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It is a common notion that arithmetic and computation alone constitute mathematics. Let me quote from a statement that was circulated among the faculty of Elizabeth City State University, Elizabeth City, sometime ago. "At the undergraduate level Mathematics is presented as that discipline that describes the real world by numbers and numerical relationships. A course in Mathematics consists of developing some mathematical skills and some mathematical arguments. Undergraduate courses are more heavily weighted towards the former than the latter." To my knowledge this statement is true of most of our courses. Courses such as Abstract Algebra and Geometry are exceptions. From among the computational skills, some are disappearing before our very eyes because of the introduction of the calculator and computer. Students are trained to think that every characteristic or phenomenon of the real world could be measured and depicted by a number and that they could make their decisions based on that number. In my judgement, this is a mistaken notion about the world and about mathematics. Therefore, I think there should be a change in course content (especially those of service courses) as well as the manner presentation of material in text books written for those courses.

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Mathematics at one level is a mode of viewing the world and a mode of thinking about it. Mathematics has a language of its own for presenting its ideas. But mathematics is not limited to mere abstractions of the real world. We will come to this later on.

"Number" is at the simplest level of abstraction. It is a mode of viewing objects in space. But then we see patterns and colors, hear rhythms and tones, feel heat and cold. Add to this list political, social and economic organizations of societies and all our physical and mental experiences. Capturing these in ideas, expressing them in appropriate languages and creating new ones have resulted in the creation of bodies of knowledge and have deepened our understanding of human existence on this planet Earth. Civilizations have been significantly affected because of such knowledge. Mathematics provides structures or models and theories about those structures. They may be immediately perceived or they may be removed from perception. Therefore it is left to the seeker to find the appropriate structure and apply the theory that mathematics provides to organize what he observes and get additional or perhaps more precise insights that mathematics could provide. The creation of a new structure could also be facilitated by a necessity for developing a model for a particular problem irrespective of the area.

Mathematical thinking is not having an opinion or a notion. It is pursuing an idea with valid arguments. Given a problem, mathematical thinking calls for finding the premises and finding the appropriate reasoning to arrive at conclusions. In certain cases, problem solving calls for discovering new techniques for solution. Therefore seeing, analyzing, abstracting, remembering, reasoning, synthesizing, discovering, representing and questioning, all constitute mathematical activity. Logic is the tool for reasoning. Therefore every mathematical activity rests on having this tool.

Critical thinking is an essential part of mathematical activity. Mathematics has progressed

because of challenges in terms of examples and counter examples, in terms of altered hypotheses or an additional hypotheses. What has been established as a piece of knowledge is a spring board for additional knowledge, for imagination, discovery, and creativity.

What is critical thinking in mathematics? It is not doubting the truth of postulates anymore. It is looking at the postulates themselves and the structures they generate. It includes entertaining alternatives to certain hypotheses of methods of solving a problem of arguments used in a solution. It is not questioning arbitrarily but thinking in such a way that it leads to better insights or new insights into the problem or opens a new world of knowledge or better method of solving a problem. Learning to think critically is learning when and how to question. Critical thinking calls for examining underlying principles. It should be a liberating experience for the student. A student should become an active participant instead of being a silent recipient.

Is developing critical thinking ability one of the primary goals of an undergraduate program? The answer is 'yes'. Critical thinking as I have defined is essential for a human being to conduct herself or himself as a free and responsible member of the human society. When this ability develops,

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it will pervade all areas in which a person is called upon to make decisions or to be creative. Education at the undergraduate level should not be limiting. By this, I mean, a student after completing an undergraduate program should not be left with no options but to opt for a routine unproductive life.

It is my thinking that mathematics courses, besides familiarizing a student with applications, should open her/his mind to viewing the world intelligently. This calls for the ability to think critically, ability to abstract a given situation and the ability to appreciate and wonder. All of these are mathematical activities and all of these are essential for intelligent and exciting living.

How can this be done?

Here are some suggestions for change.

(1) Text books can be written in a different style. The traditional style is definitions, formulae, examples and problems. Instead, concepts should be presented in an exploratory style so that the students arrive at generalizations or formulae themselves. Examples of solutions to problems should be written in exploratory style so that a student could arrive at a general method of solving a problem to the extent that the problems could be categorized.

(2) Study the content of the undergraduate courses and organize the topics in such a way that the course can be taught in an exploratory way, approaching the concepts in a simple straightforward manner. One of the virtues of mathematics, I think, is simplicity.

(3) Provide an opportunity to the students such as a lab program or individual study program in which the students will be able to cultivate his/her mathematical ability.

Aesthetically appealing results and problems exist—outside the regular syllabus—in Geometry, Number Theory, Graph theory and other ares of mathematics. Lab projects based on these can by written. There are sufficient examples for all levels of mathematical maturity.

What can be done in a lab program?

I. Freshman and Sophomore level students may need initial guidance such as a manual. A manual will consist of
   (a) breaking the problem into parts
   (b) asking a series of questions that will help the student to see relationships
   (c) ask open ended questions that motivate the student to ask his/her own questions to arrive at a solution.

II. A lab manual can also just give a list of problems and topics that can be explored with a bibliography so that more mature students in mathematics can work individually or in groups.

From time to time labs can be turned into seminars in which the students who have arrived at solutions or interesting results can present their findings.

The end result should be that a student appreciates pursuing an idea, sees beauty in the arguments and simplicity of relationships and is struck with wonder at unexpected relationships and results. Mathematical thinking can be better realized with appreciation of its beauty.

Bibliography

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Math Nonsence Verse

Helen Lewy
(widow of Hans Lewy)

An astronomer who was named Cecil
Was quite fond of those functions called Bessel
Said his Wifie, "I see
You love them more than me:...
And she boarded a foreign bound vessel!

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A programmer living in Crisp,
Fell in love with a Will o' the Wisp;
Said his parents (in Cheshire)
"Don't mix business with pleashire"—
But he still did his courting in LISP!

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Oh, his-tor-y may seem to you
A thing of bygone value;
It's soothing, tho', if you compare
It to an eigen-value.

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Möbius Strip Labels, Yet?

The tags on scarves, they make me sick;
They always show—they're so conspic:
You know what this is all about?
There's no inside, there's just an out!!