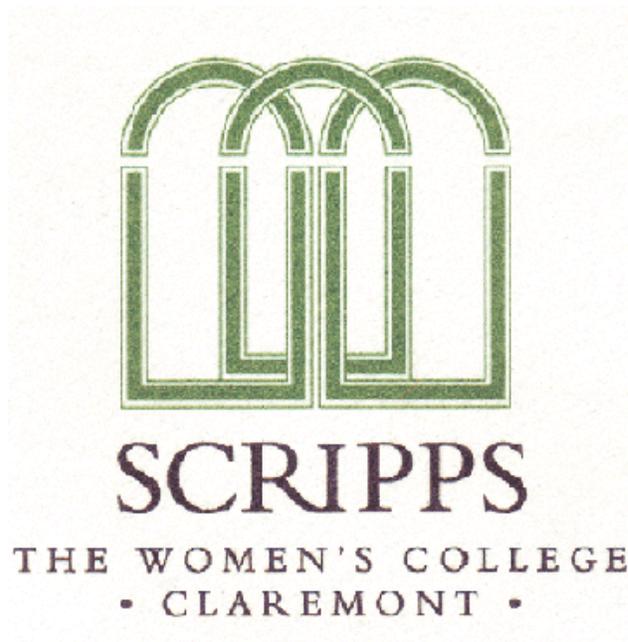2012

School Choice and Voucher Systems: A Comparison of the Drivers of Educational Achievement and of Private School Choice

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Submitted to Scripps College in Partial Fulfillment
of the Degree of Bachelor of Arts

April 20th, 2012

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Abstract

Despite promotion by well-known economists and supporting economic theory, econometric analyses of voucher systems often find that they have been unsuccessful in improving traditional measures of educational success. This paper examines a possible explanation of this phenomenon by comparing the drivers of educational achievement and of school popularity by examining private school choice. The findings of this paper indicate that there is a disconnect between school success and school popularity, which adversely effects both the demand and supply-side benefits of voucher systems. Additionally, this paper reviews matching mechanisms that seek to efficiently match students with schools based on both student and school preferences.

Acknowledgments

It is my pleasure to thank those who helped me to understand my topic and to write my thesis. I am grateful to my first reader, Latika Chaudhary, whose encouragement, guidance, and support made this thesis possible. I am also thankful to my second reader, Gizem Karaali, who was of great assistance.

Additionally, I would like to thank the Economics and Mathematics departments at Scripps College, which offered me the education that allowed me to complete this project.

Lastly, I offer my thanks to everyone who helped me in any small way throughout this project and throughout my education at Scripps. This includes a special thank you to my parents and my family, who have continuously supported me throughout my four years here.

Part I
Introduction

Academics and policy makers, alike, have been criticizing the poor quality of US public schools over the last decade. Many solutions have been proposed to improve the state of the public school system. One solution that many economists advocate is a voucher system. In general, voucher systems operate by providing each public school eligible child a voucher to cover educational costs at a school of their choice. Voucher systems are quite varied. The amount of the voucher changes from program to program, as do the range of schools that are covered by the voucher. Voucher systems both lower the cost of education and provide families with a choice as to where to send their child.

This paper poses the following question: are the factors that make a school educationally successful the same factors that parents consider when selecting a private school? The answer to this question will have an impact on the success of voucher systems. If families do, in fact, value the same characteristics that make schools educationally successful, then they will choose schools based on the correct factors. These schools will then be more likely to economically succeed over schools without these "positive" characteristics. The schools without these "positive" characteristics will either adjust to act more like the economically successful schools, or go out of business – which introduces a new phrase into the public school vernacular, eventually leaving only (both educationally and economically) successful schools. However, if families do not value the same factors that make a school educationally successful, the supply side effect will not occur and allowing families to choose which school to send their children to will not necessarily result in educationally successful schools concurrently being economically successful.

This paper also describes the mathematical methods of matching students with schools, based both on student preferences and school priorities. Student preferences refer to the school that a student most wishes to attend. School priorities refers to both any preferences given by schools to students that meet certain criteria (living within walking distance, or having a sibling that currently attends the school, for example) and schools choosing students with certain qualities (such as higher test scores). The purpose of this section is to give an idea of the most efficient way to match students with schools in the case that a voucher system is implemented.

Economic theory predicts that voucher systems will benefit students in two

ways, both of which involve taking monopoly power away from local schools: "the basic rationale behind school vouchers is that competitive markets allocate resources more efficiently than do monopolistic ones. Many observers argue that, because children are assigned to attend their local neighborhood school, public schools in the United States have monopoly power" (Rouse and Barrow 2009). First, if parents can choose to send their child to a school of their choice, then it is rational to assume that parents will select the school that they think will best fit their child. Second, allowing for school choice creates a competitive environment for public schools. The switch from a monopoly structure to a competitive market with an increased supply of schools will both increase the quantity of education and decrease the price.

Many facets of the voucher system have already been examined. The economic theory has been widely discussed, and experiments have been performed both domestically and abroad. Unfortunately, econometric analyses of these experiments have exhibited largely unfavorable results: there is generally no statistically significant increase in test scores or other traditional educational measures after the implementation of a voucher system. In this paper, I seek to examine parents' ability to properly choose schools. More specifically, I will compare the factors that parents use to select private schools with the characteristics of private schools that lead to positive educational outcomes. In order to answer my research question, I will use econometric analyses on a publicly available data set that seeks to survey every private school in the United States. I find that there is a disconnect between the factors that parents consider when they are choosing schools and the factors that make schools educationally successful. In fact, it seems that school popularity and school success are influenced by opposite criteria. The disconnect may be one possible explanation for the failure of previous voucher systems.

Part II
Literature Review

Economic Theory

The famed economist Milton Friedman was one of the first advocates of voucher systems. According to Friedman, the consequences of allowing the public school system to continue on its current path will be far and wide. He points to the creation of higher wages due to the technological progress made during the Indus-

trial Revolution as a polarizing factor. Specifically, he claims that the ratio of low skilled labor to capital has greatly increased. This shifts the supply curve of low-skilled labor out, and therefore puts downward pressure on wages of low-skilled labor. This wage differential problem is leading to an undesirable polarization of our society in which "a group of our population move[s] into Third World conditions at the same time that another group of our population becomes increasingly well off" (Friedman 1995).

The education system seems to be a major contributor to the polarization, and hence, the solution to the polarization lies partially within a dramatic restructuring of the education system. However, "such a radical reconstruction can be achieved only by privatizing a major segment of the education system – i.e., by enabling a private, for-profit industry to develop that will provide a wide variety of learning opportunities and offer effective competition to public schools" (Friedman, 1995). Friedman points to the fact that only 10% of children (in 1995) are enrolled in private schools and, therefore, are receiving a superior education. A much more robust private enterprise system is needed in order to create change.

Vouchers seem to be a way to spark this robust private enterprise system into creation, if they meet certain conditions. Friedman states "vouchers can promote rapid privatization only if they create a large demand for private schools to constitute a real incentive for entrepreneurs to enter the industry" (Friedman 1995). That is, vouchers must be available to all students and be of a large enough amount that they will cover the cost of private school tuition at a for-profit school. Furthermore, there will still be a sizable number of families that are willing to supplement the voucher amount to pay for an even better education, sparking innovation within the market. This innovation will trickle down to other schools due to competition. The result of this voucher system and the privatization of schooling would be the production of "a new, highly active and profitable private industry that would provide a real opportunity for many talented people who are currently deterred from entering the teaching profession by the dreadful state of so many of our schools" (Friedman 1995).

Friedman's argument clearly stems from solid (and basic) economic theory. But, we must also consider whether or not increased competition leads to higher productivity and lower costs in school systems in practice. Caroline Hoxby examines the effects of increased competition by looking at the effects of increased school district choice within small geographic areas. Hoxby postulates that "the incentives that schools have to be productive are generally increased by Tiebout choice because it gives households more information and leverage in the principal-

agent problem that exists between them and the people who run their local schools"
(Hoxby 2000). Tiebout choice refers to a paper by the economist Charles Tiebout,
originally published in 1956. In this paper, Tiebout introduces the idea that peo-
ple "vote with their feet" and choose places to live based on the bundle of public
services and taxes, including schooling, based on their preferences. Therefore, in
this context, Tiebout choice is choosing between alternate school districts based on
preferences and taxes. The school districts which Hoxby compares have different
political jurisdictions, but are located close enough to each other geographically
that students have a choice between them by being able to move. In essence,
Hoxby agrees with Friedman's theory that increased competition will lead to in-
creased productivity. However, Hoxby is much more specific in her predictions.
She believes that it is choice between districts, rather than choice between schools
within a district, that best demonstrates competition. In fact, Hoxby points out
that "metropolitan areas with more choice among districts do not necessarily offer
more choice among schools" (Hoxby 2000). In order to define the geographic area
in which families have access to different districts, Hoxby uses geographical bar-
riers (such as streams) to create a natural experiment, which allows for a causal
estimation of the impact of school choice on performance.

