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INVITATION TO AUTHORS

Essays, book reviews, syllabi and letters are welcome. Two copies, double spaced should be sent to Alvin White, HUM. MATH. NET., Harvey Mudd College, Claremont, CA 91711. If possible, avoid footnotes and put references and bibliography at the end using a consistent style. If you use a word processor please send a diskette in addition to the typed paper. *The Newsletter* is assembled using Microsoft Word 4.0 and PageMaker 4.0 on a Macintosh. It is possible, however, to convert from other word processing systems. Clean typed copy can be scanned (but not dot matrix). Your essay should have a title, your name and address, and a brief summary. Your telephone number (not for publication) would be helpful. Essays and communications may be transmitted by electronic mail to the editor at AWHITE@YMIR.BITNET.

COVER

THIS FUNCTION OBEYS

For this drawing, I chose the **function** of technology as the focus of my attention because it seems most suited to the idea of **obedience**. The "technology man", dressed in prison stripes and bowing low before his human master, becomes the 20th and 21st century slave who perfectly **obeys** all commands whether or not they are wise ones.

Supported by a grant from the EXXON EDUCATION FOUNDATION

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FROM NEWSLETTER #1

Dear Colleague, August 3, 1987

This newsletter follows a three-day **Conference to Examine Mathematics as a Humanistic Discipline** in Claremont 1986 supported by the Exxon Education Foundation, and a special session at the AMS-MAA meeting in San Antonio January 1987. A common response of the thirty-six mathematicians at the conference was, "I was startled to see so many who shared my feelings."

Two related themes that emerged from the conference were 1) teaching mathematics humanistically, and 2) teaching humanistic mathematics. The first theme sought to place the student more centrally in the position of inquirer than is generally the case, while at the same time acknowledging the emotional climate of the activity of learning mathematics. What students could learn from each other, and how they might better come to understand mathematics as a meaningful rather than an arbitrary discipline were among the ideas of the first theme.

The second theme was focused less upon the nature of the teaching and learning environment and more upon the need to reconstruct the curriculum and the discipline of mathematics itself. The reconstruction would relate mathematical discoveries to personal courage, relate discovery to verification, mathematics to science, truth to utility, and in general, to relate mathematics to the culture in which it is embedded.

Humanistic dimensions of mathematics discussed at the conference included:

a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished versions to be "merely technical."
b) An appreciation for the human dimensions that motivate discovery — competition, cooperation, the urge for holistic pictures.
c) An understanding of the value judgments implied in the growth of any discipline. Logic alone never completely accounts for *what* is investigated, *how* it is investigated, and *why* it is investigated.
d) There is a need for new teaching, learning formats that will help wean our students from a view of knowledge as certain, to-be-received.
e) The opportunity for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems and to participate in controversy over mathematical issues.
f) Opportunities for faculty to do research on issues relating to teaching, and to be respected for that area of research.

This newsletter, also supported by Exxon, is part of an effort to fulfill the hopes of the participants. Others who have heard about the conferences have enthusiastically joined the effort. The newsletter will help create a network of mathematicians and others who are interested in sharing their ideas and experiences related to the conference themes. The network will be a community of support extending over many campuses that will end the isolation that individuals may feel. There are lots of good ideas, lots of experimentation, and lots of frustration because of isolation and lack of support. In addition to informally sharing bibliographic references, syllabi, accounts of successes and failures, . . . , the network might formally support writing, team-teaching, exchanges, conferences, . . .

Please send references, essays, half-baked ideas, proposals, suggestions, and whatever you think appropriate for this quarterly newsletter. Also send names of colleagues who should be added to the mailing list. All mail should be addressed to

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This issue contains some papers and excerpts of papers that were presented at the conferences.

FROM THE EDITOR

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The change of name from Newsletter to Journal recognizes the maturation of this publication. The new name has been recommended by many. Patricia Kenschaft of Montclair State College, NJ suggested that the new name would be more accurate and useful. Additional referees and associate editors are being recruited.

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If your library will add the Newsletter-Journal to its collection, I'll send copies to the library. This will make it easier for people to locate essays that appear. The library should write and request to be on the mailing list.

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This issue is a festival of Humanistic Mathematics. Saunders Maclane comments on several items in issue #6. Sherman Stein muses about a definition of Humanistic Math.

The poems presented during the exciting Poetry Reading in Baltimore are included here. John S. Lew shares his long involvement with Mathematical References in Literature.

The study by Sherry Turkle and Seymour Papert on Epistemological Pluralism is reprinted here with the kind permission of the authors and Signs: Journal of Women in Culture. Although Computer Culture is studied, the issues considered, I think, relate very much to the teaching, learning and doing of mathematics. The tension between concrete thinking and formal abstraction brings the personal to the center of mathematics (and computer programming). The relation between the person and the facts and techniques is an aspect of humanistic mathematics. I think that the issues raised by Turkle and Papert are in the mainstream of humanistic science.

The essays by Elena Marchisotto and Harald Ness each relate mathematics and the humanities.

In a coincidence that is not unusual for this journal, the papers by Ruth Hubbard and Neal Koblitz are both about the texts that students confront, and efforts to create the textual material that is more appropriate for students. Professor Koblitz also reflects on other aspects of education reform.

The last essay by a student, Elizabeth Miller, was written for a course in Mathematics and Culture. Professor Kathleen Shannon is sharing Ms. Miller's insights.

THREE COMMENTS ON ETHICS

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Newsletter #6 from the Humanistic Mathematics Network leads me to make several comments.

Peter Hilton and Jean Pederson, on page 99, recall Henry Whitehead's advice "never to accept in your own work a result which you could not yourself prove" This was appropriate advice at that time, especially for young differential topologists, a field in which Henry excelled. But there are other considerations-PH may recall a furious argument between two mathematicians only one of whom (I. Kaplansky) supported the Whitehead principle.

How is it today? There is a vast literature, not always carefully written, on differential topology; it would demand much time to check everything one might use. On finite group theory, the classification of finite simple groups now runs to about 10,000 pages, some due to Geoffrey Mason not yet published (it is held that they can be simplified). Using the classification theorem, one may be able to prove other interesting results about finite simple groups by showing that they hold in every case. If I find such a new result, does PH think I should check all 10,000 pages before publishing?

Incidentally, Professor S. Abhyankhar (Purdue) has just presented in our seminar such a result depending on the classification theorem.

On page 45, Robert P. Webber hopes to design an ethics course for the junior or senior level, and asks for material, complaining that he has not found much. I suggest that this is in the nature of the situation. Those cases of fraud in science tend to arise in other sciences (for example, cold fusion or genetics engineering) where the financial rewards can be much greater. Doctored data is not prevalent in mathematics, because the custom is to write out the proof right there in the published paper. In one class session, one can consider the ethical problem of giving correct proofs (and that of acknowledging errors). I doubt that this - or other items - can fill up a course. There are better versions of math for poets.

On page 49, Gian-Carlo Rota, under the rubric "The Story of a Misunderstanding", mounts a covert attack on Logical Positivism and its ilk. To be sure, there is too much adulation of Wittgenstein today, but logical positivism died long ago (Alonzo Church, in a review in the Journal of Symbolic Language, killed the second edition of A.J. Ayer's "Logic, Truth and Language"). But something positive is left -Carnap and others pointed out that many of the traditional questions of metaphysics, prominent in Germanic literary philosophy, are simply meaningless questions. Rota does not fully describe his own predilection for some of these philosophies. Many of them fall under the famous description: A philosopher is a blind man in a dark cellar looking for a black cat - which is not there.

The precision which we enjoy in mathematics should and can transfer to aspects of philosophy.

TOWARD A DEFINITION OF 'HUMANISTIC MATHEMATICS'

Sherman Stein
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Teachers of the socalled ''humanities'' have a good thing going for them. The very word "humanities" suggests that in a humanities class the student will become, I assume, a more compassionate person, a better citizen, and a wiser parent. Of course this is nonsense. My colleagues in the humanities departments have never struck me as more caring, more involved in improving the human condition, more empathetic with the downtrodden than my colleagues in, say, physics, chemistry or mathematics. Nor do they seem to get along with each other any better than do the scientists.

When you stop to think about it, why should they be expected to be finer people just because they have mastered counterpoint or the structure of a novel or the history of classical Athens? I don't see why, any more than the unique skills of a plumber or a carpenter provides a special insight into the human condition.

So, when I agreed to participate in the first "humanistic mathematics" conference, in Claremont, I was leery of the word "humanistic". It gave off pleasant connotations, but I was anxious to find out what it meant, especially in the context of mathematics. (What, after all, would be its antonym?) As it turned out, the conference proceeded quite well without ever stopping to define the key word in its description.

Since then, every so often, I have pondered the meaning of "humanistic mathematics". I've thought that it must be what was common to all the contributed talks at the conference, on such varied topics as teaching the nature of mathematics, the role of history in a mathematics course, or how to pose questions. This approach, while logically appealing, did not lead me to a definition.

Instead, I decided to capture the underlying mood of the meeting. There certainly was something that united those teachers who attended the conference. I believe that they wanted to influence their students in ways that go beyond just improving their mathematical skills. Ideally, when they prepare a lecture they give at least as much attention to what they expected their students to do afterwards as they do to what they themselves will do in class. They try to keep in mind that, at minimum, they want to improve their students' ability to think independently, and to communicate through both the spoken and the written word. Even at a research university they retain an active interest in teaching, even of undergraduates, even of freshmen and sophomores, even those who will not go on to graduate —level math, even those who are not math majors. They feel a responsibility to provide a liberal arts student an opportunity to appreciate the substance and beauty of mathematics.

What I am describing is "humanistic mathematics instruction", not ''humanistic mathematics.'' (I don't think one can point to any particular theorem and say it is or is not humanistic.) The definition of course could be adjusted to fit other disciplines. So, even in the humanities we can find both humanistic and non—humanistic instruction.

Yet there is something about mathematics that makes it an especially suitable vehicle for humanistic instruction: the student need not lean on authority or received wisdom: every step can be checked and independent exploration can be carried out. This is not true of many disciplines, where the beginner must accept key assertions on faith for they depend on historical events which grow more remote each year or on experiments too complex or expensive to be performed for each class.

So it should not come as a surprise that a brass plaque fastened to a brick wall along the central quadrangle of the University of Virginia displays these words:

William Holding Echols, 1859-1934, Professor of Mathematics.

By precept and example he taught many generations of students with ruthless insistence that the supreme values are self respect, integrity of mind, contempt of fear and hatred of sham.

Professor Echols may well serve as a model of the humanistic teacher, who, without sacrificing the beauty or substance of his subject, sees it also as a means of developing qualities in students that will serve them after they graduate, even if they never see another theorem, proof, or algorithm.

POEMS TAKEN FROM "AN EVENING OF MATHEMATICAL POETRY"

Held on January 10, 1992
National Joint Mathematics Meetings
Baltimore, Maryland

ODE BY AN INVETERATE PAPER GRADER

(with apologies to Edgar Allen Poe and his pet bird?!)

John H. Hodges
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Once upon a midnight dreary,
As I pondered weak and weary,
Over proofs both short and longer,
In styles that could be stronger,
Trying to keep track of pages,
When not numbered, mixed in stages,
If not stapled, they got shuffled,
Frustrated sighs and barely muffled.
As I tried to judge if valid,
Gradually my face went palid,
As I searched both earth and sky
For quantifiers on both x and y,
Will I find them, I implore?
QUOTH THE RAVEN "NEVERMORE"!

LIMERICKS 1: MOTLEY MATHEMATICAL MUSINGS

(with apologies to the great mathematicians named)

by John H. Hodges

1. Ahmes was an Egyptian scribe, (Rhind Papyrus,
With practical problems he did vibe, 1650 B.C.)
He wrote on papyrus,
Inspired by Osiris,
About bread and beer to imbibe.

2. In Miletus Thales worked we're told, (~ 600 B.C.)
To create a method, new and bold:
'Tis not enough to just "grove" it,
You really must prove it,
If for eons you want it to hold.

3. Pythagoras was a secretive man, (~ 585-500
Founded a club that he also ran, B.C.)
The whole numbers they studied,
by irrationals were muddied,
But their work still makes you a fan.

4. Archytas was a versatile chum, (428-347
A leading citizen of Tarentum, B.C.)
Taught Eudoxus how to add,
Friend of Plato, made him glad,
Found mean proportionals too, by gum!

5. Eudoxus was a brilliant man, (~ 408 -
If he can't do it nobody can, 355 B.C.)
He promoted proportions,
Rescued Pythagorean fortunes,
His method of exhaustion is gran(d).

6. Eratosthenes from old Cyrene, (~ 230 B.C.)
Whose frames yield two proportions mean,
With calculations ringed the earth,
And long before Magellan's birth,
His sieve leaves primes caught in between.
(And so we're glad he made the scene.)

7. Chou-pei was a famous treatise, (~206 B.C.
Said the Greeks, "It may have beat us," - 222 A.D.)
But Pythagoras will live, or earlier
No proof did "Chou" give,
So there is no "Chou-pei's Theorem" to greet us.

8. Menelaus and (Claudius) Ptolemy, (~ 100
Both loved trigonometry, A.D.)
The first worked mainly on the sphere,
The second made Hipparchus clear,
In his "great" book of astronomy.

9. Good old Pappus wrote a guide, (~ 300
To the math that gave Greeks pride, A.D.)
If he hadn't known it,
Then no one had shown it,
His " Collection" was both deep and wide.

10. Theon was a "commentator", (~ 390
Not a true originator, A.D.)
But his comments on Euclid,
Made geometry lucid,
So he wasn't just an idle spectator.

11. Proclus was a Neoplatonist, (5th
Of Euclid's work he gave the gist, Century A.D.)
His "Eudemian" summary
Gave us Geometry's history,
Which otherwise would be lost in the mist.

12. Li Yeh liked to work with digits, (1248-
But negative numbers gave him fidgits, 1259 A.D.)
So, in numerals, with dash,
He inserted a slash,
To change them from posits to negits.

13. Yang Hui lived in ancient China, (1261 A.D.)
"Pascal's Triangle" was his line-a,
He wrote it down first,
In an energy burst,
And so he could claim that, "It's a mine-a ."

OH, AN ALGEBRAIST AND FOUR OTHER MATH-NONSENSE VERSES

Lewy, Helen
(widow of mathematician Hans Lewy)
70 Whitaker
Berkeley, CA 94708-1737

Oh, an algebraist known to myself,
Used to boast that each week without fail,
He'd invent a new matrix for fun -
But the poor fellow landed in jail.

Oh, to tackle a lemniscate's boundry or so,
Is a thing that requires lots of nerve;
It's a terrible fate to be caught unawares
On the incorrect side of the curve!

Oh, derangement dropped in on my typist -
A venus with eyes of obsidian -
When she mused if the space on her space bar
Was, - or was not, - non-Euclidian!

A math prof who lived in Gilette
Was dating a girl friend called Bette
Said she, "Tell me Dove,
All you know about love!"
To which he replied, "Empty Set"!

The equivalent impact on barroom discourse,
Let us say, to a half dozen beers,
In mathematics, is merely the question, "What
if The Jacobean disappears?"

MATHOETRY
IS THERE AN ANSWER

Russell Smith Ashman
P.O. Box 734
Harrisburg, PA 17108

MATHOETRY

8 Arith, Alge, Geo, and Trig;
6 Raised in homes strict and firm,
8 Family of learning now quite big;
8 Making children in classrooms squirm.

8 Arabic numbers now we have,
6 To them we owe a debt,
8 From here began our science math;
8 Let's give them thanks, but not our wrath.

8 Alge Arab then started small,
6 Uniting broken parts;
8 Letters and symbols serve their needs,
8 As scholars make their feeble starts.

8 Geo enhanced its mathy lines;
6 As scholars aptly stared;
8 Some ancient lands laid out their fields;
8 Geo, thru time, has surely fared.

8 Alge soon married Geo Math;
6 Family size not big;
8 They had but one child - famous now;
8 They quickly called it baby Trig.

8 Specialist Trig now known to all,
6 Loved objects, three-sided all,
8 And opened with this new found key,
8 A universe that all could see.

8 Our calculus, the troubled child,
6 Sure makes you stop and think;
8 Integral values slowly sink
8 Differences that provide no link.

8 Arith, Alge, Geo, and Trig,
6 Live but misunderstood,
8 Family confusing, now quite big;
8 Oddly, they are serving our good.

IS THERE AN ANSWER

8 Celestial bodies are the dots,
8 Of an imaginary line,
8 Extending into outer space.

8 Space, a geometric object
8 Of pure and solid nothingness,
8 Defines itself with vague symbols.

8 Numbers are limited symbols,
8 Which measure the unlimited,
8 Impenetrable lines of space.

8 Math and space are inseparable;
8 Space suffocates math, even while
8 Math correctly describes its strengths.

8 An empirical mind grapples
8 Vainly with non-empirical
8 Limitations which it admits.

8 Is math a foolish child of space,
8 Or is space a fulsome nickname
8 For our modern mathematics?

A NEW SOLUTION TO AN OLD PROBLEM

Eleanor Ninestein
608 Merrick Court
Fayetteville, NC 28311

The topologist's child was quite hyper
'Til she wore a Moebius diaper.
The mess on the inside
Was thus on the outside
And it was easy for someone to wipe her.

MATH BUFFET

Colin Day
303 Whaley Street
Columbia, SC 24201

Math, the smorgasbord of the Mind run loose
try the arithmetic appetizer or the geometry juice
or the spicy smoked algebra meats
with tangy cold calculus cheese
and to wash them all down, perfection in a bottle
ein rein Rein Wein (Klein)
and for dessert we'll play game theory pies
until our sucrose sweet complexity dies

A MATHEMATICIAN'S NIGHTMARE
BEAUTIFUL NUMBERS

JoAnne Growney
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Bloomsburg, PA 17815

A MATHEMATICIAN'S NIGHTMARE

Suppose a general store:
Items with unknown values
And arbitrary prices,
Rounded for ease to
Whole-dollar amounts.

Each day Madame X,
Keeper of the emporium,
Raises or lowers each price:
Exceptional bargains
And anti-bargains.

Even-numbered prices
Divide by two
By half themselves
And half a dollar more
To keep the numbers whole.

Today I pause before
A handsome beveled mirror
Costing twenty-seven dollars.
Shall I buy or wait
For fifty-nine long days
Until the price is lower?

The price-changing scheme of this poem is derived from the Collatz Conjecture, an unsolved problem that has stolen hours of sleep from many mathematicians. Start with any positive integer: If it is even, take half of it; if it is odd, increase it by half and round up to the next whole number. Collatz' Conjecture asserts that, regardless of the starting number, iteration of this increase-decrease process will eventually lead to the number one.

BEAUTIFUL NUMBERS

"Ah, you are a mathematician."
they say
with fear or admiration
or a giggle of disdain.

"Then," they say,
"you must be able -
I could use you -
to balance the checkbook."

Reconsider, please!
Numbers and I
are more than
merely useful.

Some numbers, perhaps, are not lovely,
But there is then,
the number of my fingers
Which count all other numbers,
And there is seven,
the number of days
with which I count my life,
Wonderful four,
the number of my children,
Unsurpassed one,
for all of us together.
A million stars are not
more beautiful than one.

AN ENGLISH TEACHER CONFRONTS ALGEBRAIC NOTATION

Bonnie Sunstein
Writing Center
Rivier College
Nashua, New Hampshire

"The factorial function f(n) = n!, n=0,1,2,... is used in the expression of certain important probability distributions."

Beaver and Beaver, Study Guide to accompany Mendenhall, Introduction to Probability and Statistics, Seventh Edition

Behold the exclamation,
That ancient punctuation;
Stiff and straight, surrounded by black brackets.

Flanked by ranks of numbers,
O'er the bridge it lumbers;
Needing multiplying in those jackets.

This army cloaked in Greek
bodes outcome dank and bleak,
And doesn't aid solution or decision.

An enemy invasion,
Each set and each equation;
Is spitting hot ellipses toward division.

Oh, where's Anticipation?
And joyous confirmation?
The exclamation stands as a memorial.

Once delight and sweet surprise,
Its verbal pleasures soothed our eyes.
But now it signals us for a factorial (!)

RANDOM RHYMES FOR NORMAL DISTRIBUTION

Our World is full of strange and odd mismatches -
Whatever fate and circumstance dispatches
Our job appears in Stats
To join the these and thats
So when we sense an itch we'll have the scratches.

A family of Chi-Square relations
Whose nominal, neat observations
Made curves toward a mound
Degree-freedom bound
With infinite sum calculation.

If Pearson had answered his lust,
And let his Eugenics Chair rust;
He'd violate labels
On data and tables
For f and t measures "robust".

GOD-137
RADICAL-2

Robert Wilson
120 North Woodrow
Little Rock, AR 72205

GOD-137

Sometimes one is some
unless some is none
For one to be none
just can't be done
Yet sometimes some is square
and sometimes square is cube
and someone is three.

√2

Continuous
Associated
Transformation

+ . 014213562. . .

A DOZEN CLUES TO TRIGONOMETRIC FUNCTIONS

Shrinivar S. Dalal
Embry-Riddle Aeronautical University
Daytona Beach, Florida 32114

Santa Only Hopes
Christmas And Hanukkah
Turn Out Alright.

Chris

Some Old Helicopters
Crash After Hovering
To Orlando Airport.

Robert S.

Sometimes Only Heroes
Can Alter History
To Our Advantage.

Eric

Some Old Horse
Caught Another Horse
Taking Oats Away.

Chris G.

Some Overly Helpful
Certified Aviators Have
Trouble Operating Aircrafts.

Phillip

Scuds of Hussein
Caused Aerial Havoc
To Our Allies.

Alberto

Silly Old Hitler
Caused Awful Headaches
To Our Allies.

Robert P.

Some Old Horses
Can Always Haul
Tons of Alfalfa.
Scott

Sperry Our Hungry
Cat Attacked Harold
Today Over Anchovies.
Christopher

Selfish Oscar Had
Candy And Hated
To Offer Any.
Eugene

Sam Our Hopeless
Calculus Assistant Had
Trouble Operating Algebra.
David

Stupid Old Hussein
Caused Awful Heartaches
To Our Atmosphere.
Candy

Legend: S = Sine
C = Cosine
T = Tangent
A = Adjacent Side
O = Opposite Side
H = Hypotenuse

CARTOGRAPHIC CLAUSTROPHOBIA MATHEMATICAL CONFESSION

Frank Bernhart
63 Crimson Bramble
Rochester, NY 14623

CARTOGRAPHIC CLAUSTROPHOBIA (A FOUR COLOR HISTORY TOUR)

ONE color, TWO color, THREE-and-FOUR,
in coloring maps, do WE-need-more?
The map of England, Guthrie tried,
With EXACTLY four he's SATIS-fied.
So he told DeMorgan, who THEN told Cayley,
And plenty of others - the news spread daily.

PLANE maps or SPHERE maps - it don't matter,
Makes no difference, 'cept in patter.
But torus and Klein sack, please refuse,
All higher surfaces you MUST NOT use.
Abstain from topology, draw simple lines,
Let the map be described by finite signs
And using the Kuratowski planarity rule,
You may trade the map for its planar dual.

TWO color, THREE, FOUR-and-FIVE,
said Percy John Heawood, not now alive,
are always enough, and I'll attemp'
to repair the gap I found in Kempe.
For years he labored, with great agility
To make four suffice, with high Probability,
Accompanied by false proofs, dare we mention?
The authors could hold a crackpot convention.

Now, finally, thankfully, the answer's FOUR,
in colorful cartography- no NEED for more.
And what team broke the century-long spell?
None other than Haken and his FRIEND Appel.
The easiest direction, discharging, entails
pages and pages of tiny details.
The other direction, reduction, is cuter:
here billions of steps were left to computer.

DISCHARGE-ing, REDUCE-ing, what's that get?
An awkward and large unavoidable set,
Kempe's methods were extended, as though on a leash
considerably lengthened by Birkhoff and Heesch.
Plus a tribe of coworkers who nobly increased
The list of reductions without help in the least
from computers. Some people say "It's a cheat
Your proof is in part an unprintable feat."

ONE year, TWO years, THREE-and-FOUR,
Better throw in a dozen more.
The years roll by, the proof is firm,
just a few little errors to MAKE them squirm!
But all are soon fixed, better THINGS to do.
A nice short proof is NOT in sight,
Better to avoid all colors, and PAINT things white!

****

DIRECTIONS: read like 'Higgledy-piggledy, My Black Hen', or 'The Wonderful One-Hose Shay', contrasting smooth and staccato phrases.

MATHEMATICAL CONFESSION

I'll tell you why to salute Robert Bly,
(who wrote "Iron John" to help stuck men move on);
He files 'mathematician' as subspecies "magician"
or mythologist/cook (page 228 in the book).

The Cook belongs to the innermost ring,
with Warrior, Trickster, wild Man, King,
also Grief-man and Lover, whose archetypal power
can act to bring 'deep manhood' to flower.

Should emotional body be crippled in Youth,
the Mythologist/Cook may haply survive,
Use intellectual energy to trace out the truth,
and then ascend to keep sane and alive.

With energies invisible he can converse,
And become a mathematician or worse!
His wise ascension: Perhaps that is why
the naive man gets born and beholds the sky.

To communicate notions he will need the emotions,
and somehow recover the feelings of lover,
to project X,Y,Z (do students agree?)
and now you see why we still *need Robert Bly!*

****

DIRECTIONS: Read evenly and quickly, with slight pause after each line, a bit longer between verses. Each line should gradually decrease in pitch and stress; slower on last syllable.

ODE TO A Φ?

Barry W. Brunson
Western Kentucky University
Bowling Green, KY 42101

A thing as lovely as a ϕ
I think that I shall never $\left\{\begin{matrix}\text{see}\\ \text{spy}\end{matrix}\right\}$. [Choose one.]

If you want to make ϕ rhyme with "see",
But if you're really determined to be
Consistent, admit
Though it doesn't quite fit,
That a disk measures r^2 times "pea".

Each of ξ, π, ϕ, χ ψ will claim
an identical rhyme for its name.
Before you get hot,
Check Liddell and Scott[1];
Give them, not the author, the blame.

TO MY LITTLE ONE

Peggie A. Smith
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Department of Mathematics, MB4203
4200 Connecticut Ave., NW
Washington, DC 20008

The Hindu Arabic alphabet to thee is sounding letters a to z, Or learning phonics at the age of three while sitting on your daddy's knee. But soon you will learn there are deeper meanings to discern the universe. All its wonders leave a lifetime of phenomena for an ebullient mind to blunder. Even the Greeks have their own set of letters of the alphabet, although you have not learned these as of yet...there is still time.

a is acceleration due to gravity
b is a y-intercept in coordinate geometry
c is celsius or centigrade
d is the distance from A to B
e is the base of the exponential function y = ex
f is force due to gravity, g
h is the height of a pyramid in Africa
i is the imaginary unit
j is a vector with magnitude and direction
k is constant to a degree
l is the length of the Nile before the flooding season
m is the slope of a line of course
n is the unknown who has no voice, but oh,
o is zero who is greater than negatives and less than positive;
but still part of reality
p,q r, and s are logical statements in Boolean algebra used to analyze complex electrical circuits in the design of electronic digital computers
t is the time it will take two planes to become 3600 km apart if they both leave Dulles at 2 p.m., one flying south at 450 km/hr and the other flying north at 675 km/hr
U is now units once ones
V is velocity which intrigued Newton and Einstein before I was born
W is work there is much to be done
x,y, and z are special ones to me because they take me into 3-D and never leave me hanging in space.

Oh I could go on til infinity; but time is of the essence you see and I must explain the true beauty of it all! It lies in the fact that each letter is pithy and none is constant, not even k!

MATH ILLUSION

Ruth Rendle
29 Edison Terrace
Sparta, NJ 07871

I had so much to say
In the middle of the night.
I must defend my view;
That picture was so right.

Myself agreed completely.
Discoveries could be made.
Pythagoras was brilliant;
The groundwork had been laid.

Circle moved about the grid;
Ellipse evolved with ease.
Hyperbola could follow;
My mind began to tease.

The order and the wonder
Of a system without end
With interlocking pieces
That one could pilot and bend.

The beauty of true answers,
The ideas I would save
Tomorrow to collect the props
For the speech I never gave.

PROOF PANTOUM
LINGUA NON-FRANCA

Melodi Goff
1512 George Avenue
Jefferson City, TN 37760

PROOF PANTOUM

In Pantoum form set down the proof
since ancient Euclid noted:
the numbers prime, they're not finite -
it's clear by contradiction.

Assume for now primes count to "zed"
and label them p-subscript:
$\{p_1, p_2$ ..... until $p_t\}$
and now we take their product.

And one to this and call it ipt
(to keep the form, though awkward).
Now what of this can we deduct
regarding some prime number?

By other theorem we have word
some prime will ipt dispart.
Now think with me, you will concur
this prime is not some p-sub.

If ipt is prime, or no, our start
is now undone, it's unroof'd,
and prime not p, exists - sweet nub:
our primes are found in-finite.

LINGUA NON-FRANCA

Mathematicians all, we must confess
to our peculiar merriness:

Greek letters in company
with Arabic numerals ...
Imaginary numbers doing
what the Real cannot, because
we need it done ...
and the English alphabet
to add cultural diversity ...
Symbols used and reused
(context is everything)
and a shorthand we devise (iff)...
Wielded with the power and
freedom of God -
"Let 'this' be 'that'" and "There exists..."

Now present it to the world,
with a straight face claiming
"It is perfectly clear,
with the briefest contemplating."

MATHEMATICS LITERATURE
THE MAJESTIC OAK STRONG IN MATH

Florentin Smarandache
P.O. Box 42561
Phoenix, AZ 85080

MATHEMATICS LITERATURE

Imagine that these poems had been created
By an-electronic device, though you are not
Too far! Than what would you have thought?
If in the most sophisticated labs the
Scientists are producing human
Embryos, we are producing souls. According
To mechanic procedures spiritual states
Are being made. Programmed algorithms
In a sophisticated language are producing
poems
On a conveyor belt. The writer wearing
A white overall is watching the bracket
Of its ordinator when these are creating
These logic sentiments

It is infant literature for adults
Or vice-versa. Linear verses tore by
Non-linear images, metaphoric equations
Of the insulant abstract systems of thoughts
Breathing of a second...

