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# What is Mathematics and Why Don't They Know That?

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I have become increasingly dismayed regarding the image the populace has of mathematics. I guess my first awareness of this was when I began teaching a survey course for liberal arts students. I begin my course by asking the students to write a brief essay on what, in their minds at the time, is mathematics. Invariably, I get statements about the "important" uses of mathematics in making change, balancing checkbooks, and similar mundane tasks or vague statements about how extremely important mathematics is without any indications of how or why. Even though they had perhaps nine or ten years of study of mathematics in school, their idea of mathematics was very limited. Perhaps one might have expected that, and I have tried, occasionally with success, to instill in these students a broader view of mathematics, and an understanding of mathematics as an integral part of our culture, and the part mathematics plays in the development of culture. This year, I had an opportunity to teach an upper division mathematics course at a nearby liberal arts institution. The class consisted almost exclusively of mathematics majors. Before delving into the course of study, I thought we would begin with a discussion of the meaning of mathematics. The entire contribution of this class to my intended class discussion was that mathematics "is the science of numbers". Now this is getting serious, folks. These are mathematics majors, nearing graduation from college, on their way to teaching high school math or entering graduate school, whose only concept of mathematics is that it is the science of numbers.

Why this paucity of knowledge of what mathematics is? John Lucas, in his paper presented at the 1989 MAA-AMS joint meeting in Phoenix implicitly, but correctly, I believe, laid the blame on the way mathematicians taught. This can be illustrated with a discussion I had with a bright student in one of my calculus classes. He came to my office several times with questions. The nature of his questions made clear that there was a lack of understanding of basic mathematical concepts. When I queried him on the meaning of these, he said he wasn't used to thinking about mathematics in that way. Then followed this conversation. "What did you do in your high school math classes?" The. teacher did a couple of problems on the board, and then we did a bunch of problems." "And then you checked your answers in the back of the book?" "Yes." "Did you ever talk about what any of this meant?" "No." A letter that appeared in the "From Our Readers" section of UME Trends, January, 1990, shows that this is not a local problem. This letter, written by a community college instructor, states,"If a student can be given an equation such as 2X - 7 = -6 and came back with X = 1/2 then they (sic) know algebra,...If I can teach a student to 'get' X = 1/2 using symbol manipulation, then I have fully succeeded." Wrong, Mr. Brown, absolutely wrong! You have not succeeded if the student doesn't know what it means to have the solution of the equation or cannot articulate this meaning. We cannot assume that students construct meaning to this exercise. I have asked students in classes from algebra to calculus and statistics (yes, and even differential equations) what a solution of an equation is. Very few can tell me. They have learned to manipulate symbols to come up with what they call, "the answer", but they have no concept of what this means.

Not only are we, as teachers of mathematics, responsible for the development of this myth that mathematics is symbol manipulation, but we add to misunderstanding in other ways. In a class I was visiting recently, the result of simplifying an expression was 21/6. In response to a student question, "Is this as far as we can go?", the instructor replied, "No, we can actually get the number if we want to." He then proceeded to describe how to use the calculator to "actually get the number". Obviously, to him 21/6 is not a number, but the decimal approximation from the calculator is "a number". It is apparent that this person learned his mathematics in a small white cave in the mountains of the island of Samos in the 6th century B.C. Current thought is to accept irrationals as numbers, Jerry.

Perhaps, since the title of this paper includes the query, "What is Mathematics?", an attempt should be made to answer that. Granted that as with the three blind dudes and the elephant, there is reason for divergent views, and granted that Courant and Robbins have clearly answered this extensively with a book of 500 pages, perhaps, in the spirit of the nature of the language of mathematics, we can come up with something more precise than the former and more concise than the latter. I would like to nominate, at least as a starting point, the definition found in the Lawrence University catalogue.

> Born of man's primitive urge to seek order in his world, mathematics is an ever-evolving language for the study of structure and pattern. Grounded in and renewed by physical reality, mathematics rises through sheer intellectual curiosity to levels of abstraction and generality where unexpected, beautiful, and often extremely useful connections and patterns emerge. Mathematics is the natural home of both abstract thought and the laws of nature. It is at once pure logic and creative art.