Using OLS estimation (and controlling for a variety of other factors), Hoxby
finds results that support the creation of a universal voucher system in two different
ways: increased student achievement as a result of increased Tiebout choice as well
as increased productivity as a result of increased Tiebout choice. In support of
the former result, Hoxby states:

> As increase from 0 to 1 in the index of Tiebout choice generates 8th-
> grade reading scores that are 3.8 points higher, 10th-grade math scores
> that are 3.1 points higher, 12th-grade reading scores that are 5.8 points
> higher, and math knowledge scores that are 2.7 points higher. In short,
> test scores rise by one-quarter to one-half of a standard deviation. In
> addition, such an increase in choice generates educational achievement
> that is 1.4 grads higher and income at age 32 that is about 15 percent
> higher. (Hoxby 2000).

Hoxby does note, however, that a "standard deviation in the choice index is 0.27"
(Hoxby 2000), which means that variation in the above mentioned student achieve-
ment measures can only partially be explained by variation in Tiebout choice.

Hoxby's results concerning productivity are perhaps even more important than
her results on student achievement. Hoxby finds that "an increase in choice among

districts lowers per-pupil spending with no loss – in fact, a gain – in student achievement" (Hoxby 2000). More specifically, "an increase from 0 to 1 in the index of Tiebout choice among districts raises productivity by between one-fourth and six-tenths of a standard deviation" (Hoxby 2000). Productivity is measured by several different educational outcomes, such as test scores and income later in life. These results imply that there is no trade off between lower spending and higher student achievement, which has strong public policy implications. According to the results of Hoxby's study, the focus should be on increasing competition at the school district level rather than increasing spending in the school system.

Voucher Experiments

Clearly, the evidence from the above economic theories – as well as studies performed to test the theory – is quite convincing. But, one must consider the success of vouchers in practice. There have been many small-scale voucher systems implemented both in the United States as well as abroad. Many of these voucher systems use lotteries to award vouchers to low-income students, although not all existing and past programs have been structured in this way.

One such program is the Milwaukee Parental Choice Program. This program allowed low-income students to use a voucher in the amount of $6501 to attend any school of their choice. Although participation in the program is now much higher, an econometric study was performed on the program when there were 12 schools and 800 students participating. The study created comparison groups by placing students that applied for a voucher and were able to attend the school of their choice by using the voucher in one group and students who applied for a voucher but did not win the lottery in a second group. The study found that "the yearly gain of being selected for the program rang[ed] from 0.06 [standard deviations] to 0.11 [standard deviations] in math and from -0.03 [standard deviations] to 0.03 [standard deviations] in reading, although the impacts in reading are never statistically different from zero" (Rouse and Barrow 2009).

Similar analysis performed on the Cleveland Scholarship and Tutoring Program, which awarded vouchers to students from low-income families and allowed them to redeem the vouchers at both sectarian and non-sectarian schools, found low to no results in educational achievement from the vouchers. A study of the D.C. Opportunity Scholarship Program, again, returned similar results. All three of the above mentioned programs were publicly funded.

The Chicago Public School system includes open enrollment, which consists

of a lottery that allows public school students to "apply to gain access to public magnet schools and programs outside of their neighborhood schools, but within the same school district" (Cullen et al, 2006). In this system, students may enter a lottery for multiple schools and are not obligated to attend a school for which they win a lottery. In fact, students that lose the lottery are sometimes able to enroll in the school. OLS estimation shows that "winning any lottery increases the probability that a student attends the school for which the lottery is held by 28.0 percentage points" (Cullen et al, 2006). This nature of this program gives two natural comparison groups: lottery winners and lottery losers.

In terms of traditional measures of educational success, the open enrollment system exhibits discouraging results:

> Comparing lottery winners and losers, we find little evidence that winning a lottery provides any benefit on a wide variety of traditional achievement measures, including standardized test scores, graduation, attendance rates, course taking patterns, and credit accumulation. (Cullen et al, 2006).

However, the open enrollment system finds some positive results in terms of non-traditional measures. For example, student reported school quality (such as number of computers and disciplinary rates) are statistically significantly better for lottery winners than for losers. Furthermore, "we observe statistically significant reductions in the percentage of students behind bars when comparing lottery winners to losers" (Cullen et al, 2006). While the open enrollment system is similar to voucher systems, it is important to remember that the two have many differences and therefore are not directly comparable. Furthermore, the open enrollment system does not create competition among districts, so these results cannot be used as a counter argument to Hoxby's theory. The results from the Chicago Public School open enrollment program may give important insights into a voucher system, but does not mimic the voucher system.

Voucher systems have been implemented internationally, as well. Perhaps the best example of this is the education system in Chile. In the early 1980's, the Chilean government implemented a statewide voucher system. Under this system, every family received a voucher to attend (almost) any private or public school that they wished. Currently, "about 94 percent of all schools (public, religious, and secular private) are voucher funded" (McEwan et al 2008). This voucher system created three categories of schools: municipal (public) schools that receive a per student subsidy and may not turn away any student; private schools that

receive the same per student subsidy, but may also charge additional tuition and select their student body; and private schools that are not voucher funded. Approximately 74 percent of private schools, which represented about 45 percent of all schools, operated as for profit institutions (McEwan et al 2008).

Despite the fact that this voucher system encompassed nearly the entire educational system in Chile (unlike the smaller domestic experiments described above), evidence shows that improvements in the educational system were mixed, at best. According to Carnoy, "an evaluation based on nationally standardized (although not strictly comparable) achievement tests in Spanish and mathematics found that the scores of fourth-graders declined between 1982 and 1988" (Carnoy 1997). However, test scores increased by 1990: "one estimate of the means of the 1990 national test showed an increased of 9 percent in Spanish and 11 percent in math, bringing these scores to about the same level as they were in 1982" (Carnoy 1997). This increase cannot necessarily be considered a success of the voucher system due to the fact that the aggregate result was test scores of comparable levels to those of the pre voucher system.

Despite convincing economic theory, actual voucher experiments seem to have mixed results. However, it should be noted that many of these experiments are not reflective of the ideal explained by Friedman in which every school is voucher eligible and there are no restrictions on operation, hence creating a free market scenario. Furthermore, several of the domestic experiments create only inter-district choice, and hence are not applicable to Hoxby's conclusions concerning intra-district choice.

Part III
Methods

In order to investigate my research question, I used the Private School Universe Survey (PSS), which surveys all schools in the United States that meet the criteria set by the National Center for Education Statistics (NCES) for private schools. The survey is taken every other school year. The first available data is from the 1989-90 school year and the most recent data is from the 2007-08 school year.

I used five of the available data sets. These are: the 1999-2000, 2001-2002, 2003-2004, 2005-2006, and 2007-2008 surveys. I chose to use multiple surveys in order to increase the amount of data to examine. However, I did not feel it was useful to use all available data sets, due to differences in how the survey was

conducted (in terms of questions asked) as well as differences in social attitudes over time towards private school that I felt would be difficult to model empirically. In order to take into account the effect that time would have on private school selection, I created a variable for year, as well as a variable for unemployment by year at the state level. The purpose of the unemployment variable is to serve as an indicator of the state of the economy, which will likely effect a family's decision as to whether or not to send their child to private school.

Part of my analysis dealt with the percentage of students who chose to attend a four-year college after high school graduation. Therefore, in order to make my dependent variables meaningful, all regressions were run on only those schools that offer the 12th grade.

Part A of my results section seeks to answer the part of my research question that deals with the characteristics of schools that lead to positive educational outcomes. I chose to use OLS estimation with college acceptance rates as my dependent variable. College acceptance rate is defined as the percentage of students that attended a four-year college, out of those that graduated with a diploma in the previous year. The exact question wording is "Of those that graduated with a diploma last year, approximately what percentage went to four-year colleges?"

This variable, admittedly, is not a perfect indicator of a school's educational success. Of course, there are many exogenous reasons why a student would choose not to attend a four year college that are not correlated with the educational success of the school. I have tried to eliminate one of these, the monetary cost of attending college, by including the unemployment variable as a proxy for the state of the economy. Several of the five data sets that I used include variables for the percentage of students who chose to attend trade school or a two-year college following graduation. However, I felt that these were not valid indicators of a school's educational success and chose not to use them as part of the dependent variable.

There are many advantages to obtaining a bachelor's degree from a four-year college. These include a better chance of finding employment and higher lifetime earnings. While there are other career paths that do not necessitate a bachelor's degree, I feel that attending a four-year college directly after graduating from high school shows that the high school has prepared the student well. Not attending college, attending a trade school, or attending a two-year college may imply that the school has ill prepared the student and therefore has not been educationally successful.

