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It is said that the motto which adorned the doors of Plato's Academy advised, "Let no one unversed in mathematics enter here." For the Greeks, the study of mathematics furnished the finest training field for the mind. It occupied an esteemed place in the curriculum of Plato's Academy. No person was considered educated if he did not know mathematics.

Mathematics has not retained such a dominant position in modern education. However the increasing importance of the discipline in the twentieth century because of the computer and related technologies has generated renewed interest in its study. Today's academies universities and colleges - are requiring the study of mathematics for more students than ever before. In 1983, the largest public university system in the country - The California state University System - established a mathematics course as a graduation requirement for all students at any of its nineteen campuses. Liberal arts colleges throughout the United States are reinstating mathematics requirements for all majors. Mathematics plays a central role in the curriculum of most universities and colleges throughout the world today.

With the exception of specialized universities like MIT and CalTech, Mathematics Departments nationwide are considered service departments, offering the majority of their courses to students in fields other than mathematics. Such departments as Engineering, Computer Science, Business, the Physical Sciences, and the Social Sciences include a core of mathematics courses which are essential to meet the mathematical needs of their majors.

A conference was held at Williams College in Williamstown, Maryland, in 1982, to evaluate the mathematical needs of students in other disciplines and the implementation of curricula to meet those needs. Papers were presented by mathematicians and mathematics educators identifying requirements for specific fields: Isaac Greber for Engineering, William Scherlis and Mary Shaw for Computer Science, Stanley Zionts for Business, Jack Lockhead for the Physical Sciences, and Robert Norman the Social Sciences.

In Engineering, the required mathematics core represents topics that are regarded as fundamental mathematics as well as those topics which students are expected to know in order to solve engineering problems in other courses. They include the understanding of limits, functions, complex variables, integral equations, and the calculus of variations. The central analytical tool of the engineer is the ability to derive, solve, and understand differential equations. Other mathematics courses which are becoming more important to engineering majors with the advent of the computer are probability and statistics, Boolean algebra, and numerical methods.

The requisites for Computer Science are many. The modes of thought which characterize mathematics are crucial to prospective computer scientists. Probably the most important contribution mathematics makes is teaching these students how to reason abstractly and problem solve.

Until the 1950's the role of mathematics in Business Programs was minimal. About that time, the discipline called Management Science was incorporated into business schools, and more and more

quantitative techniques were employed in business classes. Mathematical applications to management problems are abundant. Models that include linear programming and computer simulation are widely used in industry. Students enrolled in Business Programs need a mathematics background which includes algebra, beginning differential and integral calculus, matrix algebra, linear programming, and simulation, all with applications in the field.

Mathematics is the language of the Physical Sciences and has traditionally been at the center of all programs in those fields. Calculus is essential to the study of the physical sciences. What students in those disciplines need most from their mathematics preparation is a basic understanding of variables and functions and how to express them in mathematical language.

Mathematics preparation for the Social Sciences is somewhat different from that for prospective physical science, mathematics, or engineering majors. In fact, unlike these disciplines, there is no generally accepted body of mathematics that every social science student is expected to know. Yet social scientists, when questioned, will indicate a wide range of mathematics that they find useful in their fields. These include probability, manipulative algebra, computing, statistics, calculus and differential equations, combinatorics, linear algebra, sets and relations.

In each of the above-mentioned fields the validity of prerequisite courses in mathematics is unquestioned to provide the appropriate background for students to pursue subsequent coursework in their disciplines. The service function of mathematics is clearly defined and well justified.

However, **teaching** mathematics as a service course has some built-in liabilities. The experience of teaching mathematics to students for whom the subject is not their major field is often less satisfactory than teaching mathematics majors. Non-mathematics majors approach the subject from a perspective that is different from those for whom mathematics is their major field. Non-majors are more interested in what mathematics can do (within the limited focus peculiar to their discipline) than what mathematics is. They generally study mathematics only because it is useful in preparation for their fields, and this sometimes impairs their enthusiasm for the subject.

Is utility **sufficient** as a motivation for learning? Some mathematicians (like C.F. Gauss who has been credited with boasting that pure mathematics is useless) will ask if utility is even a **necessary** condition for study. Others (like Philip Davis and Reuben Hersh) will question if utility can be measured:

> . . .the meanings of the expression "mathematical utility" embrace aesthetic, philosophical, historic, psychological, commercial, scientific, technological, and mathematical elements. Even this does not include all possible meanings. . . One can distinguish between utility within the field itself and utility to other fields. Even with these subdivisions, the notion of utility is exceedingly slippery (1981, p. 80).