As the artificial flowers these poems
Are imitating the natural flowers

THE MAJESTIC OAK STRONG IN MATH

Innocent, diaphanous
The Spring presents itself
As the exam...

Young carnations
Bashfully as pupils
And ivory butterflies
Students in biology

Strong in math
The majestic oak
Keeps its arms raised
The white bindweed
Gets its flower diploma

COLLABORATION

John Klippenstein
Mathematics Department
The University of British Columbia
Vancouver, B.C . Canada V6T 1Y4

Spiderlike I search for facts
To anchor my webs of though,
Strung out to capture the ways of the world

Resting alone where web meets world,
Waiting for some gossamer vibration,
I feel no need for any other.

Watching the earthbound ants,
I disdain their plodding gait,
their stolid struggle, shouldering immense burdens.

When a dream of spider ant collaboration
Building a vaster, finer web
Or a subtler colony
Hidden from probing eyes
Built of web strong walls
Impervious to snout and spade,

Sent me sailing down to earth
Eager to learn the language of the ants

MATH 101

Fred Gass
Department of Mathematics and Statistics
Oxford, Ohio 45056

MCGUFFEY AUDITORIUM
DEC 21, 7.30 P.M.

They are scattered
In criminologically sound
Array, a lattice of energy
Humming with short-answers.

Page Two: they turn,
Fold and rustle
Like a rookery unsettled.
Heads bob and peer
Until, hovering,
I have eased them
Back into normal
Distribution.

Wood groans, the
Radiators sigh.

Twisted or taut,
Even the hair agonizes,
Pencils poise, dowsing
For partial credit.
Eyes maunder across
Page and wall.

Most of them have fallen
Into puzzlement.

Cataleptic
They stare.

GEOMETRY FOR ME

David Henderson
Department of Mathematics, White Hall
Cornell University
Ithaca, New York 14853-7901

Logic can only go so far -
after that I must see-perceive-imagine.
This geometry can help.

I may reason logically thru theorem and propositions galore,
but only what I perceive is real.

If after studying I am not changed -
if after studying I still see the same -
then all has gone for naught.

Geometry is to open up my mind
so I may see what has always been behind
the illusions that time and space construct.

Space isn't made of point and line
the points and lines are in the mind.
The physicists see space as curved
with particles that are quite blurred.
And, when I draw, everything is fat
there are no points and that is that.
The artists and the dreamer knows
that space is where an image grows.
For me it's a sea in which I swim
a formless sea of hope and whim.

Thru my fear of Infinity and One
I structure space to confine
my imagination away from the idea
that all is One.

But, I can from this trap escape -
I can see the geometry in which I wander
as but a structure I made to ponder.

I can dare to let go the structures and my fears
and look beyond
to see what is always there to see.

But, to let go, I must first grab on.
Geometry is both the grabbing on
and the letting go.
It is a logical structure
and a perceived meaning -
Q.E.D's and "Oh! I see!"'s
It is formal abstractions
and beautiful contraptions.
It is talking precisely about that
which we know only fuzzily
But, in the end, and, most of all,
it is seeing-perceiving
the meaning that
I AM.

THIS THING CALLED MATHEMATICS

Vatsala Krishnamani
Department of Mathematics and Statistics
P.O. Box 194
Middle Tennessee State University
Murfreesboro, Tennessee 37132

Freshmen fear it
Sophomores shove it in
Seniors survive it
Researchers unravel it
Mathematicians mold it
All admire it
This thing called Mathematics

When you start, it is a mystery
When you know a little, it is intriguing
When you know more, it is amazing
When you know more and more, it is never-ending

At the top, it is sometimes lonely
But from being fun, it never stops
This thing called Mathematics

You cannot live without it, some cannot live with it
Some love it, some learn to love it
Frightening to those not understanding it
Friendly to those handling it wise
Mystifying to those viewing from a distance
It is sometimes yielding, sometimes tense
This thing called Mathematics

If you chase it, it will hide
If you hide, it will chase you
Yes, it plays a game with you!
By the time you think you mastered it
You realize you do not know enough of it
You keep wondering what is
This thing called Mathematics?

2

The circles and the spheres
The a's and the b's
The x's and y's
The Pluses and the minuses
The derivatives and the integrals
Are not the only ones
In This thing called Mathematics

FOR
If there are no Physics
Right from the basics
It is there in all sciences
And in Arts and Humanities too
Sans math, everything is a zoo!
It is very much in the real world
This thing called Mathematics

The farmers and businessmen use it
The politicians and artists use it
Philosophers use it
The only distinction is
How one does it

Not who does it
This thing called Mathematics

Poets like Yeats quoted it
Writers like Carroll did it
Kovalevskaya and Noether crowned it
With Newton, Lagrange, and Hardy,
Fermat, Ramanujam and Cauchy.
You and I do it
This thing called Mathematics

3

It knows no race
It knows no sex
It knows no Rich or Poor
It knows no Religion, that way it is better
It speaks a language, universal
That certainly is something special
In this thing called Mathematics

It welcomes any color
Black or white or Brown or any other
Or a shade in between, it does not matter
In real equal opportunity, a trend setter
Professors teach, students learn
Sometimes it could be the other way around
In this thing called Mathematics!

People call it pure, with theorems proved
They call it applied, widely acclaimed
All agree it is unsurpassed
But what is in a name?
For it stays the same
Unaffected by the hue and cry around
This thing called Mathematics

The zero is not nothing
It definitely is something
With no beginning and no end
The infinity makes it supreme, a super find
where do you see it? Where do you catch it?
There is no match for it
All those in between are special too
In this thing called Mathematics

4

How many different strings, all making one tune!
What a harmony in variety and a boon!
It can make Peace and it can make war
What one chooses, there is no bar
But we only wish with fingers crossed
We always make more peace with it
This thing called Mathematics

What a wonder, What a puzzle!
What a miracle, What a dazzle!
What a gift, What a creation!
What a hard work, What a recreation!
Who could describe it without exclamation!
What a surprise, What a tool!
What a fair deal, How cool!

What a help, What a friend!
What a marvel, What a blend!
What a paradox, What a chance!
What a music, What a dance!
What a melody, What a trance!
If only you could fall in love with
This thing called Mathematics

Times were different in the Sixties
We often look back at the Seventies
We just have been in the Eighties
We are now going into the Nineties
Marching forward to the turn of the century
But where would we be if there were absolutely
No thing called Mathematics?

TEACHER'S GIFT
CAUSE AND EFFECT
ODE TO A TRIANGULAR MATRIX

Dan Kalman
The Aerospace Corporation
Post Office Box 92957
Los Angeles, CA 90009-2957

TEACHER'S GIFT

Confined you are, have always been,
by bonds unfelt; by bars unseen.
But not so I, I soar on wings
of thought, and thinking, dream these things:
two worlds made one yet ever two
apart; a labyrinth traced clear through
from end to end; a tone more pure
than Circe's voice; a keep secure
from even time's travail; a brightness
that confers the pain of sight
so keen it pierces to the heart.
To this and more I am conveyed.

Come, break those chains. Take up the blade
by Euclid forged, and polished since
by ev'ry soul who saw its glint
in reason's fire, and passed from hand
to hand down all the age of man
until at last here now we two.
Hold out your hand, I give it you.
Your fetters can't withstand its aim.
Here. Mathematics is its name.

CAUSE AND EFFECT

To my students it's anathematical
To study anything mathematical.
I want to go on a sabbatical.

ODE TO A TRIANGULAR MATRIX

Oh, thou three corner array,
noblest of matrices:
Thy divine figure reveals itself
reflected in multiple zeros
Exclaiming your secretes in proud display.

Where the common matrix
guards its determinant
as a potent talisman
never to be revealed,
save after careful incantation,
arcane mutterings,
unending and errorless calculation,
You, oh forthright soul of linearity
requiring merely a show of sincerity,
a token computation,
willingly exhibit your psyche's key elements
worn in a bold slash,
yea, a sash,
and whose product, your determinant
is offered for our education.

The chaotic matrix
whose aspect is disordered
has at its core,
its very kernel,
a confused maze of conflicting directions.
Lest we come to know its true
meaning, worth, rank,
it hides this kernel from our sight,
misleading and confusing us,
annihilating enemies in secret alleyways
and under cover of night.

But you, three sided paragon,
disdain such rank duplicity;
declaim your true intentions;
show every multiplicity
the measure of your heart, your soul,
your innermost dimensions.

And there is no mistake about the values that you hold.
With characteristic candor that is striking to behold,
and without undue modesty, your honesty is tangible:
your values worn for all to see with pride on your diagonal.

And you and all your fellows faithful ever do remain
you multiply together and your offspring are the same,
and in all your combinations, too, your virtues are unchanged.

Indeed, you are a model for the race of matrix kind.
In simple fact.
In artful grace.
In guileless art.
In graceful form.
You show this humble student all that he could hope to find.

GOD LOVES A CURVE!
HEART BEAT
INSTANTS

Edward E. Chipman
830 N. Shore Dr. NE
St. Petersburg, FL 33701

God loves a curve!
Oh, yes - He gives straight lines, as in geometry,
Horizontals for measurement,
Perpendiculars for elevators,
Angles, and the like.
Straight lines are for utility,
Delineate the object, but leave me cold.

God loves a curve,
And curves give grace and easy beauty:
Hills and human forms,
Marbles, apples, or a rose;
And that winding railroad track -
These all entrance the eye as nothing straight,
When looking back or forward we do see
The rushing symmetry of motion;
Or when heavenward gazing we
Do witness sweeping arcs of flight;
Or waterborne, the billowing tack
Of sails exulting in the breeze.

Yes, God loves a curve! -
And I the moon at night
In scimitar reflection or full-orbed smile.

But most of all I love a curve

In love's full grace,
Caught in your smile!

For which I wait.

HEART BEAT

"Ninety-two million beats a second" -
So was the report
From the Boulder, Colorado, lab
some 20 years ago.
A vibration rate and measuring of time.

And now we are told a new type of watch
Measures distinctions,
Elements of time,
In fractions so minute
The normal mind just boggles at the thought!
Cannot conceive, or doubts this as a fact;
And raises questions how such things can be.
Or else accepts them without a quibble.

Such is the trust in science,
Or scientific men
Who previously have shown us miracles
Of thought and planning
Put to work in deed.
That furnish in utilities and ease
A lift to common life, release from toil,
And greater culture opportunities.

"Ninety-two million beats a second"
And now much more,
The measurings of human mind
Attuned to Nature's laws, discovering
The intricacies built into cosmic structures
By Nature's God, the primal Cause.
And Man's the privilege of searching out,
Discovering, bringing to light
The ways and workings, orderings,
So long hid from human gaze or knowing.

"Have dominion" - so the first command
To ancient man from his early God -
"Over what is in the sea and in the air
And on the earth." - Gen. 1:28
"Subdue it all," And this he does
In this our day, to our amaze.

Yet there remains, for his subduing,
The raw passions of his nature,
To where truth shall overcome deceit,
Lover conquer his hurtful hates,
Justice replace entrenched wrongs,
Kindness mark his dealings with his fellows,
And love and fellowship with his Maker
Be the heart-beat of his living.

INSTANTS

Time is composed of instants
Late or soon
They carry weight of their events,
Light or heavy, trivial or momentous,
They spell together the duration
Of a lifetime;
And of all history in man's count,
Pre-history also
Whose moments, years, through science
estimates
Do come to mount
In measurements of space and time
To lengths tremendous -
Acceptable but inconceivable
In their vast spans.

Time is composed of instants,
Which in their beat and fast repeat
Are measured in the millions, billions too,
Within the move of seconds.
These distances,
Or small or great in their expanse
And temporal existence,
Do boggle and intrigue the mind,
Infusing awe and wonder, causing us to
ponder
On all the beauty, order, systems,
Creativity; of which we
You, I, all men are a fine part
Within the fathomless seas of heaven.

So, should not we
In these much later days - so well equipped
With all the instruments of science
To inform our minds, -
Enlarge our comprehension,
Increase perception - should not we
Do well to hold, assent to
And in humility confess, declare
With great Cicero:
"The beauty of the world -
Its orderly arrangement of all things celestial
Makes us confess there is
An excellent and eternal nature (N)
Which by all mankind
Ought to be worshiped and adored!"

$(X - 2)(X - 12) = X^2 - 14X + 24$
AN ALGEBRAIC POEM

Thomas F. Mulcrone
Renee Anne Viosca
Mathematical Sciences
Loyola University
New Orleans, LA 70118

There's x unknown
that one takes 12 from.

Could x be the measure of
the months of your life?

That same unknown drops 2,
say the misdirections,

The differences
of our relation.

Multiply the x less 12
by the x minus 2

And you get, let be,
your life's number.

Square that x, which
I know not of you.

From the result subtract
the 14 times x,

Which is about sum
Of our bad days,

Now add 24, the 2 happy years
I have loved you.

Behold! You have again
your life's number .

That taken 12
and dropped 2,

Double-negative law renewed,
are negative no more!

To the x squared
Of your perfection,

Is added the 24
of my affection.

A STRANGE BIFURCATION

Lilia N. Apostolova
Institute of Mathematics
Bulgarian Academy of Sciences
Sofia 1090 Bulgaria

A strange bifurcation
holds in the world
but the earth things
are only limited.

The devil is only limited
the night is limited
a new morning is in a hurry
a new light is in a hurry.

And the attractor burns
again in the eternity
nothing frightens yet
holds in the world.

A strange bifurcation takes care of the life,
the clear dreams won't die away.

SIMPLE GROUPS
THE TEST SONG OF G. BERNARD RIEMANN

Saunders MacLane
Department of Mathematics
The University of Chicago
5734 University Avenue
Chicago, IL 60637

What are the orders of all simple groups?
I speak of the honest ones, not of the loops.
It seems that old Burnside the orders has guessed,
Except for the cyclic ones, even the rest.

Groups made up with permutes will produce some more,
For A_n is simple if n exceeds 4.
There is Sir Matthew who came into view,
Exhibiting groups of an order quite new.

Still others have come on to study this thing;
Of Artin and Chevalley now we shall sing.
With matrices finite they made quite a list.
The question is: Could there be others they've missed?

Suzuki and Ree then maintained it's the case
That these methods had not reached the end of the chase.
They wrote down some matrices just four by four
That made up a simple group; why not make more?

And then came the opus of Thompson and Feit
Which shed on the problem remarkable light:
A group when the order won't factor by two
Is cyclic or solvable. That's what is true.

Suzuki and Ree had caused eyebrows to raise,
But the theoreticians they just couldn't faze.

Their groups weren't new if you added a twist,
You could get them from old ones with a flick of the wrist.

Still some hardy souls felt a thorn in their side,
For the five groups of Mathieu all reason defied;
Not A_n, not twisted, and not Chevalley,
They called them sporadic and filed them away.

Are Mathieu groups creatures of Heaven or Hell?
Zvonimir Janko determined to tell.
He found out what nobody wanted to know:
The masters had missed 1 7 5 5 6 0.

The floodgates were opened, new groups were the rage,
And twelve or more sprouted to greet the new age;
By Janko, and Conway, and Fischer, and Held,
McLaughlin, Suzuki, and Higman and Sims.

You probably noticed the last lines don't rhyme.
Well, that is quite simply a sign of the time;
There's chaos, not order, among simple groups,
And maybe we'd better go back to the loops.

THE TEST SONG OF G. BERNARD RIEMANN

Let us go then, you and I
With courses so confused both low and high
Like a snowstorm frozen on a city
Let us chalk them up, on a sleazy backboard
Boards that twist upon a tedious argument
Of dubious intent
To lead us to a wholly shaky question...
Oh, do not ask, "Why is it?"
Let us prove it lest we miss it.

In the halls the teachers go and come
Walking on Cantor's continuum

The orange fog that covers up the notes we take
The orange smoke that kept away for Artin's sake
Came smoothly in to cover every space and form
And finding no compactness, took the norm
Curled twice round all of Eckhart
Then fogged again a differential form.

And indeed there won't be time
For the smoke to clear from all those prime ideals
There won't be time, there won't be time
To prepare all knowledge for those tests we meet
To deal with Galois at those evening meals
There won't be time to study and compute
What May and Narasimhan want us to impute
Before they drop that question on our plates
No time for all those needed group revisions
Before the taking of our cake at tea.

In the halls the teachers come and go
Following Smale's unstable flow

And would it have been worth it, after all
After the cups, the cookies, and the tea
There with the variables, both the bound and free
Would it have been worth while
To seize upon a resolution with a smile
To deform the complex sphere upon the plane
Or roll it toward some universal adjoint

For they might say, setting red pencil by the end
"That is not what we meant at all
That is not true, at all."

(with apologies to T.S.E.)

122 Morningside Drive
Ossining, NY 10562

3 February 1992

Professor Joanne S. Growney
Department of Mathematics and Computer Science
Bloomsburg University
Bloomsburg, PA 17815

Dear Professor Growney

In January 1992 I met you and Professor Alvin M. White at the Baltimore AMS-MAA meeting - during an evening poetry-reading at the Convention Center. A few days later, I sent you my bibliography on mathematics in literature. Thus you know that I have taken both a long interest in mathematics and a long interest in poetry.

Your little quiz in the American Mathematical Monthly for February 1992 suggests that mathematics and poetry are "fundamentally similar". To me this seems false - unless "similarity" has so broad a meaning that it has little significance. At their best, both mathematics and poetry display esthetic qualities and involve technical difficulties, hence require concentration and sensitivity. But so do other arts, other professions, and even championship sports. Via your quiz, you suggest a thesis that stretches the resemblance a bit too much.

Moreover, connecting poetry with truth requires ignoring many witnesses to the contrary — including Plato, whose Republic would have banished poets. Much major poetry seeks only to express a particular emotional view, or even just to fix a momentary impression. In what sense does one find truth in "The Canterbury Tales"? In "Hamlet"? In "Paradise Lost"? Like Samuel Johnson, one may feel that Shakespeare presents characters free from the accidents of a particular place and time, but this is an undemonstrable FEELING. Tolstoy violently criticized Shakespeare, and one has no way to refute Tolstoy.

Your quiz items prompt a few specific comments.

(3) This is a metaphor for the feeling of wonder, but other things can excite wonder.

(4) This says that mathematics resembles those disciplines usually called "the arts", or maybe that mathematics should itself be considered one of "the arts". But it no more implies a "fundamental similarity" between mathematics and poetry than it implies such a similarity between figure-skating and poetry.

(5) Voltaire's "Candide" concludes that one "masters chaos a little" even when one merely cultivates one's garden.

(6) The Marquis de Sade tried to argue logically that a rational person need observe no moral restraints. Thus also disciples of Sade "practice absolute freedom".

(7) Our "intellectuals" flub many subjects.

(8) Poetic comparisons often violate common sense.

(10)Swinburne is not a major poet, but he survives through a verbal facility that lends some interest to his vagueness.

(11)Many things can become habits.

(16)This is a high compliment, but it is hardly a specific characterization

(17)Nemerov's statement describes poetry, and it describes mathematical ELEGANCE, but it does not describe MATHEMATICS. Birkhoff's long proof of the pointwise ergodic theorem is mathematics. The later short proofs are elegance, but Birkhoff's version was already mathematics.

Doubtless, mathematics strikes some beginning students as a tedious sequence of statements and reasons. Thus mathematicians must keep telling the world that their discipline, to a great extent, is an ART - in which logic is the MEDIUM, as paint and canvas to Rembrandt, or notes and musical instruments to Mozart, or words to Shakespeare. In all such cases, one assumes that facility with the medium is a prerequisite for high achievement. But a relationship of this sort is something less than a "fundamental similarity". Indeed, anecdotal evidence suggests that mathematicians and scientists show more affinity for music than for poetry. Maybe poetry uses words and concepts in ways too different from mathematics, whereas a sequential but nonverbal art meets easier comprehension.

An occasional mathematician may enjoy the ego-trip of writing poetry and thus "being a poet", but few mathematicians acknowledge enough relation between their work and poetry to bother reading the great poets.

I insist that Wallace Stevens is one of the great 20th-century poets. I have read much of his work; I have read a lot about him. I insist that his writing shows no specific influence of mathematics - but very considerable influence of 20th-century PHILOSOPHY. A major preoccupation of his work is to recount his search for reasoned moral and esthetic doctrines. His poetry weighs and develops concepts in an area that is NOT mathematics, but has enough the semblance of reason so that one might think his poems would interest mathematicians. I initially understood something of his work because I had read some philosophy of science, and perceived him trying similar approaches in his own domain. Yet I have never met a mathematician who has seriously read Stevens.

To me, the most notable mainstream literary works by mathematicians are the unfinished autobiography of Sonia Kovalevsky and the short stories of Bertrand Russell. But no one seems to have read those either.

Sincerely,

John S. Lew

MATHEMATICAL REFERENCES IN LITERATURE

John S. Lew
Mathematical Sciences Department, IBM Research Division
IBM T. J. Watson Research Center, Yorktown Heights, NY

TABLE OF CONTENTS

1. INTRODUCTION

Calculate insurance rates

Many years ago, my father, an actuary, began collecting references to actuaries in literature. None of his material saw print until much later (E. A. Lew 1968); but well before this, from my college reading, I had begun to find things he had overlooked. (This disappointed me; one expects one's father to be omniscient.)

Then, a few years after my graduation, the eminent critic Clifton Fadiman published two anthologies, "Fantasia Mathematica" (1958) and "The Mathematical Magpie" (1962), containing short stories, poems, and other literary excerpts with mathematical references. These books reprinted considerable science fiction. I had no complaint about that, since previously I had read most of his selections in that area; but I remarked that Fadiman too had overlooked well-known things, items that should not have escaped a literary personage. I began remembering more carefully what I would have chosen instead.

Fortunately, this secret contest with my father and Mr. Fadiman never caught the attention of a psychoanalyst. At some point I began recording my finds on index cards. In the 1970's I transferred my growing list to a computer file, standardized the references, and began inviting others' contributions. The core is still my own curiosity, but this bibliography now includes many suggestions, and its final part names all those who helped me. I thank them all here. Some are eminent scholars. Some are personal friends. One is my wife, who contributed her knowledge of science fiction - and her professional services as a librarian.

Despite this long quest, I cherish no thought that my list approaches completeness. It has gained breadth by swallowing several other lists, but still it reflects only my discoveries and those of finitely many other people, none of whom can possibly have read everything. Indeed, I shall be most grateful if those who study this accumulation, and who have further proposals, will write and tell me. But first they should consider the following paragraphs. To limit this bibliography, and avoid listing every old book that contains a mathematical term, I have adopted my own private, arbitrary, and doubtless wrongheaded criteria. Thus it may interest both casual readers and prospective

contributors to know just what I claim here to include, or not to include.

Many distinguished mathematicians have produced essays or books giving broad views of their subject, or fine examples of its discoveries, or popular introductions to its study. My office shelves hold numerous such works, and I would never claim to equal these -but my list does not include them. I do not disparage the writers; in their prose style, they need take second place to no other profession. But, in my bibliography, I have been collecting mostly reactions to mathematics by people other than its practitioners. Here I say "mostly" because now and then mathematicians make reference to their domain in some traditional literary form: novel, short story, poem. Of these, a few are too good to omit; so I list the professionals when they do non-professional things. Thus I admit Bertrand Russell's little fantasy "The Mathematician's Nightmare"; after all, its author only won the Nobel Prize in Literature.

Likewise, biographers have produced some fine lives of eminent mathematicians, but collecting these would dictate a quite different area for my search; so I omit them. Again, I make a small exception: I allow autobiographies. Even dedicated professionals may offer something beyond technical matters when describing their own lives. In particular, I admit Sofya Kovalevskaya's "A Russian Childhood", a small work whose narrative, at least in translation, recalls the great Russian novels of the 19th century.

When fans of popular music buy a respectable quantity of some classical recording, the music business calls this work a "crossover", and similarly when a popular recording achieves a more serious following. In something like this sense, I have been hunting "crossovers". However, my objective needs a few more qualifications.

In his two anthologies, Clifton Fadiman reprints considerable light verse on mathematical concepts. In my poetry section, I exclude these and collect primarily works with more ambitious goals or better-known authors. Indeed, in the 19th century, the British journal "Punch" and its competitors published untold volumes of light verse. Thus, if one allowed this genre, then, conscientiously, one would need to search these old journals for versified British academic humor (or humour) on mathematical topics. Having collected this material, one would need also to decide how much was worth listing. (Once, years ago, I wrote a perfect Italian sonnet on the definition of continuity, but a journal editor declared it totally unfunny.) I prefer to avoid the light-verse category rather than contemplate such a bibliographic project. Any volunteers?

Through such considerations, I have preferred to set rules for inclusion (and subrules for exceptions) that serve my purpose, then to intrude fewer personal judgements of merit in adding things to the list. Thus, inclusion does not always mean quality. Most emphatically, this is not a "Recommended Reading" list. Some works here I would praise whether or not they had mathematical references; others may have little value except to illustrate a theme, and my parenthetical remarks on them may betray a lack of admiration. Still, I do not altogether reject subjectivity. From major literary figures I admit rather trivial items (though, in Dante and Shakespeare, even such brief remarks have some point.) On the other hand, scientific mumbo-jumbo is an standard element of science fiction. Hence generally I exclude such fiction when it has no further mathematical relevance, though I bend this rule slightly to include marginal items when a prior source has mentioned them. I try to incorporate my sources; the last section acknowledges them all.

The Table of Contents shows my subdivision of this list. Then a short initial section notes a few anthologies that contain many later items, items some of which are otherwise not too accessible. Later entries cite these books in giving published sources. However, I have usually tried to find earlier published versions. Thus entries for some works may mention an original date and a later reprinting,

or an early source and an anthology. Here "partly in Collection X" means that I have read a more complete version, while "excerpt in ... " means that I have not. I have lumped general non-fiction and short stories because some works straddle the boundary; otherwise, my categories should need no explanation.

As its title suggests, this list collects primarily mathematical references in printed material. However, two late sections gather a few songs and films. Despite video rentals, a longer list of relevant old films requires either lifelong dedication to the movies, a good bit of luck, or further help from others. For this project it was my luck that, some years ago at 2:00 a.m., a TV station in my area chose to broadcast the film "Are You With It?". Otherwise I might have known it only from my father's memories of the play. With luck, readers may have encountered other films suitable for my list.

Yes, but what does it all mean? I claim that the cited works dramatize some Western intellectual history. Ancient astronomers were mathematicians as well; a fable of Aesop concerns such a one in a field, so preoccupied watching the stars that he fell down a well. This fable remains the stereotype; different eras have added different things.

Some writers and artists of the Renaissance (e.g., Chaucer, Leonardo) knew far more mathematics than others. (Clearly, Latin and Greek were more prized attainments.) But, to such people, mathematics was not a totally separate universe; what they knew made appearances in their work. Then the intellectual triumphs of Newton persuaded artists of 18th-century Europe to value a rational, quantitative style even in their own productions. Newton himself came to symbolize this outlook. Certain writers even studied his works! And references to him show great respect, mingled occasionally with the impish humor that aims shafts at anyone on a pedestal.

Late in the 18th century, important European writers came to feel that rationalism left too little space for human emotions. Admittedly, not everything called rational makes sense or ever has; but, as a conscious estrangement, rather than mutual ignorance, the famous "two-cultures" split may be little older than Rousseau. To Blake, Newton the hero became Newton the villain; later this view, much sophisticated, reached the universities, where it prompted G. H. Hardy's 1930's remark on the word "intellectual": "There seems to be a new definition which certainly doesn't include Rutherford or Eddington or Dirac or Adrian or me." No doubt Blake's view helped animate the F. R. Leavis counterattack when C. P. Snow, in 1959, first deplored the separation of "the two cultures". Poetry, for Leavis, was the true source of "finer awareness"; to him, supposedly, the literature alone of a time and place could embody the important aspects of its culture.

My list documents this split, but also suggests that writers had begun bridging the gap by the time of the confrontation. By then, major 20th century authors, some with mathematical training, had made literary uses of mathematics going well beyond the tired stereotypes. This is both good and bad news. Mathematicians in modern novels now commit the same range of sins as do their innumerate neighbors. It seems the literary world has conceded that mathematicians are human.

Our society needs more young people studying mathematics, and this development may encourage them. Probably most young people have not yet had time to try all the interesting sins. Literature now assures them that they can pursue a mathematical career without foreclosing the opportunity. That is a change from the ancient astronomer in the well, and I call it an improvement. In any event, those reading my list may find that mathematical things have won literary attention from more writers than they might

have supposed, including figures of great eminence in the world's eyes.

A final footnote. My very quick historical sketch reflects a book by (naturally) a mathematician, namely, Alfred North Whitehead's "Science and the Modern World" (The Macmillan Company, 1925). My source for Hardy's remark is C. P. Snow's "The Two Cultures" (Cambridge University Press, 1959). F. R. Leavis published his response in "Two Cultures? The Significance of C. P. Snow" (Chatto and Windus, 1962); for his general views, see "New Bearings in English Poetry" (University of Michigan Press, 1960). This bibliography itself, under appropriate headings, gives detailed references for all other cited works.

2. COLLECTIONS

(DB) Joan Digby and Bob Brier (eds.) - "Permutations: Readings in Science and Literature", Quill (William Morrow), New York 1985

(Fl) Clifton Fadiman (ed.) - "Fantasia Mathematica", Simon and Schuster, New York 1958

(F2) Clifton Fadiman (ed.) - "The Mathematical Magpie", Simon and Schuster, New York 1962

(M) Robert Edouard Moritz (ed.) - "Memorabilia Mathematica", Macmillan, New York 1914 (brief quotations about mathematics, some from literary sources)

(N) James R. Newman (ed.) - "The World of Mathematics", Simon and Schuster, New York 1956

(RW) Ernest Robson and Jet Wimp (eds.) - "Against Infinity", Primary Press, Parker Ford, PA 1979 (mathematical poetry)

3. NOVELS

Edwin A. Abbott - "Flatland" (1884), Barnes and Noble, New York 1963, partly in Collection N (Polygonal beings inhabit a planar world.)