This section contains a total of six models. I first regressed four-year acceptance

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rates on only those variables that I felt would, and should, greatly affect the success of the school. These were: whether or not the school had a religious orientation, whether or not the school had a library, the student teacher ratio, and the number of students in the school. In the second regression, I added in variables to control for the racial and gender composition of the student body. Subsequent regressions took into account other explanatory variables. These included: the length of the school day, the region in which the school is located, the community type in which the school is located, and the unemployment rate. The variables of greatest interest are those that appeared in the first regression. The additional variables added on in the subsequent regressions are meant to create a more accurate model. However, schools are not able to change the values of many of these additional explanatory variables.

Section B of my results section seeks to answer the second part of my research question, which considers how parents select private schools. The challenge here is to create a dependent variable that reflects school popularity. I chose to use the difference in 12th grade acceptance rates between years. This variable was constructed from two different variables, both appearing in each of the five data sets. These variables are the number of students enrolled in 12th grade around the first of October and the number of students enrolled in the 12th grade around the first of October in the previous year. The dependent variable that I created is the annual percentage change in the 12th grade enrollment. For example, in 1990 this variable would equal (Enroll1990-Enroll11989)/Enroll1989*100, which is the percentage change in 12th grade enrollment from the previous year.

As is the case with the first dependent variable, this variable is also not a perfect representation of school popularity, which is quite difficult to measure directly. However, I believe that performing analysis on percentage change in 12th grade enrollment will give a valid indication of the factors that a family considers when they choose to send their child to a private school with particular characteristics. There are clearly many other exogenous factors, other than school characteristics, which determine whether a family will send their child to a particular private school. One such factor is the price of tuition. However, the variable unemployment will help to account for this factor, as it serves as a proxy for the state of the economy. The quality of local public school is also likely to be a significant factor in private school popularity. I have controlled for this by adding explanatory variables for community type and region. In later regressions, I have controlled for this further by including an explanatory variable for city.

Like the previous section, section B also contains six models. I ran regressions

on the percent change in 12th grade enrollment in the same manner as for the first dependent variable, college acceptance rate. The preliminary regression only took into account several variables that represented school characteristics. Again, these were: whether or not the school had a religious orientation, whether or not the school had a library, the student teacher ratio, and the number of students enrolled. I built the model by including variables to control for the gender and racial composition of the student body, the state of the economy, the length of the school day, and the location of the school.

Section C of the results contains a comparison of the results of the sixth regression from section A (college acceptance rate as the dependent variable) and the sixth regression from section B (change in 12th grade enrollment as the dependent variable). The purpose of this section is to draw conclusions about the differences in the factors that determine school educational success and school popularity. These differences may help to explain why past voucher systems have failed, as well as create valuable insight into how new voucher systems should be constructed moving forward.

Part IV
Results

A. Regressions with College Acceptance Rate as the dependent variable

Please see Appendix A at the end of this paper for the tables containing the regression outputs for this section.

Regression 1

The first regression that I ran created the following model:

$(collegeacceptancerate)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} + b4(number\ of\ students)_{it} + b5(school\ day\ length)_{it} + d_t + \epsilon_{it}$

In this model, religion is a dummy variable representing whether the school has a religious orientation (1=yes, 0=no), library is a dummy variable representing whether or not the school has a library (1=yes, 0=no), school day length is the number of minutes in the school day, and d_t captures year fixed effects. My data set covers 6,742 schools on average over 5 years for a total of 33,709 observations.

In this regression, every variable is significant at the .01 level with a p-value of 0.000. The results indicate that schools with a religious orientation send approximately 5.06% more students to a four-year college than those without a religious orientation. The results also indicate that schools with a library send approximately 15.3% more students to four year colleges than schools without libraries. The student teacher ratio also has a significant effect – for every 10% increase in the student teacher ratio, the percentage of students sent to four-year colleges decreases by 1%. Further, the number of students enrolled in the school also has a significant effect: for every increase of 100 students in enrollment, this regression indicates that there is a 4% increase in the percentage of students sent to a four-year college. The data also shows that for every increase of ten minutes in the length of the school day, the number of students attending a four-year college increases by about a half of a percent.

Regression 2

Some of this effect may have to do with the racial and gender composition of the student body. I controlled for these factors by including explanatory variables for percentage of Asian students, percentage of Black students, percentage of Hispanic students, percentage of American Indian students, and percentage of male students. The following model results from the second regression:

$$(collegeacceptancerate)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} + b4(numberofstudents)_{it} + b5(schooldaylength)_{it} + b6(ethnic)_{it} + b7(male)_{it} + d_t + \epsilon_{it}$$

The variables are defined as they were in the first model. The ethnic variable represents a set of variables controlling for the racial composition of the school. The male variable is the percentage of students that are male.

After including the race and gender variables, religion (whether or not the school has a religious orientation) is no longer statistically significant at the 1% level, but is significant at the 10% level. Further, the coefficient on library drops from about 15 to about 13. The other variables remain statistically significant and have coefficients within the same range as before.

Regression 3

Could the benefits of a longer school day and number of students enrolled have a limit? I ran the regression including quadratic variables for length of the school day and for the number of students enrolled to examine the limits of these variables. The new model is as follows:

$$(collegeacceptancerate)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} +$$
$$b4(numberofstudents)_{it} + b5(squarednumberstudents)_{it} + b6(schooldaylength)_{it} +$$
$$b7(squared\ school\ day\ length)_{it} + b8(ethnic)_{it} + b9(male)_{it} + d_t + \epsilon_{it}$$

All variables are defined as they were in the second regression. The variable squared school day length represents minutes in the school day squared and the variable squared number of students represents number of students enrolled squared.

Neither school day length nor squared school day length are significant in this regression. This indicates that including only school day length may generate a more accurate model. However, both number of students and squared number of students are significant, indicating that a model with a quadratic iteration of this variable is more accurate than a linear only version. Including the quadratic variables further decreases the coefficient on library to 10.75. The coefficient on student teacher ratio also changes from -.096 to -.047. The coefficients on both number of students and squared number of students indicate that the percentage of students attending four-year colleges is maximized when the number of students enrolled is 1,754. The statistical significance on student teacher ratio decreases.

Regression 4

The fourth regression adds in the effects of unemployment, as an indicator of the economic condition. The amended model is as follows:

$$(collegeacceptancerate)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} +$$
$$b4(numberofstudents)_{it} + b5(squarednumberstudents)_{it} + b6(schooldaylength)_{it} +$$
$$b7(unemployment)_{it} + b8(ethnic)_{it} + b9(male)_{it} + d_t + \epsilon_{it}$$

All variables are defined as they were in the second regression. The variable unemployment is the unemployment rate for the second of the two years spanned by the sample for the state in which the school is located.

The statistical significance and coefficients on most variables remains similar to the third regression. In addition, unemployment is also statistically significant; the coefficient indicates that as unemployment rises by 1%, the number of students sent to a four-year college will decrease by about 2.81%. This means that as the economy improves, more students enroll in college right after high school graduation. There are several possible explanations for this result. A better economic condition would make it more likely that families are able to afford to send their children to college. In addition, a lower unemployment rate may make students more confident in their ability to find a high paying job after college graduation,

hence making the investment in four year college seem like an attractive option.

The coefficient on the variable religion changes sign. The regression now implies that schools with a religious affiliation send about 0.87% less students to a four-year college than schools without a religious affiliation.

Regression 5

In the fifth regression, I take into account the location of the school by adding in explanatory variables for region and for community type. Region is divided into four categories: northeast, midwest, south, and west. Community type is divided into three categories: central city, urban fringe, and rural. I also added in a variable for the percentage of minority students enrolled in the school. The resulting model is:

$(collegeacceptancerate)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} + b4(numberofstudents)_{it} + b5(squarednumberstudents)_{it} + b6(schooldaylength)_{it} + b7(unemployment)_{it} + b8(region)_i + b9(community)_i + b10(ethnic)_{it} + b11(male)_{it} + d_t + \epsilon_{it}$

All of the variables are defined as they were in previous regressions. The region variable represents the set of variables for region. The community variable represents a set of variables for community type.

The variables that were statistically significant in the fourth regression remain statistically significant in this regression. In addition, the coefficients remain quite similar. One of the biggest differences is a decrease in the coefficient on unemployment from -2.81 to -2.01. This variable remains significant at the 1% level and also retains its sign, which still implies that the unemployment rate and students attending a four-year college are inversely correlated. In addition, the variable religion retains its negative sign in this regression. The set of variables representing region and community type are all statistically significant as well.

Regression 6

The final regression further specifies the region by substituting a variable for the city in which the school is located for the set of variables representing region. The new model is as follows:

$(collegeacceptancerate)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} + b4(numberofstudents)_{it} + b5(squarednumberstudents)_{it} + b6(schooldaylength)_{it} + b7(unemployment)_{it} + b8(city)_i + b9(community)_i + b10(ethnic)_{it} + b11(male)_{it} + d_t + \epsilon_{it}$

All variables are defined as they were in the previous regressions. The variable city represents the set of variables for the city in which the school is located. This final model includes 19,547 observations, for a total of 3,909 schools on average.

