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Breaking Glass: Exploring the Relationship Between Kinetic Energy and Radial Fracturing in Plate Glass

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Breaking Glass: Exploring the Relationship Between Kinetic Energy and Radial Fracturing in Plate Glass

A Thesis Presented
by

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To the Keck Science Department
Of Claremont McKenna, Pitzer, and Scripps Colleges
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Table of Contents

Abstract	Page 1
Introduction	Page 1
Materials and Methods	Page 2
Results	Page 4
Discussion	Page 11
Works Cited	Page 13
Appendix	Page 14

Abstract

When glass breaks from the impact of an object, it exhibits a distinctive shattering pattern comprised of two different regions. This pattern was investigated using experimental impacts and predicted using Young's Modulus. Results were not as expected, and it is likely that there exists error in some measurements. Further investigation of this topic is recommended.

Introduction

As demonstrated in Ron Cooper's 1969 art piece "Ball Drop,"[1] dropping an object on a sheet of glass produces two distinct areas of breakage. There exists an inner area of radial breakage extending from the point of collision, surrounded by a break around the circumference, beyond which there is random (or at least less symmetrical) fracturing. From this, the question arose as to whether this breakage was predictable, specifically if the radius of the radial shattering was related to the kinetic energy of the falling object. To determine this, an experiment was designed in which a ball was dropped from various heights and the resulting radial fracturing measured. The prediction was made that there would be a roughly quadratic relationship between the radius and the height the object was being dropped from, but even if this were not the case that at the very least the radius would be strictly increasing as the height increased. In addition, the Young's Modulus of the glass used was calculated to attempt to model the crack propagation in the material. Young's Modulus is a physical constant for a given material that defines its stiffness when bent. In this case, it was necessary to find Young's Modulus in order to calculate the speed of sound in the material, a crucial part of modeling the crack propagation.

Materials and Methods

To test whether glass fracturing was predictable, it was decided that the best approach would be to drop a solid, spherical object onto sheets of glass. The glass used was procured from local hardware stores, and was theoretically of uniform thickness and size. However, as this glass was obtained on different days from two different stores, it is possible that some variation occurred. The glass used measured 61 [cm] by 45.5 [cm] and was 0.2 [cm] thick. A solid metal ball, of mass 0.5175 [Kg] was dropped onto the glass. Additional materials included two meter sticks, a large piece of cardboard to protect floors and aid in cleanup, and protective gloves and goggles.

The first step was a test break, to discover at what height the mass needed to be dropped from to break the glass. After several attempts at gradually increasing heights, this turned out to be roughly 70[cm]. Knowing this, and also wanting to extrapolate a relationship between kinetic energy and breakage, it was decided that the fracturing should be measured at three different heights: 75[cm], 100[cm] and 125[cm].

The procedure followed for each break was fairly simple. After placing the glass on the cardboard on the ground, the ball was dropped onto the center of the glass from the designated height. Interestingly, the glass rarely broke in a perfectly circular manner. To compensate for this, the radius of the circle was measured in two different places and these two measurements were averaged together to achieve the “radius” of the radial shattering. After measuring, the broken glass was properly disposed of. Five drops were done for each height. See appendix 1 for examples of breakage and measuring.

To calculate the Young's Modulus, additional materials were required. To begin with, a smaller piece of glass 13.25[cm] by 45.5[cm] was cut. Additionally, a clamp and masses with a hanger were necessary. The glass was placed on the edge of a counter with a large box securing one end, while the other end was free over the floor. The distance between the end of the table and the free end of the glass was measured, both as part of calculating the Modulus and additionally that over multiple tests the results would be consistent. After attaching the clamp to the end of the glass and hanging the mass hanger from it, the height from the free end of the glass to the floor was measured, and considered to be the zero height for the later calculations. Weights were placed on the hanger while the height of the free end of the glass from the ground was measured. Rough measurements were made until the glass broke, at which point finer measurements were made in 0.1[Kg] increments on a second piece of glass, which was arranged so that the conditions would be identical to the first test. See appendix 2 for a diagram of this experiment.

