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Quantum Foundations with Astronomical Photons

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Quantum Foundations with Astronomical Photons

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May, 2017
Abstract

Bell’s inequalities impose an upper limit on correlations between measurements of two-photon states under the assumption that the photons play by a set of local rules rather than by quantum mechanics. Quantum theory and decades of experiments both violate this limit. Recent theoretical work in quantum foundations has demonstrated that a local realist model can explain the non-local correlations observed in experimental tests of Bell’s inequality if the underlying probability distribution of the local hidden variable depends on the choice of measurement basis, or “setting choice”. By using setting choices determined by astrophysical events in the distant past, it is possible to asymptotically guarantee that the setting choice is independent of local hidden variables which come into play around the time of the experiment, closing this “freedom-of-choice” loophole.

Here, I report on a novel experimental test of Bell’s inequality which addresses the freedom-of-choice assumption more conclusively than any other experiment to date. In this first experiment in Vienna, custom astronomical instrumentation allowed setting choices to be determined by photon emission events occurring six hundred years ago at Milky Way stars. For this experiment, I selected the stars used to maximize the extent over which any hidden influence needed to be coordinated. In addition, I characterized the group’s custom instrumentation, allowing us to conclude a violation of local realism by 7 and 11 standard deviations. These results are published in Handsteiner et. al. (Phys. Rev. Lett. 118:060401, 2017).

I also describe my design, construction, and experimental characterization of a next-generation “astronomical random number generator”, with improved capabilities and design choices that result in an improvement on the original instrumentation by an order of magnitude. Through the 1-meter telescope at the NASA/JPL Table Mountain Observatory, I observed and generated random bits from thirteen quasars with redshifts ranging from $z = 0.1 - 3.9$. With physical and information-theoretic analyses, I quantify the fraction of the generated bits which are predictable by a local realist mechanism, and identify two pairs of quasars suitable for use as extragalactic sources of randomness in the next cosmic Bell test. I also propose two additional applications of such a device. The first is an experimental realization of a delayed-choice quantum eraser experiment, enabling a foundational test of wave-particle complementarity. The second is a test of the Weak Equivalence Principle, using our instrument’s sub-nanosecond time resolution to observe the Crab pulsar at optical and near-infrared wavelengths. Using my data from the Crab Pulsar, I report a bound on violations of Einstein’s Weak Equivalence Principle complementary to recent results in the literature. Most of these results appear in Leung et. al. (arXiv:1706.02276, submitted to Physical Review X).
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1 Introduction: Bell’s Inequality

The probabilistic nature of quantum mechanics led Einstein, Podolsky, and Rosen (EPR) in 1935 [1] to question whether a more complete description of nature exists, in which knowledge of some hidden initial conditions would allow the states of physical systems to be predicted deterministically rather than probabilistically [2]. Additionally, the nonlocality of quantum entanglement drew skepticism on aesthetic grounds. After all, the nonlocal inverse-square law of Newton was eventually subsumed by Einstein’s local theory of general relativity, which turned out to be a fuller, time-dependent description of the gravitational force. Some physicists wondered if quantum mechanics, too, would be replaced by a more complete, manifestly local theory.

Put more formally, we say that a physical theory is \textit{realist} if the state of a physical system exists independently of measurement. We say that a theory is \textit{local} if the state of a system is a function only of the events in its past light cone. Does there exist some extension of quantum theory that would restore realism and locality?

The work of J. S. Bell and Clauser, Horne, Shimony, and Holt (CHSH) in developing what is colloquially known as “Bell’s inequality” allowed this question to be resolved experimentally, by predicting an experimentally testable discrepancy between any local realist physical theory and the predictions of quantum mechanics given several experimental assumptions. For a system of two identical particles, each of which could be observed in one of two states, the CHSH-Bell inequality sets an upper limit on the maximum amount of correlation allowed between the two particles if the system obeys a set of local and realist physical laws. Since then, a slew of increasingly careful experiments [3–10] have measured statistically significant violations of local realism while relaxing crucial experimental assumptions.

The amount of correlation in Bell’s inequality is quantified by a parameter $S$ which can be measured for any pair of two-state particles. The definition of $S$ is best conceptualized with the concreteness of a physical system: for example, consider two polarization-entangled photons in the singlet state:

$$\frac{1}{\sqrt{2}} (|H, V\rangle - |V, H\rangle). \quad (1.0.1)$$

It is possible to make a measurement of the polarization of one of the photons with a polarizing beamsplitter, which transmits photons polarized horizontally with respect to the beamsplitter, and reflects photons polarized vertically with respect to the beamsplitter. Detecting whether a photon gets transmitted or reflected is therefore equivalent to making a measurement of its polarization angle in the basis defined by the orientation of the
Figure 1.0.1: A basic schematic of a test of Bell’s inequality using polarization-entangled photon pairs. Alice and Bob are experimenters on either side of the experiment, equipped with beamsplitters, which can be rotated to various different angles $a$ and $b$. By counting transmissions (+1) and reflections (−1) with the coincidence monitor CM, they can measure the value of correlations in the photon pairs produced by the source $S$. [Figure: “Scheme of a two channel Bell test”, reproduced from Wikipedia under a Creative Commons license]

beamsplitter. For the sake of easy bookkeeping, we assign the score +1 to transmissions and the score −1 to reflections.

However, we are not limited to one beamsplitter orientation. The polarization of a photon can be measured not only in the $\{|H\rangle, |V\rangle\}$ basis but along any two orthogonal polarization directions $\{|H'\rangle, |V'\rangle\}$, obtained by rotating the $\{|H\rangle, |V\rangle\}$ basis by some angle $\theta$. Measurement in any orthogonal basis can be experimentally implemented based on how the beamsplitter is oriented. Regardless of orientation, experimenters will always be keeping score, observing transmissions or reflections through the beamsplitter in order to measure the photon’s polarization.

Now, suppose two intrepid experimenters named Alice and Bob on either side of the experiment make measurements of both entangled partners, as shown in Fig. 1.0.1. They record their scores $A$ and $B$ based on whether their individual photons were transmitted or reflected at their respective measurement stations. Alice and Bob are allowed to measure photons in different bases obtained by rotating $\{|H\rangle, |V\rangle\}$ to angles $a$ and $b$ respectively. Together, it is possible for them to experimentally measure the correlation function $E(a,b)$, which is simply the expected value of the product of Alice and Bob’s scores, given their angle choice $a$ and $b$. For example, if we take $a = b = 0$, which corresponds to Alice and Bob both choosing to make measurements of their respective photons in the $\{|H\rangle, |V\rangle\}$ measurement basis, we can see from Eq. 1.0.1 that between their measurement outcomes, they will see exactly one transmission and one reflection. Hence the product of their scores will always be $E(0, 0) = -1$. If Bob rotates his detector to $b = 90^\circ$, then his measurement basis would be $\{|V\rangle, -|H\rangle\}$ such that
photons that look horizontal to Alice’s beamsplitter look vertical to Bob’s beamsplitter. Hence, $E(0, 90^\circ) = +1$. In general, the correlation function $E(a, b) = \langle AB \rangle = 2p(A = B|a, b) - 1$, with a maximum value of +1 corresponding to perfect correlation and a minimum value of −1 corresponding to perfect anticorrelation.