Interestingly, the variable religion switches signs in this regression and becomes much more statistically significant (it now has a p-value of 0.001). The regression now implies that schools with a religious orientation send about 2.18% more students to a four-year college than those without a religious orientation. The statistical significance of student teacher ratio also increases, and it is now significant at the 1% level. This regression now shows that as the student teacher ratio increases by ten, about 1.07% less students go to a four-year college. The variable unemployment remains significant at the 1% level, and the coefficient again increases to about -1.32. The other variables have a similar statistical significance and coefficients as they did in the fifth regression.

B. Regressions with Percent Change in 12th Grade Enrollment as the dependent variable

Please see Appendix B at the end of this paper for the tables containing the regression outputs for this section.

Regression 1

I ran regressions on the second dependent variable in the same manner as for the first dependent variable. This model includes 34,583 observations, and covers 6,917 schools on average. The first regression created the following model:

$(change12thgradeenrollment)_{it} = \alpha + b1(religion)_{it} + b2(library)_{it} + b3(studentteacherratio)_{it} + b4(number\ of\ students)_{it} + b5(school\ day\ length)_{it} + d_t + \epsilon_{it}$

The following variables are statistically significant at the .01 level: religion (whether the school has a religious orientation), library (whether the school has a library), and student teacher ratio. The variables school day length and number of students are not statistically significant. The regression implies that school popularity decreases by 7.44% if the school has a religious affiliation. Further, popularity decreases by 16% if the school has a library. Popularity also increases as the student teacher ratio increases. However, there is an obvious correlation between the student teacher ratio and the way in which popularity is measured in this model. If more students are enrolled in the 12th grade than in the previous year, and the number of teachers employed stays constant, then the student teacher

ratio will clearly increase. The same argument could be made for a correlation between popularity and the number of students enrolled. However, this variable is not significant in this regression.

Regression 2

In the second regression, variables for the racial and gender composition of the student body are included. The model is similar to that of section A, regression 1.

In the second regression, religion, library, student teacher ratio, and number of students are all significant. The coefficients on religion and library are similar to those of the first regression. The statistical significance of both student teacher ratio and number of students can be attributed to the same argument made for student teacher ratio in the first regression.

Regression 3

In the third regression, I include quadratic variables for both the school day length as well as the number of students enrolled. This model is similar to the third regression's model in section A. I reran the analysis on percent change in 12th grade enrollment.

In this regression, religion and library remain statistically significant with similar coefficients to those in the first and second regressions. Neither school day length or squared school day length are statistically significant, which is expected because school day length was not significant on its own in the previous regression. However, number of students and squared number of students are both statistically significant. The coefficients indicate that the school receives the lowest boost in admission rates when the number of students enrolled is 1,232. There is no absolute maximum boost in admission rates based on the number of students enrolled suggested by this model.

Regression 4

The fourth regression takes into account unemployment. Due to the statistical significance in the previous regression, this model retains the quadratic variable for number of students but uses only a linear model for school day length. The model is similar to that of the fourth regression in section A.

The new variable, unemployment, is not significantly significant (p-value of 0.863). Since I am using unemployment as a representation of the state of the

economy, this model implies that the economic condition does not affect whether or not families choose to send their children to private schools. The coefficient on the variable religion decreases to -5.62. This change implies that schools with a religious affiliation are about 5.62% less popular than schools without a religious affiliation. The coefficients and statistical significance of the other variables remains similar to that of other regressions.

Regression 5

The fifth regression includes variables for both region and community type. The new model is similar to that of the fifth regression in section A.

The additions of region and community type do not change the regression output by much. The statistical significance and coefficients on the variables remains similar to that of the previous model.

Regression 6

The sixth regression uses city instead of region as a geographic control. The model is similar to that of the sixth regression in section A. This final model includes 20,026 observations, and covers an average of 4,005 schools.

The coefficient on religion increases to -7.17 and is no longer significant at the 5% level – however, it is significant at the 10% level. The coefficients and statistical significance on the other variables remains similar to that of the previous regression. The variable percentage male (which is the percentage of students that are male) also becomes statistically significant in this regression. The variable implies that schools are about 20.89% less popular as the percentage of males increases by 1%. This result may indicate that all female schools are popular. The small amount of change in the last couple of regressions implies that the results are robust.

C. Interpretation of Coefficients

For both of the dependent variables, I believe that the sixth and final model can be considered the most accurate. This model includes explanatory variables for region, race, and gender. Further, the region variable in this model controls at the city level rather than at the broader level, like the other regressions. Further, the model retains explanatory variables for community type. While the specific city will mostly determine community type, I still believe that including these variables

are important, since there could be fluctuations in community type within a city.

Although the argument behind Tiebout choice is that families choose where to live based on the bundle of services (including schools) and taxes that match their preferences, we do not interpret the coefficients on the city and community type variables. The purpose of this paper is to examine which variables influence school success and popularity, rather than to find which cities and community types have the most successful and popular schools. Further, a similar assumption can be made about the variables representing race. Since these variables are for the racial composition of the entire school (which is likely correlated with city and community type), families have little control over this variable as well. They may not even have complete information on the racial composition of the private schools to which they plan on sending their children. Therefore, the coefficients on these variables are also not of interest to us. Likewise, families do not have control over the unemployment rate. There is an interpretation included in section A above (and the variable is not statistically significant in section B), but we will not consider the variable unemployment in this section.

The final regression with college acceptance rate as the dependent variable has several variables that are both significant and of economic interest. The first of these is the variable religion, which represents whether or not the school has a religious orientation. The model indicates that schools with a religious orientation send just over 2% more students to four-year colleges. Much of the previous literature indicates that parent involvement in education leads to a variety of positive educational outcomes. It is likely that religious families are more involved in their child's education than non-religious families, which may explain this result.

The model also indicates that schools with a longer school day will send more students to four-year colleges (for every additional 10 minutes in the school day, about .35% more students will be sent to a four-year college). The likely interpretation here is somewhat obvious: if students spend more time in school, they will learn more and hence be attractive candidates to colleges. Students (and families of these students) who choose to attend schools with a longer school day likely put more emphasis on academics, and therefore may be more likely to attend a four-year college after high school graduation.

The variable library (whether or not the school has a library/media center) seems to be a large factor in the number of students who go to a four-year college. The model indicates that schools with a library send about 10% more students to a four-year college than those without a library. This is an intuitive result. Schools with libraries provide more opportunities for students to read, conduct

research, and have access to computers and the internet. All of these things will likely stimulate intellectual curiosity (and therefore make students more eager to attend a four-year college) as well as make the students more attractive candidates to four-year colleges.

The student teacher ratio also impacts the percentage of students attending a four-year college. As the student teacher ratio decreases by 10, about 1% more students attend a four-year college. Students that attend schools with a lower student teacher ratio likely receive more individualized attention from teachers. A lower student teacher ratio facilitates learning and may have the same two-folded result as the above variable, library. A lower student teacher ratio both stimulates intellectual curiosity, as well as helps the students learn more, which makes them more attractive candidates to four-year colleges.

The number of students attending the school seems to matter as well. The highest percentage of students are sent to a four-year college when the number of students enrolled is about 2,133. This implies that schools that are too small may not be able to offer a wide breadth of educational opportunities to students. This is further explained by economies of scale. A minimum number of students would need to enroll in a school in order for the school to justify staffing a computer lab, for example. In addition, smaller schools cannot offer the same variety of courses as larger schools since they would not have enough students to fill the wider selection of courses. Schools that are too large may not be able to offer enough individualized help to students. This especially applies to college counseling help, meaning that very large schools likely do not have the ability to meet with students one on one to offer counseling services. Without the individualized pressure to apply to a four-year college, students may neglect to take the initiative themselves.

The results also imply that the percentage of males attending the school has a large, negative impact on the percentage of students who will attend a four-year college (for every increase of males by 1%, the percentage attending a four-year college will decrease by about 25%). This result is likely reflective of the success of all female schools. I do not think that it would be correct to assume that mixed gender schools with a high level of females send more students to four-year colleges. Over the last few decades, women have been entering four-year colleges (and two year colleges) in larger numbers than men. Women also make up over 50% of college graduates in the United States. Researchers are finding similar patterns in other developed countries and developing countries. This pattern is likely influencing the model.

Now, we will switch to an examination of the significant variables in the fi-

nal regression with percentage change in 12th grade enrollment as the dependent variable. The variable religion is both statistically significant (at the 10% level) and economically important. The model indicates that schools with a religious affiliation are about 7.11% less popular than schools without a religious affiliation. This is likely explained by cultural attitudes – the result implies that families are less likely to choose a religious school over a non-religious school.

The model implies that the length of the school day is inversely correlated with school popularity. Specifically, popularity decreases by about 1.7% for every additional 10 minutes in the school day. If we assume that students have some say over which private school they attend, we can assume that students favor schools with a shorter school day. Families may also prefer schools with a shorter school day if they want to enroll their child in extracurricular activities.