Measuring the mathematical utility of a course is often not a realistic task. However, even if one determines that what is presented in a mathematics class is useful for some purpose, the learning experience is incomplete if utility is the **only** focus. A fixed goal of learning a specified syllabus may be pursued, and perhaps attained. But if at no time the issues of how and why the goal is important are discussed in the classroom, the students get no perspective on the mathematics being taught. They focus only on the mechanics of the discipline. They have no real interest in concepts, in learning what mathematics is in addition to what it does. For students in Engineering, Computer Science, Business, and the Physical and Social Sciences, prerequisite mathematics coursework should serve, in addition to its utilitarian goal of preparation for work in the field, to provide them with some context of how the mathematics they find useful has come to

be, how it relates to disciplines other than theirs, and how it affects their lives.

These goals become even more important when mathematics course requirements are extended to students in non-math related fields: the humanities, the arts, etc. As Lynn Arthur Steen indicates in the January, 1986 issue of Focus, the newsletter of the Mathematical Association of America: "For students in the arts and humanities, mathematics is an invisible culture - feared, avoided, and consequently misunderstood.n These students see no utility in learning mathematics. The biggest challenge in teaching liberal arts students is enabling them to recognize mathematics as a creative, human endeavor, as a "humanistic discipline".

Professors are frequently unprepared to meet such a challenge. From experience, they know that mathematics is an ever-changing process which permeates our personal as well as professional lives, but they often don't know how to teach that to students. Although they recognize the intrinsic value of mathematics, worthy of study as a major field, necessary in providing the essentials for other disciplines, appropriate as a component of liberal education, they have not been trained to convey these ideas to students in the classroom. Courses which emphasize the humanistic aspects of mathematics are not part of the traditional curriculum for a Ph.D. in mathematics. Most mathematicians teaching in colleges and universities have never even studied the pedagogy of their discipline. They teach mathematics as they do mathematics. They are excellent mathematicians, but sometimes they lose sight of the need to present their subject in its human context.

This is not true of the mathematics community in general. Mathematicians like Morris Kline, Paul Halmos, Lynn Arthur Steen, Anneli Lax, Alvin White, R.J. Moore, Reuben Hersh and Philip Davis are just some of many who have written extensively about teaching mathematics humanistically. The journals regularly include articles with this perspective. For example, the American Mathematical Monthly (April, 1982) featured an article in the section entitled "The Teaching of Mathematics" edited by Mary and Robert Wardrop which places mathematics squarely in the center of human development:

> Mathematics has played a central role in the development of modern civilization. It has been essential not only to the growth of science and technology, but has had profound effects on philosophy and other forms of thought as well (Page 270).

Mathematics has a place in history. It is part of the human experience. There are few who will deny that this historical perspective affords all students good reason to learn mathematics. Yet for a variety of reasons this aspect of mathematics is often ignored in the teaching of the discipline. Students, in huge lecture halls facing blackboards strewn with Greek symbols, often get no evidence that mathematics is a growing, living, creative process, changing with time. Presentation of topics to comply with crowded syllabi and thick, heavy textbooks cluttered with formulas, theorems and proofs, most times reinforce the students' belief that mathematics is a discipline existing outside of time and human activity. This view distorts mathematics and discourages interest in the subject. As Davis and Hersh point out in Descartes Dream:

> A detemporalized mathematics cannot tell us what mathematics is, why mathematics is true, why it is beautiful, how it comes to be, or why anybody should care a fig about it. But if one places mathematics squarely within human time and experience, it becomes a warm and rich source of possible meanings and actions. Its ultimate mystery is never dispelled, yet it is exhibited as one of the primary creations of the human intellect (1986, p. 201).

When students see mathematics as a human endeavor, they have more than utilitarian reasons to understand why they should learn it. At the annual meeting of the Mathematical Association of America in San Antonio, Texas, in January, 1987, Anneli Lax of the Courant Institute so indicated: "Humans retain what they learn best if they can put new material into a human context, e.g. connect it to past experience, future aspirations, previously acquired mental structures."

Mathematics instructors need to help students recognize mathematics as a human process, integral to life, not fragmented from it. Part of this involves a fostering and understanding of mathematics in relation to other disciplines. According to Jean Piaget, all learning is interdisciplinary. This basic fact has often been ignored in the teaching of mathematics. For too long students have studied mathematics as a closed unchanging subject in isolation from other disciplines.