The CHSH-Bell correlation parameter $S$ is a linear combination of these correlation functions where Alice’s detector takes on two angles $a_1$ and $a_2$, and where Bob’s detector takes on any two angles $b_1$ and $b_2$. The inequality applies to all local hidden-variable theories regardless of the angles $a_k, b_\ell$ chosen, but allowing Alice’s detector to take on $a_1 = 0^\circ$ and $a_2 = 45^\circ$ and allowing Bob’s detector to take on either $b_1 = 22.5^\circ$ or $b_2 = −22.5^\circ$ optimally captures quantum correlations while still imposing a limit on classical ones.

$$S \equiv E(a_1, b_1) + E(a_1, b_2) + E(a_2, b_1) - E(a_2, b_2) \quad (1.0.2)$$

Why does this particular combination of correlation functions and positive/negative signs capture quantum correlations but not classical ones? First consider the situation quantum mechanically. In quantum mechanics, overall phase shifts of a quantum state do not change the value of any observables. In addition, the singlet state we are analyzing can be shown to be rotationally invariant up to an (unobservable) overall phase. In an orthogonal basis $\{|H\rangle, |V\rangle\}$ obtained by rotating $\{|H\rangle, |V\rangle\}$, the singlet state is still represented as $1/\sqrt{2} (|H\rangle^\prime |V\rangle^\prime - |V\rangle^\prime |H\rangle^\prime)$ up to an overall phase factor. For these two reasons, the correlation function $E(a, b)$ does not depend on the absolute values $a$ and $b$ but rather only on the absolute difference $|a - b|$. Thus $E(a_k, b_\ell)$ is not a function of $a_k$ and $b_\ell$ separately, but rather $E(|a_k - b_\ell|)$. In the definition of $S$, the first three terms have angle differences $|a_1 - b_1| = |a_2 - b_1| = |a_1 - b_2| = 22.5^\circ$. This angle difference is closer to zero than $90^\circ$, so we expect these three terms in $S$ to be of the same magnitude and somewhat negative, since a $0^\circ$ angle difference gives perfect anticorrelation. The final term $E(a_2, b_2)$ will be somewhat positive because $|a_2 - b_2| = 67.5^\circ$, which is closer to the $90^\circ$ required for perfect correlation. Subtracting the value of $E(a_2, b_2)$ from the other contributions maximizes the value of $|S|$. An example of this playing out with experimental data is shown in Fig. 1.0.2. The singlet state (as written in 1.0.1) with the choice of angles separated by $22.5^\circ$ as described above leads to the quantum mechanical prediction of

$$S_{\text{singlet}} = -2\sqrt{2}. \quad (1.0.3)$$

It turns out that this value of $|S|$ is in fact that largest achievable via quantum mechanics [11].

In a local realist theory, the predictions work out quite differently. An intuitive explanation is as follows: realism requires each photon to assume
Figure 1.0.2: In tests of Bell’s inequality, the four correlation functions $E(a_k, b_\ell)$ are measured for $k, l \in \{1, 2\}$. The red bars show the contributions to $S$ from the correlation functions $E(a_1, b_1)$, $E(a_1, b_2)$, and $E(a_2, b_1)$, and the green bar shows the contribution to $S$ from $E(a_2, b_2)$. All four contributions to $S$ add up constructively such that the experimental value of $S$ exceeds the local realist bound, shown with a dotted line. The basis angles $a_k, b_\ell$ are also depicted geometrically. This figure is reproduced from [10].

a particular polarization, and locality means that the photon pairs cannot make a coordinated change of polarization after they are in flight away from each other. In contrast with the quantum mechanical case, these requirements make it impossible for a pair of local realist photons to “hedge their bets” on which setting choice will be used to measure them. It is impossible for these photons to exhibit anticorrelations in three different sets of measurement bases and correlations in a fourth, all at the same time. The upshot of this is that in a local realist theory, $|S| \leq 2$ [2]. The discrepancy between the allowable values of $S$ in a local realist theory ($|S| \leq 2$) versus in quantum mechanics ($|S| = 2\sqrt{2}$ for the singlet state) is what allows experiments to reject local realist theories. Below, I prove that $|S| < 2$ for a local realist physical theory.

1.1 Proof of the Bell-CHSH Inequality

Following [12] in setting up this problem, consider a Bell test which analyzes a pair of photons that obey a set of local realist rules. Recall that in a Bell test, we assign the score $+1$ to transmissions through a polarizing beam splitter oriented at a given angle and $-1$ to reflections. We denote the outcome of the measurement on each side of the experiment as $A, B \in \{\pm 1\}$. Even though in reality we may observe that $A$ and $B$ take values of $\pm 1$ seem-
ingly at random, the premise of realism is that this apparent randomness is an illusion of averages, arising simply as a result of some hidden variable $\lambda$ taking on different values drawn from some normalized distribution $\rho(\lambda)$. In our proof, we model the measurement outcomes $A$ and $B$ as single-valued functions of the measurement settings chosen, and assume that each photon has access to an otherwise-unobservable “hidden-variable” $\lambda$ drawn from some probability distribution $\rho(\lambda)$.

$$A = A(a_k, b_\ell, \lambda) \quad B = B(a_k, b_\ell, \lambda) \quad (1.1.1)$$

Note that in this formalism, the correlation function $E(a_k, b_\ell)$ can be expressed as

$$E(a_k, b_\ell) = \langle AB \rangle = \int \rho(\lambda) A(a_k, b_\ell, \lambda) B(a_k, b_\ell, \lambda) \quad (1.1.2)$$

If the measurement setting $b_\ell$ is determined outside of the past light cone of the measurement of $A$, then the outcome $A$ cannot depend on the choice of $b_\ell$. As we will discuss later, this is the assumption of locality (which is shorthand for local relativistic causality). Then our model of the particles’ behavior is simplified to

$$A = A(a_k, \lambda) = A_k \quad B = B(b_\ell, \lambda) = B_\ell \quad (1.1.3)$$

where we introduce the shorthand notation $A_k, B_\ell$. Then $|S|$ can be expressed as

$$|S| = |E(a_1, b_1) + E(a_1, b_2) + E(a_2, b_1) - E(a_2, b_2)| \quad (1.1.4)$$

$$= \left| \int \rho(\lambda) (A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2) \, d\lambda \right| \quad (1.1.5)$$

$$\leq \max_{\lambda} \{|A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2|\} \int \rho(\lambda) \, d\lambda \quad (1.1.6)$$

Integrating over the normalized probability distribution $\rho(\lambda)$ leaves

$$\leq \max_{\lambda} \{|A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2|\} \quad (1.1.7)$$

We can write $A_1 B_1 + A_1 B_2 + A_2 B_1 - A_2 B_2 = A_1 (B_1 + B_2) + A_2 (B_1 - B_2)$. Since $B$ is either $+1$ or $-1$, it is the case that for any $\lambda$, either $B_1 = B_2$ or $B_1 = -B_2$. Then one of the quantities $B_1 + B_2$ or $B_1 - B_2$ will be $\pm 2$ and the other will vanish. Since $A$ is either $+1$ or $-1$, we conclude that

$$\max_{\lambda} \{|A_1 (B_1 + B_2) + A_2 (B_1 - B_2)|\} = 2, \quad (1.1.8)$$

implying $|S| \leq 2$. 

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1.2 Experimental Implementation of Bell Tests

A typical test of Bell’s inequality involves experimentally measuring each of the four correlation functions in Eq. 1.0.2, and computing $S$. To measure the correlation functions, it is necessary to prepare many copies of the two-photon singlet state. For each copy of the state, the detectors are oriented randomly to some joint setting $(a_k, b_ℓ)$, and scores are recorded. Suppose that $N_{kℓ}$ pairs of photons are measured in the joint setting choice $(a_k, b_ℓ)$. For each of these “runs”, there are four possible measurement outcomes, with $A = \pm 1, B = \pm 1$. The number of times that each outcome occurred can then be denoted by $N_{kℓ}^{++}, N_{kℓ}^{+-}, N_{kℓ}^{-+}, N_{kℓ}^{--}$. Then, the value of the correlation function is

$$E(a_k, b_ℓ) = \frac{N_{kℓ}^{++} + N_{kℓ}^{--} - N_{kℓ}^{+-} - N_{kℓ}^{-+}}{N_{kℓ}}$$ (1.2.1)

Hence, the raw outcome of Bell test experiments is a set of sixteen numbers $N_{kℓ}^{AB}$ for all $k, ℓ, A, B$. Statistically analyzing how much the experimental value of $|S|$ exceeds 2 enables experimentalists to reject local realist theories with quantifiable statistical confidence.