Jeffrey Archer - "Not a Penny More, Not a Penny Less", Doubleday & Co., New York 1976 (A mathematician leads a team to dupe a financier who swindled them .)

Isaac Asimov - "The Foundation Trilogy", Doubleday & Co., New York 1951 (intermittent references to the basic premise: statistical equations for broad historical development)

Isaac Asimov - "Foundation's Edge", "Foundation and Earth", Doubleday & Co., New York 1982, 1986 (resp. fourth and fifth novels in the "Foundation" series)

Desmond Bagley - "The Spoilers", Doubleday & Co., New York 1970, Chaps. 2.4, 3.4, 5.1 (A professional gambler exploits probability paradoxes.)

Samuel Beckett - "Molloy" (1951), Grove Press, New York 1955, excerpt in Collection F2 (The narrator, with ever more compulsive requirements, distributes 16 sucking-stones among his four pockets.)

Saul Bellow - "The Adventures of Augie March" (1953), Viking Press, New York 1960, p . 188 (Augie s resourceful friend Manny Padilla is a demon equation-solver and future mathematical physicist.)

Hermann Broch - "The Unknown Quantity", Collins, London 1935, Howard Fertig, New York 1975 (A young physics Ph.D. in Germany ponders the meanings of mathematics and life.)

Dionys Burger - "Sphereland", Thomas Y. Crowell Co., New York 1965 (Flatland inhabitants discover curved space.)

James Branch Cabell - "Jurgen" (1919), Dover Publications, New York 1978, Chap. 32, partly in Collection Fl (Jurgen's private session with Queen Dolores imparts mathematics by the sensuous approach.)

John Dickson Carr - "The Case of the Constant Suicides" (1941), Collier Books, New York 1977, Chap. 20 (Dr. Fell, though scorning geometry, uses it to explain a murder.)

John Dickson Carr - "Dark of the Moon", Harper & Row 19??, Berkley Medallion Books 1967, p . 61 (griping about motion problems)

John Dickson Carr - "The Three Coffins", Gregg Press, Boston 1979, 1935 British title - "The Hollow Man", Chap. 3 (The first victim's secretary is a mathematics student.)

Lewis Carroll - "Alice's Adventures in Wonderland" (1865) and "Through the Looking Glass" (1872). For passages with mathematical significance, see "The Annotated Alice", introduction and notes by Martin Gardner, World Publishing Co., New York 1963.

Arthur C. Clarke and Gentry Lee - Rama II, Bantam Books, New York 1989, Chap. 55 (General O'Toole uses a prime-number curiosity to create a 50-digit identification number.)

James Gould Cozzens - "Guard of Honor" (1948), Harcourt Brace Jovanovich, New York 1964, Part 2, Section 2 (A young mathematician, during WWII, breaks the U.S. Navy code in an idle hour.)

Daniel Defoe - "A Journal of the Plague Year" (1722: Weekly mortality figures for various London districts document the onward march of the bubonic plague.)

Daniel Defoe - "Robinson Crusoe" (1719-1720), 2/9 from start (The mechanical arts, like mathematics, require mainly perseverance and reason.)

Don DeLillo - "The Names", Alfred A. Knopf, New York 1982, Chap. 7 (esoteric nature of pure mathematics)

Don DeLillo -"Ratner's Star", Alfred A. Knopf, New York 1976 (A mathematical prodigy at a satirized scientific institute helps decode a mysterious message from outer space.)

William De Morgan - "Joseph Vance" Henry Holt, New York 1906, Chap. 9 (Joey sees that an equilateral triangle has equal angles.)

A. K. Dewdney -"The Planiverse: Computer Contact with a Two-Dimensional World", Poseidon Press, New York 1984 (Intelligent beings inhabit a scientifically consistent 2-dimensional universe.)

Norman Douglas - "South Wind" (1917), Modern Library, New York 1925, Dover Publications, New York 19??, Chap. 28 (A young professor is calculating how soon the volcanic eruption will bury the town.)

Arthur Conan Doyle - "The Valley of Fear" (1915), Chap. 1 (Holmes, to Watson, sketches the mathematical career of Prof. Moriarty.)

H. F. Ellis - "The Vexations of A. J. Wentworth, B. A.", Little, Brown, and Co., Boston 1950 (comic misadventures of an accident-prone mathematics master at an English prep school.)

Ford Madox Ford - "Parade's End" (1924-1928), Vintage Books, New York 1979 (The much-abused hero is a vastly knowledgeable statistician.)

C. S. Forester - "Randall and the River of Time", Little, Brown, and Co., Boston 1950, especially Chaps. 3-5, 13, 18, 19 (Naive mathematics student, veteran of WWI, accidentally discovers and kills the lover of his unappreciative wife.)

Johann Wolfgang von Goethe -"Wilhelm Meisters Wanderjahre" (1829), Chap. 10, English translation: "Wilhelm Meister's Travels" (Thomas Carlyle, translator), Houghton Mifflin, Boston 1889; excerpts in Collection M, pp. 36 & 121 (Children's participation in singing spurs their respect for numerical accuracy.)

Rebecca Goldstein - "The Mind-Body Problem", Random House, New York 1983 (The wife of a mathematical genius encounters the value system of his rarefied world.)

Hermann Hesse - "Das Glasperlenspiel" (1943), American title - "Magister Ludi" (The "glass bead game" of the German title has some unexplained resemblances to mathematics and music.)

Charles Howard Hinton - "An Episode of Flatland", Swann Sonnenschein & Co., London 1907, long excerpt in "Speculations on the Fourth Dimension", Dover Publications, New York 1980

James Joyce - "A Portrait of the Artist as a Young Man" (1916), Chap. 5 (The professor 's lecture distinguishes ellipse and ellipsoid.)

James Joyce - "Ulysses" (1922), U.S. 1934 edition pp. 18, 662, 683 (brief joke on algebra, remark on age ratios, remark on large numbers)

Norton Juster - "The Phantom Tollbooth", Random House, New York 1961, partly in Collection F2 (The city of Digitopolis and its king, the Mathemagician, play large roles in the fantastic adventures of Milo.)

Rudyard Kipling - "Kim" (1901), Chap. 9 (Kim wins mathematics prize on completing secondary school.)

Arthur Koestler - "Darkness at Noon" (1941), Part 1, Section 14 (Politics spins open formulas in x, where x = the masses, but history finds the meaning of this x.)

D. H. Lawrence - "Sons and Lovers" (1913), Modern Library, New York 1929, Chap . 7 (Paul, convalescing, tries to teach Miriam algebra.)

Richard Llewellyn - "How Green Was My Valley", Macmillan, New York 1941, excerpt in Collection Fl (Mother asks why mathematical bathtubs have multiple outlets - and who owns the decimal point.)

William John Locke - "The Morals of Marcus Ordeyne", John Lane Co., New York 1906, pp. 244-245 (The protagonist teaches elementary mathematics, though thinking it useless and oppressive.)

Jack London - "Martin Eden" (1909: Martin works day and night to get an education, including algebra, trigonometry, and physics .)

Jack London - "The Sea Wolf" (1904), Chap. 10 (Wolf Larsen, in lonely mathematical study, has simplified the techniques of navigation.)

George Malcolm-Smith - "Slightly Perfect", Random House, New York 1941, basis for stage and film musical "Are You With It?" (A sobersided real

man inspired this fictional tale wherein a similarly-named actuary misplaces a decimal point - and leaves his job to join a carnival.)

Russell McCormach - "Night Thoughts of a Classical Physicist", Avon Books, New York 1982 (An elderly German physicist, during WWI, regrets the increasing mathematical abstractness of theoretical physics.)

Herman Melville - "The Confidence-Man: His Masquerade" (1856) Chap. 36 (trigonometric simile)

Nicholas Meyer - "The Seven Per Cent Solution", E. P. Dutton & Co., New York 1974 (Moriarty 's criminal eminence is just Holmes' cocaine induced paranoia.)

Robert Musil - "The Man Without Qualities" (1930-1943), Coward McCann, New York 1953, Chaps. 10, 11, 13, 17, 28 (The protagonist, like the author, leaves a promising mathematical career for some undefined broader goal .)

Vladimir Nabokov - "Pnin" (1957), Atheneum, New York 1963, Chap. (brief satire on mathematical linguistics)

Joyce Carol Oates - "Wonderland", Vanguard Press, Inc., 1971, excerpt in American Mathematical Monthly 88 (1981) p. 604 (Character tells himself he can't understand calculus.)

Flann O'Brien - "The Dalkey Archive" (1964), Penguin Books, New York 1977 (atomic physics in Irish brogue)

Flann O'Brien - "The Third Policeman" (1967, written 1940), Plume Books, New York 1976, Chap . 6 (earlier and better version of preceding.)

Robert Pease - "The Associate Professor", Simon and Schuster, New York 1967 (A harried professor at a NYC college can no longer solve anything but the assigned problems .)

Thomas Pynchon - "The Crying of Lot 49", J . B Lippincott, New York 1966, Chap. 5 (brief speculation: delerium tremens = dt's)

Thomas Pynchon - "Gravity's Rainbow", Viking Press, New York 1973, especially p. 55 (stochastic vs. deterministic view of life)

Thomas Pynchon - "Vineland", Little, Brown, and Co., Boston 1990, pp. 204-217 and 342 (A young group-theorist at a California college, a leader in a 1960's rebellion, meets a fate like that of Galois - another leader shoots him; the number 2. 71828 seems "real natural".)

Erik Rosenthal - "The Calculus of Murder", St. Martin's Press, New York 1986 (A young mathematician and part-time detective solves a differential equation to explain the timing of a murder.)

Erik Rosenthal - "Advanced Calculus of Murder", St. Martin's Press, New York 1989 (The 15-year-old theft of an unpublished theorem motivates a murder at an Oxford operator-theory conference.)

Dorothy Sayers - "The Documents in the Case" (1930), Avon Books, New York 1971, p. 58 (Mathematicians, through habitual abstraction, gain a cheerful detachment from the world's vicissitudes.)

Lynne Sharon Schwartz - "Rough Strife", Harper & Row, New York 1980, especially pp. 50, 77, 89, 148 (Mathematician and art historian negotiate troubled marriage.)

George Bernard Shaw - "An Unsocial Socialist" (1884), W. W. Norton, New York 1972, Chap. 6 (Mathematics needs postulates; life offers few.)

Charles Sheffield - "The McAndrew Chronicles", Tom Doherty Associates, New York 1983 (The mathematics of black holes provides the rationale for a space drive.)

Nevil Shute - "No Highway", William Morrow, New York 1948 (Wherein separating nascent important research from mere eccentric elaboration means literally life or death.) See Amer. Math. Monthly 86 (1979) p. 305

James Park Sloan - "The Case History of Comrade V.", Houghton Mifflin, Boston 1972 (A patient in an East-European mental hospital is either a schizophrenic or a politically imprisoned mathematician.)

Tobias Smollett - "Peregrine Pickle" (1751), Everyman's Library, London 1939, Chapters 24, 35, 83 (Geometric arguments prove youth's folly; a treatise on the cycloid comforts a storm-tossed traveler; fortunetelling, not De Moivre's actuarial science, fetches the multitude.)

Alexander Solzhenitsyn -"The First Circle", Harper & Row, New York 1968 (A Stalinist work camp for "unreliable" scientists houses the protagonist, a mathematician, together with other technical specialists .)

John Steinbeck - "The Moon Is Down", Viking Press, New York 1942, Chap. 2 ("He was an arithmetician rather than a mathematician.")

Laurence Sterne - "Tristram Shandy" (1759-1767), Chap. 3, partly in Collection N, p. 734 (geometrical studies of Uncle Toby.)

Rex Stout - "And Be a Villain", Viking Press, New York 1948 (A mathematician, one of the murder suspects, discusses the investigation in probabilistic terms.)

Rex Stout - "Death of a Doxy", Viking Press, New York 1966 (The culprit, a mathematics teacher, takes the pseodonym Thales.)

Jan Struther - "Mrs . Miniver" (1940), Harcourt Brace, New York 1966, excerpt in Collection F2 (geometry problem as psychological metaphor.)

Jonathan Swift - "Gulliver's Travels" (1726), Part 2, partly in American Mathematical Monthly 92 (1985) p. 326 (education in Brobdingnag); Part 3, partly in Collection N (scientific excesses of Laputa; Grand Academy o f Lagado .)

Anthony Trollope - "The Last Chronicle of Barset" (1867), Oxford Univ. Press, London 1946, Chap. 63 (Unlettered Jane, aged 16, has read only Latin, Greek, Euclid's "Elements", and Wood's "Algebra".)

John Updike — "Roger's Version", Alfred A. Knopf, New York 1986 (Young computer scientist tutors youngster in mathematics, urges modern cosmological argument from design to prove God's existence.)

S . S . Van Dine - "The Bishop Murder Case", Charles Scribner 's Sons, New York 1930 (Homicidal reenactments of nursery rhymes stalk a New York enclave of mathematicians and chess enthusiasts.)

Jules Verne - "From the Earth to the Moon" (1865) and "A Trip around It" (1870), Book 1, Chaps . 2, 4, 7, 8, and Book 2, Chaps. 4, 8 (geometric communication with extraterrestrials, mathematical considerations for the trip, orbital recalculations en route.)

Jules Verne - "Journey to the Center of the Earth" (1864), Chaps . 2 & 3 (transposition cipher.)

Jules Verne - "The Mysterious Island" (1870), Part 1, Chap. 14 (Geometry and simple apparatus fix the castaways ' latitude and longitude.)

Sylvia Townsend Warner - "Mr. Fortune's Maggot", Viking Press, New York 1927 (about 1/3 from end), partly in Collection N. (A "simple" tropical islander can't understand mathematical abstractions.)

H. G. Wells - "Joan and Peter", Macmillan, New York 1918, Chap. 7.3, partly in Collection Fl (The usual teaching of elementary arithmetic confuses any students but the mathematically self-sufficient.)

H. G. Wells - "The Undying Fire", Macmillan, New York 1919, Chap. 1.3 (By Satan's reckoning, Job's descendants, through geometric increase, are now the whole human race.)

Thornton Wilder - "The Eighth Day", Harper & Row, New York 1967, Part 2 (Ashley, in New Orleans, uses his mathematical gift in gambling.)

4. PLAYS

W. H. Auden - "For the Time Being", The Summons, III (1944), in Marvin Halverson - "Religious Drama I", Meridian Books, Inc., New York 1957 ("The Kingdom of Infinite Number")

Thomas Dekker - - "The Honest Whore", Part 2 (1605), Act 1, Scene 3, excerpt in Collection Fl (moralized circles and squares)

Bob Elliott and Ray Goulding - Unpublished skit (Belligerent intruder on Bob-and-Ray broadcast states everything in probabilities.)

Christopher Fry - "The Lady 's Not for Burning", Oxford Univ. Press, New York 1950, Act 2, p .50 (Mathematical relations obsessed Jennet's dead father, an alchemist.)

W. S . Gilbert and Arthur Sullivan - "The Pirates of Penzance" (1879), Act 1 ("many cheerful facts about the square of the hypotenuse".)

Norman MacOwan - "The Infinite Shoeblack", London 1929, New York 1930 (A poor and moralistic actuarial student meets a sexually adventurous young woman.) See "The New York Times Theater Reviews 1920-1970", The New York Times & Arno Press, New York 1971, Vol. 2, 5 May 1929; Vol. 3, 18 Feb. 1930.

Sam Perrin and George Balzer (book), Harry Revel (music), Arnold B. Horwitt (lyrics) - "Are You With It?", musical comedy based on George Malcolm-Smith's novel "Slightly Perfect", New York premiere 10 Nov. 1945. (Having misplaced a decimal point, an actuary leaves his job and joins a carnival.) See George Jean Nathan - "The Theatre Book of the Year 19451946", Alfred A . Knopf, New York 1946 . See also "The New York Times Theater Reviews 1929-1970", The New York Times & Arno Press, New York 1971, Vol . 5, 12 Nov . 1945 .

Elmer Rice - "The Adding Machine" (1923), Scenes 2-4, in "Three Plays", Hill & Wang, New York 1965 (Repetitive arithmetic symbolizes the dehumanization of an urban clerical worker yclept Mr. Zero.)

William Shakespeare - "Henry V" (c. 1598-1599), Act 1, prologue (metaphorical use of positional notation.)

William Shakespeare - "The Taming of the Shrew" (c . 1594), Act 1,
Scene 1 & Act 2, Scene 1, partly in Collection M, p . 190 (mere mention.)

William Shakespeare - "The Winter's Tale" (c. 1611), Act 1, Scene 2 (brief joke on positional notation.)

George Bernard Shaw - "Back to Methusaleh" (1922), Part 5 (In 31920 A.D., mathematical contemplation is the main activity of adult humans.)

George Bernard Shaw - "Major Barbara" (1905), Act 3 (Mathematics not being his field, Prof Cusins must ask whether 3/5 exceeds 1/2.)

George Bernard Shaw - "Mrs. Warren's Profession" (1898), especially Act 1 (Mrs. Warren's daughter is a Cambridge mathematics student.)

Tom Stoppard - "Rosencrantz and Guildenstern Are Dead", Grove Press, New York 1967, Act 1 (The title characters try to explain Guildenstern's having thrown 90 consecutive heads.)

John Webster - "The Duchess of Malfi" (c.1613), Act 1, Scene 1 & Act 2, Scene 2, partly in Collection Fl (geometrical images in lewd jokes.)

5. GENERAL NONFICTION AND SHORT STORIES

Goodman Ace - "Like Is a Many-Splendored Thing", in Martin Levin (ed.) - "The Saturday Review Sampler of Wit and Wisdom", Simon and Schuster, New York 1966 (Poor, lonely parallel lines, doomed never to meet!)

Samuel Hopkins Adams - "The One Best Bet", in "Average Jones", Bobbs-Merrill 1911, Arno Press, New York 1976 (A deduction from similar triangles prevents the assassination of a governor.)

Joseph Addison - "The Vision of Mirzah" (1 Sept. 1711), in "CollectedEssays" (the stochastic process of life.)

W. H. Auden - "Squares and Oblongs", in W. H. Auden, Karl Shapiro, Rudolf Arnheim, and Donald A. Stauffer - "Poets at Work", Harcourt, Brace, and Co., New York 1948 (Pure mathematicians, unlike modern poets, meet no public attacks on their alleged incomprehensibility.)

St. Augustine - "Sermon 41", Art. 23, excerpt in Collection M, p.379 (religious numerology.)

Nigel Balchin - "God and the Machine", in "Last Recollections of My Uncle Charles", Rinehart & Co., New York 1951, also in Collection Fl. (A checkers-playing computer disillusions its creator by cheating.)

Hilaire Belloc - "First and Last", Methuen & Co., London 1911, specifically "On Cheeses" (nested brackets as a literary device), "The Old Gentleman's Opinions" (puzzled silence over noneuclidean concepts.)

Ambrose Bierce - "The Devil's Dictionary" (1906), Dover Publications, New York 1958, specifically entries for "hash", "logic".

Jorge Luis Borges - "Labyrinths", New Directions, New York 1962, especially "The Garden of Forking Paths" (time as infinite branching), "The Lottery in Babylon" (life as an infinite lottery), "The Library of Babel" (the set of all alphabetic n-tuples), "Death and the Compass" (mathematical patterns in a detective tale), "Partial Magic in the Quixote" (self-reference in great literary works.)

Edmund Burke - "A Philosophical Enquiry into the Origin of Our Ideas of the Sublime and Beautiful" (1757), Part 3, Section 2, partly in Collection M, p. 66 (Geometric proportion, by this reasoning, is not a cause of beauty.)

Jeffrey Burke - "All You Need to Know", Harpers, Vol 257, No . 1543 (Dec. 1978) p. 93 (If beauty = truth, then "mathematics" proves that truth = beauty.)

E. M. Butler - "Ritual Magic", Noonday Press, New York 1959, pp. 67 & 77 (Genuine lore of black magic: the principal angels, by the book "Lemegeton", have vast numbers of attendants; the demon Asmodeus grants, among other things, skill in arithmetic and geometry.)

Lewis Carroll - Miscellaneous prose, especially "The Purse of Fortunatus", in "Sylvie and Bruno",

also in Collection F2 (Moebius strip); "Eternity: a Nightmare", in "Sylvie and Bruno Concluded", Chap. 16, also in Collection Fl (algebraic geometry: an infinite task); "Knot IX: a Serpent with Corners", in "A Tangled Tale", also in Collection N (hydrostatic paradoxes); "What the Tortoise Said to Achilles", in Collection N (a logical infinite regress.)

Leslie Charteris - "The Percentage Player", in "The Saint to the Rescue", Manor Books, New York 1968 (The Saint foils a mathematically adept gambler.)

Geoffrey Chaucer - "A Treatise on the Astrolabe" (c. 1390: a manual for his son using concepts of astronomy, geometry, and proportion.)

John Cheever - "The Geometry of Love", in "The World of Apples", Alfred A. Knopf, New York 1973 (Lines and angles represent human relationships.)

G. K. Chesterton - "George Bernard Shaw", Hill and Wang 1956, Chap. 5, "The Critic" (Poetry dismays logicians because it uses words differently.)

Winston S. Churchill - "If Lee Had Not Won the Battle of Gettysburg", Scribner's Magazine, Dec . 1930, pp . 587 -597, also in J . C. Squire (ed .) - "If It Had Happened Otherwise", 1931 (Growing militarism yields agreement by miraculous transmutation, as an increasing function reaches negative values through infinity; e.g., $f(t) = -1/t$.)

Robert M. Coates - "The Law" (1947), in Collections F2 and N (Congress repairs the Law of Averages when it fails.)

Dante Alighieri - "Vita Nuova" (1292-1300), Chaps. 12 & 29; English translation: Mark Musa - "Dante's Vita Nuova", Indiana Univ. Press, Bloomington, IN, 1973 (The center of a circle is Love's image for himself; properties of the number 9 link it mystically with Beatrice.)

Arthur Conan Doyle - "The Complete Sherlock Holmes", specifically "The Adventure of the Dancing Men" (1904: substitution cipher), "The Final Problem" (1894: first appearance of Prof. Moriarty), "The Musgrave Ritual" (1894: trigonometric note.)

Corey Ford - "The Wonderful World of Figures", in "Corey Ford's Guide to Thinking", Doubleday and Co., New York 1961, also in Collection F2.

Benjamin Franklin - "On the Usefulness of Mathematics", excerpts in Collection M.

W. S. Gilbert - "An Elixir of Love", The Graphic, Dec. 1869, also in Peter Haining (ed .) - "The Lost Stories of W . S . Gilbert", Robson Books, London 1982 (This tale, the original form of "The Sorcerer", contains a curate who proves human equality by algebra.)

Johann Wolfgang von Goethe - "Maximen und Ref lexionen", Nos. 1277 & 1280, in American Mathematical Monthly 92 (1985) p. 130 (mathematicians' intolerance of the qualitative.)

Oliver Goldsmith - "An Enquiry into the Present State of Polite Learning in Europe", J. Dodsley, London 1774; Garland Publishing, New York 1970, Chap. 13 (The meanest intellects, given but the will, might understand mathematics.)

Robert Graves - "The Abominable Mr. Gunn", in "Collected Short Stories", Penguin Books, Harmondsworth, England, 1968, also in Collection F2 (A sarcastic teacher suppresses a mathematically quick student.)

J. B. S. Haldane - "On Being the Right Size", in "Possible Worlds", Harper & Brothers, New York 1928, also in Collection N (An animal's size determines its shape.)

O. Henry - "The Handbook of Hymen" (1906), partly in Collection N, p. 1487 (A woman's two suitors offer her poetry and "statistics".)

Charles Howard Hinton - "Speculations on the Fourth Dimension", Dover Publications, New York 1980, specifically "A Plane World" (1884 anticipation of Burger's "Sphereland"), "An Unfinished Communication" (1885 tale of 2-dimensional time.)

Richard Hughes - "The Vanishing Man", in "A Moment of Time", Harold Ober Associates, 1926, also in Collection F2 (Returning from a fourth dimension is harder than walking into it.)

Aldous Huxley - "Young Archimedes", in "Young Archimedes" (collection), Harper & Brothers, 1924, also in Collections Fl and N (Selfish adults exploit a young prodigy.)

Kurd Lasswitz - "The Universal Library" (1901), in Collection Fl (Every sequence of alphanumerics is a book in this library.)

Stephen Leacock - "Caroline ' s Christmas", in "Nonsense Novels", Dodd, Mead, and Co., New York 1959 (running joke: "The Good Book" = Euclid's "Elements".)

Stephen Leacock - "Common Sense and the Universe", in Collection N (Modern science narrowly survives Leacock's explanations.)

Stephen Leacock - "Literary Lapses", John Lane Co., New York 1920, specifically "A, B, and C: the Human Element in Mathematics", also in Collection F2 (the private lives of those familiar characters in algebra problems); "Aristocratic Education" (geometry amended by the House of Lords); "Boarding-House Geometry" (Euclidean axioms for lodgers); "How to Avoid Getting Married" (the importance of one's fiancee's knowledge of quadratic equations.)

Stephen Leacock - "Mathematics for Golfers", in Collections F2 & N (The average golfer's excuses for a bad game permit a good one about once in 29 million years.)

Eliphas Levi -"The History of Magic" (Arthur Edward Waite, translator), William Rider & Son, London 1922, especially Introduction and Book I (mathematical mysticism.)

Jack London - "The Siege of the Lancashire Queen", in "Tales of the Fish Patrol" (Capturing some poachers becomes a geometry problem.)

Russell Maloney -"Inflexible Logic" (1940), in Collections Fl and N (Six typewriting monkeys produce error-free world literature.)

W. Somerset Maugham - "A Writer's Notebook", Doubleday & Co., Garden City, N. Y. 1949, p. 265 (Do theorems have real content?)

W. Somerset Maugham - "The Portrait of a Gentleman" (1923-1929), in "Collected Short Stories", Penguin Books, New York 1977 (A poker-players' manual in a second-hand shop conveys well the personality of its author, an actuary.)

H. L. Mencken -"The Educational Process", in "Heathen Days", Alfred A. Knopf, Inc., New York 1943 (recollections of past algebra-teachers.)

H. L. Mencken - "A Mencken Chrestomathy", Alfred A. Knopf, New York 1956; specifically "The Scientist" (1919), p. 12; "The Metaphysician" (1940's), pp .13 -14; "Caveat Against Science" (1927), pp. 330-333; "The Eternal Conundrum" (1931), pp. 333-337.

Edward Page Mitchell - "The Tachypomp" (1873), in Collection Fl (Our hero, to win his sweetheart's hand, must invent a device reaching arbitrarily high speeds.)

David Osselton - "Monkeys and Shakespeare: A Dissent", Harpers, Feb. 1985, p . 22, from "Making a Monkey of Shakespeare", New Scientist, 1 Nov. 1984 (Random typing takes far too long to produce Shakespeare.)

Samuel Pepys - "Diary", entry for 4 July 1662 (Pepys starts learning "mathematics" - his first attempt being the multiplication table.) See American Mathematical Monthly 91 (1984) p. 52.

Plato - "Timaeus", partly in Collection DB, pp. 211-216 (geometry and mysticism.)

Edgar Allan Poe - "Essays and Reviews", The Library of America, New York 1984, especially "A Few Words on Secret Writing" (1841: cryptography essay), "Maelzel's Chess Player" (1836: impeachment of reputed chess automaton, using deduced properties of artificial intelligence.)

Edgar Allan Poe - "Poetry and Tales", The Library of America, New York 1984, especially "Eureka" (1848: metaphysics of the universe, making appeals to algebra, geometry, physics), "The Gold Bug" (1843: substitution cipher), "The Murders in the Rue Morgue" (1841: praise of that highest "analysis" combining logical with psychological acumen), "The Purloined Letter" (1845: algebra vs poetry), "The Thousand-and-Second Tale of Scheherazade" (1845: mathematically fluent birds and bees.)

John Reese - "The Symbolic Logic of Murder" (1960), in Collection F2 (Boolean algebra gets its man.)

Bertrand Russell - "The Collected Stories of Bertrand Russell", Simon and Schuster, New York 1972, specifically "The Perplexities of John Forstice" (A symposium of great men, including a mathematician, state their philosophies of life.), "The Mathematician's Nightmare" (1955), also in Collection F2 (The various integers have different shapes, colors, and personalities.)

Bruno Schulz - "The Comet", Part 2, in "The Street of Crocodiles", Penguin Books, New York 1977, 1934 Polish title - "Cinnamon Shops" (A comet, with mathematical certainty, nears a collision with the earth.)

Richard Steele - "Sir Roger and Sir Andrew" (19 Sept.1711), in "Collected Essays" (All major human enterprises demand accurate numbers.) Rex Stout - "The Zero Clue", in "Three Men Out", Viking Press, New York 1954 (Nero Wolfe interprets a mathematical clue.)

Lytton Strachey - "Hume", in "Portraits in Miniature", Harcourt, Brace, and Co., New York 1931 (The pure mathematician, among all creatures, represents the ultimate in detachment.)

J. L. Synge - "O'Brien's Table", in "Science: Sense and Nonsense", Jonathan Cape, Ltd., 1951, also in Collection F2 (What, legally, is a surface?)

Henry David Thoreau - "A Week on the Concord and Merrimac Rivers" (1849), excerpt in Collection M, p. 189 (the poetry of mathematics.)

James Thurber - "Many Moons", Harcourt, Brace, and Co., New York 1943 (The King's three advisors include the Royal Mathematician.)

Mark Twain - "Life on the Mississippi" (1883), Chap. 17, partly in Collection F2 (facetious extrapolation of the river's future length.)

William Hazlett Upson - "A. Botts and the Moebius Strip" (1945), in "The Best of Botts", McKay & Co., New York 1961, also in Collection Fl (Painting just the "outside" of a Moebius belt keeps an officious lieutenant out of Botts's hair.)

William Hazlett Upson - "Paul Bunyan versus the Conveyer Belt" (1949), in Collection F2 (The Moebius strip rolls again!)

Voltaire - "Philosophical Dictionary" (1764), specifically "Atoms" (questionable utility of mathematical abstractions), "Nature" (Nature is not a mathematician but obeys mathematical laws.)

