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Abstract: How does mathematics connect with the search for solutions to the climate emergency? One simple connection, which can be explored in an introductory differential equations course, can be found by analyzing the energy generated by solar panels or wind turbines. The power generated by these devices is typically recorded at standard time intervals producing a data set which gives a discrete approximation to the power function $P(t)$. Using numerical techniques such as Euler’s method, one can determine the energy generated. Here we describe how we introduce the topic of solar power, apply Euler’s method to determine the energy generated, and provide a variety of lesson extensions which engage students in an exploration of policy issues related to climate change. I use these examples in my course on mathematical modeling and sustainability. There I not only want my students to understand how mathematics can be used to examine important real-world issues like climate change, but I also want to empower them to use their mathematical skills to help create solutions. To that end, the course has a community-based field component in which the students assist a community partner by analyzing a sustainability problem of relevance to the partner organization.

1 Introduction

The climate emergency [1] presents an enormous challenge to humanity world-wide. In brief, humanity’s extraction and use of huge amounts of fossil fuels over the two-hundred and fifty years since the start of the Industrial Revolution has released large amounts of CO$_2$ and other climate warming gases into the atmosphere. These gases trap heat in the earth’s atmosphere, causing the climate to warm and bringing with it a host of side-effects (drought, wildfires, heat, and other extreme weather events) that have dangerous impacts
on human society.

If humanity is to lessen the impacts of fossil fuel induced climate change, we need to transition from carbon intensive energy sources to renewable energy such as solar and wind that do not release atmospheric warming gases. In determining how much energy can be generated by renewable sources, one encounters a simple but fundamental differential equation: the rate of change of energy $E(t)$ equals power $P(t)$:

$$\frac{dE(t)}{dt} = P(t).$$

The equation can be applied equally well to analyze the energy generated by solar panels or wind turbines. My campus has a (small) solar panel array, so I focus on the former. The power function associated with solar panels is determined by data generated by the solar panel array at discrete times and is not given in analytic form (Figure 1) where we have linearly interpolated between the data points to create a continuous graph. Hence rather than finding the energy function by analytically taking the anti-derivative of $P(t)$, one uses numerical techniques such as Euler’s method. Solving for the energy function associated with solar panel data provides a compelling real-world application of numerical methods, one which could be included in any elementary differential equations course. At a more basic level, by rephrasing the problem as one of numerical integration, this example could also serve as an application of Riemann sums in an integral calculus course.

![Figure 1: Plot of solar power (watts) as a function of time generated by Bryn Mawr College’s solar array on January 17, 2014.](image)

There are a host of policy issues associated with renewable energy that one can discuss when teaching this material. Examples include state laws that determine how much renewable energy must be included in the electricity grid. These requirements in turn impact the price of SRECs (solar power renewable energy credits) that consumers can get paid for their solar production and which, by offsetting the cost of installing solar panels, help make the panels more affordable. What rules will states and the Federal government impose on where solar and wind installations can be located and what types of environmental and regulatory reviews must proposed installations undergo before
receiving approvals to build? As the demand for renewable energy grows, there is a pressing need to improve the electric grid infrastructure so electricity can be moved from locations of generation, such as the sunny Southwest or windy Midwest, to urban areas of high use. How can the government change the regulatory process to facilitate the improvement of this vital infrastructure?

A powerful stimulus to the adoption of renewable energy is the 2022 Inflation Reduction Act (IRA) that budgets some $369 billion for a host of Energy Security and Climate Change programs over the next decade [2]. One component of the IRA is a 30% rebate for solar (and other) renewable energy projects. Previous versions of the solar credits were in the form of tax rebates; hence non-profit organizations such as state and local governments, schools, colleges, universities and religious institutions, who do not pay taxes, were not able to benefit. The IRA specifies that non-profits can now benefit from the rebate which they will receive as direct payments from the Internal Revenue Service [3].

1.1 Community-Based Projects

In addition to including this equation in my differential equations course, I also incorporate it in my Math Modeling and Sustainability course. In that course, I not only want my students to understand how mathematics can be used to examine important real-world issues like climate change but want to empower them to use their mathematical skills to help create solutions. To that end, the course has a community-based field component [4] (called Praxis [5] at my college). The students work in teams to assist a community partner by analyzing a sustainability problem of relevance to the partner organization. They end the semester by reporting their findings to the partner and making recommendations for action. In many cases, the students’ recommendations have been implemented - providing tangible evidence that they can make a difference. Students commented:

\[\text{The end results of all the projects were pretty satisfying; it made you feel like you were making a contribution and that you might actually be able to affect something.}\]

\[\text{I liked that the projects we worked on were meaningful and that this course was extremely applied in nature. It was nice to do something that affected our college and/or community directly.}\]

Not only have the students learned about the connection of differential equations to renewable energy policy, they have had the opportunity to impact policy in their local community.

With the grave dangers posed by the climate emergency and the challenge of making significant progress on the issue, it is easy for students (and the rest of us too) to feel overwhelmed. The term ‘climate change anxiety’ [6] has been coined to refer to the more severe forms of this response. Research suggests that collective action, by fostering a sense of hope and community connection, can provide an antidote to climate change

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1The fine print of the IRA states that if non-profits finance their solar panel purchase by tax-exempt bonds, then the rebate is reduced.
anxiety [7].