Bell conceded, however, that a simple experimental test of local realism might be flawed. A pathological hidden-variable theory might be able to reproduce quantum mechanical correlations, for example, if the “settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocities less than or equal to that of light” [2]. Three major experimental assumptions in the spirit of this concern have been identified [2, 13, 14]: the no-communication (or locality) assumption, the fair-sampling assumption, and the freedom-of-choice (or setting independence) assumption. If these assumptions are not valid, local realist mechanisms can exploit loopholes that are left open by making those assumptions in order to engineer a violation of Bell’s inequality. Since the pioneering experiment of Freedman and Clauser in 1972 [15], improved electronic and photonic technologies have allowed for technologically-advanced Bell experiments to be performed. These experiments are often referred to as having “closed loopholes” in tests of local realism, by removing the need for one or more of the assumptions of no-communication, fair-sampling, and freedom-of-choice.

1.3 No-Communication (Locality) Assumption

The first experiment testing Bell’s inequality relied on a primitive atom-cascade source of entangled photons, measuring correlations for each possible setting choice $(a_k, b_ℓ)$ one after another [15]. While this initial experiment violated local realism, it made the assumption that no information
about setting choices was transmitted to the other side of the experiment. To eliminate the need for this assumption, Bell famously insisted that the joint measurement basis for the two-particle system should be set while the particles were in flight, in order to maintain a space-like (i.e. causal) separation between detection events. The first experimental effort to address this loophole was undertaken by Aspect, et al. in 1982 [4] using acousto-optic modulators (AOMs). An AOM uses rapid acoustic pressure waves to vary the refractive index of a medium. This can modify the path taken by each entangled photon, sending Alice’s photon on different paths to two sets of detectors, fixed at angles $a_1$ and $a_2$ (or in Bob’s case, $b_1$ and $b_2$). More recent experiments have used ultrafast electro-optical modulators (EOMs, also called Pockels cells), which induce changes in the index of refraction of a transparent material by applying high DC electric fields instead of acoustic standing waves. The digital-electronic control of an EOM allows for faster switching speeds than an AOM, and allows experiments to employ an external, low-predictability source of randomness to dictate the setting choice, rather than relying on the periodic (and hence predictable) behavior inherent to an AOM. In this way, the choice of measurement basis can be toggled from, say, $a_1$ to $a_2$, with sub-microsecond latency. In that amount of time, however, light (and potentially setting-choice information) can travel about 300 meters. Thus, many modern Bell’s inequality experiments which close the locality loophole are conducted over distances of kilometers or more [5, 7–10].

1.4 Fair-Sampling Assumption

The fair-sampling assumption states that the subensemble of detected photons is a representative sample of the entire ensemble of emitted photons. This assumption leaves open the possibility that the detected subensemble of photons is biased towards exhibiting a Bell violation, a loophole exploited in a local hidden variable theory developed by Pearle [13]. In experiments that address the fair-sampling assumption, a more stringent form of the CHSH inequality (the Clauser-Horne inequality [16]) which takes non-detections and single detections into account is often used [6, 8, 9]. Since non-detections and single detections reduce violations of the Clauser-Horne inequality, the development of highly-efficient and low-latency detector technology, such as superconducting nanowire single photon detectors (SNSPDs) and transition edge sensors (TESs), has allowed for stronger tests of local realism.

These specialized detectors, along with quantum random number generators to generate setting choices with low latency, allow for so-called “loophole-free” Bell tests [7–9, 17] that do not make the no-communication or the fair-sampling assumptions. It is especially difficult to close the fair-sampling loophole at the same time as the locality loophole, because the speed of
causality (compared to the speed of electronics) necessitates long distances over which to send entangled photons, and thus increases the probability of losses in flight.

1.5 Freedom-of-Choice Assumption

Recent theoretical work by Hall has revived interest in a third major assumption, dubbed the *freedom-of-choice* assumption [5, 14, 18]. Intuitively, this is the assumption that the setting choices for a particular experimental run are independent of the source of entangled photons or anything that can possibly affect the measurement outcomes. For example, suppose that there were some hidden correlation between the production of a photon pair and the choice of setting. If the photon source were able to successfully predict which setting \((a_k, b_\ell)\) would be chosen in a particular experimental run, it would be able to engineer a violation of Bell’s inequality. For example, it could emit classically-polarized photons exhibiting the right correlations as measured in the \((a_k, b_\ell)\) basis.

In his 2010 work [18], Hall began by allowing \(a_k \in \{a_1, a_2\}\) and \(b_\ell \in \{b_1, b_2\}\) to be possible setting choices that Alice and Bob may take, with \(\rho(\lambda)\) being the distribution of all underlying hidden variable at the entangled photon source. If we relax the assumption that the probability distribution and the setting choices are mutually independent, then there might be up to four probability distributions \(\rho_{a_k,b_\ell}(\lambda)\) for \(\{k, \ell\} \in \{1,2\}\). The probability distributions’ dependence on the setting choices can conceivably arise from a hidden-variable mechanism predicting or influencing the settings chosen, given information in the past light-cone of the experiment.

Hall introduced a metric of measurement independence [18] as

\[
M = \max_{k,\ell,k',\ell'} \int d\lambda \ |\rho_{a_k,b_\ell}(\lambda) - \rho_{a_{k'},b_{\ell'}}(\lambda)|
\]  

(1.5.1)

where \(k,\ell,k',\ell' \in \{1,2\}\). Intuitively, this metric quantifies the maximum distance between any two of the four probability distributions \(\rho_{a_k,b_\ell}(\lambda)\). Note that if the four distributions are all identical, then \(M\) vanishes: Alice’s and Bob’s setting choices and the probability distribution of hidden variables at the source are independent. In the other extreme case where \(\rho_{a_k,b_\ell}(\lambda)\) does not overlap at all with \(\rho_{a_{k'},b_{\ell'}}(\lambda)\), the area under \(|\rho_{a_k,b_\ell}(\lambda) - \rho_{a_{k'},b_{\ell'}}(\lambda)|\) is 2 (because each distribution is normalized). In this case \(M = 2\). The normalized “fraction” of measurement independence is then defined as \(F = 1 - M/2\). Remarkably, by giving up only a modest fraction of measurement independence \((F = 14\%\) by this metric), Hall came up with a local-realistic hidden variable model that was able to reproduce the singlet-state correlation demonstrated in Eq. 1.0.3.
In this context, recent Bell tests have tried to enforce measurement independence by reducing the possibility that the distribution of local hidden variables can be affected by some setting choice. A recent Bell test conducted in 2015 by Shalm et al. [8] used setting choices taken from the bitstreams of various pre-2007 video clips [8]. However, even this approach is susceptible to exploitation, since digital video clips are stored in bits on a hard drive which are, in an Einstein-locality sense, are accessible to local hidden variable theories. A hidden variable theory could, for example, read the hard drives which dictate setting choices just before the beginning of the experiment. Scheidl et al.’s 2010 experiment [5], as well as recent “loophole-free” Bell tests as described earlier [7–9, 17], used quantum random number generators to generate setting choices outside the past light cone of the emission event. To the extent that these quantum random number generators truly bring a fresh, fundamentally unpredictable bit into existence in the span of a few nanoseconds as these experiments assume, this scheme does close the freedom-of-choice loophole. However, on causal grounds alone, anything in the past light cone of the source and a random number generator could be influencing both.