At the San Antonio meeting, Lax spoke about the effect of a fragmented approach to mathematics:

The isolation of mathematics from the rest of life causes further fragmentation of mathematics itself into arithmetic, algebra, geometry, etc., and these subdisciplines get cut up into sections or modules or skills to be mastered, tested and forgotten.

Attempts to free mathematics from this isolation and fix it firmly as an integral part of the human experience are currently being pursued in universities throughout the country. The NEXA program at San Francisco State University, for example, uses an interdisciplinary approach to instruction which involves faculty from different departments team-teaching mathematics classes. There are concerted efforts nationwide to integrate real-life applications into mathematics instruction. There is renewed interest in teaching what George Polya described as "the most characteristically human activity . . . problem solving" (Polya, p. ix).

But perhaps the most significant effort to revitalize the teaching of mathematics is being undertaken today by a network of mathematicians and mathematics educators devoted to what is described as "the humanistic dimensions of mathematics". Alvin White of Harvey Mudd College hosted a conference in Claremont, California, in March 1986 to try to develop a concrete definition for the phrase "mathematics as a humanistic discipline" and to discuss appropriate pedagogical goals for teaching relative to that definition. Participants in the conference assigned a variety of different, but ideologically complementary meanings to the phrase which gave rise to a series of educational objectives. In a monograph discussing the conference, White wrote:

> The concept of "mathematics as a humanistic discipline" is not as well defined as a geometric series or a triangle, but it is more evocative. Many mathematicians who have heard the phrase are not troubled by the lack of a succinct definition, but are excited by the richness of the fruitfulness that they anticipate. . .the concept, even if ill defined, challenges traditional ways of teaching and learning mathematics at all levels.

The thirty-six conference participants identified two pedagogical goals which result from defining mathematics as a humanistic discipline:

1) teaching mathematics humanistically - altering the nature of the teaching and learning environment, and

2) teaching humanistic mathematics - reconstructing the curriculum and the discipline of mathematics itself.

The first goal, teaching mathematics humanistically, seeks a student-centered classroom, placing the student in the position of inquirer rather than passive learner. It encourages the development of a community of learners with both professor and

students learning from each other. To teach mathematics humanistically, the professor must recognize the emotional climate of the learning environment and provide appropriate access for students to engage in participatory learning within that environment.

The second goal, teaching humanistic mathematics, strives "to integrate the humanistic elements of science with the mathematics" (White, 1976, p.245) in the classroom. It advocates curricula which relates mathematical discoveries to the human beings who made them. It encourages the exploration of the relation of mathematics to other disciplines, and to the culture in which it is embedded. To teach humanistic mathematics, the professor must allow the students to experience the curriculum within the context of its past, present, and potential impact upon the world in which they live.

With the dual themes of teaching mathematics humanistically and teaching humanistic mathematics in mind, the conferees outlined fourteen desirable objectives for the classroom. Among them are to provide students with

- an appreciation of the fundamental interrelationships of all knowing.
- a curriculum which relates mathematics to other areas, such as science, technology, humanities, and ethical issues, including personal ethics.
- an understanding of the human dimensions that motivate discovery, such as competition, cooperation, the urge for holistic pictures in contrast to pieces.

The thread which unifies all the pedagogical goals outlined at the conference is perhaps best expressed by Alvin White in his article Beyond Behavioral Objectives:

> . . . our guidelines and teaching objectives should not have as their major target or focus the mastery of facts and techniques. Rather the facts and techniques should be the skeletal framework which supports our objective of imbuing our students with the spirit of mathematics and a sense of excitement about the historical development and the creative process. The concepts and relationships of mathematics should be presented as the building blocks of this magnificent edifice created by the human imagination (White, 1975, p. 850).

No mathematician would deny these goals in teaching. Not only are they not controversial, they are not even new ideas. Early in the twentieth century, curriculum interest groups called "humanists" led by Charles William Eliot and William Torrey Harris recognized mathematics as "an important part of our Western cultural heritage" (Stanic, 1986, p. 192). Mathematics educators, predominantly at or near the University of Chicago, advocated breaking down the barriers between mathematics and other disciplines and between the various areas within mathematics. They promoted the teaching of mathematics within the context of human experience.

Certainly, those who have read Morris Kline, who have studied Piaget, who have focused on the teaching of mathematics as well as the doing of mathematics, are most likely pursuing these objectives in their classrooms. But what is obscuring those goals for others?