Whether or not the school has a library is negatively correlated with school popularity, as well. According to this model, schools with libraries are about 16.7% less popular than schools without libraries. This is an unintuitive result as we might originally hypothesize that parents willing to pay for private schools would look for those schools with as many amenities, including libraries, as possible. This issue could be a possible topic for further research.

Both student teacher ratio and the number of students have an impact on school popularity, according to the model. However, please see section B for an explanation of why the coefficients on these variables may not be accurate due to the way that school popularity is measured.

The results seem to indicate that the factors that lead to positive educational outcomes are not valued by families when they choose to which private school to send their child. Schools with a religious orientation, a library, and a longer school day seem to lead to a higher percentage of students attending a four-year college. However, families look for schools without a religious orientation, without a library, and with a shorter school day when they are choosing a private school to which to send their children. This disconnect may explain some of the previous "failures" of voucher systems. Previous studies on the success of voucher systems have indicated that test scores (another measure of educational success) do not increase with statistical significance when a voucher system is put into place. One possible explanation is given by this paper: families either do not value the same qualities in private schools which lead to educational success or are unaware of what these attributes might be.

Although not flawless, the dependent variables used in this study were the best available representations of both school educational success and of school

popularity from a robust data set. This research could be expanded by finding a different measure for school popularity other than the one that is used in this paper. If popularity is not highly correlated with student teacher ratio and the number of students enrolled in the school, then these variables could be considered for comparison purposes as well. By the same token, similar models could be constructed for a different dependent variable representing school success.

Part V
The Mathematics of School Choice

1 Overview

By viewing the problem of school choice as a matching problem, we can consider mathematical solutions to find optimal matches between students and schools. This problem must find a balance between the competing issues of efficiency and of stability.

In the context of private schools, we can consider n applicants for each institution, and q spaces available at each institution. In this problem, applicants must choose which schools to apply to and the schools must select which applicants to select. The issue is further complicated by the fact that not all applicants that are accepted by a particular school will decide to attend. Further, certain matching systems encourage applicants to misrepresent their true preferences.

At first, the solution to this problem may seem obvious: assignments should simply be made in accordance with preferences. However, complications soon arise. Consider the following anecdote - each of two schools have room for only one additional student, call these schools A and B. Consider also two students, α and β. School A prefers student α and school B prefers student β. However, the students have the opposite preferences: student α prefers school β and student β prefers school A. Whose preference should be considered first? The school or the student?

In order to examine the answer to this question, we first consider a set of definitions:

- *Violation of priorities (*adapted from Aksoy et al 2011)*:* We say that the priority of student $i \in I$ is violated for school $s \in S$ under a matching M if there exists some $i' \in I$ and $s' \in S$ such that the following conditions are true:

- The matching M assigns i to s and i' to s'

- i prefers attending s' over attending s

- s' prefers student i over student i'

- *Stability (*adapted from Aksoy et al 2011*)*: A match is called stable if the following two conditions are met:

    - The matching does not violate any priorities (as defined above).

    - There is no unfilled school that a student would prefer to attend over the school that they are matched with.

    - For example, let students α and β both prefer school A over school B. Let both schools A and B prefer student α over student β. The student school matching $(\alpha, B), (\beta, A)$ is unstable because student α prefers school A over her current assignment and school A prefers student α over its currently enrolled student, student β.

- *Justified Envy:* This is the result of a school matching mechanism that does not produce a stable matching.

- *Efficiency:* This term refers to a matching system in which a high number of students receive a top choice school.

- *Preference Index* (adopted from Aksoy et al 2011)*:* The preference index is calculated by summing the deviation from a student's top choice school across all students. Let the function $\gamma_i(M(i))$ represent the ranking of the school that match M assigns a student. For example, if school s_3 is student i_1's third choice, and match M assigns student i_1 to school s_3, then $\gamma_1(M(i_1)) = 3$. The preference index, then, would be the summation of all $\gamma_i(M(i))$ across all students. Therefore, the preference index, $\mu(i) = \sum_{i \in I} [\gamma_i(M(i)) - 1]$

- *Pareto Efficient:* A match is Pareto efficient if no student can be made better off without harming another student in an alternate match.

- *Strategy Proof:* All students benefit from revealing their true preferences and cannot be matched with a "better" school by individually lying about their preferences.

This question introduces the competing priorities of efficiency and stability. In general, efficient matching systems favor student preferences whereas stable matching systems favor school preferences. If we assume that schools exist for the benefit of students, then students' preferences should receive the first priority. However, if the success of voucher systems hinges on the schools' ability to exist in a free market, then the schools should be able to make decisions about admissions as they choose. In addition, the word priority can be interpreted in a number of ways when we consider the schools' preferences. Sometimes, students are given priority because they live within walking distance of the school or they have a sibling that is already enrolled. In other cases, preferences are determined by a random lottery system. This question will be returned to as different matching mechanisms are examined.

2 Gale-Shapley Student Optimal Stable Matching Mechanism

Gale and Shapley assert that the following condition should always be avoided by matching mechanisms:

> "An assignment of applicants to colleges will be called unstable if there are two applicants α and β who are assigned to colleges A and B, respectively, although β prefers A to B and A prefers β to α."

Here, the terms "college" and "school" can be used interchangeably. The reasoning behind this principle is that if this type of matching did occur, both the students and schools would be unhappy. The students could simply indicate to the schools that they would like to switch assignments, and the schools would be happy to oblige. Hence, the original matching would soon change.

Further, Gale and Shapley define another element of matching mechanisms. This time, this condition is quite desirable:

> "A stable assignment is called optimal if every applicant is at least as well off under it as under any other stable assignment."

It is not necessarily clear that every problem will have an "optimal" solution. However, it is clear that there can only be one optimal solution for each.

Proof: By way of contradiction, consider two solutions, x and y, both considered optimal. Assume that these two solutions are not identical, which implies that there must be at least one match that is different between these two solutions.

Let this be match q. Therefore, we can say that $q_x \neq q_y$. Assuming that students and schools both have strict preferences (i.e., students do not rank two schools equally and schools do not rank two students equally), at least one student and one school must be not as well off in this match as in the other match. Therefore, one of the two matches is not optimal.

In order to simplify this matching problem and to examine the above conditions, we can consider a simplified version of the problem, in which there are an equal number of students as schools and each school can accept exactly one student. Of course, this scenario is highly unlikely in a student-school matching problem. However, if we consider this as a marriage matching problem, this scenario seems to fit.

Therefore, consider a scenario is which there are an equal number of men and women. Each person ranks the members of the opposite sex in order of their preference for a marriage partner. Now, we can consider whether or not it is possible to find a stable set of "marriages." The idea is that if it is possible to find a stable set of marriages, it will also be possible to find a stable set for the student-school matching problem.

Gale and Shapley assert that there is, in fact, a stable set of marriages by giving a procedure to find the stable set.

"Theorem 1: There always exists a stable set of marriages."

- *Proof:* The proof of this theorem, as given by Gale and Shapley, is as follows: To prove existence, we give an iterative procedure for actually finding a stable set of marriages. Each male proposes to his first choice female. Each female who receives multiple proposals rejects all but her favorite suitor. However, the females do not yet accept their favorite from the first round, but rather keep his name in mind incase someone that they prefer comes along later. This is called keeping the suitor on her string.

- In the second stage, the males that were rejected propose to their second choices. Each female again rejects all but one suitor from the group of males that proposed to her in the second round and from the male currently on her string. The following n rounds proceed in the same manner until every female has received a proposal. Each female is then required to accept the male on her string. Since no male is allowed to propose to the same female more than once, every female will receive at least one proposal.

- The above described matching mechanism is stable.

Proof: By way of contradiction, assume not. Assume that male x in fact prefers female n to his actual wife, l. Then, male x must have proposed to female n at some point prior to his proposal to female l. Female n must have rejected him in favor of who she is currently married to (or male x would not have had the opportunity to propose to female l). Therefore, there is no instability.

Further, it is not necessary for the number of males to be equal to the number of females. If there are more females, f, than males, m, then the procedure ends when m females have been proposed to. If there are more males than females, then the procedure ends when every male is either on a females string or has been rejected by all females. This procedure also has symmetrical properties if it is the females who are doing the proposing. It turns out that the group that the procedure will be optimal for is also the group that is proposing.

This procedure can easily be extended to school-student matching. Let the students be the males and the schools be the females (i.e., the students are "proposing" to the schools). Then, all students apply to to their first choice school. Then, the schools place the number of applicants that they have room for on their waiting list (similar to the females' string) and rejects the remaining applicants. The students that were rejected in the first round then apply to their second choice school and the schools can bump students from the first round off of their waiting list to replace them with the new applicants, if they desire. This procedure repeats for n rounds, until every student is on a waiting list, or has been rejected by every school that he/she has applied to. At this point, each school accepts every student on their waiting list. Since this procedure is analogous to that described for the marriage matching problem, the stability property is inherited. We can call this the deferred acceptance procedure.