Ben Ames Williams - "Coconuts" (1926), in collection F2 (The well-known problem of the monkey and the coconuts diverts a commercial architect from his financial self-interest.)

Angus Wilson - Early parody of Virginia Woolf, in "Diversity and Depth in Fiction: Selected Critical Writings", Secker and Warburg, London? 1983, also in John Bayley - "Life-enhancing world views", Times Literary Supplement, 16 Sept. 1983, p. 978 (Square root of pi troubles reverie of Mrs. Green.)

William Wordsworth - 1802 Preface to William Wordsworth and Samuel Taylor Coleridge - "Lyrical Ballads", Barnes and Noble, New York 1963 (Mathematicians and scientists, like poets, ultimately seek pleasure in their creations.)

6. REMARKS ON OTHERS' VIEWS OF MATHEMATICS

John Carey - "John Donne: Life, Mind, and Art", Oxford Univ. Press, New York 1981, Chap. 4 (Donne knew little mathematics, but his metaphors use what he knew.)

Carl H. Grabo - "Newton among Poets: Shelley's Use of Science in Prometheus Unbound", Cooper Sq. 1968.

Diane Johnson - "Dashiell Hammett", Random House, New York 1983, especially pp. 18, 169, excerpts in American Mathematical Monthly 92 (1985) p. 444 (Hammett's interest in mathematics.)

D. O. Koehler - "Mathematics and Literature", Mathematics Magazine 55 (1982) pp. 81-95 (more extended comments on fewer works, including particularly the novels of Thomas Pynchon.)

Thomas Babington Macaulay - "Lord Bacon", Edinburgh Review, July 1837, also in "Critical and Historical Essays", excerpts in Collection M (Bacon's ignorance of mathematics.)

David Eugene Smith - "Thomas Jefferson and Mathematics", Scripta Mathematica 1 (1932) pp. ??, excerpts in American Mathematical Monthly 91 (1984) pp. 56, 72.

G. Otto Trevelyan - "The Life and Letters of Lord Macaulay", Harper and Brothers, New York 1875, Vol. 1, p. 91, excerpt in American Mathematical Monthly 89 (1982) p. 312 (Macaulay's youthful loathing for mathematics.)

Garry Wills - "Inventing America: Jefferson's Declaration of Independence", Doubleday & Co., Garden City, NY 1978, Part 2 (Newtonian mathematics and science inspired Jefferson's era.)

7. SCIENCE FICTION SHORT STORIES

Note: Science fiction routinely involves some technical vocabulary; the following stories attempt a bit more.

Isaac Asimov - "The Feeling of Power" (1957), in Collection F2 (A computerized world rediscovers hand arithmetic.)

Isaac Asimov - "Living Space", in "Earth Is Room Enough", Doubleday & Co., New York 1957 (Alternatives to our time-line provide homes for the expanding population.)

Isaac Asimov - "The Missing Item", in George Scithers (ed.) - "Isaac Asimov's Masters of Science Fiction", Davis Publications, New York 1978 (An accurate computation explodes a religious cult.)

James Blish - "FYI" (1961), in Collection F2 (Only transfinite arithmetic can save us if anyone should start a major war.)

James Blish - "The Glitch", in Philip Strick (ed.) - Antigrav: Cosmic Comedies by S. F. Masters", Taplinger Publishing Co., New York 1976 (The ultimate computer develops the ultimate bug.)

Ben Bova - "A Slight Miscalculation", in Isaac Asimov and J. 0. Jeppson (eds.) - "Laughing Space", Houghton Mifflin, Boston 1982 (A mathematical earthquake prediction contains just one small error.)

Miles J. Breuer, M.D. - "The Appendix and the Spectacles", in Collection F2 (An appendectomy without an incision, through a 4th dimension, leaves an old pair of spectacles inside the patient.)

Miles J. Breuer, M.D. - "The Captured Cross-Section" (1929), in Collection Fl (A 4th-dimensional being abducts the hero's fiancee.)

Miles J. Breuer, M.D. - "The Gostak and the Doshes", in Groff Conklin (ed.) - "Great Science Fiction by Scientists", Collier-Macmillan, New York 1962 (Patter about 4-dimensional rotations "explains" a trip to an alternate world.)

Arthur C. Clarke - "The Nine Billion Names of God" (1953), in Collection F2 (The Universe will end when all God's names are printed out.)

Arthur C. Clarke - "The Pacifist" (1956), in Collection F2 (A perverse computer rejects all military problems.)

Arthur C. Clarke - "Superiority" (1951), in Collection Fl (Wherein the more advanced weapons lose an interstellar war.)

Mark Clifton - "Star Bright" (1952), in Collection F2 (Precocious grasp of mathematics shows a toddler's evolution beyond homo sapiens.)

A. J. Deutsch - "A Subway Named Moebius" (1950), in Collection Fl (Disappearing trains plague the Boston subway network when a new route gives it "infinite connectivity".)

Bruce Elliott - "The Last Magician" (1952), in Collection Fl (An emulator of Houdini traps himself in a Klein bottle.)

George Gamow - "The Heart on the Other Side", in Isaac Asimov and J. 0. Jeppson (eds.) - Laughing Space", Houghton Mifflin, Boston 1982 (Making right shoes only, a mathematician undertakes to invert half, and inverts himself too.)

Martin Gardner - "The Island of Five Colors" (1952), in Collection Fl (Each of five tribes has a common boundary with all others.)

Martin Gardner - "No-Sided Professor" (1946), in Collection Fl (If we have one-sided surfaces, then why not ... ?)

Randall Garrett - "On the Martian Problem", in George Scithers (ed.) -"Isaac Asimov's Masters of Science Fiction", Davis Publications, New York 1978 (Modifying relativity rationalizes time-travel.)

Tom Godwin - "The Cold Equations", in Robert Silverberg (ed.) - "The Science Fiction Hall of Fame", Vol. 1, Doubleday & Co., New York 1970 (Celestial mechanics requires a rescue ship, carrying urgently needed antitoxin, to jettison a stowaway.)

J. B. S. Haldane - "The Gold-Makers" (1932), in Groff Conklin (ed.) -"Great Science Fiction by Scientists", Collier-Macmillan, New York 1962 (International financiers destroy a mathematician whose work yields a cheap gold-extraction process.)

Donald Hall - "The Wonderful Dog Suit", in Judith Merrill (ed.) - "lOth Annual Edition, the Year's Best SF", Delacorte Press, New York 1965 (A dog disguise lets a gifted child escape responsibilities.)

Robert A. Heinlein - "And He Built a Crooked House" (1940), in Collection Fl (An earthquake near a modern house collapses the eight cubical rooms into a hypercube.)

Raymond F. Jones - "The Person from Porlock", in Groff Conklin (ed.) - "A Treasury of Science Fiction", Crown Publishers, New York 1958 (Continual blocks to his research, including a subtly false proof of its impracticability, drive an engineer almost to breakdown.)

Norman Kagan - "The Mathenauts", in Judith Merrill (ed.) - "10th Annual Edition, the Year's Best SF", Delacorte Press, New York 1965 (Only mathematicians have the mental flexibility to pilot faster-than-light craft.)

C. M. Kornbluth - "Gomez", in Anthony Boucher (ed.) - "A Treasury of Great Science Fiction", Vol. 1, Doubleday & Co., New York 1959 (Puerto Rican prodigy discovers, and suppresses, a unified field theory.)

Henry Kuttner and Catherine L. Moore (pen name Lewis Padgett) - "Mimsy Were the Borogoves", in "The Best of Henry Kuttner", Nelson Doubleday, Inc., Garden City, NY, 1975 (Toys from the future educate two children into 4-dimensional awareness.)

Stanislaw Lem - "The Cyberiad", Avon Books, New York 1976, especially "How the World Was Saved" (literalness of computers), "Trurl's Machine" (the world's dumbest computer), "The First Sally (A) or Trurl's Electronic Bard" (mathematical poetry), "The Second Sally or the Offer of King Krool" ("simulation" of big game hunting), "The Third Sally or the Dragons of Probability" (the quantum theory of dragons.)

Stanislaw Lem - "The Star Diaries", Avon Books, New York 1977, especially "The Seventh Voyage" (Our hero traverses a region where the universe has time loops.)

H. Nearing, Jr. - "The Hermeneutical Doughnut" (1954), in Collection F2 (A toroidal pocket in space elucidates a passage from Ezekiel.)

H. Nearing, Jr. - "The Hyperspherical Basketball", Fantasy and Science Fiction, Dec. 1951 (A 4-dimensional ball wins an administration-faculty basketball game.)

H. Nearing, Jr. - "The Mathematical Voodoo" (1951), in Collection Fl (A dunce blossoms as a mathematician when the professor lectures to a voodoo doll.)

H. Nearing, Jr. - "The Poetry Machine", Fantasy and Science Fiction, Fall 1950 (A poetry-writing computer develops the artistic temperament.)

Larry Niven - "Convergent Series" (title story), in "Convergent Series" (collection), Ballantine Books, New York 1979 (An infinite geometric sequence foils a particularly nasty devil.)

M. C. Pease - "Devious Weapon", Astounding Science Fiction, Apr. 1949 (Self-reference paradox defeats sophisticated military computer.)

Green Peyton (pen name of G. Peyton Wertenbaker) - "The Ship That Turned Aside", in Groff Conklin (ed.) - "The Classic Book of Science Fiction", Bonanza Books, New York 1982 (A side-slip in a 4th dimension brings an ocean liner to an uninhabited land.)

Arthur Porges - "The Devil and Simon Flagg" (1954), in Collection Fl (Even the Devil can't prove Fermat's Last Theorem.)

Arthur Porges - "Problem Child", in Judith Merrill (ed.) - "10th Anniversary Edition, the Year's Best SF", Delacorte Press, New York 1965 (A seemingly retarded child is a mathematical prodigy.)

Kim Stanley Robinson - "The Blind Geometer", Isaac Asimov's Science Fiction Magazine, Aug. 1987 (A blind mathematician defeats conspirators who think his unpublished ideas have military applications.)

Hilbert Schenck - "The Geometry of Narrative", Analog Science Fiction/Science Fact, Aug. 1983 (The characters in this story analyze its structure by geometric analogies.)

Donald Wandrei - "The Monster from Nowhere" (1935), in Groff Conklin (ed.) - "The Best of Science Fiction", Crown Publishers, New York 1946 (The only survivor of an Andean expedition brings back a captive 4-dimensional predator to New Jersey.)

Stanley G. Weinbaum - "The Brink of Infinity", in "A Martian Odyssey and Other Classics of Science Fiction", Lancer Books, New York 1962 (Trapped by a lunatic, a mathematician must solve a mathematical riddle in ten questions.)

8. POEMS

Note: Systematically including mathematical light verse would be too large an enterprise.

Anonymous - "Marmaduke Multiply's Merry Method of Making Minor Mathematicians", Munroe and Francis, Boston 1841, Dover Publications, New York 1971 (Historical interest may warrant including this illustrated children's classic, whose rhymed couplets present the multiplication table.)

Robert Bridges - "The Testament of Beauty", Oxford Univ. Press, New York 1929, Book 4, lines 665 & 852 (Science, thru' infinitesimals, spanneth immensities; Pythagoras, behind all things, saw Mathematick.)

Samuel Butler - "Hudibras" (1663), Part 1, Canto 1, partly in Collection Fl (Hudibras's irritating talents include mathematical skill.)

Lewis Carroll - "The Humorous Verse of Lewis Carroll", Dover Publications, New York 1960, especially "Four Riddles" from "Phantasmagoria" (Yet what are all such gaieties to me . . .), "The Beaver's Lesson" from "The Hunting of the Snark" (versified arithmetic), "The Mad Gardener's Song" from "Sylvie and Bruno" ("double rule of three".)

Geoffrey Chaucer - "The Canterbury Tales" (c. 1387-1400), Prologue to the Parson's Tale (trigonometric note); the Franklin's Tale (The magic of algebra removes all rocks from the Brittany coast.)

Marion Cohen - "The Weirdest Is the Sphere", Seven Woods Press, New York 1979.

Marion Cohen - published poems in various magazines: "The Infinite Loop", "The One-Dimensional Man", "Space Is Relative", "Ram Dass Poem", "Fifth Grade Science Class", "House Poem", "Changing the Topology of the House", "The Way It Is Once and for All", "Some Simple Questions", "The Middle" (San Fernando Poetry Journal, Northridge, CA); "Waiting for the Franklin Institute", "The Mathematician and the Common Cold" (South Street Star, Philadelphia?); "The Two-Dimensional Man", "The Horror of Odd and Even" (Space and Time, New York, NY); "They Put Cantor Away" (Reflect, Norfolk, VA and Tempest, Miami, FL); "What Makes Things Tick" (Sojourner, Cambridge, MA.)

Samuel Taylor Coleridge - "A Mathematical Problem", in "The Complete Poetical Works of Samuel Taylor Coleridge", Vol. 1, Oxford Univ. Press, London 1912, partly in Collection M, p. 213 (versified construction of an equilateral triangle.)

J. V. Cunningham - "Meditation on Statistical Method", in Mark Strand (ed.) - "Contemporary American Poets", New American Library, New York 1969 (Some important things just don't "average out".)

Dante Alighieri - "Il Convivio" (The Banquet, 1304-1307), Tractate 2.14 (Jupiter); English translation: William Walrond Jackson - "Dante's Convivio", Oxford, Clarendon Press 1909 (The circle cannot be squared because its boundary is curved.)

Dante Alighieri - "Paradiso" (1321), Cantos 15 & 33, the latter partly in Collection M (The Primal Intellect generates all knowledge, as unity generates all numbers; our relation to God surpasses human comprehension - like circle-squaring.)

John Donne - "The Complete English Poems", Penguin Books, New York 1971, specifically "Lovers' Infiniteness", "The Primrose" (numerology of love), "Elegy upon the untimely death of the Incomparable Prince Henry" (geometrical metaphors), "Obsequies to the Lord Harrington" (geometry), "Of the Progress of the Soul" (geometry.)

William Empson - "Collected Poems", Harcourt, Brace, and Co., New York 1956, especially "The World's End", "High Dive", "Letter I", "Letter V". (Appended notes explain quite non-trivial mathematical references.)

John Gower - "Confessio Amantis" (1386-1393), Book 7, lines 145-202, partly in Collection M, p. 293 (Aristotle's subdivision of mathematics.)

A. E. Housman - "When First My Way to Fair I Took", in "Collected Poems of A. E. Housman", Henry Holt and Co., 1940, partly in Collection Fl (Time, in adding wealth, inevitably subtracts youth.)

Robinson Jeffers - "The Beginning and the End and Other Poems", Random House, New York 1963, specifically "The Great Wound" (the paradoxical power of modeling), "The Silent Shepherds" (the ultimate inadequacy of modeling.)

Robinson Jeffers - "The Double Axe and Other Poems", Liveright, New York 1977, specifically "The Inhumanist", Parts 9 and 36 (Mathematics itself is a profound metaphor.)

Vachel Lindsay - "Poems about the Moon, I: Euclid", from "The Congo and Other Poems", Macmillan, New York 1914, cited part in Collection Fl (A diagram of a circle is just a picture of the moon.)

Andrew Marvell - "The Definition of Love", partly in Collection Fl (geometrical metaphors.)

Edna St. Vincent Millay - "Euclid Alone Has Looked on Beauty Bare", from "The Harp-Weaver" (1923), also in Collection Fl.

Katharine O'Brien - "Three Haiku: What Is Mathematics", American Mathematical Monthly 88 (1981) p. 626; "Bilateral Convolution", American Mathematical Monthly 93 (1986) p. 399.

David Petteys - "Spaces (for Samuel Beckett)", in Collection DB, p. 190 (The center of an unbounded universe may even be your heap of manuscripts.)

Hyam Plutzik - "An Equation", in George P . Elliott (ed .) - "15 Modern American Poets", Holt, Rinehart, and Winston, New York 1956 (The beauty of a mathematical curve inhabits a realm beyond human suffering.)

Alexander Pope - "The Dunciad" (1728, 1743), Book 2, line 285, Book 4, lines 31-34, partly in Collections F2 and M (The reign of Dulness suppresses all human arts but mad mathematics.)

Kenneth Rexroth - "The Collected Longer Poems of Kenneth Rexroth", New Directions, New York 1968, specifically "The Phoenix and the Tortoise", Part 4; "The Dragon and the Unicorn", Part 1; "The Heart's Garden, the Garden 's Heart", Part 2.

Kenneth Rexroth - "The Collected Shorter Poems of Kenneth Rexroth", New Directions, New York 1966, especially "An Equation for Marie", "Fundamental Disagreement with Two Contemporaries" (mathematical jargon in dispute with surrealists), "Inversely, as the Square of Their Distance Apart" (gravitation as symbol for love), "OTTFFSSENTE" (rhapsody on the word "dozen"),

"Phronesis, III", "The Place", "Theory of Numbers", "A Lemma by Constance Reid" (A prose passage from an early Reid book is an unrhymed poem in Rexroth ' s rearrangement.)

Muriel Rukeyser - "The Dam", in "The Collected Poems of Muriel Rukeyser", McGraw Hill, New York 1978, pp. 95-98 (The mathematical beauty of the result outweighs the corruption of the builders.)

Carl Sandburg - "The Complete Poems of Carl Sandburg", Revised and Expanded Edition, Harcourt Brace Jovanovich, New York 1970, specifically "Tentative (First Model) Definitions of Poetry" (metaphors from mathematics), "The People, Yes", Part 36 (thoughts about zero), "Arithmetic" (also in Collection Fl), "Number Man" (homage to Bach), "Atlas, How Have You Been?" (musings about the world's shape.)

Wallace Stevens - "The Collected Poems of Wallace Stevens", Alfred A. Knopf, New York 1961, specifically "The Motive for Metaphor" (x = the exciting unknown), "Six Significant Landscapes" (moralized geometry.)

John Updike - "Midpoint and Other Poems", Alfred A. Knopf, New York 1969 (The long autobiographical title poem includes some scattered mathematical symbolism.)

William Wordsworth - "The Prelude" (1830), Book 2, lines 203-205 (mere mention), Book 5, lines 65-114 (poetry vs. geometry), Book 6, lines 115-167 (charms of geometry), excerpts in Collections F2 and M.

9. REAL MATHEMATICIANS IN LITERARY WORKS

Note: A few mathematicians, such as Newton, have achieved symbolic status in Western literature.

John Barth - "The Sot-Weed Factor", Part 1, Chap. 3, Grosset and Dunlap, New York 1960 (A long reminiscence by Burlingame describes his Cambridge friendship with Newton.)

William Blake - "Collected Poems", specifically "You don't believe ...", "Mock on, mock on ...", (both 1790's), also in Collection DB, pp. 181-182 (Newton and others symbolize the 18th-century rationalism that Blake attacks here.)

George Gordon, Lord Byron - "Don Juan" (1819-1824), Canto 10, partly in Collection N, p. 1094 (Adam, Newton, and their respective apples.)

Karel Capek - "The Death of Archimedes", in "Apocryphal Stories", Penguin Books, Harmondsworth, England 1975, also in Collection Fl ("Don't spoil my diagram!" he told the soldier.)

John Dos Passos - "U.S.A.", Vol. 1, "The 42nd Parallel" (1930), Modern Library, New York 1937, specifically "Proteus" (short life of Steinmetz, whose mathematics tamed alternating current.)

Friedrich Durrenmatt - "The Physicists", Grove Press, New York 1964 (Two spies, claiming to be Newton and Einstein, penetrate an exclusive mental institution housing a brilliant modern physicist.)

John Gay - "Three Hours after Marriage", London 1717 (caricature of Newton). See Jacob Bronowski - "The Ascent of Man", Little, Brown, and Co., Boston 1973, p. 236.

Arthur Koestler - "Pythagoras and the Psychoanalyst", in Collection Fl (How Pythagoras mastered his psychological problems - and discovered no theorems.)

Charles Lamb - "From the Latin of Vincent Bourne", in Collection DB, pp. 183-184 (The lessons of Newton's first schoolmistress were the foundation for his algebraic work.)

Walter Savage Landor - "Barrow and Newton", in "Imaginary Conversations", Vol. 4, J. M. Dent & Co., London 1891 (Barrow and Newton are made to discuss Bacon's "Essays".)

Alexander Pope - "Epitaph on Newton", in Collection N, p. 1094

Alfred Renyi - "Dialogues on Mathematics", Holden-Day, San Francisco 1967 (Classical dialogues with Socrates, Archimedes, and Galileo expound the nature of mathematics.)

Muriel Rukeyser - "Gibbs", in "The Collected Poems of Muriel Rukeyser", McGraw Hill, New York 1978, pp. 187-190 (Here Gibbs exemplifies the detached quest for mathematical understanding.)

Friedrich von Schiller - "Archimedes und der Schuler", excerpt in Collection M, p. 137.

J. L. Synge - "Euclid and the Bright Boy" in "Science: Sense and Nonsense", Jonathan Cape, Ltd., 1951, also in Collection F2 (criticism of the definition of a point.)

James Thomson - "To the Memory of Sir Isaac Newton", in Collection DB, pp . 175 - 180.

Hugh Whitemore - "Breaking the Code", Fireside Theatre, Garden City, NY 1987 (Broadway play about career and homosexuality of Alan Turing.)

William Wordsworth - "The Complete Poetical Works of Wordsworth", Houghton Mifflin, Boston 1932, specifically "The Prelude" (1850), Book 3, lines 60-63 (Newton), "The Excursion" (1814), Book 8, lines 220-230, excerpt in Collection M, p.137 (Archimedes).

William Butler Yeats - "Among School Children", in "Collected Poems", Macmillan, New York 1956 (The poem cites "golden-thighed" Pythagoras - probably evoking reincarnation or cyclical history.)

10. AUTOBIOGRAPHICAL MEMOIRS

Note: Biographies of mathematicians might make a long list, so this section collects only relevant autobiographies - of mathematicians and others.

Henry Adams - "The Education of Henry Adams", Massachusetts Historical Society, Boston 1918 (The author's confessed ignorance of mathematics spawns a grand historical theory in mathematical jargon.)

Winston S. Churchill - "My Early Life", Charles Scribner's Sons, New York 1930, Chap. 3, partly in Collection F2 (The author desperately crams enough mathematics to enter Sandhurst.)

Philip J. Davis - "The Thread: A Mathematical Yarn", Birkhauser, Boston 1983 (The author, during his travels, unearths the origins of Chebyshev's first name.)

G. H. Hardy - "A Mathematician's Apology", Cambridge Univ. Press, London 1940, also in Collection N.

Pamela Hansford Johnson - "Important to Me", Charles Scribner's Sons, New York 1974, Chap. 12 (Neither a good grammar school nor a patient spouse can dent the author's incomprehension of trigonometry.)

Sofya Kovalevskaya - "A Russian Childhood" (1889), Springer-Verlag, New York 1979 (the author's first fifteen years.)

Bertrand Russell - "My Mental Development", in Collection N.

Edward O. Thorp - "Beat the Dealer", Vintage Books, New York 1966 (The author's strategy for blackjack makes him persona non grata in Las Vegas.)

S. M. Ulam - "Adventures of a Mathematician", Charles Scribner's Sons, New York 1976.

Norbert Wiener - "Ex-Prodigy", M.I.T. Press, Cambridge, MA, 1964 (the author's first 26 years.)

Norbert Wiener - "I Am a Mathematician", M.I.T. Press, Cambridge, MA, 1964 (the author's professional career.)

11. FILMS

Louis de Rochemont and Alfred Werker - "Walk East on Beacon", 1952 (Russian agent pursues military secrets of refugee mathematician.) See "The New York Times Film Reviews 1949-1958", The New York Times & Arno Press, New York 1970, 29 May 1952.

Jack Hively - "Are You With It?", 1948, film version of listed stage musical, same title, by Perrin, Balzer, Revel, and Horwitt (Donald O'Connor, as a mortified actuary expiating a misplaced decimal point in a carnival job, displays numerical talent mostly through his dancing.) See "The New York Times Film Reviews 1913-1968", The New York Times & Arno Press, New York 1970, Vol. 3, 15 Apr. 1948.

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EPISTEMOLOGICAL PLURALISM: STYLES AND VOICES WITHIN THE COMPUTER CULTURE

Sherry Turkle and Seymour Papert

1. EPISTEMOLOGICAL PLURALISM

The prevailing image of the computer represents it as a logical machine and computer programming as a technical, mathematical activity. Both the popular and technical culture have constructed computation as the ultimate embodiment of the abstract and formal. Yet the computer's intellectual personality has another side: our research finds diversity in the practice of computing that is denied by its social construction. When we looked closely at programmers in action we saw formal and abstract approaches; but we also saw highly successful programmers in relationships with their material that are more reminiscent of a painter than a logician. They use concrete and personal approaches to knowledge that are far from the cultural stereotypes of formal mathematics.[1]

The diversity of approaches to programming suggests that equal access to even the most basic elements of computation requires accepting the validity of multiple ways of knowing and thinking, an epistemological pluralism. Here we use the word epistemology in a sense closer to Piaget's than to the philosopher's.[2] In the traditional usage, the goal of epistemology is to inquire into the nature of knowledge and the conditions of its validity; and only one form of knowledge, the propositional, is taken to be valid.[3] The step taken by Piaget in his definition of "epistemologie genetique" was to eschew inquiry into the "true" nature of knowledge in favor of a comparative study of the diverse nature of different kinds of knowledge, in his case the kinds encountered in children of different ages. We differ from Piaget on an important point, however. Where he saw diverse forms of knowledge in terms of stages to a finite end point of formal reason, we see different approaches to knowledge as styles, each equally valid on its own terms.

The barriers to acknowledging such pluralism are great, historically rooted in domains that go far beyond computation. The formal, propositional way of knowing, has been recognized traditionally as a standard, canonical style. Indeed, philosophical epistemology has generally taken it as synonymous with knowledge. Where concrete approaches to knowledge have been recognized at all, it has most often been as inferior ways of knowing, the kinds of knowing adopted by necessity by those who have not yet mastered the canonical style. Thus Jean Piaget recognizes in young children ways of thinking that do not conform to the canon but which are too coherent and efficacious to be branded simply as "wrong." He casts children's concrete thinking as a stage in a progression to a formal style.[4] Similarly, Claude Levi-Strauss recognizes "bricolage," a "science of the concrete," but relegates it to primitive societies, a manifestation of the "savage mind."[5]

More recently, concrete ways of thinking have been recognized in contexts that are not easily dismissed as inferior. Ethnographers of science studying the daily life of the laboratory have found that scientific discoveries are made in a concrete, ad hoc fashion, and only later recast into canonically acceptable formalisms.[6] Scientific biography reveals that Nobel

laureates relate to their materials in the concrete and tactile style of Levi-Strauss' bricoleurs.[7] Psychologists investigating adults' mathematical thinking find that they use an effective and down-to-earth style very different from the abstract and formal math they were taught at school.[8] Feminist scholars have documented the power of concrete, contextual reasoning in a wide range of domains.[9]

With such contributions has come a growing convergence of intellectual commitments to a revaluation of the concrete; but in general, the ethnographers, psychologists, and feminist scholars who have contributed to this revaluation have not seen computation as relevant to their concerns. Here we present evidence which points towards the possibility of new intellectual alliances.

In our research on programming styles, the computer has emerged as an important actor in the revaluation of the concrete, a privileged medium for the growth of alternative voices in dealing with the world of formal systems. The conventional route into formal systems, through the manipulation of abstract symbols, closes doors that the computer can open. The computer, with its graphics, its sounds, its text and animation, can provide a port of entry for people whose chief ways of relating to the world are through movement, intuition, and visual impression. At the heart of the new possibilities for the appropriation of formal systems is the computational object, on the border between an abstract idea and a concrete physical object. In the simplest case, a computational object such as an icon moving on a computer screen can be defined by the most formal of rules and is thus a mathematical construct, but at the same time it is visible, almost tangible, and allows a sense of direct manipulation that only the encultured mathematician can feel in traditional formal systems.[10] The computer has a theoretical vocation: it can make the abstract concrete; it can bring formality down-to-earth.

We have studied computers and the cultures that grow up around them in a wide variety of settings ranging from video game arcades to research laboratories of artificial intelligence. In this paper we draw particularly on a long-term line of research on how people enter the culture of programming. Using clinical methods inspired by the Piagetian and psychoanalytic traditions, we built up case studies of children using computers in grade school settings and college students taking a first programming course. We saw many manifestations of the concrete approach, favored in our study by more women than men. We were also able to observe people reacting poignantly to what they felt as a pressure to conform to an officially imposed style.[11] Although the computer as an expressive medium supports epistemological pluralism, the computer culture often does not. Our data points to discrimination in the computer culture that is determined not by rules that keep people out but by ways of thinking that make them reluctant to join in. Moreover, the existence of diverse styles of *expert* programming supports the idea that there can be different but equal voices even where the formal has traditionally appeared as almost definitionally supreme: in mathematics and the sciences.

Evelyn Fox Keller has remarked on the difficulty that people face when they try to understand what it might mean to do science in anything other than the formal and abstract canonical style. Describing such a style in the work of geneticist Barbara McClintock, Keller notes that this is the "less accessible aspect" of a scientist's relationship to nature.[12] In this essay we describe people learning to program who are having experiences with formal systems that are in many ways analogous to those of the bricoleur scientist or mathematician. One way the computer contributes to the revaluation of concrete approaches in the domain of formal systems is by giving more people access to (and an experience of) them.

The computer forces general questions about intellectual style to reveal an everyday face.[13] Even schoolroom differences in how children program computers raise issues that come up in a more abstract form in scholarly debates about scientific

objectivity. The computer makes ideas about non-canonical scientific voices more concrete and therefore appropriable because we can relate them not only to the science of the scientists but to our own thinking.

Here we focus on descriptions of a concrete way of knowing; the formal, canonical style is well known and well defended. Yet, our discussion of concrete approaches is implicitly a discussion of formal ones; it contributes to the deconstruction of the canonical style as the only way to think. It also situates it: the supervaluation of the formal approach owes much of its strength within computation to the support it gets in other intellectual domains. Formal thinking, defined as synonymous with logical thinking, has been given a privileged status which can be challenged only by developing a respectful understanding of other styles, where logic is seen as a powerful instrument of thought but not as the "law of thought." In this view, "logic is on tap not on top." As a carrier for pluralistic ideas about approaches to knowledge, the computer may hold the promise of catalyzing change not only within the computer culture but in the culture at large.