The IRA provides new opportunities and possibilities for renewable energy projects. I hope that future student projects will partner with non-profit organizations, including schools and universities, to carry out cost-benefit analysis of renewable energy projects leading to more adoption of such systems.

I discuss the energy – power differential equation and its link to issues of renewable energy in Section 2. In Section 3, I give an overview of the Math Modeling and Sustainability course, hoping to interest other educators in teaching such a course. I discuss the community-based component of the course and how I handle the logistics and challenges of working with community partners (Section 4). I conclude with examples of student projects (Section 5 and Appendix 1: List of Sustainability Projects). While I hope that this article encourages faculty to eventually teach a community-based math and sustainability course, a good initial step would be to include some of the material discussed here in an existing course.

2 How Much Energy Does a Solar Panel Array Generate?

Question: Examine the graph of the power generated by a solar panel array over the course of a day (Figure 1). How much energy did the solar panels produce?

![Figure 2: Students visiting the college’s solar panel array.](image)

In teaching this unit, I use an approach based on constructivist learning principles [8] in which I try to connect the new information I am presenting with students’ existing knowledge. I start by asking students what they know about solar panels and what experience (if any) they have had with them. We sometimes take a class “field trip”, walking across campus to see and touch the college’s small solar array (Figure 2). We explore the website associated with the college’s solar panels which shows the power being generated by the panels throughout the day and for different days throughout the year.
To get students thinking about the units of energy that arise in solar panel systems, I have them bring a home energy-bill to class. Students who do not have a bill can use a sample bill (Figure 3). I ask them to study the bill and determine how much electricity was used during the month, the units used for energy, and the cost of the electricity. In this way, the students discover that the relevant unit of energy is a kilowatt-hour (kWh). They also realize that an electricity bill is a complicated document as there are several components that go into making up the total charge.

Our next goal is to understand what kilowatt-hour (kWh) means. I bring out a 100-watt (W) lightbulb and remind them of things they might have learned in a physics class: that 1 watt = 1 joule/second and that a joule is a unit of energy. So watt is the rate of change of energy which is called power. We recall that “rate of change” is a short-hand for derivative, thus

\[ P(t) = \frac{dE(t)}{dt}. \]

I make the analogy to water; watt is analogous to the rate at which water flows (gallons/second) (see Chapter 2 of [9]). It is unfortunate, and a cause of endless confusion, that the clue for rate of change, the term “per second”, is hidden inside the definition of watt.

To further clarify the relationship between power and energy, we start with power, in watts or kilowatts, and determine the energy used in simple scenarios for which the power is constant or piecewise constant via the equation:

\[ \text{Energy} = \text{Power} \times \text{Time}. \]

This equation lays the foundation for later solving the differential equation when the power is not constant which we will do by mean of numerical methods.

**Question 1a.** If a household is using 3 kW (kilowatt) of power continuously from 1pm to 5 pm (see Figure 4), how much energy is used?
Question 1b. What is the area = height x width under the power curve for $1 \leq t \leq 5$? Give the units for this area that you get by multiplying the units for the height by the units for the width.

![Power vs Time](image)

**Figure 4:** Constant power function.

Question 2a. If the household uses 2 kW of power from 1pm to 3pm, then 4 kW from 3pm to 7pm and 1 kW from 7 pm to 9 pm (see Figure 5), how much energy does it use?

Question 2b. What is the area = height x width under the power curve for $1 \leq t \leq 9$? Give the units for this area that you get by multiplying the units for height by the units for the width.

![Power vs Time](image)

**Figure 5:** Piecewise linear power function.

From these simple problems, the students see that the unit of energy on their electrical bills, kilowatt-hours, can be thought of as the power (kW) being used multiplied by the hours of usage. Geometrically, this energy can be interpreted as the area under the power graph with units given by kW x hours = kWh.

Now we move on to the general case of determining the energy generated when the power graph is not linear or piecewise linear (Figure 1). For many students, interpreting a graph is a challenging skill. To make the point that a graph can contain important and relevant information about the world, I have the students do a number of warm-up
exercises:

*Question 3.* Estimate the time of sunrise and sunset on January 27, 2014 in Bryn Mawr, Pa.

*Question 4.* Why are there some dips and wiggles in the graph of Figure 1? What was the weather like that day?

*Question 5.* At 9 am, how much power is being produced by the solar panels?

*Question 6.* What is the maximum power that the solar panels generated during the day? At what time of day did that maximum occur?

*Question 7.* If the panels had been able to produce that maximum amount of power from sunup to sunset, how much energy would they have produced?

Then we move on to the main problem:

*Question 8.* Estimate the total energy that the solar panels produced over the course of the day.

Hint: Apply the idea that energy is the area under the power curve. Initially, I encourage students to be creative in figuring out this area. While students who have been exposed to calculus will often employ the traditional rectangle method, with a little coaxing students employ a wide range of geometrical shapes in their estimates. Students share out their various answers and are impressed and intrigued by the wide-range of methods that their classmates have used. As an extension exercise, I ask if they think that their estimate is greater than or less than the exact amount, thereby introducing the idea of upper and lower bounds.