What, then, can we do to overcome this vast misconception of mathematics? First of all, while I don't advocate complete agreement on a definition of mathematics, I do promote discussion in the mathematics community on what is the view of our discipline we would like to promote and how we can bring that about. An obvious starting point is in the teaching of mathematics - at all levels. We need to think seriously about what mathematics we should teach and how we should teach it. The course that we laughingly call "college algebra", and is a requirement at many institutions, has degenerated into a course primarily in symbol manipulation. Is this a reasonable and desirable demand in an age when machines can manipulate the symbols? It seems incongruous to me that people who insist that we must utilize the latest technology in our mathematics classrooms are still teaching classes and advocating classes that stress symbol manipulation. Are we replacing how to manipulate symbols with how to push buttons to get the machines to manipulate the symbols for us? I agree with Raymond Wilder who said, long ago, Mathematics was born and nurtured in a cultural environment. Without the perspective which the cultural background affords, a proper appreciation of the context and state of present-day mathematics is hardly possible." It behooves us to provide all our students with the culturological perspective of Wilder's reference.

The Humanities, I believe, have come up with a particularly insightful document from which we can benefit. The National Endowment for the Humanities, in their document "50 Hours", have come up with a core curriculum for college students.

18 hours: Cultures and Civilizations

I. The Origins of Civilization: a one-semester course that considers the beginnings of civilization on various continents. 3 hours.

II. Western Civilization: a one-semester course that considers the development of

Western society and thought from Periclean Athens through the Reformation. 3 hours.

III. Western Civilization (continued): a one-semester course that considers the development of Western society and thought from the Reformation into the twentieth century. 3 hours.

IV. American Civilization: a one-semester course that traces major developments in American society and thought from colonial times to the present. 3 hours.

V and VI. Other Civilizations: two one-semester courses to be chosen from the following civilizations of Africa, East Asia, Islam, Latin America, South Asia. 6 hours.

12 hours: Foreign Language: a two-year requirement; it is recommended that students fulfill this requirement by taking more advanced courses in a language they have studied in high school.

6 hours: Concepts of Mathematics: a one-year course focusing on major concepts, methods, and applications of the mathematical sciences.

8 hours: Foundations of the Natural Sciences: a one-year laboratory course that focuses on major ideas and methods of the physical and biological sciences.

6 hours: The Social Sciences and the Modern World: a one-year course that explores ways in which the social sciences have been used to explain political, economic, and social life, as well as the experience of individuals, in the last 200 years.

I am not in a position to make recommendations regarding the entire proposal, but I think it is very well done and has great possibilities. I do recommend, however, that we study the mathematics component and seriously consider its basic tenets.

The mathematics component is described.

Concepts of Mathematics: 6 hours

A one-year course focusing on major concepts, methods, and applications of the mathematical sciences. Students will explore such topics as shape, quantity, symmetry, change, and uncertainty and consider such fundamental dichotomies as discrete and continuous, finite and infinite. Theoretical advances from the ancient to the contemporary will be considered, as well as applications in such areas as business, economics, statistics, science, and art. Students will be introduced to ways in which computers pose and help solve theoretical and practical problems.

I will attempt to acquaint you with the flavor of the proposal with a few quotes. In discovering mathematics, "It is at once the most speculative and the most practical of disciplines, with ancient roots in Pythagorean mysticism as well as in Babylonian commerce and Egyptian surveying." "Students without knowledge of the range, diversity, and power of mathematics are, as Mark Van Doren once put it, 'ignorant of a mother tongue'. "In response to their quote of a National Research Council's Everybody Counts, "Today's world is more mathematical than yesterday's, and tomorrow's will be more mathematical than today's", they have stated, "To participate in a world when discussion about everything from finance to environment, from personal health to politics, are increasingly informed by mathematics, one must understand mathematical methods and concepts, their assumptions and implications." Based on Lynn Steen's statement, "Minimal mathematical and statistical literacy is crucial, but the level of this literacy is too low to warrant claim that it can represent mathematics in a core curriculum. Numeracy should be required as a prerequisite skill, not as a core subject.", they conclude, "Entrance requirements can ensure that students have adequate preparation in high school. If remediation is necessary, it should not be addressed by the core." This is consistent with a complaint I have of many survey of mathematics courses and textbooks. They should not contain remedial material, and college credit should not be given for remedial work.

There is a statement in the report, however, that makes me wonder if these people really understand the situation.

> "A required course of studies - a core of learning - can ensure that students have opportunities to know the literature, philosophy, institutions, and art of our own and other cultures. A core of learning can also encourage understanding of mathematics and science..."

Do they not realize that mathematics and science are important integral parts of our culture? Apparently not. It seems we have a bigger job than just educating our students.

**REFERENCES:**


Cheney, Lynne; "50 Hours"; National Endowment for the Humanities; Washington, D.C., 1989

Lawrence University Catalogue

Lucas, John F.; "Humanistic Thinking and Mathematics", paper presented at Humanistic Mathematics Contributed Paper Session; MAA-AMS meeting, Phoenix, January 1989.

UME Trends; January 1990

Wilder, Raymond L.; "Evolution of Mathematical Concepts: An Elementary Study"; John Wiley & Sons; 1968.