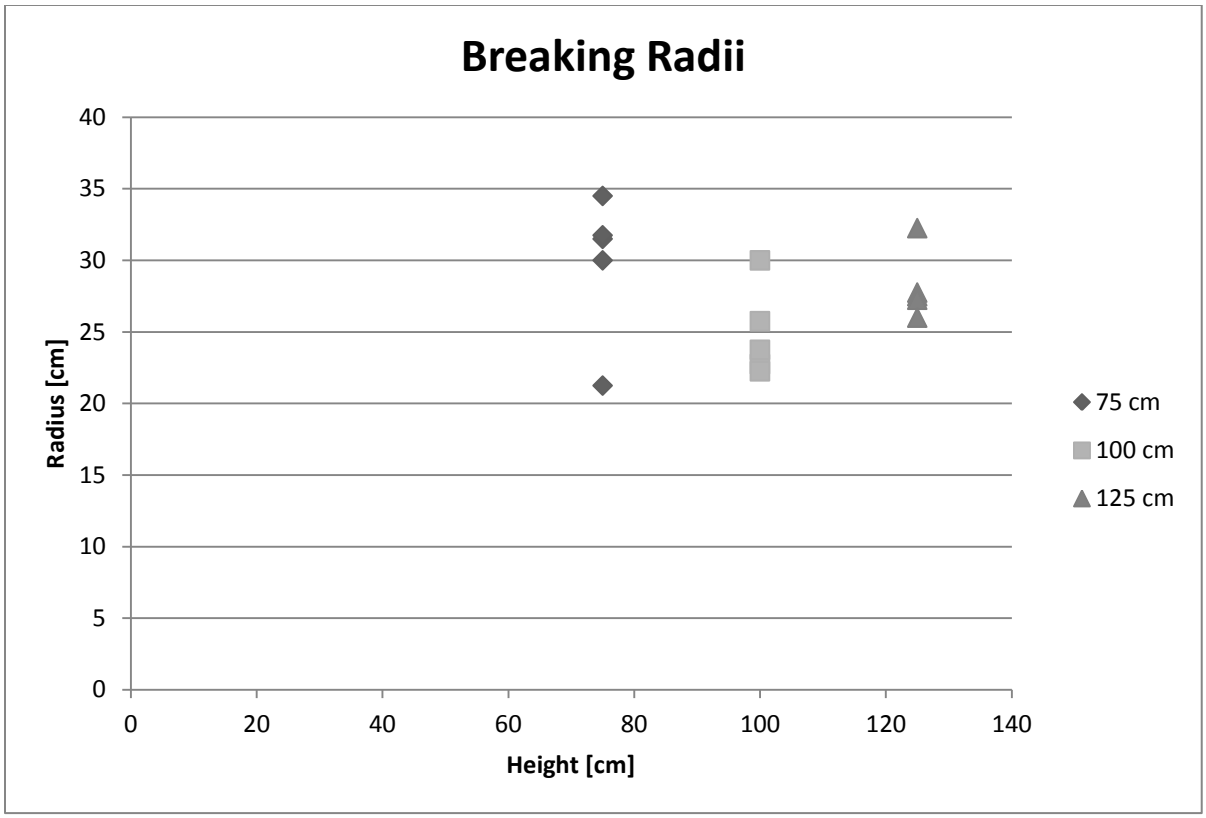
Results

Except for two outlying data points, all of the breaks were fairly consistent for each height.

75 [cm] drop	100[cm] drop	125 [cm]
33 x 36	21 x 24.5	33 x 31.5
32 x 31	22.5 x 25	28 x 27.5
30 x 30	21 x 23.5	27 x 28
20.5 x 22	26.5 x 25	27 x 27.5
32 x 31.5	32 x 28	24 x 28