2. PERSONAL APPROPRIATION

Consider Lisa, eighteen, a first-year Harvard University student in an introductory programming course. Lisa fears that she will find the course difficult because she is a poet, "good with words not numbers." In high school, she had always scorned teachers who had insisted that mathematics is a language. Yet, now, her first encounter with the computer has made Lisa ready to reconsider this proposition and with it her characterization of herself as someone "bad at math." Lisa starts well, surprised to find herself easily in command of the course material; but as the term progresses she reluctantly decides that she "has to be a different kind of person with the machine." The pressure to do so is not from the computational medium. She says she can no longer resist pressure from her teachers to think in ways that are not her own.

Lisa wants to manipulate computer language the way she works with words as she writes a poem. There, she says, she "feels her way from one word to another," sculpting the whole. When she writes poetry, Lisa experiences language as transparent, she knows where all the elements are at every point in the development of her ideas. She wants her relationship to computer language to be similarly transparent. When she builds large programs she prefers to write her own, smaller, building block procedures even though she could use prepackaged ones from a program library; she resents the opacity of prepackaged programs. Her teachers chide her, insisting that her demand for transparency is making her work more difficult; Lisa perseveres, insisting that this is what it takes for her to feel comfortable with computers.

Two months into the programming course, Lisa's efforts to succeed are no longer directed towards trying to feel comfortable. She has been told that the "right way" to do things is to control a program through planning and black-boxing, the technique that lets you exploit opacity to plan something large without knowing in advance how the details will be managed. Lisa recognizes the value of these techniques — for someone else. She struggles against using them as the starting points for her learning. Lisa ends up abandoning the fight, doing things "their way," and accepting the inevitable alienation from her work. She calls her efforts to become "another kind of person with the machine" her "not-me strategy," and begins to insist that the computer is "just a tool." "It's nothing much," she says, "just a tool." Lisa's growing sense of alienation does not stem from an inability to cope with programming but from her ability to handle it in a way that comes into conflict with the computer culture she has entered.

A classmate, Robin, is a pianist. Robin explains that she masters her music by perfecting the smallest "little bits of pieces" and then building up. She cannot progress until she understands the details of each small part. Robin is happiest when she uses this

tried and true method with the computer, playing with small computational elements as though they were notes or musical phrases. Like Lisa, she is frustrated with using prepackaged programs. She, too, has been told her way is wrong: "I told my teaching fellow I wanted to take it all apart and he laughed at me. He said it was a waste of time, that you should just black box, that you shouldn't confuse yourself with what was going on at that low level."

Lisa and Robin came to the programming course with anxieties about not belonging (fearing that the computer belonged to male hackers who lived in "a world apart") and their experiences in it only served to validate their fears.[14] Although carefully designed and imaginative, the Harvard University course taught that there is only one right way to approach the computer, a way that emphasizes control through structure and planning. There are many virtues to this computational approach (it makes sense when dividing the labor on a large programming project, for instance) but Lisa and Robin have intellectual styles at odds with it. Lisa says she has "turned herself into a different kind of person" in order to perform and Robin says she has learned to "fake it." Although both women got good grades in this programming course, both have had to deny who they are in order to succeed.

Lisa and Robin's experiences make it clear that the computer can be a partner in a great diversity of relationships, that the computer is an expressive medium that different people can make their own in their own way. Yet those who wish to approach the computer in a non-canonical way are discouraged by the dominant computer culture, eloquently expressed in the ideology of the Harvard University course. They are asked to change their style to suit the fashion when they begin to interact with the official computer world, committed to a formal, rule-driven, hierarchical approach to programming.[15] Like Lisa and Robin, their exclusion from the computer culture is perpetuated not by rules that keep them out, but by ways of thinking that make them reluctant to join in. They are not computer phobic. They do not need to stay away because of fear or panic; but they are computer reticent. They want to stay away because the computer has come to symbolize an alien way of thinking. They learn to get by and to keep a certain distance. One of its manifestations is the way they neutralize the computer through language which denies the possibility of using it creatively (recall how Lisa dismisses it as "just a tool").

In this way, discrimination in the computer culture takes the form of discrimination against approaches to knowledge, most strikingly against the one preferred by Lisa and Robin, an approach we call bricolage.

3. BRICOLAGE

Levi-Strauss used the term bricolage to contrast the analytic methodology of Western science with what he called a "science of the concrete" in primitive societies.[16] The bricoleurs he describes do not move abstractly and hierarchically from axiom to theorem to corollary. Bricoleurs construct theories by arranging and rearranging, by negotiating and renegotiating with a set of well known materials.

Levi-Strauss' descriptions of the two scientific approaches, divested of his efforts to localize them culturally, suggest the variety of ways that people approach computers. For some people in our study, what is exciting about computers is working within a rule-driven system that can be mastered in a top-down, divide-and-conquer way This is the "planner's" approach taught in the Harvard programming course. This approach decrees that the right way to solve a programming problem is to dissect it into separate parts and design a set of modular solutions that will fit the parts into an intended whole. Some programmers work this way because their teachers or employers insist that they do. For others, it is a preferred approach; to them, it seems natural to make a plan, divide the task, use modules and subprocedures.

Lisa, Robin, and others like them in our study offer examples of a very different style. They are not drawn to structured programming; their work at the computer is marked by a desire to play with the elements of the program, to move them around almost as though they were material elements — the words in a sentence, the notes in a musical composition, the elements of a collage.

The bricoleur resembles the painter who stands back between brushstrokes, looks at the canvas, and only after this contemplation, decides what to do next. For planners, mistakes are missteps; for bricoleurs they are the essence of a navigation by mid-course corrections. For planners, a program is an instrument for premeditated control; bricoleurs have goals, but set out to realize them in the spirit of a collaborative venture with the machine. For planners, getting a program to work is like "saying one's piece"; for bricoleurs it is more like a conversation than a monologue. In cooking, this would be the style of those who do not follow recipes and instead make a series of decisions according to taste. While hierarchy and abstraction are valued by the structured programmers' planner's aesthetic, bricoleur programmers, like Levi-Strauss' bricoleurs, prefer negotiation and rearrangement of their materials.

For instance, Alex, nine years old, is a classic bricoleur. He attends the Hennigan Elementary School in Boston, the scene of an experiment in using computers across the curriculum. There, students work with Logo programming and computer controlled Lego construction materials. The work is both frequent enough (at least an hour a day) and open-ended enough for differences in styles to emerge.

When working with Lego materials and motors, most children make something move by attaching wheels to a motor that makes them turn. They see the wheels and motor through abstract concepts of what they are for: the wheels roll, the motor turns. Alex goes a different route. He looks at the objects concretely, without the filter of abstractions. He turns the Lego wheels on their sides to make flat shoes for his robot and harnesses one of the motor's most tangible features: the fact that it vibrates. When a machine vibrates it tends to travel, something normally to be avoided. When Alex runs into this phenomenon, his response is to make his robot (stabilized by its flat "wheel shoes") vibrate and thus move forward. When Alex programs in Logo he likes to keep things similarly concrete.

Learners are usually introduced to Logo programming through the "turtle," an icon on a computer screen that can be commanded to move around the screen and leave a trace as it goes. So, for example, the turtle can be told to move forward a hundred steps and turn ninety degrees with the commands FORWARD 100 RIGHT 90. Four such commands would have the turtle drawing a square. Programming occurs when a set of commands such as REPEAT 4 [FORWARD 100 RIGHT 90], are defined as a procedure: TO SQUARE. Alternatively, a subprocedure TO SIDE might be defined and repeated four times.

Alex wants to draw a skeleton. Structured programming views a computer program as a hierarchical sequence. Thus a structured program TO DRAW SKELETON might be made up of four subprocedures: TO HEAD, TO BODY, TO ARMS, TO LEGS, just as TO SQUARE could be built up from repetitions of a subprocedure TO SIDE. Alex rebels against dividing his skeleton program into subprocedures; his program inserts bones one by one, marking the place for insertion with repetitions of instructions. One of the reasons often given for using subprocedures is economy in the number of instructions. Alex explains that doing it his way was "worth the extra typing" because the phrase repetition gave him a "better sense of where I am in the pattern" of the program. He had considered the structured approach, but prefers his own style for aesthetic reasons: "It has rhythm," he says. In his opinion, using subprocedures for parts of the skeleton is too arbitrary, preemptive, and abstract. "It makes you

decide how to divide up the body and perhaps you would change your mind about what goes together with what. Like, I might decide to think about the two hands together instead of each hand with the arms."[17]

In his own way, Alex has resisted the pressure to believe the general superior to the specific or the abstract superior to the concrete. For Alex, thinking about hands as a subset of arms is too far away from the reality of real hands, just as taking a motor that was most striking as a vibrating machine and using it to turn wheels in the standard fashion was too far away from the real motor he had before him. While the structured programmer starts with a clear plan defined in abstract terms, Alex lets the product emerge through a negotiation between himself and his material.

Anne, also nine years old, is another bricoleur programmer. Her favorite hobby is painting and she has become expert at using sprites in programs that produce striking visual effects.[18] A sprite is a second Logo icon, a turtle that can be set in motion. Once you give a sprite a speed and a heading, it moves with that state of uniform motion until something is done to change it, just like an object obeying Newton's first law.

In one of Anne's programs, a flock of birds (each bird built with a sprite) flies through the sky, disappears over the horizon, and reappears some other place and time. If all the birds were red, then it would be easy to make them disappear and reappear. The command SETCOLOR :INVISIBLE would get rid of them and SETCOLOR :RED would make them reappear. Yet Anne wants the birds to have different colors, and so making the birds reappear with their original color is more complicated.

One method for achieving this end calls for an algebraic style of thinking: you make the program store each bird's original color as the value of a variable, then you change all colors to invisible and recall the appropriate variable when the bird is to reappear. Anne knows how to use this algorithmic method, but prefers one that allows her to turn programming into the manipulation of familiar objects. As Anne programs, she uses analogies with traditional art materials. When you want to hide something on a canvas, you paint it out, you cover it over with something that looks like the background. Anne uses this technique to solve her programming problem. She lets each bird keep its color, but she makes her program hide it by placing a screen over it. Anne designs a sprite that will screen the bird when she doesn't want it seen, a sky-colored screen that makes the bird disappear. Anne is programming a computer, but she is thinking like a painter.

Thinking like a painter does not prevent Anne from contributing a significant technical innovation to her fourth grade computer class. She is familiar with the idea of using two sprites to form a compound object. Her classmates and teachers have always done this by putting the sprites side by side. Anne's program is like theirs in using two sprites, one for the screen, one for the bird, but she places the sprites on top of each other so that they occupy the same space. Instead of thinking of compound objects as a way of getting a picture to be bigger, she thinks of compound objects as a way of getting sprites to exhibit a greater complexity of behavior, an altogether more subtle concept.

Thus, Anne's level of technical expertise is as dazzling in its manipulation of ideas as in its visual effects. She has become familiar with the idea of data structures by inventing a new one - her screened bird. She has learned her way around a set of mathematical ideas through manipulating angles, shapes, rates, and coordinates in her program. As a bricoleur, her path into this technical knowledge is not through structural design, but through the pleasures of letting effects emerge.

As in the case of Alex, Anne does not write her program in "sections" that are assembled into a product. She makes a simple working program and shapes it gradually by successive modifications. She

starts with a single black bird. She makes it fly. She gives it color. Each step is a small modification to a working program that she has in hand. If a change does not work, she undoes it with another small change. She "sculpts." At each stage of the process, she has a fully working program, not a part but a version of the final product.

Anne is perfectly capable of producing a program with well delineated subprocedures, although her way of creating them has little in common with the planner's approach.[19] Devotees of structured programming would frown on Anne's style. From their point of view, Anne should design a computational object (for example, her bird) with all the required qualities built into it. She should specify, in advance, what signals will cause her bird to change color, disappear, reappear, and fly. One could then forget about "how the bird works"; it would be a black box. Anne's work dramatizes the feature of bricolage that was so salient for Lisa and Robin: the desire for transparency. Like Lisa and Robin, she enjoys keeping open the possibility of renegotiating their exact form. This means staying in touch with that form at all times. The bricoleurs in our study tend to prefer the transparent style, planners the opaque, but the program's authorship is a critical variable in this preference. Planners want to bring their own programs to a point where they can be black-boxed and made opaque, while bricoleurs prefer to keep them transparent; but when dealing with programs made by others, the situation is reversed. Now, the bricoleurs are happy to get to know a new object by interacting with it, learning about it through its behavior the way you would learn about a person, while the planners usually find this intolerable. The planners' more analytic approach demands knowing how the program works before interacting with it. They demand the assurance that comes from transparent understanding, from dissection and demonstration.

Despite the dominant ideology of the computer culture which privileges the structured, hierarchical, planner's style, Anne's case makes it clear that the difference between planners and bricoleurs is not in quality of product, it is in the process of creating it. In describing bricoleur programmers, we have made analogies to cooks and painters. Bricoleurs are also like writers who don't use an outline but start with one idea, associate to another, and find a connection with a third. In the end, an essay "grown" through negotiation and association is not necessarily any less elegant or easy to read than one filled in from an outline, just as the final program produced by a bricoleur can be as elegant and organized as one written with the top-down approach.

Do programmers graduate from bricolage when they develop greater expertise? Will Anne become a structured programmer in junior high? Our observations suggest that with experience, bricoleurs reap the benefits of their long explorations, so that they may appear more "decisive" and like planners when they program on familiar terrain. Also, of course, they get better at "faking it." Still, the negotiating style resurfaces when they confront something challenging or are asked to try something new. Bricolage is a way to organize work. It is not a stage in a progression to a superior form. Interviews with computer scientists and their graduate students turned up highly skilled bricoleurs, most of them aware that their style was "countercultural." Indeed, there is a culture of programming virtuosos, the hacker culture, that would recognize many elements of the bricolage style as their own.

Within feminist scholarship there is a substantial body of literature that challenges the notion that human reason best expresses itself within terms of Western male gender norms.[20] For example, Carol Gilligan's work on moral reasoning calls into question the idea of one privileged, mature way of thinking. Gilligan's description of two approaches to moral reasoning is analogous in to our contrast between the formal, canonical approach to programming and the concrete style of the bricoleur.[21] In the first, justice is like a mathematical principle: to solve a problem you set up the right algorithm, the right black box, you crank the handle, and the answer comes out.[22]

In the second, a contextualized argument is like a concrete argument, one needs to stay in touch with the inner workings of the arguments, with the relationships and possibly shifting alliances of a group of actors whose interests need to be negotiated.

Despite Anne's high level of achievement, theorists of structured programming would criticize her style for the same kind of reasons that a stage theorist of moral development would classify the most impressively articulate contextual thinker at a lower intellectual level then his or her "formal" colleague. In both cases, criticism would center on the fact that neither is prepared to take the final step to abstraction. Gilligan challenges this standard hierarchy; she uses her observations of moral reasoning through concrete situations to reject Lawrence Kohlberg's stage theory with its determinate end point to development, an end point in abstract, universal principles.[23] If one branch of the development of moral reasoning moves towards the primacy of "justice," of the formal and analytic, Gilligan insists on equal respect for a different branch of development which leads toward increasingly sophisticated ways of thinking about morality in concrete terms of care through relationship and connection.

Gilligan is concerned with both morality and epistemology when she says: "the moral problem [for women] arises from conflicting responsibilities rather than from competing rights and requires for its resolution a mode of thinking that is contextual and narrative rather than formal and abstract."[24] Her language expresses a primary concern with the character of the morality which, as she says, requires a certain mode of thinking. This emphasis on the character of the morality (rather than the mode of thinking) is even more marked in recent writing where she redescribes Kohlberg's theory as being about only one side of moral reasoning. In this view, Kohlberg is talking about justice, thus leaving the other side of morality, namely care, to her.[25]

This compromise which splits off the *content* of moral judgments from the *mode of thinking* about them blunts the force of Gilligan's observations as a challenge to something more general than moral reasoning; but that challenge is central to our argument. Kohlberg's theory of the development of moral judgment mirrors Piaget's theory of the development of intelligence *per se*. Both express the value-laden perspective on intellectual growth that has dominated Western philosophy. Piaget sees a progression from egocentric beginnings to a final, "formal stage" when propositional logic and the hypothetico-deductive method "liberate" intelligence from the need for concrete situations to mediate thinking.[26] In this vision, mature thinking is abstract thinking. We disagree: for us, formal reasoning is not a stage, but a style. Gilligan's materials on the countercultural style of moral reasoning, like the countercultural style in programming, challenges the existence of hierarchical stages: for although Piaget would place the "concrete" Anne squarely in the preformal stage, her level of achievement undermines his assumptions about the superiority of the analytic and formal.

Thus, observation of programmers at work calls into question deeply entrenched assumption about the classification and value of different ways of knowing. It provides examples of the validity and power of concrete thinking in situations that are traditionally assumed to demand the abstract. It supports a perspective which encourages looking for psychological and intellectual development within rather than beyond the concrete and suggests the need for closer investigation of the diversity of ways in which the mind can use objects to think with rather than the rules of logic.

4. OBJECTS

Sooner or later in building objects with Lego, students at the Hennigan School where we met Alex run into the need for gears.[27] Looking at their work

provides a good example of alternate styles applied to working with the same problem, formal styles that use rules and concrete styles that use objects.

The motors in the construction set turn at a high speed with low torque. A car built by attaching these motors directly to the wheels will go very fast, but will be so underpowered that the slightest slope or obstruction will cause it to stall. The solution to the problem with Lego cars is the same as that adopted by designers of real cars: use gears. Yet in order to use them effectively, children need to understand something about gear ratios.

If a small gear drives a larger gear, the larger gear will turn more slowly and with greater torque. It is the relative and not the absolute size of the two gears that counts. But when we interview children, we find that some of them reason as if the size of only one gear matters, as if they were following a set of rules such as "large gears are slow and strong" and "small gears are fast and weak." Without the notion of relative size, such rules fail. Other children, and in our study, predominantly the girls, are less articulate and more physical in their explanations. They squirm and twist their bodies as they try to explain how they figure things out: and they get the right answer.[28]

Theorists who look at intellectual development as the acquisition of increasingly sophisticated rules would say that children run into problems if the rules they have built are not yet good enough.[29] The idea of "closeness to objects" enables us to consider a different kind of theory. Our observations suggest that the children who did so well did not have better rules, but a tendency to see things in terms of relationships rather than properties, access to a style of reasoning which allowed them to imagine themselves "inside the system." They used a relationship to the gears to help them think through a problem.

This "reasoning from within" may not be adequate for all problems about gears, but for the kind of problem encountered by the children in our project, it was not only adequate, but much less prone to the errors produced by a too-simple set of rules. Relational thinking puts you at an advantage: you do not suffer disaster if the rule is not exactly right.

We have defined bricolage as a style of organizing work that invites descriptions such as negotiational rather than planned in advance, what Warren McCulloch called "heterarchical" rather than hierarchical.[30] The story of the children and their gears serves to introduce another characteristic displayed by many bricoleur programmers. We call this characteristic proximality or closeness to the object. There is little distance between Anne and her computational objects. Like the children, who "reasoned from within" with the gears, Anne psychologically places herself in the same space as the sprites. She experiences her screens and birds as tangible, sensuous, and tactile. She is down there, in with the sprites, playing with them like objects in a collage. When she talks about them her gestures with hand and body show her moving with and among them. When she speaks of them she uses language such as "I move here." The object relations school of psychoanalysis focuses on the way development progresses by a process of internalization of the things and people of the world. They come to live within us; they become the objects with which we think.[31] When psychoanalysts talk about "objects" they usually mean people.[32] Here we extend the idea of internalized "objects to think with" to the domain of everyday relationships with artifacts. It is not enough to ask whether individuals "like" or "don't like" to program because that puts the question on too high a level of generalization. "Liking" to program depends on forging a personally meaningful relationship with a computational object, a relationship that "fits." In forging this relationship, there are several dimensions of choice. People can choose among computational objects. For example, in the version of Logo used by Anne there was a choice between sprites and turtles. Some prefer the turtle, its static nature, the fineness in the way it

draws. For others, these same qualities are reasons to reject the turtle as constraining, even unpleasant. They prefer the sprites, which move with flash and speed.

People can (and do) choose different ways of approaching the same object. Computational objects, like turtles and sprites stand on the boundary between the physical and the abstract. You can see them, move them, put one on top of another. Yet, they are mathematical constructions. Canonical programmers treat a sprite more like an abstract entity, a Newtonian particle, while bricoleur programmers treat it more like a physical object, a dab of paint or a cardboard cut-out.

Computational objects offer a great deal to those whose approach to knowledge requires a close relationship to an object experienced as tactile and concrete. Some people are comfortable with mathematical exercises that manipulate symbols on quadrille-ruled paper. For many others, computational objects offer a physical path of access to the world of formal systems. For them, the ambivalent nature of computational objects may make possible a first access to mathematics.[33] Feminist critics have related the standard notion of scientific objectivity to the social construction of gender: objectivity in the sense of distancing the self from the object of study is culturally constructed as male, just as male is culturally constructed as distanced and objective. From this point of view, Anne's proximal style is countercultural, reminiscent of Keller's description of geneticist Barbara McClintock's intimate relationship to the objects of her scientific study. For McClintock, the practice of science was essentially a conversation with her materials. The more she worked with neurospora chromosomes (so small that others had been unable to identify them), "the bigger [they] got, and when I was really working with them I wasn't outside, I was down there. I was part of the system. I actually felt as if I were right down there and these were my friends.... As you look at these things, they become part of you and you forget yourself."[34]

Alex and Anne relate to computational objects much as McClintock related to chromosomes as does a successful computer science graduate student Lorraine who explains how she uses "thinking about what the program feels like inside" to break through difficult problems. "For appearances sake," she wants to "look like I'm doing what everyone else is doing, but I'm doing that with only a small part of my mind. The rest of me is imagining what the components feel like. It's like doing my pottery." This is in sharp contrast to programmers in the structured, canonical style who use their favorite device of black-boxing as a way to maintain distance. The idea of the black box, designed not to be touched, mediates between the structured (planning) style of organizing work and their relationship to computational objects. Structured programmers are not *among* the sprites, they act *on* the sprites.

The contemptuous comment of one fourth-grade boy who overheard a classmate talking about "being a sprite" when he programs can be interpreted from this point of view. "That's baby talk," he said. "I am not in the computer. I'm just making things happen there." The remark reflects an insistence on boundaries and the development of a world view that will fall easily into line with the canonical, objective science whose gender-based meanings Keller has delineated.

In our research we find a close relationship between bricolage, a style of organizing work, and proximality, a style of relating to the objects of work.[35] Our data is consistent with a model of styles as clusters of characteristics in which bricolage and proximality form the nucleus of one cluster ("concrete thinking") and planning and distality the nucleus of the other ("formal thinking"). These clusters are ideal types: our contention is not that the attributes in each cluster are exactly correlated but that each has internal coherency in the way that a stable culture is coherent.

So for example, closeness to objects tends to support a concrete style of reasoning, a preference for using objects to think with, and a bias against the abstract formulae that maintain reason at a distance from its objects. Conversely, a distanced relationship with objects supports an analytic, rule and plan oriented style. Our theoretical conjecture is that degree of closeness to objects has developmental primacy; it comes first. The child forms either a proximal or distant relationship to the world of things. The tendency to use the abstract and analytic or concrete and negotiational style of thinking follows.

Although closeness to objects favors contextual and associational styles of work, it does not exclude the possibility of using a hierarchical one. Planning is not always an expression of personal style. It can be acquired as a skill, sometimes because it is needed to get a particular job done, sometimes as a facade to hide rather than express individuality.

Indeed, our data suggest that we may be underestimating the degree of association between proximality and bricolage. Some people adopt elements of the canonical style because they feel a social pressure to do so. In order to attract less negative attention, Lisa said that she decided to be a different kind of person, i.e., more of a "planner," Robin says she "fakes it" and forces herself to black box, Lorraine affects the discourse of a distanced style while something very different is going on in her head. Some bricoleurs respond to the dominant ethos of the computer culture by entering into an inauthentic relationship with the computer. This can lead to a paradoxical reaction: frustrated bricoleurs appear at first sight to be extremely rigid "planners." Some turn to a "cookbook" approach — like when in third grade, we were told to divide fractions by turning "the second fraction" upside down. When denied a chance to do their "real thinking," they turn to rules that do not require them to think at all. People like Lisa and Robin "escape to conformity," a reaction that muffles the manifestation of their different voices in computing. Nevertheless, those voices are there. Recall the graduate student Lorraine, who says she tries "to look like I'm doing what everyone else is doing," in order to preserve "appearances." Her style is hidden beneath her efforts to fit in.

In our culture, the structured, plan-oriented, abstract thinkers do not only share a style but constitute an epistemological elite. We have never seen a case in which someone claims to have felt pressure to move away from the canonical style. Thus, since the phenomenon of "faking it" goes only in one direction, we conclude that true occurrence of the bricoleur/proximal combination are even more common than our raw count.

Another attribute associated with proximality and bricolage when working with computers is a tendency to anthropomorphize, to refer to the system as though it had human qualities. The anthropomorphization extends from the computational objects ("That sprite doesn't want to do what I tell it now") to the computer itself. Anne, for instance, has no doubt that computers have psychologies: "they think," she says "but can't really have emotions." She believes, however, that the computer has preferences, "He would like it if you did a pretty program." When it comes to technical things, Anne assumes the computer has an aesthetic: "I don't know if he would rather have the program be very complicated or very simple." Anne knows that the computer is just a machine, but she sees it nonetheless as a male companion, if only a limited one. Anthropomorphization, both of the computer system and its parts, does not follow from lack of technical expertise. It is a stylistic preference.[36]

Very young children are in fact uncertain whether computers should be counted as alive or not alive, and argue the question hotly, debating the computer's aliveness on the basis of its psychology, its intentions, consciousness, and feelings. By age ten, most are sure that the computer is not actually alive. However, at this point, some children, like Anne, continue to behave with and talk about the computer as if it were sentient. They brag that it is

helping them or complain that it is not. In this, they are not showing confusion about biology. They do not think that the computer is alive the way an animal is, rather that it has a "kind of life," the kind of life appropriate to a computer: it thinks.[37]

Others have a very different reaction. Once they are no longer perplexed by whether the machine might actually be alive biologically, they shy away from anthropomorphization. When they complain about the computer, they do so in objective terms: it is too slow, it does not have enough memory. Talking about the computer usually means talking about technical details.

Lise Motherwell, a researcher at the Hennigan school, did an intensive investigation of eight fifth grade students. Motherwell found she could describe children's stances towards the anthropomorphization of the computer by distinguishing two styles: relational and environmental. Relational children, like those we are calling proximal thinkers, treat the computer as much like a person as they can, while environmental children, analogous to those we describe as preferring a distanced approach, treat it like a thing.

Once they have placed the computer in the not-alive category, the environmental children tend to settle with relief into treating it as a thing. This helps them to appropriate it through a relationship that involves distance, objectivity, and control. The relational children, once having settled the question of biological aliveness, get more comfortable with the machine by making it an interactive partner. In the computer they have found something in the domain of formal systems to which they can relate with informality. Three out of the four girls in Motherwell's study were relational; three out of the four boys environmental.[38]

In Motherwell's study, as in the study of children and gears, gender seems implicated in, but not a definitive influence on, style, consistent with observations of adult computer cultures where some men are alienated from the dominant engineering style and many women work creatively within it. Again, as in our examples of Anne, Alex, Lisa, Robin, and Lorraine, the concrete style did not imply a lower quality of work. Concrete, proximal gear builders did just as well and in some cases better than the formal thinkers; children who anthropomorphize the computer are no less technically sophisticated than those who do not. The degree of concrete, proximal, and anthropomorphic thinking does not reflect expertise but preferred approach to knowledge.

5. GENDER, CLOSENESS, AND CONFLICT

Several intellectual perspectives suggest that women would feel more comfortable with a relational, interactive, and connected approach to objects and men with a more distanced stance, planning, commanding, and imposing principles on them.[39] Indeed, we have found that many women do have a preference for attachment and relationship with computers and computational objects as a means of access to formal systems. Yet in our culture computers are associated with a construction of science that stresses aggression, domination, and competition. The cultural construction of science leads to a conflict that considerably complicates our story of how women appropriate technology. In the case of computation this conflict is particularly acute. From its very foundations, science has defined its way of knowing in a gender-based language. Francis Bacon's image of the (male) scientist putting the (female) nature "on the rack," underscores the way objectivity has been constructed not only in terms of the distance of the knower from nature, but in terms of an aggressive relationship towards it (or rather towards her). And from its very foundations, objectivity in science has been engaged with the language of power, not only over nature, but over people and organizations. Such associations have spread beyond professional scientific communities; aggression has become part of a widespread cultural understanding of what it means to behave in a scientific way. Its methods are expected to involve

"demolishing" an argument and "knocking it down" to size. Here the object of the blows is not a female nature but a male scientific opponent. If science is first a rape, it is then a duel.[40]

The traditional discourse of computation has not been exempt from these connotations. Programs and operating systems are "crashed" and "killed." We write this paper on a computer whose operating system asks if it should "abort" an instruction it cannot "execute." In our ethnographic studies of the social worlds that grow up around computing, we have found that this is a style of discourse that few women fail to note. Thus, women are too often faced with the not necessarily conscious choice of putting themselves at odds either with the cultural associations of the technology or with the cultural constructions of being a woman.

When Lisa had a confrontation with her instructor about the proper way to program the computer culture and its canonical epistemology were represented by a person in authority with whom she argued. In other cases the tension comes from fears of what people might think rather than a confrontation with what someone actually thinks. Lorraine who programs by imagining "what the components feel like" ends her description of her programming style by adding: "Keep this anonymous. I know it sounds stupid." In both these reactions there is a tension between an individual and an outside agency; but the conflict is internalized: the computer culture alienates by putting one in conflict with oneself .

