank.

Relating the project to public policy modeling on pollution crisis and damage estimate:
Suppose that the tank represents the waterways built for the drainage management of a town, the waterways received an influx of pollution. Following the wastewater control standard, the town needs to regularly discharge the pollution. Due to a nearby accident, the pollution of the waterways reaches the terrible level of 50% (the polluted water containing 50% pollution). The town immediately pumps water (caring for the environment, etc.) to clear the waterways. a) How long will the pollution be reduced to the acceptable level of 25% ?

**Solution:** Let \( Q_2(t) = \frac{(435.475-t)^2}{320} = 800 \times 0.25 = 200 \), solve for \( t \), we get
\[ t_1 = \frac{17419}{400} + 80\sqrt{10} \approx 683, \quad \text{or} \quad t_2 = \frac{17419}{400} - 80\sqrt{10} \approx 182.5. \]
Since \( t_1 \) is greater than \( t_{end} \), we take \( t \approx 182.5 \).

b) The town needs to estimate the damage cost and then increase the household tax for the next year. How much damage is caused by the pollution crisis? Suppose it costs $10,000 per hour for the town.

**Solution:** We calculate the cost with the formula cost = (hours taken) \( \times \) cost per hour = 
\[ 10,000 \times \frac{182.5}{60} \approx \$ 30,416.7. \]

4.2 A skills inventory problem (Unit 3)
Find the eigenvalues of the linear system
\[
\begin{align*}
x' &= -x + y \\
y' &= cx - y
\end{align*}
\]
and discuss the shapes of the solutions to the cases $c = 4, \frac{1}{4}, 0, -9$.

![Figure 1: Phase portraits for the above 4.2 problem as $c$ varies](image)

The above Figure 1 includes all possible shapes for solutions to the system of linear ordinary differential equations: a), b) are cases with two eigenvalues, of opposite or same sign, respectively; c) has a single eigenvalue, and d) has complex roots. In c), please notice that all "S" shaped curves move towards the origin, and their tangent lines towards the $x$-axis. The red-dotted line is a tangent line of a solution curve.

### 4.3 Project of modeling a logistical growth population with weekly harvest (Unit 4)

Suppose that the population of the fish in a lake follows the logistical growth model with weekly growth parameter $k = 0.08$, and the maximum population $M = 1,000$ thousands. Suppose that the fish are harvested at the rate of 15 thousands weekly.

a) Write the differential equation for the fish population, find the critical (equilibrium) points, draw the phase portrait, and analyze the stability.

**Solution:** The ODE for this population model is $\frac{dP}{dt} = 0.08P \left(1 - \frac{P}{1000}\right) - 15$, where $t$ is measured in weeks.

Let $0.08P \left(1 - \frac{P}{1000}\right) - 15 = 0$, we get the critical points $= 250, 750$. The slope field for the situation with weekly harvest and the stationary point are shown in the following Figure 2 on the next page. Solutions with initial values close to 250 will move away from $P = 250$, so $P = 250$ is unstable. Next, solutions with initial values close to 750 move in towards $P = 750$, so $P = 750$ is asymptotically stable.

b) Suppose that the initial size of the fish population was 200 thousands, solve this equation using partial fractions, and describe the behavior of the solution.

**Solution:** The initial value problem (IVP) is given by

$$\frac{dP}{dt} = 0.08P \left(1 - \frac{P}{1000}\right) - 15, \quad P(0) = 200.$$
The Fish Population of a Lake
Weeks after Harvesting Fish

Figure 2: Slope field of the logistical growth population with a harvest component

We solve the ODE for \( P(t) \) using partial fractions, we have

\[
\ln \left| \frac{P - 250}{P - 750} \right| = \frac{1}{25} t + c.
\]

Using \( t = 0, P = 200 \), we get

\[
c = -\ln 11 \quad \text{and} \quad P - 250 = \frac{1}{11} e^{\frac{1}{25} t} (P - 750)
\]

Then we have the explicit solution and its derivative

\[
P(t) = \frac{2750 - 750e^{\frac{1}{25} t}}{11 - e^{\frac{1}{25} t}} \quad \text{and} \quad P'(t) = -\frac{220}{(11 - e^{\frac{1}{25} t})^2} < 0 \quad \text{for all} \ t.
\]