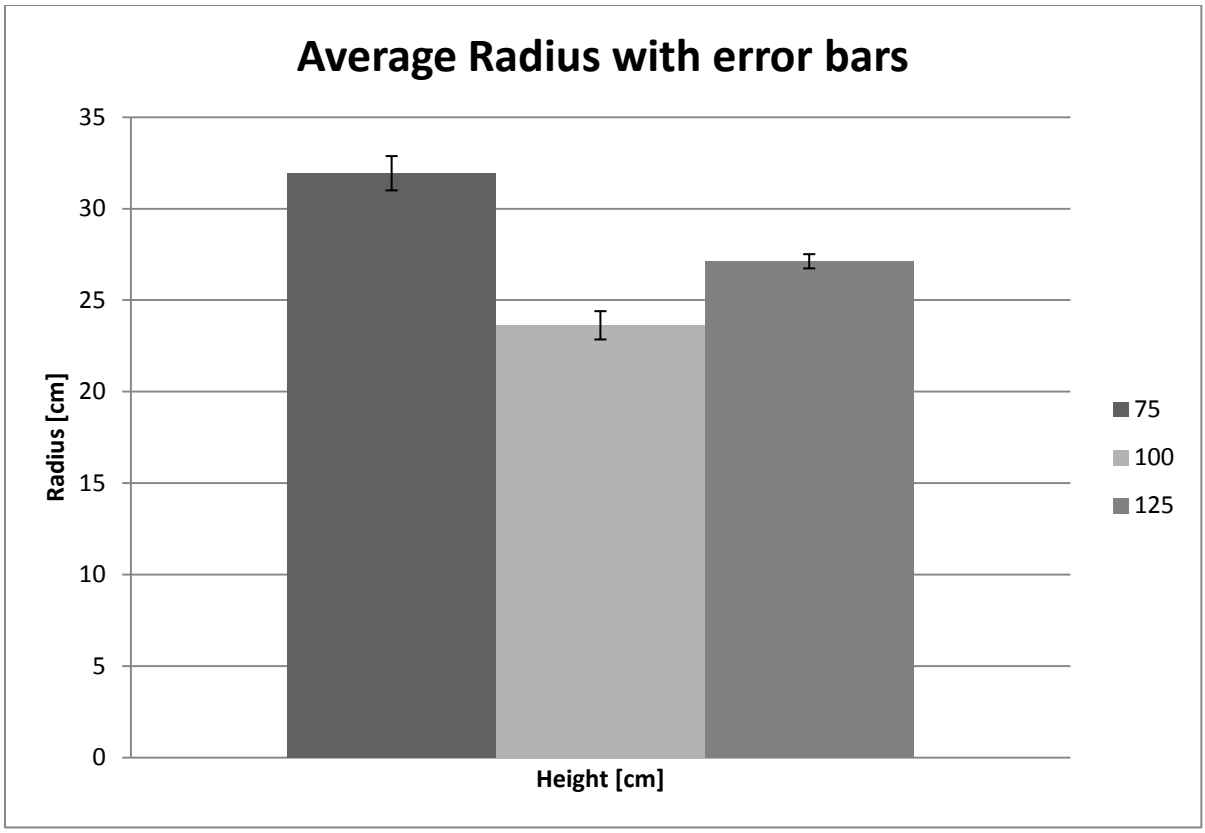
Measured radii for each drop. All measurements are in centimeters.

The above table contains the measured values for the radii in each drop. The two numbers in each cell are the measurements taken at two different angles of the shattered area, which were averaged together to obtain an average radius for each drop. Again, see appendix for an example of how these measurements were taken. These data points were then graphed to get a better understanding of how they compared both to drops from the same height and over all drops.