1.6 Settings from Space

A more elegant way to enforce measurement independence is to use setting choices from galactic sources such as Milky Way stars or extragalactic sources [19] such as distant quasars or the cosmic microwave background radiation. For example, consider generating setting choices using the color of individual photons from astronomical sources. Since the bits of information encoded in the colors of the photons are determined long before the experiment, and since the color of an individual photon is a piece of information that travels at the maximum causal speed, there is no way for a local hidden variable theory to know about the setting choice unless the four probability distributions $\rho_{ak,bl}(\lambda)$—which come into play at the time of the Bell test experiment—are instantiated in the union of the past light cones of the astronomical emission events, potentially many billions of years prior. This is illustrated in a cartoon in Fig. 1.6.1.

In April 2016 in Vienna, we performed the first in a series of “cosmic Bell” tests following this method, using the color of the photons from bright Milky Way stars to determine setting choices in a test of the Bell-CHSH inequality, while addressing the no-communication (locality) assumption [10]. We will continue to use setting choices from increasingly-distant astronomical sources to generate settings that are independent of each other as well as the photon source. This will allow us to chip away at the assumption of freedom-of-choice by progressively limiting the space-time region in which local hidden-variable theories which relax this assumption could remain viable.
Figure 1.6.1: A cartoon of the cosmic Bell scheme. Here, two different quasars located on opposite sides of the sky are used to determine the measurement settings $a$ and $b$ while the entangled-photon pair is in flight. A local theory says that the results $A$ and $B$ can only depend on events in their past light cone. To be a function of the setting on the other side, as would be required to violate Bell’s inequality, the measurement needs access to the past light cone of the quasar on the other side. Figure modified from [19].

In the remainder of this thesis, I will describe several methods of generating randomness from astronomical sources in a way that allows experiments to address the freedom-of-choice loophole. I will describe our Vienna experiment which I helped to analyze and which demonstrated $7\sigma$ and $12\sigma$ violations of local realism. In addition, I sketch another application of an “astronomical random number generator” (ARNG) to perform foundational tests of wave-particle duality in the spirit of Wheeler’s delayed-choice interferometer gedankenexperiment. Then, I describe how I designed, constructed, and validated an ARNG with an order-of-magnitude better performance for use in an extragalactic cosmic Bell test, in which the setting choices are determined by quasars billions of light years away, which emitted their light when the universe was less than a tenth of its current age. Finally, in a different application of my instrument, I observe the Crab pulsar and analyze the data to set a new, competitive upper limit on violations of the Weak Equivalence Principle.
2 Generating Bell-Test Measurement Settings from Astronomical Sources

To generate measurement settings for a Bell test from astronomical sources, we must extract a piece of information carried by astronomical photons that is determined at the time of the astronomical photon’s emission. The information carried by the photon must in addition be resistant to corruption by local hidden variable mechanisms in the vicinity of the experiment. In fact, I will later show that to address the freedom-of-choice loophole, an average of 79% of measurement settings on both sides of the experiment must be generated from uncorrupted astronomical photons.

Several observables carried by astronomical photons, which are determined at the time of astronomical photon emission, can be extracted to be turned into a random bit. The photon’s time of arrival, color (i.e. its wavelength/momentum/energy), and its polarization are set at or near the time of emission and can be measured. Though these parameters are continuous, the range of their allowed values can be partitioned into two similarly-sized subsets in order to extract random bits, and there are advantages and drawbacks to each observable and partitioning method. The scheme chosen should minimize the fraction of corrupted bits and should be practical to implement. These considerations led us to use photon colors as classical randomness sources in the first Cosmic Bell test [10], and our group intends to continue to do so.

2.1 Time of Arrival

The time of arrival of an astronomical photon is a piece of information that is obtained for free with a detection, because the time of arrival needs to be recorded to nanosecond precision in order to address the locality loophole. To generate random bits, we can map a timestamp to either a zero or one based on whether a designated digit in the timestamp is even or odd. For example, in [20], random bits are generated by looking at the nanosecond decimal place of the timestamp and checking whether it is even or odd.

One advantage of this scheme is that it is simple and the need for additional hardware is minimal. In addition, the bits generated are almost evenly split between 0’s and 1’s. Perhaps the biggest disadvantage of this scheme is that using even/odd timestamps to determine the setting choice admits the possibility that a local hidden variable theory synchronizes its “entangled” photon emissions to coincide with a particular choice of setting. Since the method for generating random settings is determined while preparing for the experiment, the entangled source could potentially obtain knowledge of which setting will be generated at each measurement station as a function
of time, if an astronomical photon were to arrive. Even though a local realist mechanism does not know whether an astronomical photon will arrive during that time period, it is in principle able to predict the setting choice on both sides of the experiment better than chance as a function of time. If even/odd nanoseconds were used and if each setting was only valid for a nanosecond, there would be perfect predictability of setting choices. If the experiment was large enough so the settings could be valid for longer with astronomical photons only arriving occasionally, there would be reduced, though still non-zero predictability. With enough predictability, by sending pairs of classically-polarized photons towards the measurement stations at just the right time, a local realist mechanism could yield Bell-violating correlations.

In addition, it is very difficult to precisely quantify terrestrial influences on the recorded timestamp of an astronomical photon’s arrival. Therefore, we would have a hard time quantifying how many setting choices on each side of the experiment are corrupted. If a less significant decimal place is chosen, then a wider variety of mechanisms which can potentially influence the setting choice (such as a fluctuating atmospheric delay due to turbulence) need to be quantified. If a more significant decimal place is chosen, it becomes harder to close the locality loophole for given-sized experiment while avoiding the predictability problem above.

To make this atmospheric argument explicit, we quantify uncertainties in arrival times due to local influences, assuming that uncertainties (not overall delays) can be exploited by a local realist mechanism. One source of uncertainty is in fluctuations in the index of refraction of air. It can be determined, over a baseline of tens of meters, to an accuracy of $\Delta n \sim 1 \times 10^{-6}$ due to turbulence [21]. If we assume that this RMS error accumulates independently and in an identically-distributed manner over the 8 km scale height of the atmosphere, then we have an overall uncertainty of

$$\Delta n_{atm} \sim \sqrt{\frac{8000 \text{ m}}{10 \text{ m}}} \times 10^{-6} \sim 10^{-5}$$

and a corresponding fundamental uncertainty in the arrival time

$$\Delta t_{turb} \sim \frac{L_{atm} \Delta n_{atm}}{c} \sim 1 \times 10^{-10} \text{ s.}$$

If the setting choice is determined by a bit less significant than the $(10^{-10})$ place, it could be that the setting choice is tampered by atmospheric turbulence. This estimate neglects the effect of dynamic weather conditions, the presence of aerosols and humidity, temperatures and pressures, etc., which all have significant effects on the index of refraction of air, and which change as a function of time along the line of sight of our instrument.
To remove some of these complications, we can use a more significant bit to determine the setting choice. But the bit we choose cannot be too significant, because information about one setting choice would travel to the other side of even the largest experimental setup such as that used in Scheidl et al. [5] within $\sim 10^{-5}$ seconds, and within $\sim 10^{-6}$ seconds for the baseline used in our initial photonic Cosmic Bell test [10].

As an experimenter trying to outwit sources of corruption, I do not know how to overcome the possibility that the entangled source emits local-realist photons synchronized with particular setting choices. In addition, the uncertainties involved in quantifying influences on the arrival timing of astronomical photons discourages me from pursuing this method of generating random setting choices during a Bell test, though it is possible to do so without any additional hardware.

2.2 Polarization

Another observable that can be used to set basis choices is the polarization of astronomical photons. The problem with this approach is the classical law of Malus: the probability of transmitting a polarized photon through a polarizer oriented at an angle $\Delta \theta$ relative to the photon’s polarization is $\cos^2(\Delta \theta)$, decreasing smoothly from 1 to 0 as $\Delta \theta$ is increased. Since this function does not sharply transition from an angle range where photons are “mostly transmitted” to another angle range where photons are “mostly reflected”, it is difficult to partition photons into two bins without many misclassifications.