In Mathematics Tomorrow, Peter Hilton outlines those forces which he believes undermine effective mathematics education. He includes the quest for instant satisfaction (ignoring the fact that education is a slow, gradual, cumulative process with rewards that are largely long-term); the view that the prime purpose of education is to guarantee a high material standard of living (affirming the utilitarian goal of learning); and the mechanical approach to teaching elementary mathematics which emphasizes rote calculation and memory dependence (compounded by the fact that many teachers are not skillful in mathematics, do not enjoy the subject

themselves, do not feel comfortable with it, and convey these attitudes to the students). He is particularly critical of standardized tests:

> These tests, superimpose a degree of artificiality on that which is already present in the curriculum. They force students to answer artificial questions under artificial circumstances; they impose severe and artificial time constraints; they encourage the false view that mathematics can be separated out into tiny water-tight compartments; they teach the perverted doctrine that mathematical problems have a single right answer and that all other answers are equally wrong; they fail completely to take account of mathematical process, concentrating exclusively on the "answer" (1981, p. 7 9) .

In this quote, Peter Hilton focuses on how standardized tests distort the teaching curriculum and promote those very practices which discourage learning: failure to make mathematics real for the student, fragmentation of the discipline, disregard for mathematics as a creative process.

But standardized tests are only a symptom of traditional ineffective approaches toward teaching mathematics. Even when the inappropriateness of standard pedagogy is exposed, significant change does not occur. What is the reason for such inertia in the system, for what Hilton calls "the remarkable stability of those practices which militate against effective mathematic education" (1981, p. 79)? According to the title of an Opinion article in the January 21, 1987, Chronicle of Higher Education "Mathematics Teachers Are Too Lazy to Change Their Ways; as a Result Teaching is Stagnant." In that article, James T. Sandefur states:

> Mathematics, like other disciplines, is evolving, but the teaching of math is stagnant. Instead of developing new courses, as professors do in other disciplines, we continue to teach the same topics that were taught to us. . . There has been little outside pressure for a change in the way we teach math. For one thing, non-mathematicians rarely know enough math to be able to make concrete proposals. Also many of them are intimidated by mathematicians who tell them that change is impossible and suggest that such a proposal only shows their ignorance. As a result, mathematicians have been able to dictate what is covered in math courses and to resist any pressure for change. (Sandefur, p. 44).

There is some truth in what Sandeful is saying. More than laziness, however, the culprit is fear. In an article in the Education journal, Alvin White discusses the reasons why he found it difficult to deviate from conventional approaches to teaching Calculus when he was considering experimenting with a humanistic alternative: "In addition to having to contend with the skepticism of my traditionalists colleagues, I also had to deal with my own self doubts" (White, 1974, p. 132). It is as Carl Rogers indicates in Freedom to Learn:

> I believe that all teachers and educators prefer to facilitate . . . [a] meaningful type of learning. . . Yet in the vast majority of our schools, at all educational levels, we are locked into a traditional and conventional approach which makes significant learning improbable if not impossible. . . It is not because of any inner depravity that educators follow such a self defeating system. It is quite literally because they do not know any feasible alternative" (Rogers, 1969, p. 5).

Mathematics professors must not only identify the problems in traditional mathematics education, but they must provide alternatives - practical ways courses can be taught, different hypotheses upon which instruction can be built, new goals for which teachers and students can strive. They must also create a climate receptive to such alternatives. If the present network of humanistic mathematicians and mathematics educators which is assembling can do this, they will succeed in ways that their early twentieth century counterparts did not. The evi-

dence to date is promising. The new "humanists" are seeking to provide concrete alternatives. They are generating interest throughout the country in teaching mathematics humanistically. They are attempting to illustrate why and how mathematics should be taught to all students.

Once mathematics is recognized as part of the human experience, the approach to its teaching will seek to demonstrate how mathematics has evolved, how it works in our present environment, and what are its possibilities for our future. It will focus on what is beautiful, creative, and imaginative in mathematics, stimulating students to want to discover more about the subject, and its relationship to other disciplines. very few persons choose mathematics as a career. Mathematics classes can and must engage the non-mathematicians without losing the prospective majors. George Polya claims that to be justified, mathematics courses must conform to two principles:

> First, each student should be able to derive some profit from his study irrespective of future occupation.
> Second, such students as have some aptitude for mathematics should be attracted to it and not get disgusted with it by ill-advised teaching (p.122).

Lynn Arthur Steen describes mathematics as an "enabling force" (Steen, 1986, p.l). Mathematics professors must help students understand mathematics in its historical perspective, with its creative present and imaginative future. It is these humanistic aspects of mathematics which will provide the approach to its teaching and the reasons for its study.

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