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The Role of Faith in Mathematics

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We typically are so involved with the fruits of mathematics that we neglect its roots! Students are generally aware that science is based upon the "scientific method." But mathematics students have little knowledge of the foundations of mathematics. Is mathematics really as firmly rooted as most people think? Are mathematical results necessarily true? Or, is there an element of faith/belief/trust involved?

We should be honest with our students and discuss metamathematics a bit. They need to be aware that mathematics is rooted in logic and the axiomatic method. Generally, students use a naive (unstated) logic, and this is appropriate. However, they should be aware that there are numerous formal logics and various philosophies as to the meaning of "proof."

We must realize that mathematicians are human—and, hence, fallible.

Suppose we prove that $\sqrt{2}$ is an irrational number, or that the cardinality of [0, 1) is uncountable. These typically are done via proof by contradiction within a mathematical system. How can we be so quick to reject this idea?

We know that a contradiction is a statement of the form "P and not P." Also, we realize that this is a false statement. Further, if one can prove a false statement, then one can prove each statement within the mathematical system; and hence the system is not useful.

Fundamental to the axiomatic method is the requirements that each set of axioms be "consistent" so that no contradiction can be proven within the axiomatic system. But, how can this be that one system is as consistent as another one is. But this still leaves room for doubt. We can try to obtain "absolute consistency" by presenting a model of the axioms within the framework of reality. But this leaves one wondering about the consistency of reality. Further, since the model needs a one-to-one correspondence with elements of the system, can we obtain absolute consistency for infinitely large sets of elements?

For example, the arithmetic of whole numbers is not absolutely consistent. So why do I trust it? Faith is not necessarily blind! Over the centuries we have come to rely on certain axiom systems. Yet we should realize that it really is a matter of convenience/faith. "If we hold to finitary or even classical methods of proof, faith cannot be banished from mathematics: we simply have to believe that PA (Peano Arithmetic) is consistent, since any proof which we could formalize will use methods or principles which are more questionable that those we use in the system itself."[2, p. 214]

Further, we need to point out that the results really do change when various axioms are used. Geometry provides nice examples of this as we move between Euclidean and non-Euclidean geometries.

We must realize that mathematicians are human—and, hence, fallible. You and I are sophisticated enough to have seen errors in the answer book, erroneous statements, and fallacious proofs. But these can really be upsetting to students. Stanislaw Ulam estimates that mathematicians publish 200,000 theorems every year. [3, p. 288] Many of these are later disallowed or thrown into doubt, and most are ignored. The discredited results are typically not the work of incompetent people. Reference [1] indicates a number of prominent mathematicians who have published flawed proofs.

Certainly a good reputation provides increased believability. This is a key reason for a teacher trying to be creditable to the students. We tend to put trust/belief in the results published in a reputable journals and in articles by noted persons. With limited time resources, can we do otherwise?

The computer brings into play another level of faith. We are fully aware of GIGO (Garbage In, Garbage Out). Yet we daily rely on computer results even when we know there are operating system "bugs," possible programming errors, and the problems involved in representing real numbers exactly. We tend to be more skeptical of computer-dependent "proofs" and results. Professors De Millo, Lipton, and Perlis remind us to stay wary in these.[1] Perhaps we each have several degrees of belief.

In conclusion, we cannot avoid the realization that mathematics involves faith. Some see mathematics as the ultimate in logical rigor, and I agree. However, we need to realize <u>all</u> of the basic assumptions, hiding nothing.

REFERENCES

1. De Millo, Richard A., Lipton, Richard J., and Perlis, Alan J., <u>Social Processes and Proofs of Theorems and Programs</u>, Communications of the ACM, Vol. 22, No. 5, May 1979.

2. Epstein, Richard L. and Carnielli, Walter A., <u>Computability: Computable Functions, Logic, and the Foundations of Mathematics</u>, Wadsworth, Inc.., Belmont, California, 1989.

3. Ulam, Stanislaw, <u>Adventures of a Mathematician</u>, Scribner's, New York, 1976.

Note: A good source of fallacious mathematics is the "Fallacies, Flaws and Flimflam" section of <u>The College Mathematics Journal</u> published five times a year by the MAA.