When Lisa first found herself doing well in her programming course, she found it "scary" because she felt she needed to protect herself from the idea of "being a computer science type." In high school, Lisa saw young men around her turning to computers as a way to avoid people: "They took the computers and made a world apart." Lisa describes herself as "turning off" her natural abilities in mathematics that would have led her to the computer. "I didn't care if I was good at it. I wanted to work in worlds where languages had moods and connected you with people." Although Robin had gone through most of her life as a musician practicing piano eight hours a day, she, too, had fears about "guys who established relationships" with the computer. "To me, it sounds gross to talk about establishing a relationship with the computer. I don't like establishing relationships with machines. Relationships are for people."

In the vehemence with which many women insist on the computer's neutrality, on its being nothing more than a mere tool, there may be something more subtle going on than a clash between culture and personal style - a clash between personal style and sense of self. Many women may be fighting *against* having a close relationship to a computer. For some, like Lorraine, it could be because they want to belong to the dominant computer culture. But for others, the experience of closeness to the object is a source of conflict with themselves.

Lisa, like Anne, placed herself in the space of the computational objects she worked with and she tended to anthropomorphize, responding to the computer as though it had (at least) an intellectual personality. In Lisa's case, her own style came to offend her because it had led her to what she experienced as a too-close relationship with a machine. When Lisa began programming she saw herself as communicating with the computer, but the metaphor soon distressed her. "The computer isn't a living being and when I think about communicating with it, well that's wrong. There's a certain amount of feeling involved in the idea of communication and I was looking for that from the computer." She looked for it and she frightened herself. "It was horrible. I was becoming involved with a thing. I identified with how the computer was going through things."

In our research we find that women express such sentiments with particular urgency. We observe that a conflict fuels their convictions. In many cases, they are most comfortable with a style of thinking in which they get close to the objects of thought. The

computer offers them such objects; but the closer they get to them the more anxious they feel. One remedy for their anxiety is denial. The more these people become involved with the computer, the more they insist that it is *only* a neutral tool. Again, their assertion is belied by the vehemence with which it is expressed.

Lisa's conflict with her instructor would be resolved in principle by a greater tolerance for her way of thinking; but addressing *internal* conflicts about being close to computers requires more than tolerance. It requires profound changes in the culture that surrounds the computer. For instance, if the computer is a tool, and of course it is, is it more like a hammer or more like a harpsichord?

The musician Robin is not distressed by her close relationship with her piano which is also a machine. Lisa who finds attachment to the computer "unnatural" is not upset by her passion for the beautiful, heavy antique ink pens with which she writes. If Lisa had been in music school, it is most likely that she, like Robin, would not experience as threatening her sense of communicating with her instrument or her emotional involvement with it. Music students live in a culture that over time has slowly grown a language and models for close relationships with music machines. The harpsichord, like the visual artists' pencils, brushes and paints, is a tool, and yet we understand that artists' encounters with these can (and indeed, will most probably) be close, sensuous, and relational. Indeed, the best artists will develop highly personal styles of working with them.

The development of a new computer culture would require more than environments where there is permission to work with highly personal approaches. It would require a new social construction of the computer, with a new set of intellectual and emotional values more like those applied to harpsichords than hammers.[41] Since increasingly computers are the tools people use to write, to design, to play with ideas and shapes and images, they should be addressed with a language that reflects the full range of human experiences and abilities. Changes in this direction would necessitate the reconstruction of our cultural assumptions about formal logic as the "law of thought." This point brings us full circle to where we began, with the assertion that epistemological pluralism is a necessary condition for a more inclusive computer culture.

7. ROADBLOCKS AND OPENINGS FOR CHANGE

While the computer supports epistemological pluralism, the computer culture has not. In some ways, though, the computer culture is catching up with the potential of the computer. In particular, significant openings for change within the technological culture come from a new emphasis on computational objects which is making itself felt in domains as diverse as debates about which personal computers are the best and how to build artificial brains.

Its simplest manifestation is the fashion for using icons to control personal computers. In a traditional IBM-style computer system control is through typing instructions. In an iconic system, the same effects are achieved by moving screen symbols. The current technology for the act of moving something on a screen falls short of what the computer industry expects to provide quite soon, but existing systems, such as the Macintosh's "mouse" and touch sensitive screens already give a tactile sense that recalls Anne's experience of programming as collage.

Even superficial use of icons is enough to transform the perception of the computer by people who are using it in computationally simple ways. For example, it is commonplace knowledge that many writers who began to use computers reluctantly, as a necessary evil, now find they have friendlier relationships mediated by the icons, the mouse, and the cozier appearance of a Macintosh. Although these particular warmer relationships do not involve programming, their influence may mean that the

next generation of people like Lisa and Robin will come to programming courses with a different sense of who "owns" the computer.

A multiplicity of technical methods does not by itself lead to pluralism. It can simply lead to competition, to which the computer industry is scarcely averse, nor are those computer users who seem to enjoy conversations that engage them in heated debate about the merits of their favorite system. It is only when we understand the computer as a projective screen for different approaches to knowledge that we can listen to these conversations as a striving for pluralism. Different people are comfortable with different systems. When people fight about the IBM versus the Macintosh, what they may be trying to do is defend their intellectual styles. Yet, the debate is cast as an effort to prove the other side wrong; as if it would be impossible to prove both sides right.

A multiplicity of technical methods can also lead to elitism. The Logo language allowed Anne and Alex to program in their own ways, but in many educational settings Logo is defined as the computer language for children who have not reached the top stage in Piaget's hierarchy. In such settings even as sophisticated a thinker as Anne or as creative a thinker as Alex would get their liberty at the cost of having their intellects defined as immature. Similarly, the very success of the Macintosh has often been cast in terms that reflect the elitism of the dominant computer culture. The Macintosh iconic interface has been marketed as "the computer for the rest of us," with the implication that the rest of us need things made simple and do not want to be bothered with technical things.

As it happens, the popularity of icons may be settling this argument, a conclusion which has many interpretations. The designers of computer interfaces might interpret it as final proof of the technical superiority of icons. A psychologist might read it as a challenge to the concrete/formal split; perhaps most people are concrete thinkers most of the time, and formal thinking is used for acceptability or prestige or functionality. Others might simply say that icons are "easier." Individually, these positions do not support pluralism. What is important here is whether the support for icons is part of a larger shift towards an acceptance of concrete ways of thinking.

If the shift goes far it will be through the connection of the icons to something deeper, a philosophy of "object oriented programming."[42] In the traditional concept of a program, the unit of thought is an instruction to the computer to do something. In object oriented programming the unit of thought is creating and modifying interactive agents within a program for which the natural metaphors are biological and social rather than algebraic. The elements of the program interact as would actors on a stage. This style of programming is not only more congenial to those who favor concrete approaches, but it puts an intellectual value on a way of thinking that is resonant with their own. In principle it could undermine the canonical position in at least two ways: first, within the world of programming through legitimating alternative methods; second, in the larger intellectual culture, by supporting trends in cognitive theory that challenge the traditional canon.

Until recently, prevailing models of cognitive theory have bolstered the commitment of psychologists and educators to the superiority of algorithmic and formal thinking. They were given support by the cognitive theorists most influential in the computer world, the leaders of the artificial intelligence community. In the late 1970s and early 1980s, the model of AI with the greatest visibility was the rule-based "expert system" with its model of mind as a structured information processor. Critics of how computers influence the way we think cited the information processing model as demonstrating the instrumental reason and the lack of ambiguity allegedly inherent in all computational thinking about intelligence.[43] Artificial intelligence is not a unitary enterprise, however, and recently, another model has become increasingly prominent: "emergent A"I.[44]

Emergent AI does not suggest that the computer be given rules to follow but tries to set up a system of independent elements within a computer from whose interactions intelligence is expected to emerge. Its sustaining images are drawn not from the logical but from the biological and social. Families of neuron-like entities or societies of anthropomorphized subminds and sub-subminds are in simultaneous interaction from which mind-like process is expected to emerge. These models are sometimes theorized in notions of "mind as society," where negotiational processes are placed at the heart of all thinking. Those who espouse and support such models are far more inclined to find bricolage acceptable than are classical Piagetians. What concerns us here, is not making value judgments about these trends in AI, just as we are not advocating a choice between the use of icons and the use of textual instructions in computer operating systems. What does concern us is that the new trends - icons, object-oriented programming, actor languages, society of mind, emergent AI - all create an intellectual climate in the computer world which undermines the idea that formal methods are the only legitimate methods.

Thus, recent technological developments, in interfaces, programming philosophy, and artificial intelligence, have created an opening for epistemological pluralism. We began by presenting the notion of epistemological pluralism by reference to three streams of thought - feminism, ethnography of science, and psychology - which, although different in many ways, converge in reasserting the importance of objects in thinking. We close by noting the opportunity for new alliances between them and computation.

Louis Althusser writes about psychoanalysis that the important breakthrough was not any particular statement about the mind, but the step of recognizing the unconscious as an object of study that defines a new theoretical enterprise.[45] Psychology long had considered the rational and the conscious as the quintessential mental activity; Sigmund Freud shifted the ground to the irrational and the unconscious. The unconscious was not given recognition as only an important factor, but rather became an object of science in its own right. Similarly, we imagine the emergence of a science of thought which would recognize the concrete as its central object.

There is every reason to think that the revaluation of the concrete will open the computer culture to accepting the computer as an expressive medium which encourages distinctive and varied styles of use. There is every reason to think that this pluralistic computer culture could be more welcoming and nurturing to women and to men. Gilligan has said that women can protect the recognition "of the continuing importance of attachment in human life."[46] The evidence from our research on programming styles leads us to conclude with an analogous speculation. Feminist scholarship could make a crucial contribution to the (until now) predominantly male computer culture by promoting the recognition of the diverse ways that people think about and appropriate formal systems and by encouraging the acceptance of our profound human connection with our tools.

NOTES

1. Research reports that emphasize approach to programming or programming style in the sense we are using it here include Seymour Papert, Andrea di Sessa, Sylvia Weir, and Daniel Watt, "Final Report of the Brookline Logo Project," Logo Memos 53 and 54, Massachusetts Institute of Technology, Cambridge, MA, 1979; Sherry Turkle, "Computer as Rorschach," *Society* 17 (December 1980): 15-22; Sherry Turkle, *The Second Self: Computers and The Human Spirit* (New York: Simon and Schuster, 1984), especially Chapter 3; Sylvia Weir, *Cultivating Minds: A Logo Casebook* (New York: Harper and Row, 1987); Sherry Turkle, Donald Schon, Brenda Nielsen, M. Stella Orsini, and Wim Overmeer, "Project Athena at MIT," (unpublished ms., May 1988); Lise Motherwell,

"Gender and Style Differences in a Logo-Based Environment" (Ph.D. diss., Massachusetts Institute of Technology, January 1988); Idit Harel, "Software Design for Learning: Children's Construction of Meaning for Fractions and Logo Programming," (Ph.D. diss., Massachusetts Institute of Technology, June 1988).

2. Piaget's use of the term epistemology is pervasive in his writing. See, in particular, *Introduction a Epistemologie Cenetique* vol. 1-3 (Paris: Presses Universitaires de France, 1950). Alvin Goldman discusses the modern redefinition of the field of epistemology as something as close to psychology and sociology as to philosophy in *Epistemology and Cognition* (Cambridge, MA: Harvard University Press, 1986).

3. For a critical and polemical account of this history, see Paul M. Churchland, *A Neurocomputational Perspective: the Nature of Mind and the Structure of Science* (Cambridge, MA: The MIT Press, 1989).

4. See for example, Jean Piaget and Barbel Inhelder, *The Growth of Logical Thinking from Childhood to Adolescence* (New York: Basic Books, 1958).

5. Claude Levi-Strauss, *The Savage Mind* (Chicago: University of Chicago Press, 1968).

6. A sample of relevant studies in scientific ethnography is provided by Karin Knorr-Cetina and Michael Mulkay, eds., *Science Observed. Perspectives on the Social Studies of Science* (London: Sage Publications, 1983). See also Karin Knorr-Cetina, *The Manufacture of Knowledge: An Essay on the Constructivist and Contextual nature of Science* (Pergamon: Oxford, 1981); Bruno Latour and Stephen Woolgar, *Laboratory Life: The Social Construction of Scientific Facts* (Beverly Hills CA: Sage, 1979); Sharon Traweek, *Beamtimes and Lifetimes. The World of High Energy Physicists* (Cambridge, MA: Harvard University Press, 1989).

7. Evelyn Fox Keller, *A Feeling for the Organism: The Life and Work of Barbara McClintock* (San Francisco: W.H. Freeman, 1983).

8. A sample of studies on everyday thinking is contained in Barbara Rogoff and Jean Lave, eds., *Everyday Cognition. Its Development in Social Context* (Cambridge, MA: Harvard University Press, 1984). Also see, Jean Lave, *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life* (Cambridge: Cambridge University Press, 1988).

9. See, for example, Mary Field Belenky, Blythe McVicker Clinchy, Nancy Rule Goldberger, and Jill Mattuck Tarule, *Women's Ways of Knowing: the Development of Self, Voice, and Mind* (New York: Basic Books, 1986). Edited collections that focus on approaches to knowing in science include: Ruth Bleir, ed., *Feminist Approaches to Science* (New York: Pergamon, 1986) and Sandra Harding and Merrill B. Hintikka, eds., *Discovering Reality. Feminist Perspectives on Epistemology, Metaphysics, Methodology, and Philosophy of Science* (London: Reidel, 1983). An overview that highlights many of the issues we deal with in this essay is provided by Elizabeth Fee, "Critiques of Modern Science: The Relationship of Feminism to Other Radical Epistemologies" in Bleir, *Feminist Approaches.*

In this essay we situate our position by focusing on two writers, Carol Gilligan and Evelyn Fox Keller. Gilligan, with her emphasis on moral discourse might seem out of place in a discussion of non-canonical approaches to science and technology. But here we argue that key issues in the critique of science are not about *scientific* reasoning but about *reasoning*. Juxtaposing moral and computational reasoning helps us make this point. In addition, Gilligan's critical relationship to the theories of Lawrence Kohlberg is analogous to our own critical relationship to Piaget's work. We emphasize Keller because her work underscores as does ours, the importance

of relationships with objects in the development of non-canonical styles. Using Gilligan and Keller as a contrasting pair allows us to highlight two different dimensions of what we call the concrete approach to science. See Carol Gilligan, *In A Different Voice. Psychological Theory and Women's Development* (Cambridge: Harvard University Press, 1982); Evelyn Fox Keller, *A Feeling for the Organism. The Life and Work of Barbara McClintock;* Evelyn Fox Keller *Reflections on Gender and Science* (New Haven: Yale University Press, 1985).

10. See Philip J. Davis and Reuben Hersh, *The Mathematical Experience* (Boston: Houghton Mifflin, 1981) and Seymour Papert, "The Mathematical Unconscious" in Judith Wechsler, ed., *Aesthetics and Science* (Cambridge, MA: MIT Press, 1980).

11. For grade school children we worked with forty cases. Of the twenty girls in our study, fourteen preferred concrete approaches, of twenty boys there were four who followed this route. In the study of college students taking a first programming course, of the fifteen women, nine were concrete style programmers, of fifteen men, four. Because of our interest in spontaneous approaches we classified as concrete thinkers some students who finally adopted elements of the canonical approach in order to please their teachers. (See for example, the cases of Lisa and Robin, below.) In our research, the male/formal and female/concrete dichotomy was most dramatic in a predominantly white, wealthy private school in the South where traditional patterns of socialization would favor boys learning the ways of control, hierarchy, and distance and girls learning the ways of negotiation and closeness.

12. Keller, *A Feeling for the Organism,* 1980.

13. For a fuller discussion of the computer as an evocative and concretizing object see Sherry Turkle, *The Second Self: Computers and the Human Spirit* (New York: Simon and Schuster, 1984).

14. Lisa and Robin were part of a larger study of Harvard and MIT students taking introductory programming courses. The study found anxiety about an identity as a "computer person" to be an important aspect of reticence towards computers, especially among women. See Sherry Turkle, "Computational Reticence: Why Women Fear the Intimate Machine," in Cheris Kramarae, ed., *Technology and Women's Voices: Keeping in Touch* (New York: Pergamon, 1988). See also Sara Kiesler, Lee Sproull, and Jacquelynne S. Eccles, "Poolhalls, Chips, and War Games: Women in the Culture of Computing," *Psychology of Women Quarterly, 9* (1985).

15. In 1987, Turkle conducted a survey of 37 women members of a local computer society. Of these, 17 reported feeling pressure to change their preferred ways of working with the computer in order to be more acceptable to the dominant computer culture. "I got my wrists slapped enough times and I changed my ways," said one of them, a college student for whom programming on her Macintosh was a private passion until she entered MIT.

16. Levi-Strauss, *The Savage Mind* . Levi-Strauss contrasted bricolage with Western science, ignoring the significant aspects of bricolage present in the latter. Several recent writers have written in a way that begins to redress this imbalance. See for example, Paul Feyerabend, *Against Method. The Outline of An Anarchistic Theory of Knowledge* (London NLB, 1975); N.R. Hanson, *Patterns of Discovery* (Cambridge: Cambridge University Press, 1958); Ludwig Wittgenstein, *Philosophical Investigations* (New York: MacMillan, 1953). In a less formal vein, see Richard Feynman, *Surely You Must Be Joking Mr. Feynman* (New York: Norton, 1985).

17. In its ideal, the structured method would have the programmer go beyond subprocedures to make one procedure that could be given different parameters to produce arms and legs, right and left sides, even differently shaped people. This aesthetic, known as "procedural abstraction," wants to see a right arm and a left leg disappear into a generalized abstract idea of "limb." But for someone like Alex, the top priority is staying in touch with the concrete. He is aware of the importance of organizing his program in order to find his way around it, but he does so by giving it what he calls "rhythm" rather than a hierarchical structure of procedures and subprocedures.

18. Anne's program has the merit of showing in compact form a set of qualities characteristic of the bricoleur, but usually more diffusely represented. For a more detailed account, see Turkle, *The Second Self,* pp. 110-15.

19. Bricolage does not exclude the use of subprocedures; it simply does not give their *a priori* delineation the status of a privileged method. For example, a part of a holistically conceived program can be demarcated as a subprocedure at any stage of programming. Subprocedures need not be "black boxes"; they too can be developed as the program grows as a whole. Indeed, the bricoleur may use as subprocedures programs that happen to be "lying around," possibly even programs that were originally made for very different purposes.

20. See for example the writings on scientific epistemology referred to in note 16.

21. Gilligan, *In A Different Voice*. For a critical discussion of Gilligan's proposals and her reply see Linda K. Kerber et. al., "On *In A Different Voice:* An Interdisciplinary Forum," *Signs* 11, no. 2 (Winter 1986): 304-33. Its methodological criticisms of Gilligan's treatment of the relationship between "voice" and gender do not detract from how her subjects illustrate the way of thinking we call "bricolage."

22. It is an eleven-year-old, Jake, who when confronted with a moral dilemma describes it as "sort of like a math problem with humans." See Gilligan, *In A Different Voice, p.* 26

23. Gilligan, *In A Different Voice,* 30ff. Kohlberg had already been challenged on other grounds. See, for example, John Gibbs, "Kohlberg's Stages of Moral judgment: A Constructive Critique," *Harvard Education Review,* 47, no. 4 (February, 1977): 43-61. Similar issues have been raised in critiques of Jean Piaget. See, for example, Steven Toulmin, *Human Understanding* (Princeton, NJ: Princeton University Press, 1972). Toulmin argues that Piaget's experimental investigations reflect an *a priori* commitment to a Kantian position. We single out Toulmin, because unlike most of Piaget's critics he does not quarrel with the detail of how the stages are described but with the epistemological assertion of the final end point.

24. Gilligan, *In A Different Voice, p. 19.*

25. Carole Gilligan and Jane Attanucci, "Two Moral Orientations," in Carol Gilligan, Janie Victoria Ward and Jill McClean Taylor, eds., *Mapping the Moral Domain (Cambridge:* Harvard University Press).

26. Piaget and Inhelder, *The Growth of Logical Thinking from Childhood to Adolescence.*

27. These experiments with Lego and programming are undertaken in a Piagetian spirit. See Jean Piaget, *La Prise de Conscience* (Paris: Presses Universitaires de France, 1951) for experiments that deal with how mechanisms work. For a personal statement about the power of gears as an introduction to formal systems, see Seymour Papert, "The Gears of My Childhood," in

Mindstorms: Children, Computers, and Powerful Ideas (New York: Basic Books, 1980).

28. Our sample does not allow us to say that girls did systematically better than boys. Research is in progress on this point. Our present discussion is about styles of explanation (rule-driven vs. body-syntonic) not distribution of abilities.

29. For example most of those inspired by the Carnegie-Mellon schools of artificial intelligence. See, Ryszard S. Michalski et al, eds., *Machine Learning: An Artificial Intelligence Approach* (Los Altos, CA: Morgan Kaufmann, 1983).

30. Warren McCulloch, *Embodiments of Mind* (Cambridge, MA: MIT Press, 1988).

31. For an excellent overview of the object relations perspective perspective, see Jay R. Greenberg and Stephen A. Mitchell, *Ob Object Relations in Psychoanalytic Theory* (Cambridge: Harvard University Press, 1983).

32. In contrast, D.W. Winnicott has some suggestive ideas about the power of the "transitional object" - the baby's blanket, the teddy bear - that in developmental terms mediates between experience of self and non-self. In the current context, it suggests the power of the inanimate in inner life. See *Playing and Reality* (New York: Basic Books, 197 1).

33. The Logo turtle was designed to be "body syntonic," to allow users to put themselves in its place. When children learn to program in Logo, they are encouraged to work out their programs by "playing turtle." The classic example of this is developing the Logo program for drawing a circle. This is difficult if you search for it by analytic means (you will need to find a differential equation), but easy if you put yourself in the turtle's place and pace it out. (The turtle makes a circle by going forward a little and turning a little, going forward and little and turning a little, etc.) Turtles are a path into mathematics for people whose surest route is through the body. See Seymour Papert, *Mind storms*.

34. Keller, *A Feeling for the Organism*, p. 117. Keller describes McClintock's approach as dependent on a capacity to "forget herself," immerse herself in observation, and "hear what the material has to say." (page 198)

35. In the 70 cases on which we report here, 40 grade school children and 30 college students, we found these two dimensions of approach to programming in all but 9 cases. Thus, empirically, we sometimes find each aspect of the concrete approach - bricolage as a style of organization and closeness to the object - without the presence of the other. In particular, one finds people who are planners but who enjoy a close relationship with concrete objects (and who experience computational objects this way). On the pairing of planning and what they call an interactive style with the computer, see Rosamund Sutherland and Celia Hoyles, "Gender Perspectives on Logo Programming in the Mathematics Curriculum," in Celia Hoyles, ed., *Girls and Computers* (London: Bedford Way Papers/34, Institute of Education, University of London, 1988).

36. Anthropomorphization is not limited to children. It is a habit of mind shared by adults, including technical experts. When we say the computer "moves the queen" in a game of chess, the program has invited us to speak of it as though it had intentions. And programs within a computer system interact with each other in a way that supports models of the computer as composed of "agents" in communication. Computer scientists talk about a concept such as recursion with anthropomorphic metaphors: one agent "calls up" another, "wakes up" another, and "passes on a job." They sometimes even refer to the agents within a computer system as citizens of a "society

of mind." See Marvin Minsky, *Society of Mind* (New York: Simon and Schuster, 1987) and Papert, *Mindstorms*.

37. Turkle, *The Second Self*, especially Chapter 1.

38. Motherwell, "Gender and Style Differences in a Logo Based Environment." (see n.1)

39. Gilligan's work illuminates the relationship between bricolage and gender and Keller speaks to the gender meanings of the proximal style of relating to objects, be they physical objects such as gears or chromosomes or conceptual objects such as the elements of programming. A psychoanalytic perspective would place the roots of such approaches at an early stage of child development. If in our culture, women are the primary care takers for children, the earliest and most compelling experiences of merging are with the mother; differentiation and delineation take on gender meanings. See for example, Nancy Chodorow, *The Reproduction of Mothering: Psychoanalysis and the Sociology of Gender* (Berkeley: University of California Press, 1978) and Keller, *Reflections on Gender and Science*. The traces of such early experiences are culturally reinforced by continuing gender divisions of parenting roles and by the very different socialization of men and women.

40. Keller, "Baconian Science: The Arts of Mastery and Obedience," in *Reflections on Gender and Science*, pp. 33ff; Carolyn Merchant, *The Death of Nature* (New York: Harper and Row, 1980); Donna Haraway, "The Biological Enterprise: Sex, Mind, and Profit From Human Engineering to Sociobiology," *Radical History Review* 20: 206-37.

41. On values for a new computer culture, see Seymour Papert, "Technological Thinking versus Computer Criticism," *Educational Researcher* 16, no. I (January 1987): 22-30.

42. For a non-technical discussion see, Alan Kay, "Microelectronics and the Personal Computer," *Scientific American* 237 (September 1977): 230-44 and Alan Kay, "Software's Second Act," *Science 85* (November 1985): 122ff.

43. See, for example, Hubert Dreyfus, *What Computers Can't Do. The Limits of Artificial Intelligence*, 2nd ed. (New York: Harper and Row, 1979) and Joseph Weizenbaum, *Computer Power and Human Reason* (San Francisco: W.H. Freeman, 1976).

44. The reaction within artificial intelligence against abstract, propositional, rule-driven methods was given literary expression in the writings of Douglas Hofstadter. See for example "Waking Up from the Boolean Dream, or Subcognition as Computation" in Douglas Hofstadter, *Metamagical Themas: Questing for the Essence of Mind and Pattern* (New York: Basic Books, 1985), pp. 631-65. Two other manifestations of this reaction are Marvin Minsky, *Society of Mind* (New York: Simon and Schuster, 1985) and David E. Rumelhart, James L. McClelland, and the PDP Research Group, *Parallel Distributed Processing: Explorations in the Microstructure of Cognition* (Cambridge MA: MIT Press, 1986. For our more extended comments on the "two AIs," see Seymour Papert, "One AI or Many" in *Daedalus*, 117, No. I (Winter 1988): 1-13 and Sherry Turkle, "Artificial Intelligence and Psychoanalysis," in *Daedalus*, 117, No. I (Winter 1988): 241-68.

45. Louis Althusser, "Freud et Lacan," *La Nouvelle Critique* Nos. 161-2 (December, January, 1964-65).

46. Carol Gilligan, *In A Different Voice*, p. 23.

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MATHEMATICS AS A HUMANISTIC DISCIPLINE

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It is said that the motto which adorned the doors of Plato's Academy advised, "Let no one unversed in mathematics enter here." For the Greeks, the study of mathematics furnished the finest training field for the mind. It occupied an esteemed place in the curriculum of Plato's Academy. No person was considered educated if he did not know mathematics.

Mathematics has not retained such a dominant position in modern education. However the increasing importance of the discipline in the twentieth century because of the computer and related technologies has generated renewed interest in its study. Today's academies universities and colleges - are requiring the study of mathematics for more students than ever before. In 1983, the largest public university system in the country - The California state University System - established a mathematics course as a graduation requirement for all students at any of its nineteen campuses. Liberal arts colleges throughout the United States are reinstating mathematics requirements for all majors. Mathematics plays a central role in the curriculum of most universities and colleges throughout the world today.

With the exception of specialized universities like MIT and CalTech, Mathematics Departments nationwide are considered service departments, offering the majority of their courses to students in fields other than mathematics. Such departments as Engineering, Computer Science, Business, the Physical Sciences, and the Social Sciences include a core of mathematics courses which are essential to meet the mathematical needs of their majors.

A conference was held at Williams College in Williamstown, Maryland, in 1982, to evaluate the mathematical needs of students in other disciplines and the implementation of curricula to meet those needs. Papers were presented by mathematicians and mathematics educators identifying requirements for specific fields: Isaac Greber for Engineering, William Scherlis and Mary Shaw for Computer Science, Stanley Zionts for Business, Jack Lockhead for the Physical Sciences, and Robert Norman the Social Sciences.

In Engineering, the required mathematics core represents topics that are regarded as fundamental mathematics as well as those topics which students are expected to know in order to solve engineering problems in other courses. They include the understanding of limits, functions, complex variables, integral equations, and the calculus of variations. The central analytical tool of the engineer is the ability to derive, solve, and understand differential equations. Other mathematics courses which are becoming more important to engineering majors with the advent of the computer are probability and statistics, Boolean algebra, and numerical methods.

The requisites for Computer Science are many. The modes of thought which characterize mathematics are crucial to prospective computer scientists. Probably the most important contribution mathematics makes is teaching these students how to reason abstractly and problem solve.

Until the 1950's the role of mathematics in Business Programs was minimal. About that time, the discipline called Management Science was incorporated into business schools, and more and more

quantitative techniques were employed in business classes. Mathematical applications to management problems are abundant. Models that include linear programming and computer simulation are widely used in industry. Students enrolled in Business Programs need a mathematics background which includes algebra, beginning differential and integral calculus, matrix algebra, linear programming, and simulation, all with applications in the field.

Mathematics is the language of the Physical Sciences and has traditionally been at the center of all programs in those fields. Calculus is essential to the study of the physical sciences. What students in those disciplines need most from their mathematics preparation is a basic understanding of variables and functions and how to express them in mathematical language.

Mathematics preparation for the Social Sciences is somewhat different from that for prospective physical science, mathematics, or engineering majors. In fact, unlike these disciplines, there is no generally accepted body of mathematics that every social science student is expected to know. Yet social scientists, when questioned, will indicate a wide range of mathematics that they find useful in their fields. These include probability, manipulative algebra, computing, statistics, calculus and differential equations, combinatorics, linear algebra, sets and relations.

In each of the above-mentioned fields the validity of prerequisite courses in mathematics is unquestioned to provide the appropriate background for students to pursue subsequent coursework in their disciplines. The service function of mathematics is clearly defined and well justified.

However, **teaching** mathematics as a service course has some built-in liabilities. The experience of teaching mathematics to students for whom the subject is not their major field is often less satisfactory than teaching mathematics majors. Non-mathematics majors approach the subject from a perspective that is different from those for whom mathematics is their major field. Non-majors are more interested in what mathematics can do (within the limited focus peculiar to their discipline) than what mathematics is. They generally study mathematics only because it is useful in preparation for their fields, and this sometimes impairs their enthusiasm for the subject.