2.3 Color

Another way to classify photons is by their energy, which is set at the time of photon emission: Fix a central wavelength $\lambda'$ and map all “blue” detections with $\lambda < \lambda'$ to ‘0’s and “red” detections with $\lambda > \lambda'$ to ‘1’s. This scheme can be implemented using dichroic beamsplitters with appropriately-chosen spectral responses, and a separate detector for blue and red photons. The advantage of the color scheme is that possible influences on the color of photons are well-studied and characterized by models of absorption and scattering in the atmosphere, and by detector manufacturers. In contrast to effects which alter arrival times, effects on the probability distribution of photon wavelengths passing through the atmosphere and being detected by avalanche photodiodes are straightforward to model with knowledge of the spectral response of each component, as shown in Fig. 7.5.1. Instead of dynamic effects which may vary on short timescales, the effect of the atmosphere on photons shifts over the course of minutes or hours, as astronomical sources and clouds move through the sky during a night-long Bell
test. Processes that affect a photon’s detector-frame wavelength are well understood and are slowly varying, such as Rayleigh scattering, cosmological redshift, etc. It is thus straightforward to model atmospheric and detector effects on the probability distribution of incoming astronomical photons, if we make the basic assumption that atmospheric and detector effects treat each photon based only on their color (and not on some inaccessible hidden variable). This assumption—that we fairly sample astronomical photons—is necessary in any astronomical randomness scheme, and is very similar to the fair-sampling assumption that is already being made for entangled photons in Bell tests which do not close the detection loophole. To quantify the amount of color misclassification, it suffices to analyze the overlap in the probability distribution of photons detected at the blue and red detectors.

These obvious advantages lead us to use the color scheme in our implementations of cosmic Bell tests, which I will describe in Section 3. It is worth noting that the chief disadvantage of this scheme is that the fluxes of “red” and “blue” photons will almost never be in equal proportion. In our proof-of-concept of this color scheme using quasars, I find that some quasars can have up to three times the flux of red photons as blue photons. This increases the duration of an experiment because collecting robust statistics for each of the four setting choices will take longer. In a worst-case scenario, one could collect about nine times more statistics in the setting choice corresponding to “red-red” than the one corresponding to “blue-blue.” Should this become a limiting factor, a better pair of dichroics would be purchased, or different astronomical sources would be used.

2.4 At Least 79% of Settings Must be Uncorrupted

Regardless of which scheme is chosen, it is not always the case that every random bit generated by an ARNG is truly from an astronomical source. We deem “corrupt” all of the random bits which are generated from detector dark counts, background noise, or any terrestrial source of detections. In addition, any experimental implementation of a scheme to partition an ensemble of photons into zeros and ones will have some probability of classifying an astronomical photon incorrectly. In the timing scheme, for example, a timestamp which would generate a 0 could be affected by jitter in the time-tagging unit and result in a 1 being transduced with some nonzero probability. In the color scheme, a red photon with $\lambda > \lambda'$ has a nonzero probability of being detected in the detector designated for blue photons. We must assume that these misclassifications are also caused by some local mechanism and deem them corrupt.

In other words, only setting choices determined at the time of astronomical photon emission (rather than near the time of the experiment) are deemed valid. There is no way to determine which detections are corrupt on
a case-by-case basis, but I will demonstrate that to address the freedom-of-choice loophole in a CHSH-Bell test, an average of at least 79% of the bits generated on each side of the experiment must be uncorrupted.

To show this, I assume that the photons from astronomical sources are not corrupt, i.e. that they are not predictable and not able to be influenced by terrestrial hidden variable mechanisms. These valid settings contribute at most $S_{\text{valid}} = 2$ to each run because they obey the Bell-CHSH inequality that $|S| \leq 2$. The premise of the freedom-of-choice loophole is that any setting that does not use information set at the time of the astronomical photon’s emission leads to a “corrupted” experimental run whose outcome does not play by the rules of Bell’s inequality, instead only obeying the weaker algebraic bound of $|S| \leq 4$. A local realist theory exploiting the freedom-of-choice loophole seeking to engineer a Bell violation would only be able to achieve $S_{\text{valid}} = 2$ for each valid experimental run, but would be able to orchestrate each corrupt experimental run to yield $S_{\text{invalid}} = 4$. Hence, the maximum value of $S$ attainable by this model, denoted $S_{\text{free}}$, is greater than 2. Instead of having to measure $S$ in violation of the standard CHSH-Bell inequality $|S| < 2$, we now need to violate the more conservative Bell’s inequality

$$|S| < S_{\text{free}} = qS_{\text{valid}} + (1 - q)S_{\text{invalid}} = 2q + 4(1 - q) = 4 - 2q \quad (2.4.1)$$

where $q$ is the probability that both setting choices are valid for an experimental run for a given pair of astronomical sources. Note that making the freedom-of-choice assumption and taking $q = 1$ recovers the original Bell-CHSH inequality. However, since quantum mechanics predicts a maximum value of $|S| \leq 2\sqrt{2}$, the goal posts cannot be moved by too much: we must have $4 - 2q < 2\sqrt{2}$, or that

$$q > 2 - \sqrt{2} \approx 0.586 \quad (2.4.2)$$

for a Bell violation to be possible due to quantum-mechanical phenomena. Because only a single setting choice needs to be influenced or predicted to invalidate the run, one must assume that the hidden variable mechanism is stingy in its use of corrupt settings, influencing only one setting choice at a time and never corrupting both simultaneously. Hence the overall fraction of valid settings is $q = 1 - p^{(\text{Alice})} - p^{(\text{Bob})}$, where $p^{(i)}$ is the probability that a setting at the $i^{th}$ detector is invalid. Defining $q^{(i)} = 1 - p^{(i)}$, the requirement in Eq. 2.4.2 can be expressed in terms of the probabilities $q^{(i)}$ of generating a valid setting at the $i^{th}$ instrument:

$$q_{\text{Alice}} + q_{\text{Bob}} > 3 - \sqrt{2} \approx 1.59. \quad (2.4.3)$$

In terms of the average probability of obtaining a valid-setting, Eq. 2.4.3 becomes

$$\frac{q_{\text{Alice}} + q_{\text{Bob}}}{2} > \frac{3 - \sqrt{2}}{2} \approx 0.79. \quad (2.4.4)$$
In particular, if we assume that the experiment is symmetric with $p_{Alice} = p_{Bob} = p$, we find that

$$q_{Alice} = q_{Bob} > \frac{3 - \sqrt{2}}{2} \approx 0.79$$

or that at least eight out of ten detections on each side need to be of astronomical origin. While this is a very encouraging “signal-to-noise” ratio of $\approx 3.76$, we note that it is very difficult in practice to achieve a measured value of $S$ close to the quantum mechanical maximum of $2\sqrt{2} \approx 2.82$ due to imperfections in the experimental setup. To get a sense of this, experimental values of $S = 2.43$ and $S = 2.50$ were obtained in our first cosmic Bell test [10]. Furthermore, the closer the experimentally-measured value of $S$ is to the local realist bound, the more experimental runs are required to achieve a statistically-significant Bell violation.
3 Cosmic Bell Test: Settings from Milky Way Stars

In our initial cosmic Bell experiment, we performed two tests of Bell’s inequality, using two different pairs of Milky Way stars in the HIPPARCOS catalog to produce measurement settings determined in the past. I wrote the code that ultimately decided which pairs of stars were observed. I selected these pairs of stars on the basis of their brightness, distance, and location in the sky. Bright stars ensure high setting generation rates. Distant stars (the nearest star observed is 577 light years away) push back any need to invoke the freedom-of-choice assumption as described in Sec. 1.6. Finally, since Alice’s star needed to be spacelike-separated from Bob’s measurement station and vice versa, the stars chosen were roughly collinear with the measurement stations and the source of entangled photons. As a result, the stars were somewhat low in the sky at the time of observation, with altitudes of $24^\circ - 37^\circ$. The geometry of our setup is shown in Fig. 3.0.1, and did not change much over the 179 s duration of each Bell test.