2.1 Euler’s Method

To link the students hands-on experience of finding the area under a curve with a more theoretical approach to solving a differential equation, we review the Fundamental Theorem of Calculus.

If \( \frac{dE(t)}{dt} = P(t) \) with initial condition \( E(t_0) = E_0 \),

then \( E(t) = E_0 + \int_{t_0}^{t} \frac{dE(u)}{du} du = E_0 + \int_{t_0}^{t} P(u)du \).

Thus, for our simple differential equation, the solution can be found by integration.

The output of a solar panel system is often displayed via a web interface in which the power is recorded once per (5 or 15 minute) time interval. This data can be considered as a discrete approximation for an associated continuous power function \( P(t) \). However, since we do not have an analytic expression for the continuous function, but only its value at
discrete points, we cannot use analytic integration techniques to determine the energy function \( E(t) \). What to do?

At this point, I introduce numerical approaches as a way to determine the energy generated by the solar panels, specifically Euler’s method. If the power function \( P(t) \) is given at a set of times \( \{ t_0, t_1, t_2, ..., t_n \} \), then

\[
E(t_{k+1}) = E(t_k) + \frac{dE(t_k)}{dt} \Delta t = E(t_k) + P(t_k) \Delta t,
\]

where \( \Delta t \) is the time interval between measurements of the power.

Over the time interval \( (t_k, t_{k+1}) \), we are treating the power function as constant. Applying Euler method’s is similar to solving for the energy in the case of a piece-wise linear power function (Figure 4). This example illustrates a principle I stress to my students: when trying to understand the meaning of a differential equation, first consider the case of a constant derivative. Once you understand that case, you will be well on your way to understanding the general case.

One can go on to introduce more sophisticated numerical methods, such as the Improved Euler and Runge-Kutta methods, in the context of this solar panel example.

From their years of experience in elementary mathematics courses, many students have developed the misconception that functions are always expressed analytically (i.e., with a formula). Examples like the solar power function (Figure 1) and other real-world data sets can help disabuse them of this misconception and lead them to appreciate the importance of numerical methods.

Such an appreciation does not always come naturally, as the following conversation I overhead calculus students having illustrates. They were bemoaning the day’s lesson on Riemann sums which they found difficult and confusing. To make matters worse, at the end of the lesson the instructor assured them that they wouldn’t really need to use Riemann sums as they would be able to take the anti-derivative instead. The students then wondered why they were subjected to the misery of Riemann sums at all. My point here is that in applications using real-world data, there is often not an anti-derivative at hand. There are times when one really must use numerical methods.

2.2 Curve Fitting

One can also use solar power data as an application of curve fitting. On a sunny day, the power curve has a nice parabolic shape (Figure 6). In those cases, enterprising students have fit a quadratic or trigonometric function to the data and then used analytic techniques to integrate the differential equation. That approach leads into the rich field of curve fitting.

2.3 Connecting to climate change and other extensions

Question: How much can we reduce CO\(_2\) emissions if we use the electricity generated from our solar panels instead of electricity generated by traditional power plants?
9. Examine the power vs time graph generated by the solar panels (Figure 4).

a. What is the maximum power that the solar panels generated during the day? At what time of day did that maximum occur?

b. If the panels had been able to produce that maximum amount of power from sunup to sunset, how much energy would they have produced?

c. Estimate how much energy the solar panels actually produced.

d. Why are there some dips and wiggles in the graph?

Figure 4. The power (kW) produced by a solar panel installation at Bryn Mawr College on January 27, 2013. 

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10. Do you think it would be “worth it” home to install solar panels at your school (or home)? What information would you need to be able to answer this question? Present your findings to the school board (or college administration).

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This question allows us to connect our solar panel analysis to the impacts of greenhouse gas emissions on climate change. We need to know how much CO₂ is produced by generating 1 kWh of electricity at a power plant. This value depends on the mix of generating sources used (coal, gas, hydro, or nuclear) and can vary widely by state (Table 1 in Appendix 2). Vermont has the lowest rate at 0.013 lbs CO₂ / kWh electricity; West Virginia the highest at 1.956 lbs/kWh.

Although this article focuses on the differential equation aspect of the lesson, there are a wide array of additional directions in which one can go.

Discussion 1. Develop a feel for how much electricity households use. Have the students share the amount of energy on their bill by entering their amount on a class spreadsheet. Since students come from an array of home situations, the energy usage will vary. Take the class average and see how close the class average comes to the national average for a U.S. residential utility customer of about 886 kWh per month [10].

Discussion 2. Discuss the various components of the bill which generally include generation, transmission and distribution charges and how they relate to the components of the energy grid [11]. This topic provides an opportunity to discuss the need to upgrade the electric grid to provide a safe dependable conduit to handle the increased amounts of electricity that will need to flow through the grid as society electrifies [12].