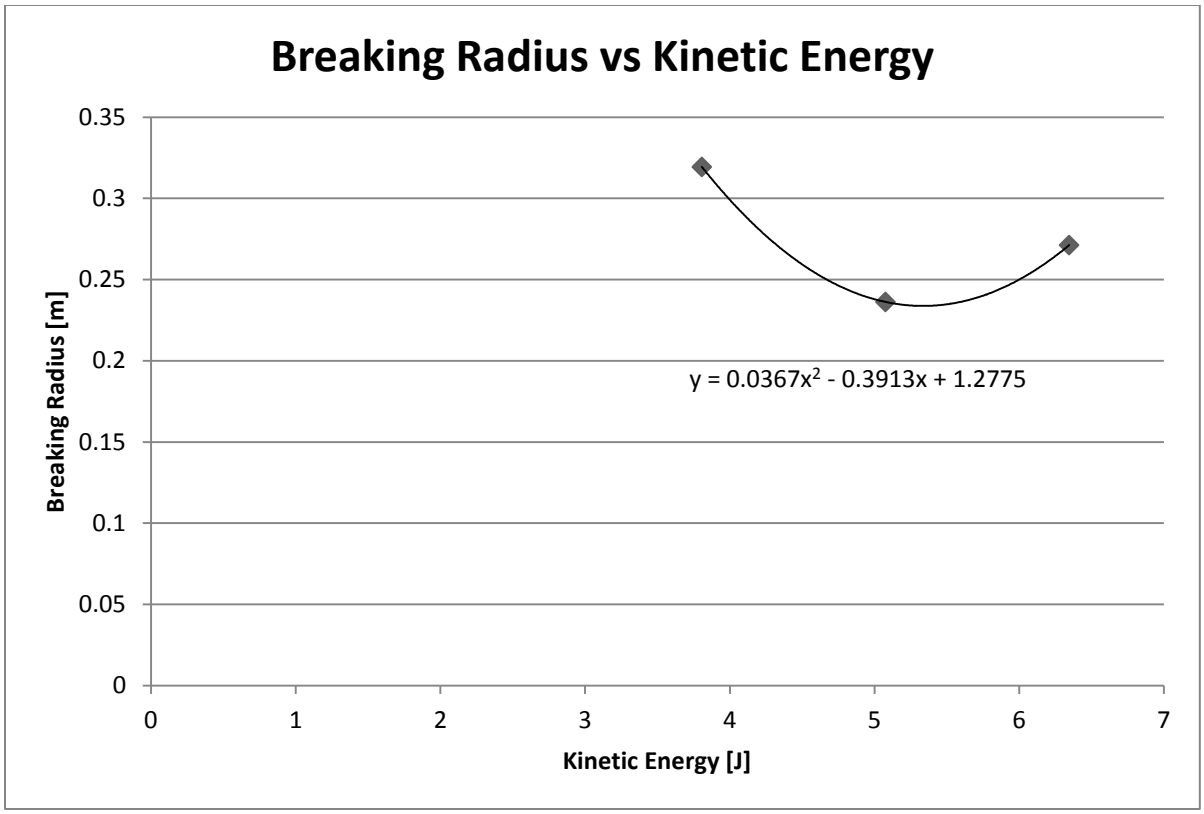


As evidenced in the table and made more obvious by this graph, each drop height has at least one data point that is a clear outlier from the others. For the 75 [cm] drop it is the (20.5 x 22) data point, for the 100 [cm] drop it is the (32 x 28) data point, and for the 125 [cm] drop it is the (33 x 31.5) data point. Including these data points in our calculations, all three drops have a standard error of the mean over 1, and in the case of the 75 [cm] drop it is even over 2. However, by removing these three points from our data sets we are able to achieve a standard error of the mean of less than 1 in all cases.

After removing these points from the data set, the average radius for each drop and the standard deviation and standard error of the mean were calculated. These average radii and their errors were then graphed.



For the 75 [cm] data, the average radius per drop is 31.9 [cm], for 100 [cm] it is 23.6 [cm] and for 125 [cm] it is 27.1 [cm]. The standard deviations for each are 1.8, 1.5 and 0.7, respectively, and the standard errors of the mean are 0.94, 0.77 and 0.39. Interestingly, from this graph it is clear that the radius decreases from the 75 [cm] to the 100 [cm] drop, before increasing again for the 125 [cm]. This is contrary to what had been predicted, which was that the radius would be strictly increasing as the height increased.