Our experiment also closed the locality loophole. This is particularly challenging since it is necessary for the setting choices to be determined while the entangled photons are in flight towards their detectors. While experimenters do not have control over the time of arrival of an astronomical photon, the probability of two detections while the entangled photons are in flight can be increased by increasing the length of time that the entangled photons are in flight. Hence, our experiment utilizes a source station and two receiver stations that are at least half a kilometer away in roughly opposite directions, as shown in Fig. 3.0.2.

The photon source $S$, located in the Institute for Quantum Optics and Quantum Information (IQOQI), generates photon pairs in the singlet state $\frac{1}{\sqrt{2}} (|H,V\rangle - |V,H\rangle)$. A 405 nm laser coupled into a Sagnac interferometer pumps a nonlinear ppKTP crystal in both clockwise and counterclockwise directions, generating superposition states of pairs of 810 nm daughter photons going in each direction. The daughter photons leave the interferometer through a polarizing beam splitter that converts which-path entanglement into polarization entanglement. The end result is that entangled pairs of photons in the singlet state are generated at a rate of 275kHz. These entangled pairs are sent to the roof of IQOQI via a fiber link, which induces a delay of 180 ns. Once on the roof, the entangled photon pairs are transmitted to measurement stations at the Austrian National Bank (OENB) and the University of Natural Resources and Life Sciences (BOKU) respectively using two pairs of small telescopes (diameter between 50 mm – 140 mm) as free-space links. Of the photons produced and distributed over the free space links, a total of $\sim 10^5$ experimental runs are conducted. The resulting coincidence counts $N_{AB}^{AB}$ for the first of our two experimental runs are reported...
Figure 3.0.1: A map of the alignment of the HIPPARCOS stars used as randomness sources, the measurement stations at OENB and BOKU, and the entangled photon source. In order to optimally address the locality loophole, each location is roughly collinear in the following order: Alice’s Star ← Alice ← S → Bob → Bob’s Star. Figure taken from our paper [10].
Figure 3.0.2: (a): Using pairs of transmitter and receiver telescopes to transport entangled photons (denoted Tx-EP and Rx-EP), a source of entangled pairs S based at the Institute for Quantum Optics and Quantum Information (IQOQI) sends entangled photons towards measurement stations A and B, located in the Austrian National Bank (OENB) and in the University of Natural Resources and Life Sciences (BOKU) respectively. At each measurement station, a 10-inch receiver telescope for stellar photons (Rx-SP) and a custom-built “setting reader (SR)” generate random bits from stars. These bits determine whether to toggle the EOM at each measurement station for making a measurement of each photon’s polarization in the basis $a_1$ or $a_2$ ($b_1$ or $b_2$). In the Vienna experiment, we referred to our astronomical random number generators as “setting readers”. (b)-(c): Two diagrams that show the geometric arrangement of the stars, the measurement stations, and the source of entangled pairs. Figure taken from our paper [10].
Table 1: From these coincidence counts $N_{kl}^{AB}$, it is possible to compute the correlation functions $E(a_k, b_\ell)$ and obtain a value of $S$, the Bell-CHSH correlation parameter. Statistical analysis can then show how much the experimentally-measured value of $S$ differs from the local-realist bound.

<table>
<thead>
<tr>
<th>$k\ell \setminus AB$</th>
<th>++</th>
<th>+−</th>
<th>−+</th>
<th>−−</th>
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<tr>
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<td>6 406</td>
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<td>1 184</td>
<td>4 537</td>
<td>5 113</td>
<td>959</td>
</tr>
<tr>
<td>22</td>
<td>18 451</td>
<td>3 512</td>
<td>3 949</td>
<td>14 196</td>
</tr>
</tbody>
</table>

in Table 1, from which one can compute $S$.

As the entangled photons are in flight, an astronomical photon must be detected at both sides of the experiment. Both ARNGs must produce an uncorrupted bit in order to generate a valid joint setting.

3.1 Vienna’s Astronomical Random Number Generator

To gather photons from astronomical light sources, a 10-inch hobby telescope couples light directly into a multi-mode optical fiber. A schematic of the Vienna ARNG is shown in greater detail in Fig. 3.1.1. Since the fiber is only $\approx 1.5$ m long, the losses in the fiber itself are negligible compared to a broadband 4% coupling loss at both the input and output fiber couplers. After an achromatic lens collimates the light, it passes through a pair of dichroic beamsplitters (Thorlabs M 254H45 and Thorlabs M 254C45). The dichroic beamsplitters either transmit or reflect a photon based on its color towards dedicated detectors for visible and near-infrared photons. Then, a broadband silvered mirror reflects each beam, which get refocused onto ID120 avalanche photodiodes for detection.

The main advantage of using multi-mode fibers to directly couple astronomical photons into the ARNG is that the ARNG does not need to be mechanically connected to the telescope as it slowly tracks a source throughout the sky. However, this may not be an optimal scheme to use with larger telescopes and dimmer targets. With a larger telescope comes a larger mount, and therefore a longer fiber which may be on the order of 10 m. The losses in fibers get amplified exponentially over longer distances, and may vary as a function of stresses on the cable that shift as the telescope moves. This introduces uncertainties in our model of the spectral response of the instrument. In addition, with only a fiber coupler at the focal plane of the telescope, a guide scope is needed to locate targets, and fine adjustment needs to be done without visual feedback purely by monitoring the ARNG’s count rates.
Figure 3.1.1: A schematic of the astronomical random number generator (ARNG) used in the Vienna experiment. A short multi-mode fiber was used to couple astronomical photons from a hobby telescope into the Vienna ARNG. Collimating the beam, splitting the beam of photons by their color, and refocusing the two beams onto ID120 detectors generates a stream of random basis choices which were implemented by the electro-optic modulator (EOM). This was a preliminary figure developed for our paper [10].
Though this approach works well enough for bright Milky Way stars, it may run into difficulties for dimmer objects such as quasars.

Soon after an astronomical detection at Alice and Bob’s measurement stations, the EOM at each polarization analyzer implements a particular setting choice with a latency of $\tau_{\text{set}} = 170\,\text{ns}$. Once the setting choice is implemented on both sides, entangled photons must be detected at both measurement stations before which-setting information can potentially reach the opposite measurement station. An additional safety buffer $\tau_{\text{buffer}}$ further shortens the validity time window in order to allow for some errors in addressing the locality loophole. These errors can be induced by factors such as imprecise estimation of the atmosphere’s index of refraction and thus the errors in the time it takes for an astronomical photon to travel through the atmosphere, or inaccurate GPS location estimates. The remaining time window after each detection after subtracting $\tau_{\text{set}}$ and $\tau_{\text{buffer}}$ are represented graphically in Fig. 3.1.2 and are roughly 2.5$\,\mu$s and 6.9$\,\mu$s at measurement stations A and B respectively.

### 3.2 Corruption Estimates and Statistical Significance

As mentioned earlier in Section 2.4, at least 79% of the setting choices used in our experiment must be uncorrupt on average. Assuming that corrupted setting choices may come from stray terrestrial photons, detector dark counts, and misclassified astronomical photons, we can quantify each of these contributions to estimate the overall fraction of uncorrupted runs $q$. Later in this thesis I will show that in the limit that the astronomical sources are bright (Milky Way stars, for example), the overall probability of corruption is dominated by misclassification of astronomical photons rather than noise. Hence, for this experiment, it is particularly important to know what fraction of photons detected in the red band are actually blue ($\lambda < \lambda'$) and vice versa, as discussed in Sec. 2.4.