Discussion 3. The cost of electricity varies quite dramatically by state (Table 1 in Appendix 2). Have students from different parts of the country share what the cost per kWh is for their bill or have students look up what the cost is in different states. Which states have the most (least) expensive electricity? Why?
In the Math Modeling and Sustainability course, my goal is for students to learn how to model real-world situations involving a range of sustainability issues and to show them that much modeling can be carried out with simple (often just algebra) math. Students voice surprise at the level of math commenting "the math involved in most of these applications was pretty basic and that . . there were more numbers than mathematics involved in our projects."

However, most of the students are new to the practices of mathematical modeling [13] and so have much to learn in this area, starting with the important role of units.

In developing and refining this course over the past decade, I was inspired by a number of sources. My initial inspiration in this area came from a faculty workshop offered by Professor Tom Pfaff of Ithaca College. He shared a number of modules he had developed which linked sustainability to standard mathematics topics [14]. I use some of these modules in the course, often starting with one on the Keeling CO\textsubscript{2} curve [15]. Students use curve fitting and differential calculus to predict what the CO\textsubscript{2} levels will be a number of years in the future.

It has been sobering to see the rise in CO\textsubscript{2} levels over my lifetime [16]. In pre-industrial (before 1780) times, the level was about 280 ppm. In 1959, the year I was born, the level was 316 ppm. When I started teaching the math modeling class in 2010, the level was 390 ppm. The CO\textsubscript{2} level has continued to rise at rate of about 2.5 ppm/year and in May 2023 reached 423 ppm.

In early versions of the course, I included a strong focus on modeling ground water flow as described in the text Mathematical Modeling in the Environment by Charles Hadlock [17]. If a gas station has underground fuel tanks that leak gasoline, as has happened just this year in my own community, in what direction will the gasoline flow? Will it impact water sources? On a larger scale, if there were to be an accident at a natural gas mining site and the polluted fluids used in the fracking process were to escape into the water table, where would they migrate and what health hazard might they create? Ground water flow is modeled in simple cases by Darcy’s Law and in more complicated cases by the Laplace partial differential equation.

The text Sustainable Energy without the Hot Air [9] by David MacKay examines whether renewable energy could be harnessed on a sufficiently large scale to meet the energy needs of society. He breaks down this seemingly overwhelming issue into manageable chunks involving different types of energy sources and energy uses. He shows how to make simplifying assumptions that lead to informative order of magnitude calculations using only elementary mathematics (Fermi type problems). Topics covered in the course include:

1. CO\textsubscript{2} Emissions. Introduction to the issue of climate change and the history of CO\textsubscript{2} emissions world-wide (Ch.1 of [9]). We examine the Keeling curve [16] and see via an engaging video how the atmospheric CO\textsubscript{2} levels have changed over time [18]. Which countries have historically been most responsible for CO\textsubscript{2} emissions? What ethical obligation do these developed countries have to less developed countries that are now facing the devastating impacts of climate change but did not themselves
contribute significant greenhouse gas emissions?

2. **Energy Usage and Carbon Footprint**  How much energy is used in our everyday life: by lightbulbs, home appliances, cars, airplanes, and various types of public transportation [9]? How much CO$_2$ is generated by these activities?

3. **Renewable Energy Generation.** How much energy can different types of renewable sources (solar, wind, hydro) generate [9]? In the United States, what is the potential for solar [19] and wind [20] generation. How much land would be needed if we want to produce renewable energy at a scale sufficient to meet national needs? How does the renewable energy generation compare to energy usage?

4. **Cost- Benefit Analysis.** We ask the question, “is it worth it to invest in …..” for a variety of energy efficient but perhaps expensive technologies. We carry out cost-benefit analysis and determine the payback time and net present value for investments in energy conserving measures. For example, is it “worth it” to purchase an electric vehicle? What is the associated pay-back time? This analysis involves determining the initial capital costs (cost of car, perhaps cost of installing an EV charger) and the yearly operating costs (costs of electricity, of maintenance) and comparing these costs to those of a fossil fuel vehicle. Students learn how to use a spreadsheet to set up and carry out these calculations.

5. **Developing a Mathematical Model Involving Multiple Variables.** We derive the Darcy’s Law model for ground water flow [17] and the equation for heat transfer across a material. This latter topic leads to an examination of the potential for high levels of insulation to reduce the heat loss/gain in a building which is a crucial factor in designing net-zero energy buildings [21].

6. **Student Mini-Presentations.** Since there might be other topics of interest to the students that I have not included in the syllabus, I include a student mini-presentation component in the course. Each class starts with a short (5-minutes, one PowerPoint slide) student presentation which can be on any sustainability related topic of their choosing. In this way, students have agency to explore and share their interests with the class and I am modeling and honoring the perspective that we can all learn from one another. Students have presented on a range of fascinating topics that introduce new ideas into the course. Students often remark in the course evaluations that these presentation were a highlight. They both enjoyed researching and presenting their own topic and also learning from their classmates’ creative presentations.

If we are to create a just, sustainable world that works well for everyone, we need to evolve our thinking and develop the perspective that everyone has something of value to share, regardless of age, sexual orientation, race, nationality, socio-economic status or academic pedigree.
4 Considerations in Implementing Community Based Projects.

We describe a range of considerations that go into setting up the community projects. For other examples of how mathematics faculty teach courses with a community-based component see [22, 23].