So \( P(t) \) is decreasing and will limit to \( P = 0 \) at \( t = 25 \ln \left( \frac{2750}{750} \right) \approx 32.5 \). This means that the fish population will decrease from the initial size and will become extinct after 32.5 weeks.

c) Set (i) \( H = 8.75 \), (ii) \( H = 19.8 \), (iii) \( H = 20 \) and (iv) \( H = 21 \) in the IVP

\[
\frac{dP}{dt} = 0.08P \left(1 - \frac{P}{1000}\right) - 15 = 0, \quad P(0) = 200.
\]

How do these values affect the critical (equilibrium) points? How do these values affect the fish population size in the long term?

**Solution:** (i) The critical points for \( H = 8.75 \) are 125 and 875; (ii) The critical values for \( H = 19.8 \) are 450 and 550.

(iii) We only have one critical point \( P = 500 \), which is semi-stable. It means that for any initial value greater than 500, the population size will decrease and approach the equilibrium state 500. For any initial value less than 500, our solution will decrease to 0.

(iv) For any \( H > 20 \), we have no real critical points, and the solution will decrease to 0.
Relating the project to public policy modeling on sustainable fisheries management:
For the general logistical population with constant harvest rate $\frac{dP}{dt} = kP (1 - \frac{P}{M}) - H,$
where the factor $kP$ represents the rate of growth of a population which is following
an exponential path when the population size is small, the factor $1 - \frac{P}{M}$ represents the
constraint which staunches the exponential grow as the population size approaches the
carrying capacity $M, H$ is the constant harvest rate per unit time.

Solution: Solve for the critical points of differential equation $\frac{dP}{dt} = kP (1 - \frac{P}{M}) - H$ by letting

$$kP (1 - \frac{P}{M}) - H = 0$$

or equivalently $kP^2 - kMP + MH = 0$. Solving for $P$,

$$P = \frac{KM \pm \sqrt{k^2M^2 - 4kMH}}{2k}.$$  

If $H = \frac{kM}{4}$, then we have a unique critical point and this value is the maximal sustainable harvest rate ($H_{MSR}$), and the corresponding value $P = \frac{M}{2}$ is also the size of the population which has the highest absolute rate of growth in the absence of fishing. We call this maximum value of $\frac{dP}{dt}$ the maximal sustainable yields ($MSR$).

If $H = \frac{kM}{4}$, then $P = \frac{M}{2}$, we are at the boundary between two qualitatively different cases since the equilibrium point at $\frac{M}{2}$ is neither stable nor unstable. If $P$ starts out at $\frac{M}{2}$ to a higher value, then will return to $\frac{M}{2}$ after the harvesting, as it would be for a stable equilibrium. On the other hand, if $P$ departs from $\frac{M}{2}$ to a lower value and the population failed to regenerate itself fully, then $P$ will continue to move away from $\frac{M}{2}$ as it would for an unstable equilibrium and $P$ will continue to shrink towards zero. To fish exactly at the maximum sustainable rate of $H = \frac{kM}{4}$ would be risky, so it’s necessary to determine a safety margin, the local government may need to allow only harvesting of a small amount less than the value $\frac{kM}{4}$.

If $H < \frac{kM}{4}$, we have two critical points, the larger one is stable (call it $P_s$). The smaller value is unstable (call it $P_u$). When $H$ increases, $P_s$ decreases and $P_u$ increases, $P_u$ and $P_s$ will get closer and closer.

For $H > \frac{kM}{4}$, we will have no real critical points, and no matter how large the initial population size is, the population will decrease towards 0, this means the extinction of the population. In this case, once the population is below $\frac{M}{2}$, then $P$ will be decreasing faster and faster until reaches zero, the averted action should be taken immediately.

It can be concluded that even with a sufficiently large population, fishing exactly at the maximum sustainable rate is not practical and that a fluctuating and harvest quota below the maximum sustainable rate can reduce the risk of extinction. The above discussion may be used in the sustainable fisheries management. In the real world, however, it’s hard to compute the values of $k, M,$ and $H_{MSR}$. It may be politically difficult to set the harvest
quota low enough to ensure the long-term sustainability of the harvested population. To maintain a stable harvest and a healthy environment, science-based management, effective governance, individuals or communities that rely on fishing for a living must work together.