Example 1 (adapted from Gale and Shapley 1962):

Let the following matrix give preferences for four students and four schools:

	A	B	C	D
α	1,3	2,3	3,2	4,3
β	1,4	4,1	3,3	2,2
γ	2,2	1,4	3,4	4,1
δ	4,1	2,2	3,1	1,4

In this example, the letters A, B, C, D represent schools and the letters $\alpha, \beta, \gamma, \delta$ represent students. The first number in each matrix entry gives the student's ranking of the school and the second entry gives the school's ranking of the students.

- In the first round, students α and β both apply to school A, student γ applies to school B, and finally student δ applies to school D. School A then puts student α on its wait list, school C puts student γ on its wait list, and school D puts student δ on its wait list.

- In the second round, student β (who is the only student not currently on a wait list) applies to school D. Since this student has a higher preference than student δ, school D puts student β on its wait list and now student δ is unassigned.

- In the third round, student δ applies to school B. Student δ has a higher priority at this school than student γ, so student δ is put on the wait list at school B, and student γ is left unassigned.

- In the fourth round, student γ applies to school A and receives a place on the wait list since this student's priority is higher than that of student α.

- In the fifth round, student α applies to school B, but does not receive a place on the wait list since his/her priority is lower than that of student δ.

- In the sixth round, student α applies to school C and receives a place on the wait list, which is currently empty. This is the final step since all students have been assigned to a school.

- The final outcome results in the following student-school pairs: $(\gamma, A), (\delta, B), (\alpha, C), (\beta, D)$

Note that although no student (or school) receives their first choice, this matching is stable.

The Gale-Shapley SOSM has a number of desirable properties:

1. Like the marriage matching mechanism described above, the Gale-Shapley SOSM produces a stable matching and therefore eliminates Justified Envy. In fact, this matching is stable and Pareto dominates every other possible stable matching. The following theorem is given by Gale and Shapley:

   (a) "Theorem 2: Every applicant is at least as well off under the assignment given by the deferred acceptance procedure as he would be under any other stable assignment."

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(b) *Proof:* An inductive proof is given by Gale and Shapley. Define a school to be possible for student α if there is a stable matching that sends him/her there. We begin our examination of the procedure at a point in which no student has been turned away from a school which is possible for him/her. Let school A reject student α. This implies that school A has received a full quota of applicants, $\beta_1, \beta_2, ..., \beta_q$ that outrank student α. Now, we need to show that A is impossible for α. The fact that applicant β_i is on the waiting list of school A implies that β_i prefers school A to all other schools, other than those that rejected him/her and hence are impossible for him/her (by the assumption at the beginning of the proof).

(c) By way of contradiction, consider a stable assignment that sends student α to school A. Then, at least one student, β_i, will be sent to a less desirable school than A. However, this matching is not in fact stable because both school A and student β_i would happier with a different matching. Hence, school A is impossible for student α, and we can conclude that the deferred acceptance procedure only rejects students from schools which are impossible for them, and is therefore optimal.

2. The Gale-Shapley SOSM is strategy proof.

(a) Proof: Assume, by contradiction, that there is some student i_i that misrepresents his/her preferences in round k. Under both the falsified preference profile and true preference profile, let the student i_i not be on any school's wait list at the beginning of round k. Assume that the student is presenting falsified preferences. In round k, student i_i applies to school s_1 and gains entry to the wait list. Keep in mind that the student does not truthfully prefer this school to all others that they have proposed to. Therefore, there is at least one other school that student i_i has not yet proposed to and that he/she prefers to school s_1. Call this school s_a.

(b) Case 1: The student is put on the wait list at school s_1. Then, the student either remains on the wait list at school s_1 and is eventually matched with the school, or is kicked off the wait list during some later round. If the latter scenario occurs, then student i_i continues proposing to schools and has the opportunity to propose to school s_a in a later round. Since student i_i's actions will not impact the behavior of other

students, he/she still has the same chances of getting into school s_a as he/she did if he/she had proposed in round k. Therefore, he/she is not better off by waiting to propose to this school.

(c) Case 2: The student is not put on the wait list at school s_1. Then, the student proposes to school s_a in the next round, which they truly prefer. As in case 1, his/her chances of getting into this school are unchanged by which round he/she propose to the school during, so there is no benefit to waiting to propose to school s_a in this scenario, either.

(d) From case 1, the student may eventually be matched with school s_1 and would therefore be worse off. The best case scenario is that the student has the same chances of getting into the school which they truly preferred as they did before, so there is no benefit to falsifying preferences.

Although this method cannot be manipulated by a *single* student misrepresenting his/her preferences, it can be manipulated by a group of students misrepresenting their preferences. Let us call this group of students a coalition. Although this is the case, only a subset of the group of students who misrepresent their preferences will benefit from this misrepresentation. Aksoy et al (2011) give the following theorem asserting this:

> Theorem 3: "In the SOSM algorithm, no subset of students can improve their assignment by falsifying their preference lists."

This theorem creates the implication that there is at least one member of every coalition whose assignment is not strictly improved by misrepresenting his/her preferences (the assignment would remain the same as it was without the coalition, if not, it would not be logical for this student to participate in the coalition). Further, in order for coalitions to successfully better the assignment of any member, they need perfect information about school and student preferences. These two facts make it unlikely that a coalition would be organized by students.

Rather, coalitions can be used to the benefit of schools and school districts. Although the Gale-Shapley Student Optimal Stable Matching Mechanism has many beneficial properties, it is possible to find a student-school match that Pareto dominates the SOSM outcome. The school district does have perfect information about both student and school preferences. The school district could therefore use a computer program to find all possible coalitions and then chose to apply the coalition that has the best of whichever property the school district values

the most. Aksoy et al (2011) give specific examples of which qualities might be valuable. Examples include: largest Pareto improvement, minimizing the preference index (the preference index is the sum of deviations from a student's top school choice across all students), and minimizing the number of priority violations (hence creating a more "fair" match). Allowing the school to create these artificial coalitions does not harm any student because membership in a coalition will never negatively impact a student's final school matching. Further, allowing the school to utilize artificial coalitions to create a matching that is a Pareto improvement over the SOSM outcome rather than encouraging students to create coalitions themselves avoids the problem of students misrepresenting their preferences.

Of course, this method is not the only possibility for producing matchings that are both stable and optimal and is certainly not the only procedure used in actuality.

3 Top Trading Cycles Mechanism

Abdulkadiroglu and Sonmez define an alternative matching mechanism, called the Top Trading Cycles Mechanism. This method uses the following algorithm to find matches:

- Step 1: A counter keeps track of how many seats are still available at each school. Initially, this counter is set equal to the capacity of the school. Every student points to their favorite school and then every school points to the student who has the highest priority. This creates at least one cycle (where a cycle is defined as an ordered list of schools and students $(s_1, i_1, s_2, ..., s_k, i_k)$ where s_1 points to i_1, i_1 points to s_2, ... points to i_k, who points to s_1). Note that each school and each student can be in at most one cycle (they can each only point to one school/student per step respectively). Every student that is a part of a cycle is assigned a seat at the school that they are pointing to and is removed. The counter of each school that is part of a cycle is reduced by one. If this reduction causes the counter of the school to be reduced to zero, then it is removed.

- Step k: Each student that is still remaining points to his/her favorite school and each school that is still remaining points to the student with the highest priority. This creates at least one cycle. Every student that is in a cycle is assigned to the school that he/she is pointing to, and is removed. The counter of every school that is part of a cycle is reduced by one. The school

is removed if the counter is reduced to zero.

Assuming that the number of total available seats is at least equal to the number of students, this mechanism will terminate when all students are assigned a seat, which happens in at most as many steps as there are students. This "worst case scenario" high number of rounds occurs when all of the schools have identical preferences. In this case, the student with the highest priority is assigned his/her top choice, the student with the second highest priority is assigned his/her top choice out of the remaining seats, etc.

Example 2 (adapted from Abdulkadiroglu and Sonmez): the following schools (s_1, s_2, s_3, s_4) and students $(i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8)$ have these priorities:

School priorities:

s_1	s_2	s_3	s_4
i_1	i_3	i_5	i_6
i_2	i_5	i_3	i_8
i_3	i_4	i_1	i_7
i_4	i_8	i_7	i_4
i_5	i_7	i_2	i_2
i_6	i_2	i_8	i_3
i_7	i_1	i_6	i_5
i_8	i_6	i_4	i_1

Student priorities:

i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
s_2	s_1	s_3	s_3	s_1	s_4	s_1	s_1
s_1	s_2	s_2	s_4	s_3	s_1	s_2	s_2
s_3	s_3	s_1	s_1	s_4	s_2	s_3	s_4
s_4	s_4	s_4	s_2	s_2	s_3	s_4	s_3

Note that there are eight students and four schools. Schools 1 and 2 have two seats each whereas schools 3 and 4 have three seats each.