4.1 Identifying Projects

Before the semester starts, we line up a number of projects. Providing supervision for the field component takes extra time so we limit the class size to a maximum of 16-20 students. I like to group four students together into a project team which determines how many projects we need.

My college has a Civic Engagement Office and their staff support faculty in identifying community partners and projects. Over time, I have developed my own network of community connections so need less support from this office. While I am eager to have community partners from the surrounding region, I also consider non-academic departments at our college to qualify as community partners. Students have worked successfully with staff from the Dining, Admissions, Facilities and Transportation Departments. Our colleagues in these departments have a wealth of expertise which they enjoy sharing with students. These types of cross-college partnerships break down the perception that it is only faculty who have knowledge worth sharing.

Key aspects in choosing a partner are that: they have a project related to sustainability that has a quantitative component, are willing to guide the students in understanding and engaging with the problem, and that students can make significant progress on the problem in one semester. The level of oversight that the partner provides varies. Some very engaged and generous partners meet with their student team weekly; others might meet bi-weekly. As faculty member for the course, I also meet with the student teams several times during the semester.

4.2 Student-Project Match-Making

Early in the term, often the start of the second week, the community partners come to the class and give short (10-15 minute) presentations about their organizations and their projects and answer questions from the students. Students then fill in a survey in which they indicate in order their preferences for the various projects. By good fortune, I have almost always been able to match students with their first choice; occasionally they end up with their second choice.

4.3 Getting Started

Once the student teams are selected, we have a class meeting with the Civic Engagement staff member to discuss expectations about working with a community partner and the importance of carrying out their work in a responsible fashion. The partner organizations have been, or could start being, in a long-term relationship with the college and we want this to be a positive experience for both partners and students.
As part of developing the teams, we have students think about what skills they bring to the work and then share this information with their teammates. While they initially think of skills related to math and technology, we stress that in carrying out this type of real-world team project there are a range of other skills that are equally important including organizational, social, and communication skills. In addition to making use of their existing skills, we encourage the students to select some areas in which they would like to strengthen their skills. We share with them the Career & Civic Engagement Center’s competencies [24] as a way to generate ideas.

At this stage, I encourage the teams to set up a regular meeting time. With their busy schedules, finding this common time is often one of the hardest parts of the project. If the teams are not able to find a common time, it will make communication much harder.

The teams now reach out to their community partner to introduce themselves and set up an initial conversation to learn more about the project. Growing out of this and follow up discussions, the team develops a work plan agreement with their partner. The plan describes each individual member’s personal learning goals, the work the team will do for the community partner and the deliverables the partner will receive at the end of the semester. The team members and community partner all sign this agreement. Typically, the deliverable is a report presenting the students’ research and recommendations. Occasionally the partner requests a PowerPoint slide deck rather than a more formal report. I have been open to students creating some type of video about their findings which could be shared with the partner’s community and might have more impact than a written report. To date students have not chosen to go that route.

4.4 Ongoing work

As the semester progresses, I try to meet regularly with the teams to keep abreast of their progress. I devote class time to the projects. I have teams do presentations describing their projects and what they are finding out to the class. Some class periods, especially in the later part of the semester, are devoted to project work time. Given the challenges the teams usually have in finding common work time, they appreciate this in-class time.

Half-way or so through the semester, the students submit a written update report on their work.

If I realize that there is a mathematical topic that would be useful for the teams to learn or which would be helpful for them to learn sooner in the semester, I adjust the syllabus.

4.5 The End Game

I try to give the students some assignments that will help scaffold their final report. An important part of the report is a short (one-page) Executive Summary in which they present their key findings. I explain that they should image a company director who won’t have time to read their full-report but will read the Executive Summary. The summary should contain the key take-aways.

I have the teams submit a draft Executive Summary about two weeks before the end of the semester. They might not have finished all the work yet, but they can still make a
good initial effort at the summary. I have teams read each other’s summaries and give feedback.

The projects culminate with a presentation by the teams to their partners. Usually, the partners come to the college for the event, but we have sometimes gone to their location too. The presentations are open to all members of the college community, and we often have members of the college’s sustainability community attend and occasionally the college president.

The week before the presentation, class time is devoted to the teams giving a practice presentation and getting feedback from their peers. This practice round leads to much improvement in the final presentations.

4.6 Faculty Challenges

Some students find the lack of structure and open-ended nature of these projects unsettling. They are used to the more linear progression of classroom learning. A student from the very first iteration of the class commented:

I found that the course was a little bit chaotic. We did not actually know what we were doing some of the time and I found that there was a lack of focus. It would have been nice to have known from the start what we were supposed to do in our projects, instead of gathering information that did not pertain to what we presented.

I now proactively prepare students for this uncertainty by selling it as a feature not a flaw of the course: a chance to gain experience with the types of uncertainty that they will face in “real-world” projects.

When I first started doing the projects, I was nervous about my level of knowledge. The projects could be about many different topics and go in different directions towards areas I did not have much (any) expertise in. Over time, I have become more comfortable with this issue, realizing that my general mathematical knowledge and problem-solving abilities put me in good position to provide advice and direction on the issues that arise in the projects even if they are in areas in which I am not expert. In the same way the students are challenged by the discomfort of tackling something new, it is only fair that I as the instructor face my own version of that challenge. And I enjoy the opportunity the projects present for me to learn new material.