5 Outcomes and Student Feedback

The outcomes and student feedback to this dynamic curriculum have been impressive. Many students expressed their deep appreciation of how this class provided a solid foundation for other challenging courses such as biochemistry and mechanical engineering. Qualitative themes from feedback include appreciation for the asynchronous nature, applications to real-world situations, and ability to assume leadership roles during team exercises. A high school student wrote me a letter:

*I really enjoy the asynchronous class over the summer 2022. As someone who is interested in the applications of math, it was interesting to see how some of the equations we looked at could be used to model real-world situations, I definitely benefited from the deep and broad coverage of course topics, and our discussion forum. I remember asking theoretical questions and contributing my ideas for solving different problems almost every day, I was able to take the lead for a few units and designate roles to the people in my group on problem sets. This experience with math at a college level further confirmed my math major plan at a prestigious university.*

Figure 3 showcases the distribution for the final grades. The overall average correct for the four unit exams and a final exam was 83.6%.

![Final grade distribution based on the grading policy after withdrawals](image)
The course success rate was 89.7%. The class had a total of 29 students, 3 students withdrew from the course, and the other 26 students successfully completed with the grades A, B, or C. The distribution of the final grade is calculated according to the following process:

Tests (4 unit exams 10% each): 40%; Final Exam (accumulative): 20%; Quizzes: 30%; Class participation in discussion forum, and group work on projects and practice exams: 10%. After calculating the overall average, the final grade will be assigned according to the following scales:

\[
\begin{align*}
100 \Rightarrow & \ A^+ \\
93-99 \Rightarrow & \ A \\
90-92 \Rightarrow & \ A^- \\
87-89 \Rightarrow & \ B^+ \\
83-86 \Rightarrow & \ B \\
80-82 \Rightarrow & \ B^- \\
77-79 \Rightarrow & \ C^+ \\
73-77 \Rightarrow & \ C \\
70-72 \Rightarrow & \ C^- \\
67-69 \Rightarrow & \ D^+ \\
63-67 \Rightarrow & \ D \\
60-62 \Rightarrow & \ D^- \\
\text{below 60} \Rightarrow & \ F
\end{align*}
\]

6 Conclusion

It is the emphasis on how differential equations can be applied to real-world situations, engaging students outside the classrooms through online discussion forums, empowering them and building confidence through leading and participating in group exercises and presentations in a low-stake environment that I believe will be able to show the 21st-century learner that mastery of differential equations can be impactful across a variety of careers.

I have some final words of advice for those who wish to implement a condensed class. While some students appreciated the asynchronous nature of the course, we know that fully online classes have their challenges, and their lack of structure can be detrimental to some students’ learning processes. We clearly stated our expectations of students in the course and advised that students should be prepared to spend a minimum of 3 hours daily on reading and course assignments (required online quizzes/exams and optional practice problems, and participation in discussion forum) so that we can maintain high standards of academic integrity and scholarly practice.

We carefully monitored student’s performance and participation in their coursework and deployed early intervention strategies for struggling students. I reached out early to those students who struggled with the first week’s quizzes or who may fail the coming exams, I sent them email reminders often, such as "What you need to catch up", "Please come to see me or contact STEM learning lab for help", "Complete your work on quiz question # for improvement", "Good improvement! ".

As academic performance of the online class was more of a self-study or self-management, I help students improve their mathematics skills and guide them how to study the online class effectively. For each section or chapter, I posted the detailed review of a checklist of topics, guiding examples, practice questions, and asked students to read, then solve the practice questions, and then to participate in discussion. The consistent support, rigor and varied breadth of daily assignments motivated students to solve every problem and finish each assignment. I also applied a flexible approach to maximize the opportunities for
student success. For example, I extended the due dates of a couple of quizzes or granted additional submissions to accommodate a few students who faced barriers to complete and pass the quiz on time. I devoted extra effort and time to students who scored below a passing score on the unit exam to keep engaging with the class and to complete the course.