In Fig. 3.2.1, Panels A-D, I model the fraction of photons whose color gets misclassified by analyzing the spectral responses of our red and blue observing bands. This was my largest contribution to the analysis of our Vienna experiment [10] and became the basis for a significant fraction of the supplemental material. Here, I model each star as a blackbody characterized solely by its temperature. With this model, the fraction of detections that are misclassified, range from 1.4\,-\,2.0% over the four stars used in the Vienna experiment, the two ARNGs (one on each side), and the two possible setting choices per ARNG. In addition, I calculate that between 22% and 25% of photons incident on the top of the atmosphere in the appropriate direction get turned into settings. Rayleigh scattering and imperfect detector quantum efficiency are the primary sources of losses. Following the analysis which I describe in detail later in Section 7.2, I use experimentally measured signal
Figure 3.1.2: This 1+1D spacetime diagram illustrates the causal ordering of the events that lead to a successful experimental run. Starting from the creation of the entangled photon pair at S and a short delay in a transmission fiber to the roof of the source building at IQOQI, the photons are sent to measurement stations A and B, whose location are denoted by vertical lines. To enforce no-communication between the measurement stations, a measurement setting has to arrive at A and B sometime within the solid blue and red lines, whose past and future light cones correspond to the blue and red shaded regions. Since it takes some finite amount of time to transduce an astronomical photon detection into a rotated measurement basis, $\tau_{\text{set}} = 170$ ns further limits the valid time interval in which an astronomical photon detection generates a valid setting, and $\tau_{\text{buffer}}$ gives an additional safety margin to ensure no-communication in the face of uncertainties such as the variable index of refraction of the atmosphere. Figure taken from our paper [10].
and noise rates to compute \( q \) for the pairs of HIPPARCOS stars used in the experiment. I also compute the value of \( S_{\text{free}} \), the maximum value of \( S \) attainable by a local realist mechanism exploiting the freedom-of-choice loophole. Recall that \( S_{\text{free}} \) needs to be exceeded by experimentally-observed data to conclude a violation of local realism. These values are summarized in Table 2.

In [10], the statistical significance of our Bell violations were particularly difficult to estimate. A crude estimate of the uncertainty in the measured value of \( S \) would simply propagate \( \sqrt{N} \) counting errors in the measured values of the sixteen numbers \( N_{k\ell}^{AB} \). However, this method of statistical significance assumes that the experimental runs \( N_{k\ell}^{AB} \) are independent and identically-distributed trials. Several improved methods of computing the statistical significance of a Bell violation relax these assumptions [22], but they are designed for experiments where settings are generated in roughly equal proportions. Since our ARNGs did not give equal proportions of each joint setting choice, we find these methods also suitable for our particular experiment. We formulated a new method of calculating the statistical significance which is described in detail in the Supplemental Material of [10]. Since I had only marginal involvement with this aspect of the analysis, I will not reproduce the mathematical details here.
Figure 3.2.1: I model the number distribution of photons as a function of wavelength as atmospherically-extincted blackbodies, plotted in Panel A. The cumulative spectral response due to detector quantum efficiencies and anti-reflection coatings on optics is computed and plotted in Panel B. Combined with measured transmission curves for our dichroic beamsplitters in Panel C, I compute the number distribution of photons at each detector in Panel D. I then quantify the number of blue photons that are detected by the detector meant for red photons, and vice versa to quantify the fraction of corrupt settings. Figure taken from our paper [10].
Table 2: For both of the Cosmic Bell tests we performed, I compute the fraction of valid settings $q$ from astronomical sources, based on measured photon fluxes and the fraction of misclassifications from my model. Given that value of $q$, the maximum value of $S$ attainable by a local realist model exploiting the freedom of choice loophole, defined by $S_{\text{free}} = 4 - 2q$, is computed. Then, I compare $S_{\text{free}}$ to the value of $S$ that is experimentally measured, observing statistically significant violations of local realism in both Bell tests.

<table>
<thead>
<tr>
<th>Run</th>
<th>$q$</th>
<th>$S_{\text{free}}$</th>
<th>Measured $S$</th>
<th>Statistical Significance</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>2.356</td>
<td>2.425</td>
<td>7.54$\sigma$</td>
</tr>
<tr>
<td>2</td>
<td>0.839</td>
<td>2.322</td>
<td>2.502</td>
<td>11.93$\sigma$</td>
</tr>
</tbody>
</table>
4 Beyond Cosmic Bell: Testing Wave Particle Duality

Before I discuss the design and characterization of my improved astronomical random number generator (ARNG), I will discuss using an ARNG to improve other foundational quantum experiments. These experiments use a single ARNG rather than a pair to address a loophole similar to the freedom-of-choice loophole in tests of wave-particle duality.

The concept of testing wave-particle duality by sending single photons through a Mach-Zehnder interferometer was first proposed by John Archibald Wheeler [23] and has been realized on a laboratory scale [24]. However, in a famous gedankenexperiment, Wheeler proposed using light from a doubly gravitationally-lensed source of astronomical photons as two arms of a Mach-Zehnder interferometer of cosmological scale. The beams would pass through a set of narrowband color filters to increase the coherence length of the light. Then the light would be recombined by routing each copy of the image into the two faces of a beamsplitter. One might observe interference fringes at the output of that beamsplitter, suggesting that the light took both paths around the gravitational lens and interfered at the beamsplitter. This is indicative of single photons exhibiting wave-like behavior. If the beamsplitter in the Mach-Zehnder interferometer is removed, the light is prevented from recombining. The astronomical light then manifests itself as single photons which appear at one output or the other but never both. This suggests that the single photon behaves like a particle. Rejecting wave/particle duality would lead to one of two absurd logical conclusions. Either the choice of inserting the beamsplitter in the final moments of the light’s journey somehow retrocausally affected the light’s trajectory across the cosmos, or that the choice of inserting the beamsplitter was predictable by the light before it decided to embark on its journey.

While our group has no plans to interfere astronomical photons using a gravitational lens, here I propose a similar experiment that leads to the same logical conclusion. Instead of testing the wave-particle duality of an astronomical photon, one can use a standard tabletop Mach-Zehnder interferometer such as the one implemented in [24]. A random bit from an astronomical random number generator determines whether to insert or remove the beamsplitter only after a single photon has propagated past the first beamsplitter. Note that the choice of whether or not to remove the beamsplitter is not in the past light cone of the interferometer photon’s creation, and is in fact pre-ordained billions of years before the interferometer photon is created. In this experiment as well as Wheeler’s original gedankenexperiment, separating the decision event of inserting the beamsplitter from both the creation of the photon and its journey makes alternate explanations
of wave/particle duality increasingly implausible.

This scenario, however, still admits a local-realistic explanation as follows. Two local-hidden-variable-like surrogates of the interferometer photon could take both paths and accumulate a phase as if they were interfering waves. When they come together, these surrogates would either see a beam splitter or not. At that moment, they can decide to combine their accumulated phases and act like a wave. Alternatively, they can ignore their phases and pick one detector over the other in some deterministic or locally-probabilistic way. The choice can determined when the paths recombine rather than when the paths diverge at the input end of the interferometer. In this way, there is a perfectly local-realistic explanation for the evidence of wave-particle duality of single particles.

4.1 Delayed-Choice Quantum Eraser

However, two-photon experiments such as a delayed-choice quantum eraser cannot be explained in a local realist framework similar to the one outlined. In modern delayed-choice quantum eraser experiments [25], wave-particle duality is tested by interfering one of the entangled partners (denoted the signal photon) of a two-photon entangled state in a Mach-Zehnder interferometer. Rather than removing the beamsplitter in the Mach-Zehnder interferometer, a measurement of the other entangled partner (the environment photon) is made outside the light cone of the signal photon at the same time or after the signal photon propagates through the interferometer. Such a setup is shown in Fig. 4.1.1. This measurement of the environment photon can either erase or reveal which-path information about the signal photon, depending on the basis in which the environment photon is measured. Revealing which-path information causes the signal photon to manifest itself as a particle. If which-path information is erased, the signal photon takes both paths through the interferometer and exhibits single-photon interference, manifesting wave-like behavior which cannot be explained by local physical theories.