I also struggled with the fact that so much of what happens in the projects does not involve mathematics. Rather it involves 21st century soft-skills [25] like critical thinking, creativity, teamwork, communication, leadership, initiative, planning, carry through and social skills. Should I really be taking time away from mathematics to help students develop these (important) real-world skills? And what would my more traditional mathematics colleagues think and say about me if I did? Since I had already reached the level of full professor, at least I did not have to worry about how the Tenure and Promotion Committee would view these efforts. From this article, the reader can intuit where I ended up on this issue.

At this point I would argue that these soft skills are exactly the skills and dispositions of mind that we want our students to develop through their education [26] and that
mathematics faculty should be looking for more opportunities, such as these types of student projects, to support student learning in these areas. My science colleagues tell me that both the soft skills and the ability to apply math to open ended problems are valuable in the courses they teach.

On balance, the feedback I have received on incorporating these student projects into my teaching has been much more positive than negative. People are often surprised to learn that mathematics has something to contribute to issues of sustainability and climate change and comment that they would have enjoyed their math classes more if they had the chance to do these types of projects.

Keeping close enough watch on the projects that I can help the students make good progress on their work is a challenge. With multiple projects going on at once, it can be difficult to fully understand what the students are trying to do. When I have the teams explain and give updates, it often sounds like they have a handle on the work and are making progress. However when I read their final report and am really able to dig into the details, I often find aspects of the work that might have benefited from more of my input – if I had only realized the issue earlier. A remedy would be to have the teams submit a draft of their work before the end of the semester, but a semester is short, and the teams sometimes only pull the pieces of their project together at the very end.

Balancing the classroom component of the course, in which I have certain topics I would like to cover and give assignments and tests on, with the students’ out of class project commitments is also a challenge. It is important not to overload the students with class assignments if I want them to have enough time to carry out the community project. I have made peace with the fact that I will cover less material in the course when there is a project. I feel the benefits to the students from doing the projects, including learning how to manage their own mathematical learning, greatly outweigh the loss in coverage. These benefits are also worth the extra time and effort I must put in to run the course this way.

5 Examples of Student Projects

Here is a representative sample of student projects. For additional details see [4]. A complete list projects is given in the Appendix 1: List of Sustainability Projects.

Project 1. Geothermal System.

My township was in the final stages of planning a community recreation and environmental center. The township commissioners were curious about the possibilities of including renewable energy systems in the building and asked my students to examine the issue. While solar power would be a good option for the building, that could be added after the building was constructed. So instead, the students focused on the cost-benefit associated with a ground based (geothermal) heat pump system to heat and cool the building. They worked with the building designer and one of his heating engineers to learn how such a system works and what it would cost. They compared its energy usage to a traditional heating/cooling system as well as examining the reduction in carbon footprint.
They estimated that the geothermal heat pump system would cost an additional $600,000 but would provide a yearly savings of $50,000 in comparison to a traditional system. By using less energy, it would also significantly reduce the carbon footprint of the building. The students presented their findings at a commissioners’ meeting (Figure 7). The next month, the Commissioners voted unanimously to install the geothermal system. One of the Commissioners commented: This is a historic day for our township - an acknowledgement that the Commissioners were starting to be proactive in considering environmental factors when making their decisions.

The following summer, a new team of two students worked to support the township’s efforts to submit a $300,000 grant proposal to the state’s energy reduction program. The students worked with township officials to collect and organize data about energy savings, emissions, and economic benefits of the projects. They entered all the data into the state’s online application system. At the last minute, there was a web glitch and the online system receiving data from Mac computers failed. The students calmly transferred all the data over to a set of PC computers and succeeded in uploading the application before the deadline. The township was awarded the $300,000 grant.

In appreciation for their contributions to the success of the grant application, the Township Commissioners presented an official declaration of thanks to the students.

This project grew out of a connection I had formed with the township Recreation Director through an earlier project and illustrates the value of being on the look-out for initiatives that are taking place in one’s local community.

Project 2. Composting and Waste Management.

Our college was undertaking a composting pilot project in the dining halls. They asked the students to undertake a cost-benefit analysis of the program. The students found that there was a large reduction in the amount of trash being generated since the food waste was going into the compost bins rather than the trash. As a result, the college was able to
reduce the size of the trash dumpster that the dining hall was using. The cost of trash service depends on the number of pick-ups as well as the size of the dumpsters. The cost savings from having a smaller dumpster cancelled out the additional cost of paying for the compost pickups. Thus the compost program was cost neutral.

The company picking up the compost was based in Delaware and had a 30 mile drive each way to reach our campus. The students argued that the carbon footprint of driving a fuel-inefficient truck more than outweighed the environmental benefits of the composting.

In their final report, they recommended that the college continue and indeed expand the composting program but look to switch to a more local composting company. Both recommendations were accepted and acted upon by the college.