Is utility **sufficient** as a motivation for learning? Some mathematicians (like C.F. Gauss who has been credited with boasting that pure mathematics is useless) will ask if utility is even a **necessary** condition for study. Others (like Philip Davis and Reuben Hersh) will question if utility can be measured:

> . . .the meanings of the expression "mathematical utility" embrace aesthetic, philosophical, historic, psychological, commercial, scientific, technological, and mathematical elements. Even this does not include all possible meanings. . . One can distinguish between utility within the field itself and utility to other fields. Even with these subdivisions, the notion of utility is exceedingly slippery (1981, p. 80).

Measuring the mathematical utility of a course is often not a realistic task. However, even if one determines that what is presented in a mathematics class is useful for some purpose, the learning experience is incomplete if utility is the **only** focus. A fixed goal of learning a specified syllabus may be pursued, and perhaps attained. But if at no time the issues of how and why the goal is important are discussed in the classroom, the students get no perspective on the mathematics being taught. They focus only on the mechanics of the discipline. They have no real interest in concepts, in learning what mathematics is in addition to what it does. For students in Engineering, Computer Science, Business, and the Physical and Social Sciences, prerequisite mathematics coursework should serve, in addition to its utilitarian goal of preparation for work in the field, to provide them with some context of how the mathematics they find useful has come to

be, how it relates to disciplines other than theirs, and how it affects their lives.

These goals become even more important when mathematics course requirements are extended to students in non-math related fields: the humanities, the arts, etc. As Lynn Arthur Steen indicates in the January, 1986 issue of Focus, the newsletter of the Mathematical Association of America: "For students in the arts and humanities, mathematics is an invisible culture - feared, avoided, and consequently misunderstood.n These students see no utility in learning mathematics. The biggest challenge in teaching liberal arts students is enabling them to recognize mathematics as a creative, human endeavor, as a "humanistic discipline".

Professors are frequently unprepared to meet such a challenge. From experience, they know that mathematics is an ever-changing process which permeates our personal as well as professional lives, but they often don't know how to teach that to students. Although they recognize the intrinsic value of mathematics, worthy of study as a major field, necessary in providing the essentials for other disciplines, appropriate as a component of liberal education, they have not been trained to convey these ideas to students in the classroom. Courses which emphasize the humanistic aspects of mathematics are not part of the traditional curriculum for a Ph.D. in mathematics. Most mathematicians teaching in colleges and universities have never even studied the pedagogy of their discipline. They teach mathematics as they do mathematics. They are excellent mathematicians, but sometimes they lose sight of the need to present their subject in its human context.

This is not true of the mathematics community in general. Mathematicians like Morris Kline, Paul Halmos, Lynn Arthur Steen, Anneli Lax, Alvin White, R.J. Moore, Reuben Hersh and Philip Davis are just some of many who have written extensively about teaching mathematics humanistically. The journals regularly include articles with this perspective. For example, the American Mathematical Monthly (April, 1982) featured an article in the section entitled "The Teaching of Mathematics" edited by Mary and Robert Wardrop which places mathematics squarely in the center of human development:

> Mathematics has played a central role in the development of modern civilization. It has been essential not only to the growth of science and technology, but has had profound effects on philosophy and other forms of thought as well (Page 270).

Mathematics has a place in history. It is part of the human experience. There are few who will deny that this historical perspective affords all students good reason to learn mathematics. Yet for a variety of reasons this aspect of mathematics is often ignored in the teaching of the discipline. Students, in huge lecture halls facing blackboards strewn with Greek symbols, often get no evidence that mathematics is a growing, living, creative process, changing with time. Presentation of topics to comply with crowded syllabi and thick, heavy textbooks cluttered with formulas, theorems and proofs, most times reinforce the students' belief that mathematics is a discipline existing outside of time and human activity. This view distorts mathematics and discourages interest in the subject. As Davis and Hersh point out in Descartes Dream:

> A detemporalized mathematics cannot tell us what mathematics is, why mathematics is true, why it is beautiful, how it comes to be, or why anybody should care a fig about it. But if one places mathematics squarely within human time and experience, it becomes a warm and rich source of possible meanings and actions. Its ultimate mystery is never dispelled, yet it is exhibited as one of the primary creations of the human intellect (1986, p. 201).

When students see mathematics as a human endeavor, they have more than utilitarian reasons to understand why they should learn it. At the annual meeting of the Mathematical Association of America in San Antonio, Texas, in January, 1987, Anneli Lax of the Courant Institute so indicated: "Humans retain what they learn best if they can put new material into a human context, e.g. connect it to past experience, future aspirations, previously acquired mental structures."

Mathematics instructors need to help students recognize mathematics as a human process, integral to life, not fragmented from it. Part of this involves a fostering and understanding of mathematics in relation to other disciplines. According to Jean Piaget, all learning is interdisciplinary. This basic fact has often been ignored in the teaching of mathematics. For too long students have studied mathematics as a closed unchanging subject in isolation from other disciplines.

At the San Antonio meeting, Lax spoke about the effect of a fragmented approach to mathematics:

The isolation of mathematics from the rest of life causes further fragmentation of mathematics itself into arithmetic, algebra, geometry, etc., and these subdisciplines get cut up into sections or modules or skills to be mastered, tested and forgotten.

Attempts to free mathematics from this isolation and fix it firmly as an integral part of the human experience are currently being pursued in universities throughout the country. The NEXA program at San Francisco State University, for example, uses an interdisciplinary approach to instruction which involves faculty from different departments team-teaching mathematics classes. There are concerted efforts nationwide to integrate real-life applications into mathematics instruction. There is renewed interest in teaching what George Polya described as "the most characteristically human activity . . . problem solving" (Polya, p. ix).

But perhaps the most significant effort to revitalize the teaching of mathematics is being undertaken today by a network of mathematicians and mathematics educators devoted to what is described as "the humanistic dimensions of mathematics". Alvin White of Harvey Mudd College hosted a conference in Claremont, California, in March 1986 to try to develop a concrete definition for the phrase "mathematics as a humanistic discipline" and to discuss appropriate pedagogical goals for teaching relative to that definition. Participants in the conference assigned a variety of different, but ideologically complementary meanings to the phrase which gave rise to a series of educational objectives. In a monograph discussing the conference, White wrote:

> The concept of "mathematics as a humanistic discipline" is not as well defined as a geometric series or a triangle, but it is more evocative. Many mathematicians who have heard the phrase are not troubled by the lack of a succinct definition, but are excited by the richness of the fruitfulness that they anticipate. . .the concept, even if ill defined, challenges traditional ways of teaching and learning mathematics at all levels.

The thirty-six conference participants identified two pedagogical goals which result from defining mathematics as a humanistic discipline:

1) teaching mathematics humanistically - altering the nature of the teaching and learning environment, and

2) teaching humanistic mathematics - reconstructing the curriculum and the discipline of mathematics itself.

The first goal, teaching mathematics humanistically, seeks a student-centered classroom, placing the student in the position of inquirer rather than passive learner. It encourages the development of a community of learners with both professor and

students learning from each other. To teach mathematics humanistically, the professor must recognize the emotional climate of the learning environment and provide appropriate access for students to engage in participatory learning within that environment.

The second goal, teaching humanistic mathematics, strives "to integrate the humanistic elements of science with the mathematics" (White, 1976, p.245) in the classroom. It advocates curricula which relates mathematical discoveries to the human beings who made them. It encourages the exploration of the relation of mathematics to other disciplines, and to the culture in which it is embedded. To teach humanistic mathematics, the professor must allow the students to experience the curriculum within the context of its past, present, and potential impact upon the world in which they live.

With the dual themes of teaching mathematics humanistically and teaching humanistic mathematics in mind, the conferees outlined fourteen desirable objectives for the classroom. Among them are to provide students with

- an appreciation of the fundamental interrelationships of all knowing.

- a curriculum which relates mathematics to other areas, such as science, technology, humanities, and ethical issues, including personal ethics.

- an understanding of the human dimensions that motivate discovery, such as competition, cooperation, the urge for holistic pictures in contrast to pieces.

The thread which unifies all the pedagogical goals outlined at the conference is perhaps best expressed by Alvin White in his article Beyond Behavioral Objectives:

> . . . our guidelines and teaching objectives should not have as their major target or focus the mastery of facts and techniques. Rather the facts and techniques should be the skeletal framework which supports our objective of imbuing our students with the spirit of mathematics and a sense of excitement about the historical development and the creative process. The concepts and relationships of mathematics should be presented as the building blocks of this magnificent edifice created by the human imagination (White, 1975, p. 850).

No mathematician would deny these goals in teaching. Not only are they not controversial, they are not even new ideas. Early in the twentieth century, curriculum interest groups called "humanists" led by Charles William Eliot and William Torrey Harris recognized mathematics as "an important part of our Western cultural heritage" (Stanic, 1986, p. 192). Mathematics educators, predominantly at or near the University of Chicago, advocated breaking down the barriers between mathematics and other disciplines and between the various areas within mathematics. They promoted the teaching of mathematics within the context of human experience.

Certainly, those who have read Morris Kline, who have studied Piaget, who have focused on the teaching of mathematics as well as the doing of mathematics, are most likely pursuing these objectives in their classrooms. But what is obscuring those goals for others?

In Mathematics Tomorrow, Peter Hilton outlines those forces which he believes undermine effective mathematics education. He includes the quest for instant satisfaction (ignoring the fact that education is a slow, gradual, cumulative process with rewards that are largely long-term); the view that the prime purpose of education is to guarantee a high material standard of living (affirming the utilitarian goal of learning); and the mechanical approach to teaching elementary mathematics which emphasizes rote calculation and memory dependence (compounded by the fact that many teachers are not skillful in mathematics, do not enjoy the subject

themselves, do not feel comfortable with it, and convey these attitudes to the students). He is particularly critical of standardized tests:

> These tests, superimpose a degree of artificiality on that which is already present in the curriculum. They force students to answer artificial questions under artificial circumstances; they impose severe and artificial time constraints; they encourage the false view that mathematics can be separated out into tiny water-tight compartments; they teach the perverted doctrine that mathematical problems have a single right answer and that all other answers are equally wrong; they fail completely to take account of mathematical process, concentrating exclusively on the "answer" (1981, p. 7 9) .

In this quote, Peter Hilton focuses on how standardized tests distort the teaching curriculum and promote those very practices which discourage learning: failure to make mathematics real for the student, fragmentation of the discipline, disregard for mathematics as a creative process.

But standardized tests are only a symptom of traditional ineffective approaches toward teaching mathematics. Even when the inappropriateness of standard pedagogy is exposed, significant change does not occur. What is the reason for such inertia in the system, for what Hilton calls "the remarkable stability of those practices which militate against effective mathematic education" (1981, p. 79)? According to the title of an Opinion article in the January 21, 1987, Chronicle of Higher Education "Mathematics Teachers Are Too Lazy to Change Their Ways; as a Result Teaching is Stagnant." In that article, James T. Sandefur states:

> Mathematics, like other disciplines, is evolving, but the teaching of math is stagnant. Instead of developing new courses, as professors do in other disciplines, we continue to teach the same topics that were taught to us. . . There has been little outside pressure for a change in the way we teach math. For one thing, non-mathematicians rarely know enough math to be able to make concrete proposals. Also many of them are intimidated by mathematicians who tell them that change is impossible and suggest that such a proposal only shows their ignorance. As a result, mathematicians have been able to dictate what is covered in math courses and to resist any pressure for change. (Sandefur, p. 44).

There is some truth in what Sandeful is saying. More than laziness, however, the culprit is fear. In an article in the Education journal, Alvin White discusses the reasons why he found it difficult to deviate from conventional approaches to teaching Calculus when he was considering experimenting with a humanistic alternative: "In addition to having to contend with the skepticism of my traditionalists colleagues, I also had to deal with my own self doubts" (White, 1974, p. 132). It is as Carl Rogers indicates in Freedom to Learn:

> I believe that all teachers and educators prefer to facilitate . . . [a] meaningful type of learning. . . Yet in the vast majority of our schools, at all educational levels, we are locked into a traditional and conventional approach which makes significant learning improbable if not impossible. . . It is not because of any inner depravity that educators follow such a self defeating system. It is quite literally because they do not know any feasible alternative" (Rogers, 1969, p. 5).

Mathematics professors must not only identify the problems in traditional mathematics education, but they must provide alternatives - practical ways courses can be taught, different hypotheses upon which instruction can be built, new goals for which teachers and students can strive. They must also create a climate receptive to such alternatives. If the present network of humanistic mathematicians and mathematics educators which is assembling can do this, they will succeed in ways that their early twentieth century counterparts did not. The evi-

dence to date is promising. The new "humanists" are seeking to provide concrete alternatives. They are generating interest throughout the country in teaching mathematics humanistically. They are attempting to illustrate why and how mathematics should be taught to all students.

Once mathematics is recognized as part of the human experience, the approach to its teaching will seek to demonstrate how mathematics has evolved, how it works in our present environment, and what are its possibilities for our future. It will focus on what is beautiful, creative, and imaginative in mathematics, stimulating students to want to discover more about the subject, and its relationship to other disciplines. very few persons choose mathematics as a career. Mathematics classes can and must engage the non-mathematicians without losing the prospective majors. George Polya claims that to be justified, mathematics courses must conform to two principles:

> First, each student should be able to derive some profit from his study irrespective of future occupation.
> Second, such students as have some aptitude for mathematics should be attracted to it and not get disgusted with it by ill-advised teaching (p.122).

Lynn Arthur Steen describes mathematics as an "enabling force" (Steen, 1986, p.l). Mathematics professors must help students understand mathematics in its historical perspective, with its creative present and imaginative future. It is these humanistic aspects of mathematics which will provide the approach to its teaching and the reasons for its study.

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WHAT IS MATHEMATICS AND WHY DON'T THEY KNOW THAT?

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I have become increasingly dismayed regarding the image the populace has of mathematics. I guess my first awareness of this was when I began teaching a survey course for liberal arts students. I begin my course by asking the students to write a brief essay on what, in their minds at the time, is mathematics. Invariably, I get statements about the "important" uses of mathematics in making change, balancing checkbooks, and similar mundane tasks or vague statements about how extremely important mathematics is without any indications of how or why. Even though they had perhaps nine or ten years of study of mathematics in school, their idea of mathematics was very limited. Perhaps one might have expected that, and I have tried, occasionally with success, to instill in these students a broader view of mathematics, and an understanding of mathematics as an integral part of our culture, and the part mathematics plays in the development of culture. This year, I had an opportunity to teach an upper division mathematics course at a nearby liberal arts institution. The class consisted almost exclusively of mathematics majors. Before delving into the course of study, I thought we would begin with a discussion of the meaning of mathematics. The entire contribution of this class to my intended class discussion was that mathematics "is the science of numbers". Now this is getting serious, folks. These are mathematics majors, nearing graduation from college, on their way to teaching high school math or entering graduate school, whose only concept of mathematics is that it is the science of numbers.

Why this paucity of knowledge of what mathematics is? John Lucas, in his paper presented at the 1989 MAA-AMS joint meeting in Phoenix implicitly, but correctly, I believe, laid the blame on the way mathematicians taught. This can be illustrated with a discussion I had with a bright student in one of my calculus classes. He came to my office several times with questions. The nature of his questions made clear that there was a lack of understanding of basic mathematical concepts. When I queried him on the meaning of these, he said he wasn't used to thinking about mathematics in that way. Then followed this conversation. "What did you do in your high school math classes?" The. teacher did a couple of problems on the board, and then we did a bunch of problems." "And then you checked your answers in the back of the book?" "Yes." "Did you ever talk about what any of this meant?" "No." A letter that appeared in the "From Our Readers" section of UME Trends, January, 1990, shows that this is not a local problem. This letter, written by a community college instructor, states,"If a student can be given an equation such as $2X - 7 = -6$ and came back with $X = 1/2$ then they (sic) know algebra,...If I can teach a student to 'get' $X = 1/2$ using symbol manipulation, then I have fully succeeded." Wrong, Mr. Brown, absolutely wrong! You have not succeeded if the student doesn't know what it means to have the solution of the equation or cannot articulate this meaning. We cannot assume that students construct meaning to this exercise. I have asked students in classes from algebra to calculus and statistics (yes, and even differential equations) what a solution of an equation is. Very few can tell me. They have learned to manipulate symbols to come up with what they call,

"the answer", but they have no concept of what this means.

Not only are we, as teachers of mathematics, responsible for the development of this myth that mathematics is symbol manipulation, but we add to misunderstanding in other ways. In a class I was visiting recently, the result of simplifying an expression was 21/6 . In response to a student question, "Is this as far as we can go?", the instructor replied, "No, we can actually get the number if we want to." He then proceeded to describe how to use the calculator to "actually get the number". Obviously, to him 21/6 is not a number, but the decimal approximation from the calculator is "a number". It is apparent that this person learned his mathematics in a small white cave in the mountains of the island of Samos in the 6th century B.C. Current thought is to accept irrationals as numbers, Jerry.

Perhaps, since the title of this paper includes the query, "What is Mathematics?", an attempt should be made to answer that. Granted that as with the three blind dudes and the elephant, there is reason for divergent views, and granted that Courant and Robbins have clearly answered this extensively with a book of 500 pages, perhaps, in the spirit of the nature of the language of mathematics, we can come up with something more precise than the former and more concise than the latter. I would like to nominate, at least as a starting point, the definition found in the Lawrence University catalogue.

> Born of man's primitive urge to seek order in his world, mathematics is an ever-evolving language for the study of structure and pattern. Grounded in and renewed by physical reality, mathematics rises through sheer intellectual curiosity to levels of abstraction and generality where unexpected, beautiful, and often extremely useful connections and patterns emerge. Mathematics is the natural home of both abstract thought and the laws of nature. It is at once pure logic and creative art.

What, then, can we do to overcome this vast misconception of mathematics? First of all, while I don't advocate complete agreement on a definition of mathematics, I do promote discussion in the mathematics community on what is the view of our discipline we would like to promote and how we can bring that about. An obvious starting point is in the teaching of mathematics - at all levels. We need to think seriously about what mathematics we should teach and how we should teach it. The course that we laughingly call "college algebra", and is a requirement at many institutions, has degenerated into a course primarily in symbol manipulation. Is this a reasonable and desirable demand in an age when machines can manipulate the symbols? It seems incongruous to me that people who insist that we must utilize the latest technology in our mathematics classrooms are still teaching classes and advocating classes that stress symbol manipulation. Are we replacing how to manipulate symbols with how to push buttons to get the machines to manipulate the symbols for us? I agree with Raymond Wilder who said, long ago, Mathematics was born and nurtured in a cultural environment. Without the perspective which the cultural background affords, a proper appreciation of the context and state of present-day mathematics is hardly possible." It behooves us to provide all our students with the culturological perspective of Wilder's reference.

The Humanities, I believe, have come up with a particularly insightful document from which we can benefit. The National Endowment for the Humanities, in their document "50 Hours", have come up with a core curriculum for college students.

18 hours: Cultures and Civilizations

I. The Origins of Civilization: a one-semester course that considers the beginnings of civilization on various continents. 3 hours.

II. Western Civilization: a one-semester course that considers the development of

Western society and thought from Periclean Athens through the Reformation. 3 hours.

III. Western Civilization (continued): a one-semester course that considers the development of Western society and thought from the Reformation into the twentieth century. 3 hours.

IV. American Civilization: a one-semester course that traces major developments in American society and thought from colonial times to the present. 3 hours.

V and VI. Other Civilizations: two one-semester courses to be chosen from the following civilizations of Africa, East Asia, Islam, Latin America, South Asia. 6 hours.

12 hours: Foreign Language: a two-year requirement; it is recommended that students fulfill this requirement by taking more advanced courses in a language they have studied in high school.

6 hours: Concepts of Mathematics: a one-year course focusing on major concepts, methods, and applications of the mathematical sciences.

8 hours: Foundations of the Natural Sciences: a one-year laboratory course that focuses on major ideas and methods of the physical and biological sciences.

6 hours: The Social Sciences and the Modern World: a one-year course that explores ways in which the social sciences have been used to explain political, economic, and social life, as well as the experience of individuals, in the last 200 years.

I am not in a position to make recommendations regarding the entire proposal, but I think it is very well done and has great possibilities. I do recommend, however, that we study the mathematics component and seriously consider its basic tenets.

The mathematics component is described.

Concepts of Mathematics: 6 hours

A one-year course focusing on major concepts, methods, and applications of the mathematical sciences. Students will explore such topics as shape, quantity, symmetry, change, and uncertainty and consider such fundamental dichotomies as discrete and continuous, finite and infinite. Theoretical advances from the ancient to the contemporary will be considered, as well as applications in such areas as business, economics, statistics, science, and art. Students will be introduced to ways in which computers pose and help solve theoretical and practical problems.

I will attempt to acquaint you with the flavor of the proposal with a few quotes. In discovering mathematics, "It is at once the most speculative and the most practical of disciplines, with ancient roots in Pythagorean mysticism as well as in Babylonian commerce and Egyptian surveying." "Students without knowledge of the range, diversity, and power of mathematics are, as Mark Van Doren once put it, 'ignorant of a mother tongue'. "In response to their quote of a National Research Council's Everybody Counts, "Today's world is more mathematical than yesterday's, and tomorrow's will be more mathematical than today's", they have stated, "To participate in a world when discussion about everything from finance to environment, from personal health to politics, are increasingly informed by mathematics, one must understand mathematical methods and concepts, their assumptions and implications." Based on Lynn Steen's statement, "Minimal mathematical and statistical literacy is crucial, but the level of this

literacy is too low to warrant claim that it can represent mathematics in a core curriculum. Numeracy should be required as a prerequisite skill, not as a core subject.", they conclude, "Entrance requirements can ensure that students have adequate preparation in high school. If remediation is necessary, it should not be addressed by the core." This is consistent with a complaint I have of many survey of mathematics courses and textbooks. They should not contain remedial material, and college credit should not be given for remedial work.

There is a statement in the report, however, that makes me wonder if these people really understand the situation.

> "A required course of studies - a core of learning - can ensure that students have opportunities to know the literature, philosophy, institutions, and art of our own and other cultures. A core of learning can also encourage understanding of mathematics and science..."

Do they not realize that mathematics and science are important integral parts of our culture? Apparently not. It seems we have a bigger job than just educating our students.

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WRITING HUMANISTIC MATHEMATICS

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SUMMARY

This article suggests some guidelines for writing mathematics texts so that students can learn by reading.

INTRODUCTION

One thing that distinguishes the teaching of mathematics from the teaching of other subjects is that the teacher rarely gives the students a list of references which they are supposed to find and study on their own. Mathematics undergraduates generally study from their notes and from handouts. Even the class text is often used only as a convenient source of exercises. The teacher is considered as a necessary interpreter between the text and the student. In a survey of first year students to investigate the attitudes to and past experiences with mathematics, Landbeck (1990) found that of the 65% who reported negative experiences, most attributed these bad experiences to bad teachers. I replicated part of this survey, asking students to describe their past experiences with mathematics and again most students reported on teachers or teaching and the overwhelming majority of experiences were negative, Hubbard and Kelly (1990).

In an essay about writing in the mathematics curriculum, Gopen and Smith characterize the prose in mathematics textbooks as follows:

> "The model of good mathematical prose most available to students would seem to be their textbook; however, the writing found there is often less than effective, and the students often avoid reading it. We trace the blame for this to generations of combined efforts from two quarters: On the one hand, authors and publishers produce textbooks that do not have to be read before doing the exercises; on the other hand, teachers acquiesce by agreeing that this is the way mathematics ought to be taught. What prose there is has tended to be introductory, apologetic, and self-justifying. It implies that the real importance lies not in the students' ability to conceptualize, but rather in their ability to function."

Another factor which affects the writing of mathematics textbooks raised by Dudley, is the need for the text to be absolutely mathematically correct and complete so that it cannot be criticized by mathematical colleagues. This requirement results in texts written for mathematicians, not students.

The way in which mathematics texts are written seems to be one reason why students do not learn mathematics by reading.

TEACHING STUDENTS TO READ MATHEMATICS

Having read the literature on teaching students to become independent learners and being acutely aware that this was not what was happening in mathematics departments I decided that as a first step I would try to teach my students to read their mathematics text, Hubbard (1990). Another reason

for trying to help students to read mathematics was the fact that at the present time university students in Australia take lectures in groups of several hundred and tutorials are overcrowded or non- existent. Students are increasingly being forced to become independent learners, not for pedagogic reasons but because of the financial constraints being placed on universities. In the process of trying to help the students to read I looked very critically at the structure of their texts and at the features that made them so difficult to read.

There is a great deal of dispute about measures of readability in ordinary English text. Should an index of readability be based on sentence length, vocabulary, number of ideas in a sentence and so on. When it comes to measuring the readability of mathematics text, Howard (1977) the problem is compounded because there is symbolic as well as ordinary text and the ordinary text is not really ordinary.

To demonstrate the problems involved in measuring the readability of mathematical text, even without considering symbols, here are some definitions of "function" from standard texts followed by an ordinary sentence of about the same length, complexity, vocabulary etc.

> "A function from a set D to a set R is a rule that assigns a single element of R to each element in D."
> Thomas and Finney (1988)

> "A function is a collection of pairs of numbers with the following property: if (a,b) and (a,c) are both in the collection, then b = c; in other words,the collection must not contain two different pairs with the same first element."
> Spivak (1967)

> "A function is a rule or correspondence, relating two sets in such a manner that each element in the first set corresponds to one and only one element in the second set. In other word a functional relationship is a single-valued relationship"
> Zill (1988)

> "I am going along to see my grandmother this afternoon because she called me to say that she was not well so she may need some assistance around the house or a lift to the doctor's."

One obvious reason why the definitions are more difficult to read than the grandmother sentence is because they deal with abstractions. There are other reasons why mathematics textbooks are hard to read.

(a) The language is precise. Every word or phrase has an exact meaning which may or may not correspond to the meaning in everyday speech and writing.

(b) The writing is concise. Definitions use as few words as possible which means that there is no redundancy. If a single word is not understood or is overlooked the definition can be misunderstood or not understood at all.

(c) The development is highly sequential. Each new abstract concept is defined in terms of earlier abstract concepts so that layers of increasing abstraction are continually being created.

Added to this is the difficulty of reading the symbolic language and the fear that many students have of complicated formulae.

The conventional logical structure of mathematics text, definition - theorem - proof example, is not conducive to learning by reading. The following quote from Demana and Waits (1988) explains why this is so and suggests a different approach.

"Formal language is kept to a minimum throughout the lessons. Definitions are not given until the conceptual base and need for the definition has been developed. Ordinary or previously used language is used until a need for more precision arises. This stems from two different factors. First, we believe that mathematics is made to appear more abstract than it really is by the use of formal definitions that do not have a proper base of meaning or need. A large number of students are lost to mathematics because it is made to seem unnecessarily abstract by the use of nonmotivated definitions. Second, stating definitions first and then progressing to the problem situations is inconsistent with our use of problems as means to the new mathematical ideas. Problems are found in familiar, known contexts described with words already in the vocabulary of the students. The ideas stem from the development of solution processes and methods. Then the ideas are refined and stated precisely. This means that definitions arise naturally."

Because of the difficulty of the conventional structure, many students who cannot understand the definitions simply omit them and focus on the examples which they try to mimic in their exercises. This often degenerates into replacing one group of numbers with another group with no attempt to understand the process involved. The definitions and proofs are ignored unless they are to be examined, in which case they are memorized.

The "improvements" produced by instructional designers, such as isolated words in the margin and shaded boxes which contain important facts, encourage the student to concentrate on these words rather than on making connections between them. These devices make "finding the formula" easier but do not make reading for understanding any easier.

CRITERIA FOR WRITING READABLE MATHEMATICS

Having achieved only limited success with teaching students to read standard texts (Hubbard, 1990), I decided that some affirmative action was required to produce mathematics text that was more readable. Reading the symbolic language of mathematics is inherently difficult but by making the ordinary language part of the text more readable I hoped to reduce the problems presented by the symbols. From my observations of students' difficulties with reading mathematics, I produced a set of guidelines which I proposed to put into practice. The guidelines address two issues, language and pedagogy.

LANGUAGE GUIDELINES

Other things being equal, short sentences are easier to read than long sentences, so most sentences should contain just one concept. If a sentence starts to get complicated, break it up into two shorter ones.

If there is a simple familiar word which means the same thing as a more erudite sounding word, choose the simple word. For example determine and evaluate can often be replaced by find and examine, and consider can be replaced by look at.

Use informal language while examples are being investigated and new concepts are being developed. Students can't be expected to understand a definition if they do not understand the technical terms involved. The concept has to come first, then when it is established it can be given a name.

Use only those technical terms that are really necessary. Often technical jargon seems to be used deliberately to create the "insider/outsider" phenomenon that Tobias describes as a factor in math anxiety.

Use the active rather than the passive voice to give the text immediacy. This effect can also be obtained by using I and you instead of the formal we. These devices help to counteract the formality and impersonality of the symbolic part of the text. It helps the student to realize that other human beings like themselves are writing the material.

Make the text redundant by explaining difficult ideas twice or even three times using different words in the hope that one of the explanations will reach each student. This is just what we do when explaining something difficult verbally.

GUIDELINES FOR PEDAGOGY

Try to find contexts for the examples which are part of the everyday experience of the students so that they can relate to them easily. Alternatively where this is not possible try to invent imaginative examples which might excite the students' curiosity or sense of humor. Most "real life" examples in standard texts are very uninteresting. Introduce new concepts by way of interesting examples so that the concept and the example become associated in the student's mind. The concept is then readily recalled by reference to the example.

Instead of dealing with a topic in its entirety in one chapter, introduce it at an elementary level the first time and return to it several times always revising and building on what has gone before.

Do not assume that the reader is an empty bucket waiting to have words of wisdom poured into it. Every student already has some mathematical concepts and possibly misconceptions in her head so it is worth trying to appeal to these in the hope of finding something to attach new concepts to.

Warn students about common misconceptions or use them as counter-examples when a new idea or formula is explained.