- In step 1, the counters at schools 1 and 2 are set to two. The counters at schools 3 and 4 are set to three. We first consider s_1, who points to i_1. i_1 points to s_2, who points to i_3. i_3 points to s_3, who points to i_5. i_5 points to s_1, so this cycle is closed. We must also consider s_4, who is not in a cycle. s_4 is pointing to student i_6, so we consider this student. i_6 is pointing to s_4, so this second cycle is closed. Therefore, the two cycles that are generated in this step are: $(s_1, i_1, s_2, i_3, s_3, i_5)$ and (s_4, i_6). Students 1, 3, 5, and 6 have been assigned to a school, so they are removed. Since all schools participated in a cycle, each counter is reduced by one.

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- In step 2, the counters at schools 1 and 2 are set to one. The counters at schools 3 and 4 are set to two. We first consider school s_1, who points to i_2. Student i_2 then points to s_1, so this cycle is closed. We then consider school s_2 who points at i_4, who points at s_3, who points at i_7, who points at s_1. Since s_1 is already part of a cycle, there is no cycle generated in this step starting with school s_2. We must also consider school s_4. s_4 points at i_7, who points at s_1. Since s_1 is already part of a cycle, there is no cycle generated in this step starting with school s_4. All schools have now been considered, and there has only been one cycle generated, that is: (s_1, i_2). So, student 2 is removed and the counter for school 1 is reduced by one. This removes school 1. All other counters remain the same.

- In step 3, the counter at school 2 is set at one and the counters at schools 3 and 4 are set to two. We first consider school s_2, who points to i_4. Student i_4, in turn, points to s_3, who points to i_7, who points to s_2. Hence, this cycle is closed. School s_4 is not a part of this cycle, so we must also consider it. School s_4 points to i_8, who points to s_2, which is already part of a cycle, so no second cycle is generated. The cycle generated by this step is: (s_3, i_7, s_2, i_4). Therefore, students 7 and 4 are removed. The counters at schools 2 and 3 are reduced by one, so school 2 is removed.

- In step 4, the counter at school 3 is set at one and the counter at school 4 is set at two. Student 8 is the only student remaining. In this step, both s_3 and s_4 point at i_8. Student i_8 points at s_4. The only cycle created is (s_4, i_8). Since all students have now been assigned, the process terminates.

The following student-school matches are generated:

$(i_1, s_2), (i_2, s_1), (i_3, s_3), (i_4, s_3), (i_5, s_1), (i_6, s_4), (i_7, s_2), (i_8, s_4)$.

Note that the SOSM process would have resulted in different student-school matches. These would be: $(i_i, s_1), (i_2, s_1), (i_3, s_3), (i_4, s_3), (i_5, s_3), (i_6, s_4), (i_7, s_2), (i_8, s_2)$

Abdulkadiroglu and Sonmez give the following two propositions and proofs in relation to the above trading mechanism:

1. The top trading cycles mechanism is Pareto efficient

   (a) Proof: Let student i_i leave the algorithm at step 1. This student is therefore assigned her top preference, call this preference s_1. Any student, i_j, who leaves the algorithm at step 2 is also assigned her top choice among all of the remaining schools. The only way that she could

be made better off is if her top preference was the same as that of student i_i and there was only one seat at this school. Therefore, student i_j would be made better off by receiving a seat at school s_1. However, in order for this to happen, student i_i would be made worse off. Hence, this is not a Pareto improvement. If student i_j has a preference other than that of student i_i (and any other student that left the algorithm at step 1), then there is no way that student i_j can be made better off. We proceed in the same fashion for the remaining $k-2$ rounds of the algorithm and see that no student can be made better off without hurting someone who left the algorithm at a previous step. Therefore, the top trading cycles mechanism is Pareto efficient.

2. The top trading cycles mechanism is strategy-proof.

   (a) Proof: We first introduce the following lemma.

   (b) Lemma: "Let the set of all preferences, other than the preference of student i be $(Q_j)_{j \in I \setminus \{i\}}$. Suppose that in the algorithm, student i is removed at step T under Q_i, but at step T^* under Q_i^*. Let $T \leq T^*$. Then the remaining students and schools at the beginning of step T are the same whether student i announces Q_i or Q$_i$*."

   (c) Proof of lemma: Since step T^* occurs after step T, student i does not participate in a cycle before step T in either case. Therefore, the students and schools remaining are the same, independent of whether student i announces Q or Q_i^*.

   (d) Now, we can move to the proof of the proposition. Let student i have true preference profile P_i. We want to show that the outcome for student i is at least as good if she announces profile P_i as it would be under any other preference profile, Q_i. Let $(Q_j)_{j \in I \setminus \{i\}}$ be the announced preference profile of every other student except student i, let T be the step at which student i leaves under her falsified preference profile, Q_i, and let $(s, i_1, s_1 ..., s_k, i)$ by the cycle which student i joins at step T. This indicates that student i is matched with school s. Therefore, we want to show that her assignment under P_i is at least as good as school s. We have two cases to consider.

   (e) Case 1: $T^* \geq T$. Assume that student i truthfully reports her preference profile, P_i. By the lemma, the same schools and students remain in the algorithm at step T whether student i announces P_i or Q_i. Since

the schools' preferences are independent of whether or not student i reports her true preferences, at step T, school s points to student i_i, ..., school s_k points to student i. Now, student i points to the school that she truthfully prefers. She therefore will either point at school s, or a school that is at least as good as school s. The assignment that she receives is therefore at least as good as school s.

(f) Case 2: $T^* \leq T$. Assume that student i truthfully reports her preference profile. By the lemma, the same schools remain in the algorithm at step T^* independent of how student i presents her preference profile. Since school s remains in this step, student i will either point to school s (if there are no other schools at least as good as school s) or point to a school that she prefers over school s. Therefore, student i is at least as well off in this scenario as well.

(g) We see that in both cases, student i does not benefit from falsifying her preference profile.

4 Relationship between SOSM and Top Trading Cycles

The Top Trading Cycles Mechanism and the Gale-Shapley SOSM method can be related by coalitions. If SOSM is initially performed, often times, the TTC outcome can be obtained from the SOSM procedure through the creation of coalitions. Here is an example (adapted from Aksoy et al 2011) in which this is the case:

Example 3:

- Let students have the following preference profile:

i_1	i_2	i_3
s_2	s_1	s_1
s_1	s_2	s_2
s_3	s_3	s_3

- Let schools have the following preference profile:

s_1	s_2	s_3
i_1	i_2	i_2
i_3	i_1	i_1
i_2	i_3	i_3

- The SOSM finds the following student-school match: $(i_1, s_1), (i_2, s_2), (i_3, s_3)$. Note that this match has preference index 4 (students i_1 and i_2 are both one deviation from their first choices and student s_3 is two deviations from her first choice).

- The TTC finds the following student-school match: $(i_1, s_2), (i_2, s_1), (i_3, s_3)$. This match has preference index 2 (students i_1 and i_2 are both zero deviations from their top choice whereas student s_3 remains two deviations from her top choice). Note that this is a Pareto improvement over the SOSM matching because both students i_1 and i_2 are better off, without harming student i_3.

- We can obtain the TTC outcome by using the SOSM procedure if student i_3 modifies her preferences to indicate that s_3 is her first choice. Then, all students are assigned their "top choice" schools and the TTC outcome is the result. Note that even though student i_3 was forced to misrepresent her preferences, she was not made worse off by this procedure than she was in the original SOSM outcome.

It is not always possible to find use coalitions to obtain the TTC outcome from a SOSM procedure. Specifically, there are cases in which the TTC outcome is not a Pareto improvement over the SOSM outcome. In this case, it is never possible to find a coalition that will give the TTC outcome. Askoy et al (2011) give the following theorem:

> Theorem 4: "The outcome of a mechanism that is Pareto incomparable to SOSM cannot be obtained by a coalition adjustment to SOSM."

This theorem follows from the fact that coalition improvements are also Pareto improvements. If the TTC outcome is not a Pareto improvement over the SOSM outcome, then we cannot use coalitions to obtain the TTC outcome.

Part VI
Conclusion

Past analyses of failed attempts at implementing a voucher system have stressed the disconnect between the economic theory of vouchers, as given by economists such as Friedman and Hoxby, and the actual operation of the voucher system in

practice as the reason for the failure. This paper gives an alternative to this past explanation. Specifically, the drivers that influence positive educational outcomes are different from those that influence school choice.

This has large implications for the economic theory arguing in favor of vouchers. Since families do not seem to be selecting schools for the correct reasons, the supply side effect is not creating the correct type of competitive pressure on schools when these voucher systems are implemented. Therefore, there is no incentive for schools that are performing poorly to begin mimicking the behavior of schools that are performing academically well. Furthermore, small scale experiments lack the demand side effect. The demand side effect is dependent upon families choosing schools that will most benefit their child. The results of this paper seem to indicate that families have been unable to do this.

Further research of this question might include better defining dependent variables. School educational success has been defined in a variety of ways in the past, including: test scores, graduation rates, and highest grade level completed. The model could be rerun on any of these educational outcomes. However, a better solution might be to construct an educational success index, which gives a weight to and incorporates all of the above variables, as well as four-year college acceptance rates.