The results of this project got the Facilities Department thinking about the size and number of pickups of trash dumpsters all over campus. The next time I taught the course, students worked with facilities to examine the dumpster issue. Facilities painted lines inside the dumpsters at the \( \frac{1}{4} \), \( \frac{1}{2} \) and \( \frac{3}{4} \) full levels. Before each dumpster pick-up, students would record how full they were. The end result of their study was that although it did not seem practical to reduce the number of pick-ups per week, a number of the dumpsters could be reduced in size with little risk of the smaller dumpsters becoming overloaded. The college made these changes which resulted in savings of $1000 per month. As this project illustrates, one project can lay the ground work for the next.

Project 3. Electric Vehicles.

Our college has a “Blue Bus” shuttle system connecting to our partner college two miles away. One of the buses was reaching the end of its life and the transportation department was preparing to purchase its successor. The students worked with the director of transportation to research the cost-benefit of purchasing an electric bus. They found that the initial cost would be several hundred thousand dollars more than for a fossil-fuel bus ($450,000 to $150,000) while there would be annual savings from reduced maintenance and fuel costs. Because of the long hours the bus would be in use and the large battery used to power the bus, the college would need to purchase a high-speed charger ($50,000). If, as anticipated, future Blue-Bus purchases were also electric, this charger would service them too. The students found that there were a number of grants that the college could apply for to help cover part of the purchase cost. There were health benefits to the electric bus as well: the bus would generate no exhaust or noise pollution.

The college president and the directors of transportation and facilities attended the students’ presentation. In addition to the above quantitative benefits, the students also stressed the (less easily quantifiable) positive public relations benefits that would accrue from being an early-adopter of this innovative new technology. Although the college was initially hesitant, after a year’s delay and further investigation, the college decided to purchase the electric bus. The bus is due for delivery in late fall of 2023. For the past two summers, when the president gave her State of the College address to alumnae at reunions, she bragged about the upcoming electric Blue Bus as evidence of the college’s commitment to sustainability.

Projects 2 and 3 show that the non-academic departments of one’s institution can be great partners and provide excellent projects.
Project 4. Data Analysis.

Many of the student projects have been forward looking, modeling the potential costs and benefits of adopting a new technology. In fall 2022, we flipped this practice and instead looked backwards to see how much savings had been realized by a local town which had instituted a number of sustainability initiatives over the past decade. The student teams worked with the town’s grants administrator to analyze the impacts of energy saving (motion detectors, parking garage and street light conversion to LEDs) and waste initiatives (a pilot composting project). The administrator had detailed data on the initiatives but did not have the time to dig into the data. When the initiatives were initially proposed, a consulting company had predicted certain levels of savings. She wanted to know whether these savings were realized.

We were fortunate to have a partner who had an excellent set of data but even so there were a range of challenges for the students to access, compile and make sense of the data. And there were some pieces of data missing that added to the challenge. For the energy savings initiatives, the students found that the realized savings were slightly larger than had been predicted and totaled around $100,000! They also calculated the reduction in carbon-footprint due to the reduction in energy use.

For the composting, the students found that the 200 households taking part in the pilot project has diverted about 12% of their waste (35 tons) from trash to composting. Since the fee for disposal was about $20 less per ton for composting than for trash, there was a financial savings of some $700 in disposal fees as well as the associated environmental benefits of composting [27].

The students presented their findings at a meeting of the town’s Environmental Advisory Committee which was pleased to see concrete evidence of the benefits of the various sustainability initiatives. The committee is now exploring the possibilities of expanding the composting program to the entire town.

As a follow-up, the next semester several of the students took an independent study class with me and worked on further aspects of the analysis.

This project grew out of a talk I gave on the Math Modeling and Sustainability course at the county’s Earth Day celebration. The grants administrator was in attendance and expressed interest in having students work on her projects.

The LED streetlight conversion project echoed a student summer research project from 2017. The student worked for the Philadelphia Street Lights Department analyzing the cost-benefit of converting the city’s 120,000 streetlights to LEDs. She found that LED lights could reduce energy usage by about 50%, saving the city on the order of $5 million per year, and reducing carbon emissions by 15,000 metric tons annually. As an example of the sometimes-slow pace of government, it took until 2022 for the City to move forward with the conversion project [28].

Project 5. Developing Sustainability Plans.

A team of high school students wanted their school district to become more environmentally responsible by switching to a renewable energy provider. With our state’s deregulated electric energy market, customers can choose which electricity provider to
use [29, 30]. The rates and generating sources of electricity vary by company. The school
district’s existing electricity contract was with a company whose electricity was generated
by traditional power plants. The students wanted the district to switch to a supplier whose
electricity came from renewable sources. They asked my students to analyze the carbon
and financial savings the district could realize if they made the switch.

The school district made available their energy bills for the eight schools in the district.
Using these bills, the college team undertook a greenhouse gas inventory. They determined
the total electricity use per year by the district (9,103,600 kWh) which generated 6,763,970
lbs of CO$_2$ emissions (calculated using the PA state rate of .74 lbs CO$_2$/kWh). The students
found an electricity provider who would charge the same rate as the existing company
but would source the electricity from renewable sources. Thus, the school district had a
cost-neutral opportunity of achieving a carbon footprint reduction of some 3075 metric
tons per year.