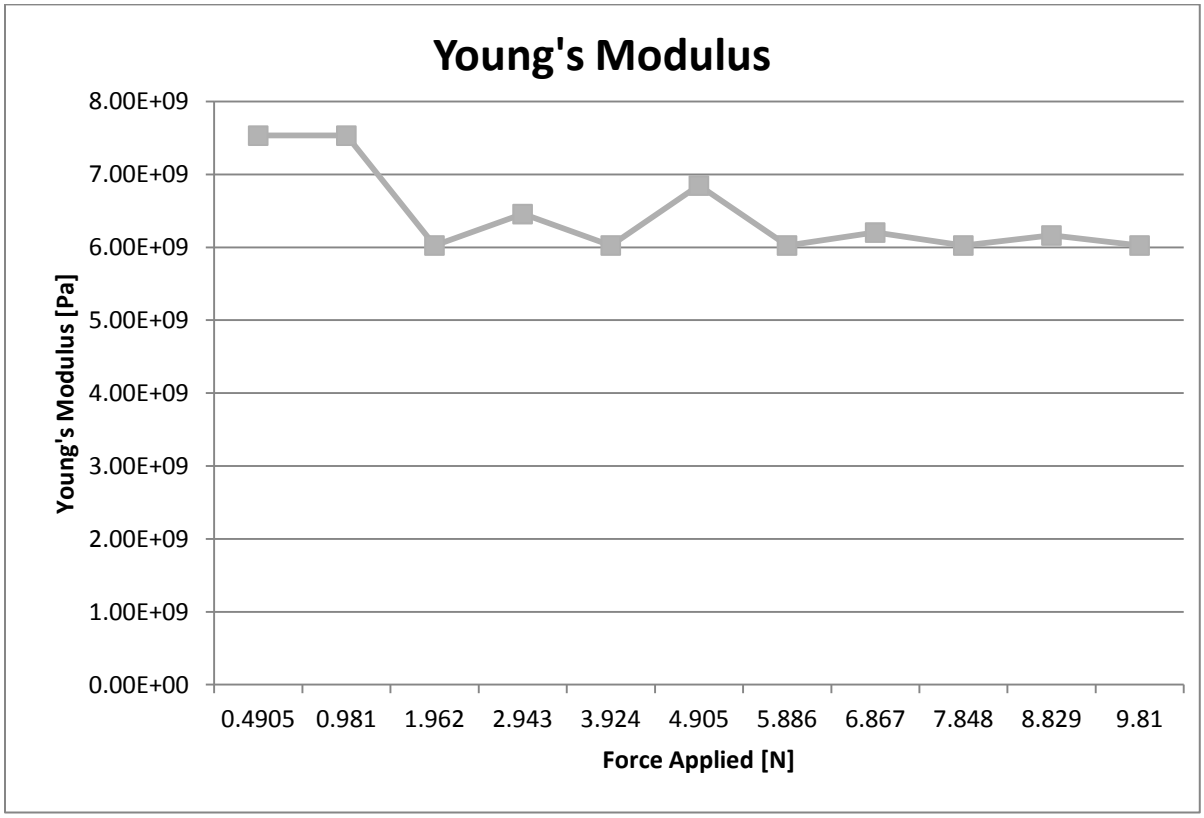
When compared to the kinetic energy of the ball, we see that while it is possible that there exists a quadratic relationship between the energy of the falling object and the radius of the radial shattering, it is not the one that was expected. Admittedly, assuming a quadratic relationship from three data points is not necessarily sound, and this experiment should be expanded in order to better understand the behavior.

It was hoped that by calculating the Young's Modulus of the glass being used that the crack propagation in the glass could be modeled. To calculate this quantity, the equation

$$E = \frac{4 * F * L^3}{\partial * w * h^3}$$

was used, where *F* is the force applied, *L* is the distance the force is from the fulcrum (in this case, the edge of the table), δ is the deflection of the end of the class from its initial

position, *w* is the width of the glass, and *h* is the height of the glass, in this case its thickness.[2] This quantity was calculated for every 0.1 [Kg] of weight that was added to the free end of the glass, yielding the following graph.



It is likely that the fluctuations in the calculated Modulus with smaller force are due to experimental measurement errors rather than actual variation in the bending of the material. The average calculated value for this glass is 6.37 [GPa]. This value is on the low end for glass, which typically has a Young's Modulus somewhere between 70 and 120[GPa][3]. However, as this was glass procured from a hardware store, it was likely not of the same quality as glass intended for industrial usage. For the Young's Modulus, the higher the value, the more brittle the material is, meaning that the glass used in this experiment was in fact more flexible than industrial glass. The consequence of this is that more energy

is required in order to break the glass, as opposed to a glass with a larger modulus. However, the fact that this material has a smaller modulus also makes the speed of sound lower within the material.

When cracks propagate in a material, the crack accelerates until it reaches a speed of one half the speed of sound in the material, at which point the crack begins branching[4] . To calculate the speed of sound in a material from Young's modulus, we use the equation

$$c = \sqrt{\frac{E}{\rho}}$$

where *c* is the speed of sound in the material and *ρ* is the density of the material[5]. The measured density of the glass used is 3274.311 [kg/m^3], which gives us a speed of sound of 1394.793 [m/s] (using the average Young's Modulus of the material). At this point, however, the calculations get a bit more complicated. We would like to calculate the distance from the source that the glass cracks before branching. To do this, we need to find the rate of acceleration of the crack through the glass. While we have the kinetic energy of the ball at the time of impact, it is likely that not all of this energy was transferred to the glass as breaking energy. Even if we were to use the kinetic energy of the ball, energy is in units of [kg m^2/s^2], while acceleration is in units of [m/s^2]. If we consider energy as an applied force through a distance, and thus have *Energy=mass*distance*acceleration*, we somehow need to account for a unit of mass when solving for the acceleration. Since we know that $d=(1/2)a*t^2$, we can substitute that in for the distance factor, but the mass remains problematic. Solving for the acceleration at this point, we have the equation