Likewise, a school popularity index could be constructed. The construction of this index might include surveying students and families about which schools in their area are the most desirable, as well as a variable representing the number of applications received by each school.

Once the dependent variables are better defined, the results of the studies must be communicated to families that participate in voucher systems. Clear communication of the factors that lead to positive educational outcomes will allow these families to have a better chance at making the best possible school selection, and therefore increase the demand side effects. To increase the supply side competitive pressure on all schools, the values of those variables that have a statistically significant and economically important effect on school academic success should be available to the general public. Possibilities include publishing this information in local newspapers, or mandating that reports containing this information be available on school websites.

If a universal voucher system were to be implemented, there would inevitably be some schools that received more applicants than for which there was space. Instead of allowing the schools to choose which students to accept on their own, better matches could be made overall by implementing a matching system. This

paper gave a review of two potential matching systems, the Gale-Shapley SOSM and Top Trading Cycles. The type of matching system mandated is a policy issue, and would still allow the government to have some control over the school system, while allowing for both demand and supply side effects.

From a theory standpoint, transitioning the school system to the private sector via vouchers seems to be an ideal solution to many of the problems that public education currently faces. However, voucher systems have failed to perform as described by the theory when put into practice. This paper explains these failures by demonstrating a disconnect between the drivers of educational success and the factors that families use to choose schools. Fortunately, the solution to this problem likely lies in increased transparency of school characteristics.

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Appendix A: Regressions with College Acceptance Rate

VARIABLES	(1) collegeper	(2) collegeper	(3) collegeper	(4) collegeper	(5) collegeper	(6) collegeper
unempl				-2.8097***	-2.0092***	-1.3220***
				[0.281]	[0.300]	[0.429]
p_asian		0.2135***	0.2755***	0.2931***	0.3325***	0.3055***
		[0.024]	[0.023]	[0.029]	[0.030]	[0.033]
p_black		-0.1025***	-0.0927***	-0.0845***	-0.1073***	-0.1168***
		[0.011]	[0.011]	[0.013]	[0.014]	[0.015]
p_hisp		-0.1986***	-0.1919***	-0.1669***	-0.1494***	-0.2223***
		[0.014]	[0.013]	[0.017]	[0.018]	[0.022]
p_indian		-0.1956***	-0.1727***	-0.2026***	-0.1394***	-0.2509***
		[0.034]	[0.034]	[0.040]	[0.041]	[0.056]
relig	5.0645***	0.8460*	0.5680	-0.8688	-0.9665*	2.1862***
	[0.455]	[0.479]	[0.465]	[0.570]	[0.578]	[0.680]
min	0.0543***	0.0448***	0.0010	0.0337***	0.0279**	0.0356**
	[0.010]	[0.011]	[0.034]	[0.013]	[0.013]	[0.014]
libr	15.2868***	13.1151***	10.7547***	11.8050***	11.5531***	10.0314***
	[0.629]	[0.641]	[0.687]	[0.809]	[0.809]	[0.760]
sttch_rt	-0.1026***	-0.0970***	-0.0471*	-0.0425**	-0.0393*	-0.1085***
	[0.011]	[0.010]	[0.026]	[0.022]	[0.021]	[0.024]
numstuds	0.0415***	0.0386***	0.0672***	0.0651***	0.0630***	0.0580***
	[0.001]	[0.001]	[0.005]	[0.004]	[0.004]	[0.001]
sqnumstuds			-0.0000***	-0.0000***	-0.0000***	-0.0000***
			[0.000]	[0.000]	[0.000]	[0.000]
per_male		-35.3584***	-32.9974***	-28.7920***	-29.4077***	-25.8532***
		[1.972]	[2.064]	[2.433]	[2.446]	[2.255]
northeast					7.7675***	
					[0.845]	
midwest					6.5059***	
					[0.826]	
south					4.5010***	
					[0.755]	
central_city					4.0019***	4.3472***
					[0.734]	[1.113]
urban_fringe					2.2277***	2.7209***
					[0.675]	[0.943]
_lyr_2001	2.3943***	2.5532***	2.4390***	7.3028***	5.8467***	4.4081***
	[0.584]	[0.609]	[0.600]	[0.770]	[0.792]	[0.929]
_lyr_2003	3.3642***	3.7956***	3.7113***	7.7775***	6.4879***	5.2330***
	[0.581]	[0.608]	[0.595]	[0.725]	[0.746]	[0.849]
_lyr_2005	3.5312***	4.5076***	4.4028***	4.0888***	4.1494***	3.4958***
	[0.579]	[0.603]	[0.591]	[1.222]	[1.230]	[1.173]
_lyr_2007	9.8132***	11.2950***	11.1891***	13.3348***	12.8350***	10.4502***
	[0.573]	[0.596]	[0.583]	[1.286]	[1.317]	[1.393]
sqmin			0.0007			
			[0.001]			
Constant	27.2375***	52.4407***	48.4218***	57.2472***	48.2681***	49.4653***
	[0.752]	[1.436]	[1.727]	[2.178]	[2.312]	[2.442]
Observations	33,709	29,820	29,820	19,547	19,547	19,547
R-squared	0.187	0.209	0.240	0.231	0.236	0.510

Robust standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Appendix B: Regressions with Change in 12th Grade Enrollment

VARIABLES	(1) change_12th	(2) change_12th	(3) change_12th	(4) change_12th	(5) change_12th	(6) change_12th
unempl				0.1689	-0.4580	-2.2549
				[0.976]	[1.523]	[2.341]
p_asian		0.2041**	0.1541*	0.1415	0.0953	0.3810**
		[0.094]	[0.092]	[0.120]	[0.150]	[0.181]
p_black		0.0893**	0.0809**	0.0798*	0.0833*	0.1270
		[0.035]	[0.035]	[0.044]	[0.047]	[0.080]
p_hisp		0.0010	-0.0048	-0.0745	-0.1063	-0.3550***
		[0.046]	[0.046]	[0.057]	[0.087]	[0.119]
p_indian		-0.1612**	-0.1800**	-0.2648***	-0.2962***	-0.3912
		[0.079]	[0.078]	[0.077]	[0.100]	[0.302]
relig	-7.4398***	-7.0447***	-6.7494***	-5.6216***	-4.7695**	-7.1109*
	[1.831]	[1.730]	[1.724]	[2.126]	[2.222]	[3.656]
min	-0.0335	-0.0988**	-0.0852	-0.1434**	-0.1386**	-0.1738**
	[0.073]	[0.048]	[0.138]	[0.065]	[0.065]	[0.077]
libr	-15.9897***	-15.4636***	-13.5573***	-14.5000**	-14.1395**	-16.6910***
	[4.237]	[4.602]	[4.708]	[6.788]	[6.421]	[4.091]
sttch_rt	0.1656***	0.1707***	0.1294***	0.1431**	0.1427**	0.5864***
	[0.037]	[0.033]	[0.041]	[0.056]	[0.057]	[0.130]
numstuds	-0.0095	-0.0156***	-0.0392***	-0.0440***	-0.0438***	-0.0446***
	[0.007]	[0.003]	[0.005]	[0.005]	[0.005]	[0.008]
sqnumstuds			0.0000***	0.0000***	0.0000***	0.0000***
			[0.000]	[0.000]	[0.000]	[0.000]
per_male		-8.1028	-10.0389	-20.8325	-20.8997	-28.2410**
		[12.022]	[11.912]	[16.966]	[16.988]	[12.007]
northeast					-2.9304	
					[5.613]	
midwest					-8.7467	
					[6.079]	
south					-3.4207	
					[5.946]	
central_city					-0.2952	1.2199
					[3.834]	[6.048]
urban_fringe					0.5508	2.3006
					[2.270]	[5.127]
_Iyr_2001	-1.4672	-1.5384	-1.4396	-1.6841	-0.6182	2.3114
	[3.051]	[3.366]	[3.354]	[4.038]	[5.541]	[5.065]
_Iyr_2003	-5.3851***	-5.1839***	-5.1074**	-5.2468**	-4.3714	-2.8085
	[1.894]	[2.012]	[2.009]	[2.460]	[3.202]	[4.627]
_Iyr_2005	-2.2217	-2.0861	-1.9895	-5.1552	-5.2240	-2.9755
	[2.135]	[2.281]	[2.277]	[3.693]	[4.015]	[6.403]
_Iyr_2007	0.1688	-2.5103	-2.4248	-5.2074	-4.5048	-0.9394
	[3.591]	[2.099]	[2.096]	[4.793]	[5.600]	[7.589]
sqmin			-0.0000			
			[0.003]			
Constant	46.0666***	51.6817***	54.9991***	62.3843***	67.8269***	74.0921***
	[4.538]	[10.539]	[10.522]	[16.367]	[22.862]	[13.213]
Observations	34,583	30,543	30,543	20,026	20,026	20,026
R-squared	0.002	0.005	0.006	0.007	0.007	0.136

Robust standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1