That same semester, a second team worked with members of a township’s Environmental
Advisory Council to undertake a similar analysis for that township.

The growing interest by local governments and educational organizations in becoming
carbon neutral [31, 32] could provide a plethora of future project opportunities for students.

These two projects grew out of contacts I made by attending meetings of a regional
parents’ group which was working to get their school districts to address climate change.
I learned about this group in conversation with one of the region’s leading grass-roots
climate activists who was knowledgeable about initiatives taking place in the region.

**Project 6. Solar Panels.**

For our final example, we link back to the differential equation which determines how
much energy is being produced by solar panels and describe a project that supported the
adoption of solar panels.

I sometimes teach my course as part of the college’s 360° initiative [33] in which several
courses are clustered together around a unifying theme. In spring 2020, I took part in the
Climate Change 360° in collaboration with faculty members teaching courses in political
science (Global Politics of Climate Change) and philosophy (Science, Technology and the
Good Life). We were slated to take a 10-day trip over spring break to Freiburg, Germany
to learn about the many green initiatives undertaken there but Covid intervened, and we
had to switch to a virtual trip. One of the interesting things we learned was that many
renewable energy projects (wind turbines, solar panels) in Germany had been funded by
citizen groups coming together via Co-op organizations. This differs from the United
States where large energy companies and financial institutions are the main drivers of
renewable energy projects.

Intrigued by the German approach, a team of students investigated what it would
take to put solar panels on the Community Recreation and Environmental Center (see
Project 1 above) and whether such a project could be funded by a citizens’ investment
fund. They learned that the laws in Pennsylvania and the United States make this type
of citizen funded initiative very difficult but that there are (complicated) potential work
rounds. They presented their findings first to the Township Environmental Advisory
Committee and then to the township commissioners. The commissioner appreciated the
students raising the issue: they had not thought about putting solar panels on township owned buildings and were intrigued.

As has happened in a number of projects, simply bringing forward an innovative idea, separate from the accompanying mathematical analysis, can often provide the impetus for positive change.

The commissioners ended up deciding that the citizens’ funding component raised too many legal issues and instead they would have the township fund the solar panel installation itself. They budgeted $1 million for the initiative!

Since they and their township staff did not have expertise in solar issues nor that capacity to conduct the needed research, the commissioners formed a Citizens Ad Hoc Advisory Committee of which I became the chair. Our committee developed a Request for Proposals (RFP) document which would form the basis for bidding to select a contractor. Before the RFP could be approved and set out to bid, several glitches arose which have delayed the process. These delays turned out to be fortuitous as in the meantime the Inflation Reduction Act passed with its provision of giving non-profits, such as the Township government, a 30% cash rebate on renewable energy projects. As a result, the township will have a significant savings when they purchase solar panels.

I am hoping that future student projects can help schools, local governments and other non-profits access the benefits of the IRA and develop their own renewable energy projects.

Let me end by encouraging you to provide your students with the opportunity to use their passion, energy and math skills to address the challenges of the climate emergency and make a positive difference in their communities.

Appendix 1: List of Sustainability Projects

We list the various community-based projects that students have undertaken in the Math Modeling and Sustainability course, organized by who was the community partner.

College Partners

Dining Services

- Trays in dining hall: Would removing trays from the dining hall cut down on food waste?
- Composting: Cost-benefit analysis.
- Freight Farming: Cost-benefit analysis of using a repurposed freight container to set up a hydroponic growing system.

Facilities Department

- Trash audits.
- Landfill or incinerator for college waste?
• Energy savings in buildings from using temperature setbacks (conservation mode).
• Pay-back time for LED bulbs.
• On/off switch for fume hoods in chemistry department labs: cost-benefit analysis.
• Energy footprint for science building renovation.
• Potential for electric Blue Bus.
• How many electric vehicle (EV) chargers would be needed if college fleet was converted to EVs?

Admissions Office
• Paperless admissions system.

Community Partners
• Energy efficient windows at retirement center.
• Math education materials on recycling for school district.
• Storm water management via rain gardens: how much run-off have been captured?
• Earth Quaker Action Team (EQAT) social action organization: social cost of carbon to convince electric utility to increase the proportion of renewable energy in their portfolio.

Local Governments
• Alternative energy system for township recreation center.
• Level of safety analysis for bike routes.
• Which city owned buildings would be best candidates for solar panels?
• How much energy and CO₂ has been saved by sustainability initiatives?
• Converting streetlights to LEDs.
• Waste/composting analysis.
• Analysis of town’s tree cover and associated benefits.
• Potential to convert vehicle fleet to EV.
• Where to put EV chargers in town?
Business

- Electric service vans – home garaging or central garage?

Public Schools

- Potential for rooftop solar and cost-benefit analysis

Appendix 2: Data on Electricity Generation by State

Here we give data on the amount of CO₂ produced per kWh of electricity generated and on the retail price of electricity (can be accessed at [34].)

<table>
<thead>
<tr>
<th>Name</th>
<th>Carbon dioxide (lbs/kWh)</th>
<th>Average retail price (cents/kWh)</th>
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<tr>
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Table 1: CO₂ production and retail cost per kWh of electricity generated by state.
References