$$a = \sqrt{\frac{2 * KE}{m * t^2}}$$

where *KE* is the kinetic energy of the ball. The equation for the velocity of the crack at any given time would then be

$$v = \frac{1}{t} * \sqrt{\frac{2 * KE}{m * t^2}}$$

(given that *v=a*t*). While this would be solvable under normal circumstances, the factor of mass provides a proverbial bump in the road.

It is possible, however, to simply ignore the factor of mass. While this is "cheating," in that we are dropping units, it does allow us to proceed in our calculations. By having v in the above equation be equal to the speed of sound in the material and solving for t, we can then plug t into the equation for a as well as $d=(1/2)a*t^2$ to solve for the distance that the crack propagates before branching. Solving for t gives the equation

$$t = \sqrt{\frac{\sqrt{2 * KE}}{c}}$$

where again *KE* is the kinetic energy and *c* is the speed of sound. Plugging this into the equations for *a* and *d* gives us the following solution for *d*

$$d = \frac{1}{2} \sqrt{\frac{4 * KE^2}{\sqrt{2 * KE}}}$$

This now gives us an equation we can use to predict the radius of shattering of the glass for a given kinetic energy. Interestingly, it is not dependent on the speed of sound in the material (as that term cancelled). Calculating *d* for the three different kinetic energies gives

us a predicted crack length of 2.3 meters for the 75 cm drop, 2.8 meters for the 100 cm drop and 3.4 meters for the 125 cm drop.

Discussion

The fact that the glass breakage does not follow the predicted pattern of being strictly increasing with respect to increasing Kinetic Energy suggests that not all of the energy from the collision is being expended in the breaking of the glass, and must be going somewhere else for higher energy collisions. Additionally, it is clear that the predicted radii of fracturing and the actual are significantly different, by approximately a power of 10. The predicted radii do however agree with the predicted pattern of breaking. Clearly the elimination of the mass term in the equation was not the correct approach, and the mass should be accounted for in some way. If more time and materials were available, it would be possible to investigate further how the energy and breaking radius relate to each other by dropping from greater heights in addition to having more data for each drop height.

While this experiment was conducted as carefully as possible, there are sources of error inherent in the data. For example, because the breaking radius was measured visually with a meter stick, lengths were rounded to the nearest half centimeter. Because of this, the data is somewhat lacking in precision. An additional source of error would be the cardboard used to protect the floors. While this did serve an important function, it is entirely possible that the compressibility of the material may have had some effect on the resulting data. For the purposes of this thesis, this problem is largely ignored, since all data was taken in the same conditions, including the cardboard. If this experiment were to be

conducted again, it is recommended that a compressible substance is not utilized in this manner.

Similarly to this experiment, working exclusively with the Young's Modulus of the material and not the impact energy would be interesting. However, this would have to be done carefully; simply increasing the thickness of the glass would possibly effect the collision in other ways. It is clear that this experiment has left room for plenty of further investigation of the behavior of breaking glass.