Resist the temptation to "tell it all", i.e. to include details which are not necessary in an elementary treatment of the subject, even at the expense of leaving a topic incomplete. Students have great difficulty recognizing main concepts from less important details. Often they learn details which are of no use without the concept to which they belong. For example, statistics students always seem to remember one-tailed and two-tailed tests even when they have no idea what is involved in testing a hypothesis. The word "tail" and the diagram that goes with it are easy to recall. The logical steps involved in testing a hypothesis are difficult to follow, so students don't learn them.

Ask questions which enable students to check their understanding as they are reading. An experienced reader constructs his own questions and if he can't answer them reads a passage again. Students need to learn to read in this critical way. So that the student does not get frustrated, when he is unsure of the appropriate response, provide the answers to these questions immediately or at the end of the chapter.

Do not provide summaries. By providing summaries you encourage the student to read and memorize without understanding. Summaries are most useful when they have been constructed by the student before or after she has worked some exercises.

WRITING TO THE CRITERIA

I have put these criteria into practice in two quite different areas. One is a distance education bridging course for adults in remote areas of Queensland who wish to prepare for university studies. Distance education students do not have a teacher to act as an intermediary between them and mathematics and to a large extent have to depend on print materials. Readable text is absolutely vital in this context.

The second is a first-year undergraduate statistics course for non-mathematics majors. What you might ask, is the point of producing yet another first level statistics text? The main point is that I wanted the students to learn about those aspects of statistics that still need to be done by humans and to relegate the routine calculations to a computer. In order to learn statistics in this way the students have to use a statistical package and some of these packages are powerful tools for investigation, animation and simulation. Since a thorough search failed to find any texts which approached the subject from this viewpoint, I had to write my own and this gave me the opportunity to apply my writing criteria.

THE BRIDGING COURSE

People who undertake mathematics bridging or remedial courses come from mathematically deprived backgrounds. If in addition, they are isolated, with only the possibility of an occasional telephone conversation with a tutor, whom they have never met, providing them with mathematics text that they can read on their own is a tremendous challenge. Furthermore, the course starts with simple operations involving money and finishes with calculus and mechanics so it is enormous. A close-knit team of four*, also separated geographically by vast distances, has written the materials according to the above guidelines.

Symbolic notation is introduced slowly and carefully. The student is shown how every new symbol is pronounced and encouraged to say the words aloud. The student is continually reminded to think about the meaning of the symbols. The questions which check the students understanding of her reading are frequent and the student is given a few lines in which to answer these questions. Students are also asked to write down ideas and rules in their own words, to construct their own summaries and to express their feelings about their progress.

Because the course is for adults we try to use examples which they could find useful. The arithmetic in the first part of the course revolves around the theme of how to manage money because everyone has to do this. But there are also examples about sport, horse racing, cooking, arts and crafts, TV soaps and farming, because these are the pursuits of country Queenslanders. We have tried to avoid sex stereotyping by having men go shopping for clothes and women ordering fencing materials.

We have also tried to broaden our students horizons by setting mathematics in a historical or cultural context. Not the picture of Descartes with a few dull biographical notes underneath, which is standard in many texts and which students don't read, but something more imaginative like the following.

"You have just climbed to the top of the leaning tower of Pisa and in your excitement you drop you camera over the edge. This distance s is meters that it falls in t seconds is given by the formula

$$s = 4.9\,t^2$$

This formula was derived by Newton, building on the work of Galileo, who was a Pisan and who dropped things (not cameras) from the top of the tower in order to calculate gravitational force.

You would like to know the speed with which your camera will hit the groud. The height of the leaning tower of Pisa is about 55 m. (The height keeps changing as the tower does more leaning)......"

Later problems about velocity and acceleration refer back to the Leaning Tower problem which the student is unlikely to have forgotten.

The central concept of function is introduced very early and revisited many times. Domain and range are introduced in diagrams showing the relationships of characters in TV soaps. Thereafter almost every module asks students to recall their concepts of function via these examples and develops the ideas in greater depth.

Students are given opportunities to investigate the relationships between equations and their graphs and to try their hand at generalizing from special cases using a simple graphing package. The Queensland Government has established Open Learning Centers in virtually all rural communities and these are equipped with PC's.

This is a brand new endeavor so I cannot report on the success of the project at this time.

A READABLE STATISTICS BOOK

The problem with writing a first level statistics text is that many of the main concepts and formulae cannot be explained in terms of elementary mathematics. Most authors are forced into stating formulae and giving students numbers to substitute into them. For non-mathematicians, substituting into a formula may help to memorize the formula but does not help to give it meaning. I would go so far as to say that substituting into formulae temporarily dulls the students mind and prevents her from thinking about the meaning of the formulae.

Since in this situation the formulae cannot be satisfactorily explained, I have left as many as possible out. Of course this is not possible in those parts of mathematics where it is necessary to operate on formulae. It is for these purposes that formulae were invented. However, contrary to popular opinion and (I suspect) the opinions of many mathematics teachers, mathematics deals with abstract concepts and the symbols are merely a convenient way to represent the concepts. If it is possible to discuss the concepts without the symbols, no harm is done.

USING THE COMPUTER TO COMPUTE

For a long time statistics textbooks concentrated on the computational aspects of the subject because without doing the calculations it is not possible to produce any results. But even though computer packages have been available for many years this has had very little effect on statistics texts, even recent ones. Some authors acknowledge that the packages exist by appending printouts at the ends of chapters, hoping this will help to sell their books. All the formulae are still there and the examples show how to substitute the numbers into the formulae. The student is still expected to do all the substitution exercises. When he is finished, he is asked to type the numbers once more into a computer package and admire the printout.

By using a statistical package which already contains suitable data sets, the student is freed from the burden of entering the data and doing the calculations and is able to concentrate on the more important aspects of the subject.

USING THE COMPUTER TO EXPLORE

Statistics is concerned with the analysis of data. The first step in this analysis is investigating the data to find out what interesting information it contains. This aspect of statistical education was almost completely neglected in the past because without a computer to produce graphs and tables it was too time consuming to carry out. In all the standard texts students are given lists of numbers and told to do this or that with them. They are not asked whether the result of doing this or that has any meaning or whether it might have been better to try doing something else.

Using a computer, students learn to choose appropriate methods of investigating data and to think about the outcomes. They can observe the effects of changing scales, changing class intervals, detecting and removing outliners, transforming data

to produce linear relationships. They can also relate the results of statistical analysis to the impressions they formed from studying the data.

USING COMPUTER ANIMATION

There a number of packages, for example those produced by Bowman and Robinson, which contain animated displays. They show the building up of probability distributions from repeated sampling and the generation of the various sums of squares in regression and the analysis of variance. These are powerful visual aids to understanding complicated abstract processes. One particular procedure in this package which allows the user to move the regression line about on the screen and gives the residual sum of squares at each stage is not only a valuable learning experience but also an interesting game.

USING THE COMPUTER TO SIMULATE

An alternative to stating formulae without proof is to use simulation to show that what the formula states actually does happen. As I stated earlier, the key results such as the central limit theorem cannot be proved in elementary statistics courses. Many innovative ways of simulating these theoretical results to make them plausible appear in the Minitab Handbook but are not referred to in the standard texts so I assume they are not widely used.

Simulations of the distributions of sample statistics help to convince students that sample means and proportions do appear to have normal distributions with standard errors that approximate the theoretical values. Simulations can show that confidence intervals do contain the parameter about the right proportion of the time and that the level of significance does lead to the rejection of the null hypothesis in about the correct proportion of cases.

Simulation is also useful in teaching probability. Because the idea of probability is a part of our culture, most students already have some concepts and misconceptions about probability. The games from which they developed their probability concepts can easily be simulated to ensure that their intuitive ideas are consistent with the mathematical theory of probability. Textbooks which introduce probability through set notation help to ensure that the connections between "real life" probability and textbook probability are not made.

USING COMPUTER OUTPUT FOR DISCUSSION AND WRITING

The interpretation of computer output provides an excellent topic for discussion among students and tutors. I have students working in pairs in tutorial classes because this encourages informal discussion. By describing their results and comparing them with the results of others, students get practice in using the technical vocabulary. This is reinforced if they also submit written reports on their findings. Drawing conclusions from their investigations and analyses gives students opportunities to clarify their ideas by writing about them.

CONCLUSION

I have digressed somewhat from my writing guidelines in order to establish two points about my writing in statistics. The first point is that my approach to statistics is sufficiently different from the standard approach to warrant a different kind of text. The second point is that I wanted to demonstrate that by using a computer, the learning of statistics can become a truly creative human activity.

Returning to the writing guidelines, it is interesting to note that in response to surveys, almost all students found my writing very readable although they were not asked for specific reasons why this was so. The only guidelines the students were given for their writing were that they must use sentences and that they should take care with spelling and grammar. Nevertheless many students reverted in their writing to the standard textbook style. For example they would frequently preface their comments with, "It can be stated that" or "It can therefore

be concluded that" instead of just saying what they think. This result is hardly surprising, as these students would have been exposed to the language of mathematics textbooks for many years. It is going to take a long time to change the way mathematics is written.

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THE PROFIT MOTIVE:
THE BANE OF MATHEMATICS EDUCATION

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1. THE TEXTBOOK SCAM

For many years both students and professors have been complaining — sometimes loudly and sometimes quietly — about calculus textbooks. The bulky, heavy tomes — often running well over 1,000 pages — are overpriced, full of wide margins, multicolored drawings and other gimmicks, and are poorly written. In fact, they are virtually unreadable. Most students, after paying 60+ dollars for their 6-pound Illustrated Encyclopedia of Calculus, end up reading nothing but the highlighted formulas and assigned homework exercises.

Why have the publishers failed to respond to all the complaints? Why are the textbooks still almost identical to one another? Why is there so little experimentation? Why can't the size — and also the price — come down? After all, isn't the big selling point of our much-vaunted Free Enterprise System its supposed ability to respond to the changing needs and demands of the consumer?

If I were a believer in conspiracy theories — which I am not — I would be tempted to hypothesize that perhaps the different textbook publishers are not really separate competing private companies, but rather are all part of a single secret government bureaucracy whose purposes are to

(1) eliminate diversity,
(2) stifle innovation, and
(3) ensure that calculus students help pay to reduce the national budget deficit.

Although at first glance this conspiracy theory does appear to fit the data, it seems to me that the true explanation lies elsewhere — in the nature of for-profit publishing.

A few years ago I had the misfortune of being on my department's textbook committee at a time when we were considering a possible change in calculus textbook. During that year large numbers of salespeople would haunt the halls of the math department, buttonholing faculty members to argue the supposed merits of their company's textbook.

Although the sales reps knew nothing about teaching calculus, they took up a lot of faculty time. Meanwhile, they were receiving salaries and expenses from their company — all of which, of course, would be passed on to the hapless students in the form of price hikes for the calculus books.

In addition, the companies would send us mounds of glossy advertisements hyping their textbooks, usually written in a way that would insult the intelligence of anyone with a post-secondary education. In extreme cases the come-ons would approach bribery. I recall one company that gave out prizes (personal computers) at random to its list of professors who had agreed to examine their textbook. The junk mail and prizes obviously cost the companies a lot of money — to be passed on to the consumer in higher textbook prices.

As a member of the department textbook committee, I made a modest attempt to influence the publishers in the direction of sanity. I wrote to Prentice-

Hall, the publisher of a textbook that at the time was the leading contender to replace our earlier choice, explaining that a major consideration in our final decision would be the cost to the student. Echoing a suggestion that had been made in the *American Mathematical Monthly*,[1] *I* urged them to consider producing a two-volume, soft-cover cheap edition, the first volume of which would suffice for most first year students.

I received a reply from the Executive Editor for Science and Mathematics at Prentice-Hall, who wrote:

> If we were to decide to publish an inexpensive version... we might be able to reduce the cost to your students to some degree. However, some financial advisors here at Englewood Cliffs would try to dissuade me from going in this direction. They would argue that students are quite comfortable paying for movies, pitchers of beer, and ski weekends. The value of a calculus book is far greater than these other expenditures.

Prentice-Hall's stereotype of students with money to burn is undoubtedly based on the youngsters living in the posh suburbs of New York like Englewood Cliffs. But it does not accurately reflect the reality at a state university such as mine, where many students find the escalating cost of textbooks to be a financial hardship. Of course, when confronted with the arrogant let-them-eat-cake attitude of the people at Prentice-Hall, we decided to stick with our earlier textbook published by one of their competitors.

2. COUNTERATTACK AGAINST THE TEXTBOOK PROFITEERS

Despite our dislike of the commercial textbooks, out of habit most of us continue to use them, almost as if we were addicted. But we *do* have a choice. With a certain amount of effort and force of will we can, to borrow a phrase from Nancy Reagan, "Just Say No" to the textbook pushers.

The way to do this is quite straightforward. One simply puts together a packet of uncopyrighted material—lecture notes, exam packets, publicly available applications modules, etc. — tailored to the needs of the particular course. Such a packet can be photocopied and distributed to the students at a small fraction of the cost of a textbook.

This "guerrilla publishing" frees us and our students from the dictatorship of the textbook industry. Like guerrilla warfare, the strategy is based upon informal networks of volunteers, relying on flexibility, adaptability and ingenuity to outwit, outflank and outmaneuver a well-financed but cumbersome and bureaucratic army.

In my own department, this past Fall for the first time we taught beginning calculus without a commercial textbook, using instead a packet of material that I prepared. The packet contains homework sets, applications handouts, practice exams, and short heuristic explanations of everything. Multi-step word problems are heavily emphasized, especially those that arise from practical applications. The student is responsible for everything in these notes, without exception, and is not responsible for topics not covered in the notes.

The material for each quarter (10 weeks) runs to a little over 100 pages — it weighs 11 ounces — and it costs the student \$6. On campus it is copied and distributed by a non-profit student agency. The objective of all concerned is to provide a service to the students, not to rip them off.

Any reader who would like to examine this material can send me \$6 (for 10 weeks' material) or \$12 (for 20 weeks), and I'll buy and mail you a copy. Feel free to reproduce any parts of it you find useful. The material is not copyrighted, and I receive no money from it. My only compensation is the feeling of satisfaction that comes from imagining the expressions of horror on the faces of the textbook tycoons as they see the dropping sales figures for their calculus books.

Of course, there's nothing special about what I've done. Anyone who has been teaching calculus for a while and has developed material that seems to work well with the students can easily make it available to colleagues for nothing more than photocopying cost. Then any math department or individual instructor can collect this material from different sources, pick and choose what's appropriate for the particular course at hand, perhaps supplement it with a few of their own desktop-published modules, and with a modest effort develop their own tailor-made not-for-profit textbook. The textbook companies' loss will be our (and our students') gain.

3. THE COMPUTER CRAZE

Another area where lust for profit has distorted educational objectives is the so-called computer revolution in education. I think it's fair to say that computers have been shamelessly oversold as a panacea for the problems of math education. In fact, one sometimes gets the impression that the computer lobby has hijacked the educational reform movement. Because of intensive lobbying by the computer industry, many educators are putting a disproportionate amount of time, energy, and resources into finding ways to integrate computers into the curriculum. The National Science Foundation, for example, in its announcement of grants to improve the teaching of calculus, stipulated that preference would be given to proposals that involve the use of computers.

Of course, it is perfectly reasonable to think that there might be some appropriate uses for computers — just as earlier for television and movies — in the classroom. However, before going whole-hog into it, there are some fundamental questions that should be asked: What are the basic deficiencies in our students' background, and how can we remedy them? How can we impart good problem-solving techniques and a sense of discipline to youngsters who have grown up in a culture that emphasizes gimmickry, easy technological fixes, and 15-second sound bites on TV? What criteria should be used to evaluate success or failure of a pedagogical approach? Can we judge the effectiveness of an educational technique by whether or not the students are entertained by it and have the subjective feeling of having learned something? How can we change students' common perception of mathematics as something formalistic and mechanical? Will computers help here, or only make matters worse? How can we encourage people to investigate a wide range of teaching methods? How can we avoid faddism and catering to the mass media?

One can hardly expect the computer companies to want to see universities seriously ponder these questions. Rather, the companies have cleverly moved to get universities hooked on the new technology — with special discount arrangements to get computers to students, grants for developing uses for computers, and even outright donations to relevant departments of the university. Who has time to think, when such attractive deals are being dangled in front of us?

4. BACK TO BASICS

These days it is easy to forget that a university is supposed to be something fundamentally different from a profit-making company. In fact, the traditional role of a research university was as a place where the faculty studied the types of basic questions that did not have enough short-term promise of profitability (or application to weaponry) to be of interest to industry and government. Now, however, university administrations routinely apply pressure on faculty to bring in grant money, to shift their department's emphasis to fields with short-term applications, and to rush to patent their ideas, so that the university can collect royalties if the patents are used.

In the area of education as well, colleges and universities easily lose sight of basic objectives. A lot of money goes for gadgets that look good in glossy brochures for parents, alumni, and prospective students. Much less money goes for released time for faculty to develop courses that meet the

needs of particular groups of students, or to hire teaching assistants to give intensive practice sessions.

Faced with public demands to improve teaching, college administrators interpret that to mean that professors should strive to be popular and charismatic, so as to get high student rating numbers. A department is judged to be cost-effective in proportion to the number of student credit-hours it services. The absence of complaints is assumed to mean that high quality education is taking place. As in any bureaucracy, the typical administration strategy is to follow the path of least resistance.

However, as math educators we do not have to accept this state of affairs. We don't have to let market forces dictate how we teach. We can resist the profiteers and hypesters. The direction of reform in mathematics education is a serious question — too serious to be determined by the profit motive.

NOTES:

[1] Rosenthal, Amer. Math. Monthly, 90, 576-579.

[2] Or by electronic mail at no cost at all. It would not be hard to organize an e-mail network for the purpose of exchanging TeX files of public domain calculus modules.

[3] In one of the many international tests that show American students in a bad light, out of six countries tested the Koreans scored best in math and the Americans scored worst. More interesting, though, was that along with the test the students were asked to agree or disagree with the statement, "I am good at mathematics." 68% of the American students agreed with this statement, and only 23% of the Koreans. The lesson is: feeling good about one's education is not the same thing as actually having learned something.

"MATHEMATICS - FROM AN ENGLISH MAJOR'S POINT OF VIEW"

Elizabeth Miller

The following paper was written by a Sophmore English major, Elizabeth Miller, in my Mathematics and Culture class last fall. I was intrigued by her view of mathematics and thought that the paper would be of interest to other members of the Humanistic Mathematics Network.

The class of which Ms. Miller was a member involved discussions of and readings about mathematics. It was a somewhat successful attempt to show students who generally avoid mathematics some of the beauty and diversity of the subject. It used The Mathematical Experience by Davis and Hersh, Adventures of a Mathematician by Stan Ulam, and The Fourth Dimension by Rudy Rucker as its major texts. I offer one or two sections of the course every semester.

The term paper assignment for the course was deliberately open ended, leaving the choice of topic entirely up to the individual student. Most students write about one of a list of suggested topics or about a particular mathematician or physicist. Ms. Miller's choice of topic was much more original as was her treatment of it. Enjoy.

Sincerely,

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THE NATURE OF MATHEMATICS

For the duration of this semester in a mathematics and culture class, I have been learning how mathematics affects and is affected by cultures of the world. Now I would like to explore the concept of mathematics as a cultural system or a subculture in and of itself, its subsequent evolution, and the multiple characteristics thereof. The nature of the subject of mathematics is generally misunderstood, though every civilization uses mathematics to some extent. Even among professional mathematicians, opinions regarding the nature of mathematics and its relationships to other aspects of culture vary greatly. These opinions can include elements of mysticism, practical utility, logic, and Platonism. Mathematics is more than a method, an art, or a language; it is a body of knowledge that serves the physical and social sciences and the fields of theology, art, and philosophy.

MATHEMATICS AS A SUBCULTURE

Mathematics has not always been considered a subculture; for example, the mathematics of the Babylonians and the Egyptians merely warranted the status of a "cultural element." The question of when the status changed format from that of cultural element to subculture has not been addressed extensively by anthropologists, but the criteria for the transformation includes 1) a unique set of traditions within the traditions of the general culture 2) laws of development and 3) the fact that it is cumulative, so as a result, it evolves (Wilder, Mathematics 13). Mathematics as a subculture can be thought of as a

system of vectors with each vector striving for growth. Different vectors impinge upon one another, diffuse, and affect others, sometimes resulting in new consolidations and new vectors. Each vector generates its own stress or force, but at the same time, it is subjected to external stress both from other vectors and from the outside culture. Thinking of mathematics as a cultural system offers a means of explaining anomalies not satisfactorily explained by philosophical or psychological methods. Culture consists of a general collection of beliefs and prejudices and the knowledge required to do the job; it is not invented or discovered, but acquired through the processes of communication. The culture of a group consists of the sum of the individual world views united by the bonds of communication.

MOTIVES OF CHANGE

Before discussing the evolution of mathematics, I think it would be appropriate to illustrate some of the motives that produced the changes. The most obvious motive for the evolution of mathematics involves social needs, i.e.: the development of calendars, commercial and financial transactions, navigation, the construction of buildings, and the design of weapons of warfare. An equally significant motive has been to provide rational organization of natural phenomena. The concepts, methods, and conclusions of mathematics contribute to the foundations of the physical sciences. The success of these fields has been dependent on the extent to which they have entered into a sound partnership with mathematics.

These aforementioned motives are completely valid, but it is a mistake to assume that mathematics is stimulated by practical considerations only. Some mathematicians have these practical considerations in mind while pursuing their ideas, while some remain totally indifferent to pragmatics. For instance, much of the mathematical knowledge pursued by the Polish mathematician Stanislaw M. Ulam was practically applied (Ulam). In contrast, it is possible to say also that the idealistic contemplation of Pythagoras and Plato have yielded more significant contributions than those provoked by purposeful acts. The field of mathematics has been molded by practical, scientific, aesthetic, and philosophical interests alike, so it would be impossible to separate the influences and contributions of any one of these forces and compare it to the others.

The drive to create is often inspired by the search for beauty. Aesthetic satisfaction can be achieved through the indispensable use of imagination and intuition incorporated in the creation of proofs and theories. If it is true that insight and imagination, symmetry and proportion, and the exact adoption of means to ends are comprehended in beauty and are characteristic of works of art, then it can be said that mathematics is art with a beauty of its own.

EVOLUTION OF A SUBCULTURE

Throughout the history of the evolution of mathematics, there exist certain identifiable stresses or forces that figure prominently in the development of the field. Environmental stress, diffusion, cultural lag and resistance, symbolic language, and selection all affect the evolution of mathematics. The idea of evolution of a subculture did not arise from the field of biology, but from that of sociology, namely through the work of Herbert Spencer (Wilder, Mathematics 20). It is also very important to establish the fact that the term "evolution" should not be confused with the term "history." History is a generalizing process and can be thought of as a "record of past events arranged in chronological order with some discussion of relations between events (Wilder, Mathematics 18)." Evolution on the other hand can be considered as a "process of change by which various forms and structures change into 'improved' forms and structures and are generally motivated by certain forces whose nature is dependent upon types of forms or structures involved (Wilder, Mathematics 18)." It is necessary to deal with the evolution of concepts as well as the history and evolution of mathematics as a cultural entity.

ENVIRONMENTAL STRESS

As in the case of most subcultures, mathematics has been subjected throughout its history to influences from the environment and virtually owes its existence to the necessities of culture. Allow me to site some examples. Counting and measuring systems arise in each culture as it advances. The Greek word "geometry" literally means "earth measure" and therefore displays its social origins (Wilder, Mathematics 54). For the Babylonians, geometry had no status as a social discipline; it served more as an accessory to arithmetic. Its function seemed to be predominantly as a compendium of formulae for calculating lengths and areas, thus satisfying a social need. For the Greeks however, geometry became a full-fledged discipline, the development of which was influenced by philosophy and astronomy. Originators include Eudoxus, an astronomer, and Parmenides and Zeno, two philosophers whose influences apparently contributed in large part to the consolidation of logic with mathematics.

Though the general feeling may be that mathematics has become more self-sufficient and less dependent in modern times upon environmental stresses for its concepts, it is true that such stresses continue to exercise influence on the development of the field. During World War II, mathematics initiated the invention of more efficient computers and accompanying theories, pursuits in operations and systems analysis, not to mention new developments in already established fields. Adventures of a Mathematician, the autobiography of Stanislaw M. Ulam, gives an in-depth account of Ulam's involvement with the construction of the hydrogen bomb. In fact, he co-authored with C.J. Everett the paper that supplied the foundation for the creation of the bomb. It appears that environmental stress and its affects on the field of mathematics function in a cyclical fashion; environmental stress yields the need for advanced or different mathematical theories which in turn give way to original environmental applications that result in new environmental stresses. An appreciation of the impact of culture upon the mathematician and his work can make significant and beneficial contributions to the profession. Knowledge of cultural influence substitutes for otherwise vague intuitions and can influence problem choice and attitudes towards the work of fellow mathematicians.

DIFFUSION

The process of diffusion also plays an integral role in the evolution of mathematics. Diffusion can be defined as cultural elements passing from one culture to another. A basic pattern of diffusion is as follows: from Babylon and Egypt to Greece and India to Arabia to western Europe (Wilder, Mathematics 48). Arithmetic, geometry, and elementary algebra in a primitive form were involved in this schema. It was the diffusion from India to Arabia that brought about the term "Hindu-Arabic" assigned to the modern numerals (Wilder, Mathematics 48). This process of diffusion is particularly evident where traders or missionaries intervened. "The Ascent of Man-Music of the Spheres" also points out the aid of the Islamic and Christian religions in the diffusion of mathematics.

Without the diffusion of mathematical methods and concepts to the natural sciences, our modern technological culture would not exist. However, there is a trade-off in that as mathematics has contributed to the advancement of other fields, those fields have influenced mathematics, by suggesting models for analysis, for example. Geographic diffusion is no more important than diffusion among fields such as mathematics and the natural sciences. Consider the history of logic. "Discovered" by the Greek philosophers, logic penetrated Greek mathematics early through the axiomatic method; Euclid's "Elements" are upheld as the prime example of logical deduction. In both philosophy and mathematics, logic passed through medieval phases until

De Morgan and Boole developed it into higher symbolic mathematical logic which in the 20th Century, is its own field.

CULTURAL LAG AND RESISTANCE

As is true in every cultural system, a degree of cultural lag or cultural resistance is inevitable. Cultural lag is the "failure of a culture to adopt or adapt to innovation" which is similar to procrastination or conservatism on the individual level. When the refusal to adopt an invention is more overt, cultural lag is more aptly designated cultural resistance (Wilder, Mathematics 25). While these terms appear to connote negativism, cultural lag and resistance do have a certain survival value, because they can denote cultural stability. The development of the numeral system, also known as the method of counting, is a good example of cultural lag. Ionic numerals, letters of the Greek alphabet augmented by three archaic letters with modifying symbols, were easy to use and sufficient for ordinary calculations. They persisted until the 15th Century, though place value numerals of the Babylonians and the Hindu-Arabic numerals were known. The Roman numeral system, though clumsy, survived past the Roman Empire. Newton's calculus symbols known as "dotage" persisted in England, though Europe adopted Leibnizian notation which was operationally more effective. Thus, cultural resistance was due to national pride. Cultural resistance has also kept the United States from converting to the Metric System.

THE SYMBOLIC LANGUAGE

Another important characteristic of mathematics is its symbolic language. Mathematics expresses quantitative relations and spatial forms symbolically. Unlike the usual language of discourse, which is a product of custom and social and political movements, the language of mathematics is carefully and purposefully designed. Because of its compactness, it permits the mind to deal with ideas which, if expressed in ordinary language, would be unwieldy and conducive to inefficiency of thought. However, the specialized symbolic language does create its own problems. The mathematical language is precise and often confusing, thus making it harder for those unaccustomed to its form to follow any sort of mathematical discussion. However, exact mathematical thinking and exact language do go together.

As cultures advance, so do their linguistics and numeral systems. Every culture seems to have yielded to the necessity of counting. Symbols in various tally forms sufficed for primitive cultures, but eventually gave way to number-words. The evolution of mathematical symbols is marked by three significant achievements including cipherization, the concept of place-value, and the invention of the zero. "Cipherization," attributed to C.B. Boyer, is the invention of efficient symbols for individual digits. The Hindu-Arabic digits represent the peak of cipherization in western culture. "Place value" is the assignment of a value to a digit according to its position. The invention of the zero is the direct result of the need for a device to indicate the idea of "no value." Prior to this invention, early Babylonians were forced to guess values by context. (Wilder, Mathematics 50)

SELECTION

Selection is inextricably associated with the evolution of mathematics and involves the choices governing such things as ciphers, bases, theories, and symbols. Mathematical selection is not necessarily equivalent to the natural selection associated with the natural sciences and Darwin's theory of survival of the fittest. In the field of mathematics, this criterion applies only to some selections. Selection on a global basis has usually been a cultural process including both individual and cultural choice. The reasons for selection differ from one case to another. While selection of a general theory may at first be primarily influenced by the eminence of the author and the status of the institution with which he or she is associated, its survival in the long run is

more dependent upon its mathematical significance. Its usefulness and especially its ability to further the development of mathematics will often determine its persistence throughout the evolution of mathematics.

ABSTRACTION AS AN INEVITABLE FUTURE

By examining characteristics of culture which include evolution, environmental stress, diffusion, cultural lag and resistance, language, and selection and applying these characteristics to the field of mathematics, it is possible to perceive mathematics as a cultural system or a subculture in and of itself. Mathematics as a subculture is inherently subject to evolution through relentless study, philosophical discussion, and pragmatic application. As a cultural system like that of mathematics grows, evolves, and becomes institutionalized, increased abstraction inevitably results. This occurs within the structure of every culture and is not restricted to scientific systems. In the field of religion, abstract theologies augment the original simple rules and rituals in an effort to meet the demands of a society growing in complexity. Individuals and their cultures will continue to evolve, as will the subculture of mathematics. Mathematical innovations will persist in broadening cultural perspectives and influencing such seemingly unrelated fields as philosophy, theology, and art. It is through this mathematical abstraction of special fields that the promotion of other fields is made possible.

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