Bibliography


1. For the method of undetermined coefficients, the assumed form of the particular solution

\[ y_p \text{ for } y'' - 4 = 4 + e^x \text{ is } \underline{\text{________________________}}. \]

2. Find the particular solution of the given differential equation

\[ y'' - 2y' - y = -8t + 6t^2. \quad y_p = \underline{\text{________________________}}. \]

3. Solve the following differential equation by the method of undetermined coefficients.

\[ y'' + 2y' = 2x + 9 - e^{-2x}. \]

The complementary solution for the differential equation: \[ y_c(x) = \underline{\text{________________________}}. \]

The particular solution for the differential equation: \[ y_p(x) = \underline{\text{________________________}}. \]

The general solution for the differential equation: \[ y(x) = \underline{\text{________________________}}. \]

4. Find the general solution of the given differential equation

\[ y'' + 2y' + 5y = 8 \sin 2t. \quad y(t) = \underline{\text{________________________}}. \]

5. Find the general solution of the given differential equation

\[ y'' - 2y' - 3y = -9te^{-t}. \quad y(t) = \underline{\text{________________________}}. \]

6. Find the general solution of the given differential equation

\[ y'' + 6y = -294x^2e^{6x}. \quad y(x) = \underline{\text{________________________}}. \]

7. Find the general solution of the given differential equation

\[ y'' + 4y = 6 \sin 2x. \quad y(x) = \underline{\text{________________________}}. \]
8. Find the general solution of the given differential equation
\[ y'' + 2y' + y = 14e^{-t}. \quad y(t) = \] 

9. Solve the given initial value problem
\[ 5y'' + y' = -4e^{-x}, \quad y(0) = 0, \quad y'(0) = -25. \quad y(x) = \] 

10. Solve the given boundary-value problem
\[ y'' - 2y' + 2y = 2x - 2, \quad y(0) = 0, \quad y(\pi) = \pi. \quad y(x) = \] 

11. Solve the given initial value problem
\[ y'' - 2y' - 3y = 3te^{2t}, \quad y(0) = 6, \quad y'(0) = 0. \quad y(x) = \] 

12. Using the method of the undetermined coefficients, solve the given differential equation
\[ y''' - 3y'' + 3y' - y = x - 7e^x. \quad y(x) = \] 

13. Using the method of the undetermined coefficients, solve the given differential equation
\[ y''' - 6y'' = 4 - \cos x. \quad y(x) = \] 

14. Solve the given initial value problem
\[ y''' - 2y'' + y' = 2 - 24e^x + 40e^{5x}, \quad y(0) = \frac{1}{2}, \quad y'(0) = \frac{5}{2}, \quad y''(0) = -\frac{11}{2}. \quad y(x) = \]
Appendix B  Quiz 8.3 - Nonhomogeneous Linear System

1. Consider the following nonhomogeneous system.

\[ X' = \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} X + \begin{pmatrix} -3t^2 \\ t + 4 \end{pmatrix} + \begin{pmatrix} -3t^2 \\ t + 4 \end{pmatrix} . \]

Find the eigenvalues of the coefficient matrix:

\[ \lambda_1 = \underline{\quad} , \quad \lambda_2 = \underline{\quad} . \]

Find an eigenvector for the corresponding eigenvalues:

\[ \mathbf{K}_1 = \underline{\quad} , \quad \mathbf{K}_2 = \underline{\quad} . \]

Find the general solution of the given system:

\[ X(t) = \underline{\quad} . \]

2. Use the method of undetermined coefficients to solve the given nonhomogeneous system:

\[ \begin{cases} 2 \frac{dx}{dt} - 4x + \frac{dy}{dt} = e^t \\ \frac{dx}{dt} - x + \frac{dy}{dt} = 6e^t \end{cases} \]

\[ X(t) = \underline{\quad} . \]

3. Solve the given initial value problem.

\[ X' = \begin{pmatrix} -1 & -2 \\ 15 & -4 \end{pmatrix} X + \begin{pmatrix} 2 \\ 2 \end{pmatrix} , \quad X'(0) = \begin{pmatrix} -5 \\ 6 \end{pmatrix} . \]

\[ X(t) = \underline{\quad} . \]