Works Cited

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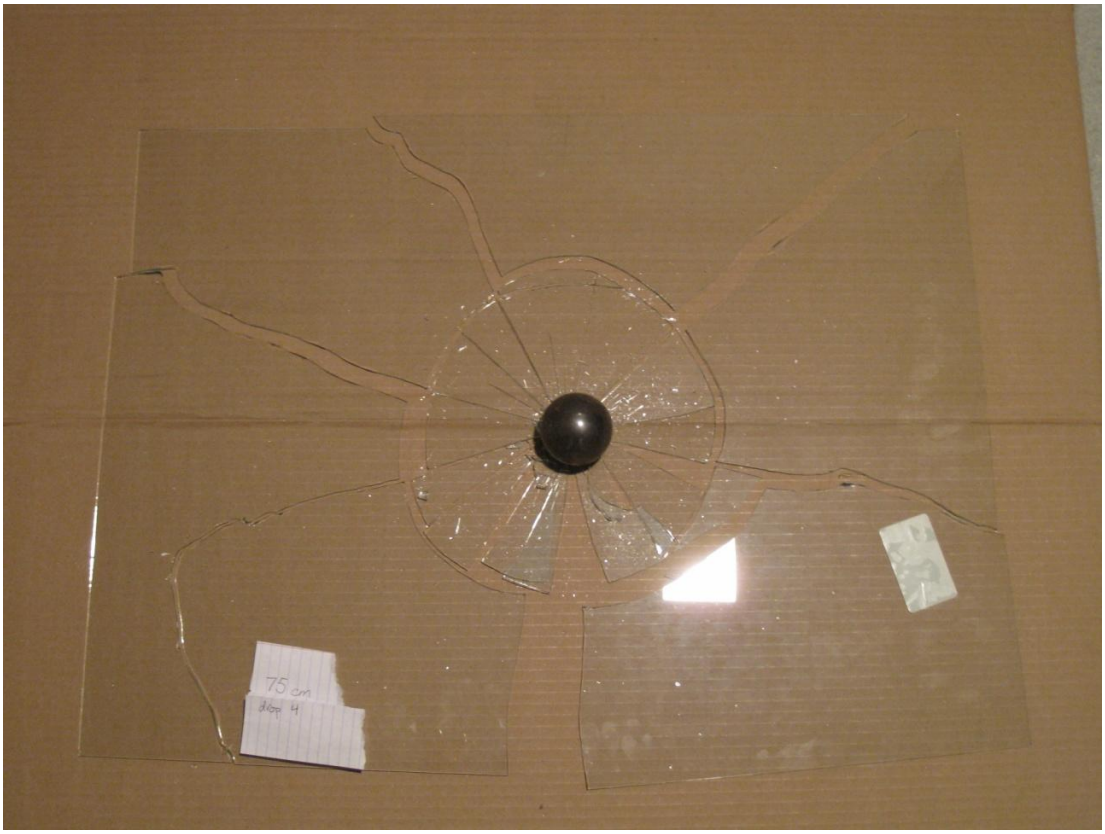
3. Callister, Jr., William D. *Fundamentals of Materials Science and Engineering* 2nd ed.; John Wiley & Sons: New York, 2005; pp A8
4. Callister, Jr., William D. *Fundamentals of Materials Science and Engineering* 2nd ed.; John Wiley & Sons: New York, 2005; pp 303 – 307.
5. http://www.engineeringtoolbox.com/speed-sound-d_82.html, Speed of Sound Formulas, The Engineering Toolbox.

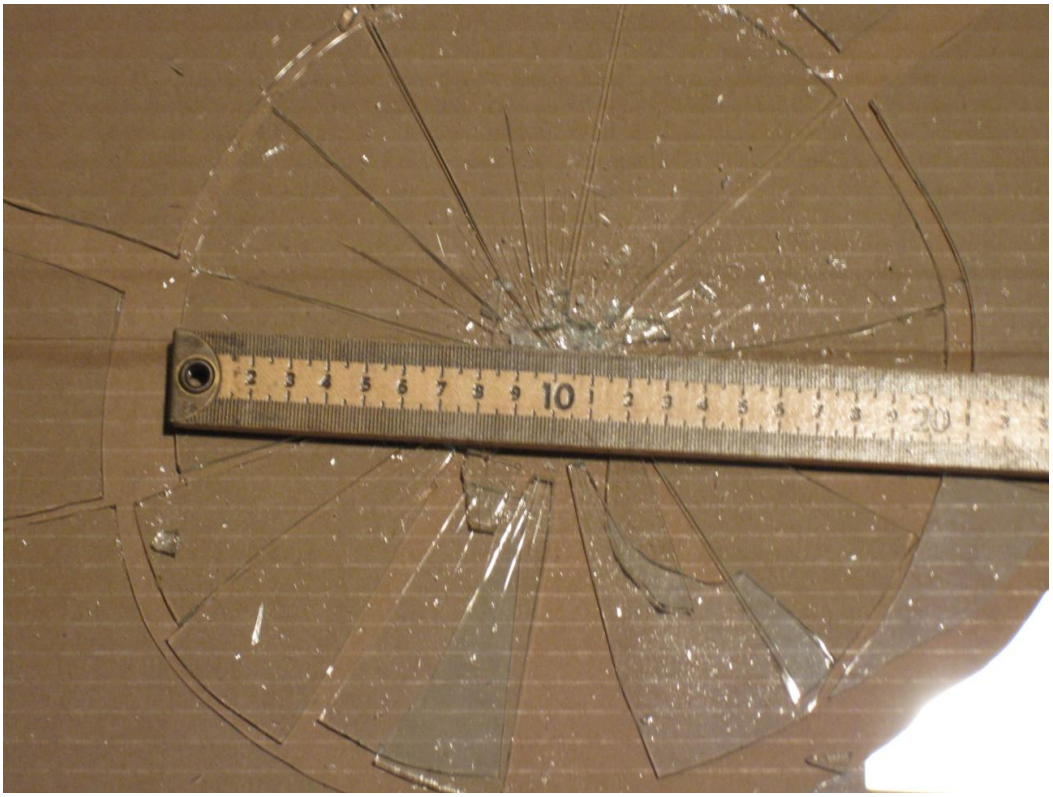
Glass was obtained from Home Depot and Lowes stores in Upland, CA

Appendix

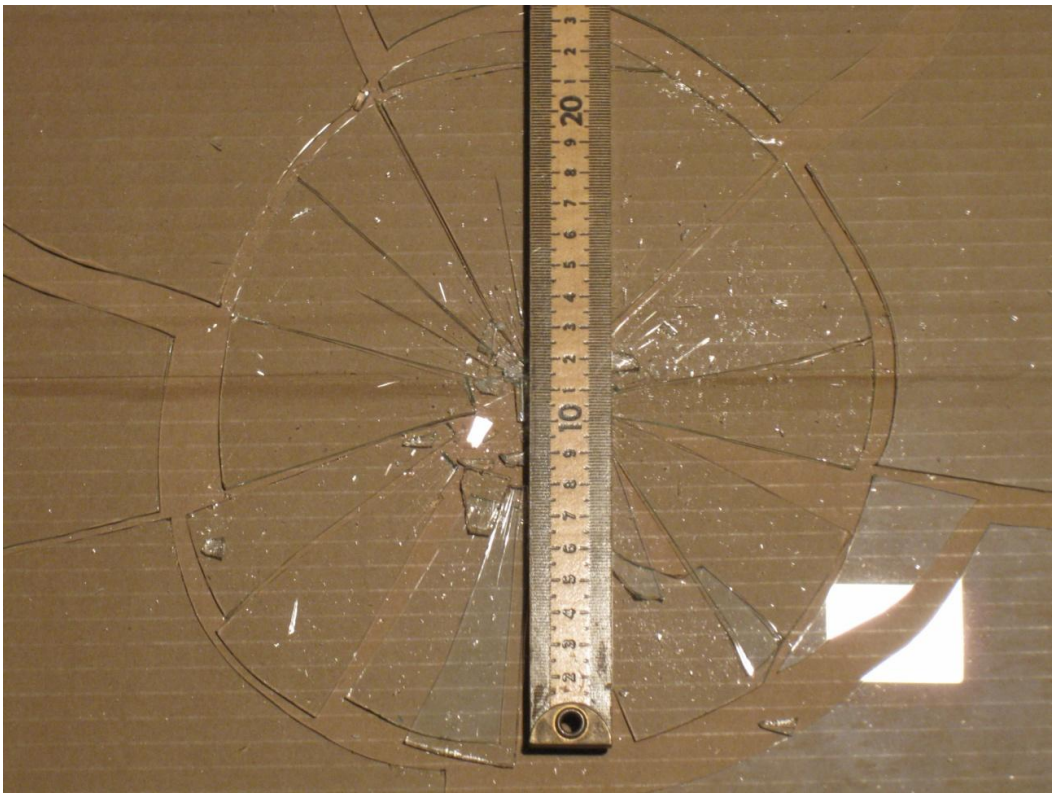
1. Measuring the radius of shattering



Example of a breakage from 75 [cm]. Note the breakage pattern, with an interior area of radial shattering and an exterior area of less well defined breakage.



Above and Below: an example of measuring the radius of shattering. Note that the breakage is not completely circular. The two radii measured will be averaged together for an estimated radius of shattering of this particular break.



2. Measuring Young's Modulus

http://www.doitpoms.ac.uk/tlplib/optimisation-biomaterials/figures/cantilever.png

Top: diagram showing how the length and height are measured for calculating Young's Modulus. Bottom: diagram showing how force is applied to the glass, creating deflection that is measured to calculate Young's Modulus. Image credit: 2