In the language of quantum mechanics, a quantum eraser experiment begins with a polarization-entangled state between a “signal” photon that will go through the interferometer and an “environment” photon that will have its polarization measured in a space-like separated region. Such a state can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_s |V\rangle_e + |V\rangle_s |H\rangle_e) .$$ (4.1.1)

When the signal photon enters the polarizing beam splitter at the start of the interferometer (MZI), its polarization state gets mapped onto its path.
Figure 4.1.1: My proposed experiment to probe wave particle duality, in the spirit of Wheeler’s “delayed-choice” experiment. In my version of a delayed choice experiment, a two-photon entangled state is produced at S. One one entangled partner is sent towards a which-way device (W) and the other toward the Mach-Zehnder interferometer (MZI). An astronomical random number generator (ARNG) chooses the polarization measurement basis for the entangled partner at W through the electro-optic modulator (EOM), potentially revealing which-path information. At the same time, the other particle at the interferometer’s final polarizing beam splitter acts as a particle or a wave accordingly, even though the choice of whether to reveal which-path information is made in a causally-disconnected way, billions of years before the experiment has been run.

(a or b) through the interferometer. The state becomes

$$|\psi\rangle \rightarrow \frac{1}{\sqrt{2}} (|b\rangle_s |V\rangle_e + |a\rangle_s |H\rangle_e).$$

(4.1.2)

If the environment photon is measured in the horizontal/vertical basis at W, either result collapses the superposition of “which-path” states, and no interference is observed at the second polarizing beam splitter. If however the environment photon is measured in the circular basis, the signal photon is put in a superposition of both paths. Before that collapse, the state can be written as

$$|\psi\rangle \rightarrow \frac{1}{2} [(|a\rangle_s + i|b\rangle_s)|L\rangle_e + (|a\rangle_s - i|b\rangle_s)|R\rangle_e],$$

(4.1.3)

with the signal photon in each term being in a superposition of both paths. After the final beam splitter, the paths will combine and interfere. In each term, the amplitude for each detector to fire becomes a function of the relative phase due to path-length differences around the interferometer. Conditioned on whether the environment photon is left or right-circularly polarized, the signal photon’s interference fringes will be 180 degrees out of phase. For both linear and circular basis choices the signal photon enters each detector with equal probability, so like with any entangled state, information
can not be sent merely by choosing a measurement basis. Interference fringes or the lack thereof can only be seen when one sorts the the signal photon’s detections into categories based on the basis choice and measurement result of the environment photon. Like in tests of Bell’s inequality, any apparent violation of causality is merely non-locality of correlations.

Any local explanation of the non-local correlations in this experiment would rely on being able to predict whether the measurement of the idler photon erases or reveals which-path information, dictating the wave-like or particle-like behavior of the signal photon. Setting the idler photon’s measurement basis with setting choices from an ARNG can be used to constrain the potential origins of this predictability. By sourcing random settings from stars or quasars, this experiment would be able to push back the origin of a local conspiracy that fakes complementarity. As with the cosmic Bell experiment, and in the same spirit as Wheeler’s original delayed-choice experiment, this would improve bounds on the scope of a local conspiracy by tens of orders of magnitude.

4.2 Feasibility of an Experiment

I will now argue that this delayed-choice experiment is feasible and within experimental reach even at my undergraduate institution. First, as shown in Fig. 4.1.1, only one ARNG is needed for this test, rather than the pair of ARNGs needed for a cosmic Bell test. Harvey Mudd already has a functioning quantum eraser setup, which merely needs to be equipped with an electro-optical modulator to enforce rapid setting choices to close the locality loophole.

Now, I compute the length of time it takes to perform the experiment as a function of the distance $L$ between the Mach-Zehnder interferometer and the which-path measurement station. To conduct the experiment with causally-disconnected choice, the setting must be generated and implemented while the environment photon is en route towards its measurement station (W in Fig. 4.1.1). Suppose that this transmission happens over a fiber link of length $L$. For simplicity, I ignore latencies due to instrumental considerations, such as the time $t_{\text{set}}$ it takes for the EOM to implement a setting choice. Let $n_{\text{ent}}$ be the rate at which entangled photons enter the fiber and $n_{\text{RNG}}$ be the rate at which random setting choices from astronomical photons are implemented. The rate at which environment photons are detected will be $n_{\text{ent}} \exp(-L/L_0)$, where $L_0$ is a constant, due to losses in the fiber. At the measurement station, the probability that at least one setting is successfully generated during the duration of the entangled photon’s travel time $\Delta t \approx$
Table 3: Using a typical single-mode fiber with $L_0 = 0.8 \text{ km}$ and a source of entangled pairs emitting $n_{\text{ent}} = 10^5$ pairs per second, we can estimate the optimal length of a delayed-choice quantum eraser experiment that maximizes the coincidence rate. The quasar APM 08279+5255 is at redshift $z = 3.911$ and emitted its light when the universe was only a tenth is current age.

<table>
<thead>
<tr>
<th>Randomness Source</th>
<th>Settings</th>
<th>Optimal Length</th>
<th>Coincidences Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milky Way Stars</td>
<td>$10^5 \text{Hz}$</td>
<td>0.71 km</td>
<td>51 kHz</td>
</tr>
<tr>
<td>APM 08279+5255</td>
<td>$10^2 \text{Hz}$</td>
<td>0.80 km</td>
<td>9.8 Hz</td>
</tr>
</tbody>
</table>

$L/c$ is $1 - \exp(-n_{\text{RNG}}L/c)$. Then the expected rate of successful runs is

$$N(L) = n_{\text{ent}} \exp(-L/L_0)[1 - \exp(-n_{\text{RNG}}L/c)] = r_{\text{ent}} \exp(-x)[1 - \exp(-xy)],$$

where $x = L/L_0$ and $y = n_{\text{RNG}}L_0/c$. The highest rate of successful runs $N_{\text{max}}$ is attained when the experiment is of size $x = L/L_0 = \frac{\log(1 + y)}{y}$, and in terms of $y$,

$$N_{\text{max}} = n_{\text{ent}}y(1 + y)^{-1 - \frac{1}{y}}.$$ (4.2.2)

With an entangled source like the one used in the Vienna experiment, it is possible to achieve emission rates on the order of $n_{\text{ent}} \sim 10^5$ Hz. I consider a standard fiber (e.g. Thorlabs 780HP fiber), which typically has $\sim 3.5 \text{ dB/km}$ of loss at 780 nm. This corresponds to $L_0 = 0.8 \text{ km}$. I use pessimistic order-of-magnitude estimates of experimental count rates that we obtained with the device described in the next section. The optimal setup parameters for a delayed-choice quantum eraser experiment using either stars or quasars are shown in Table 3.

In an experiment where as few as $\sim 10^3$ coincidences need to be recorded [25], our coincidence rate estimates are encouraging and gives us several orders of magnitude of wiggle room for experimental imperfections. I will continue to develop the details of this experiment next year during my time at IQOQI-Vienna.
5 Improved Astronomical Random Number Generator

As discussed in Sec. 1.6 and originally proposed in [19], the freedom-of-choice assumption is most conclusively addressed by using astrophysical events from extragalactic sources such as distant quasars or the cosmic microwave background as sources of randomness. At Harvey Mudd, I used my familiarity with the Vienna instrumentation to develop an astronomical random number generator (ARNG) with the ability to observe faint and distant quasars. My instrument can implement two randomness schemes: the time-of-arrival scheme, employed in [20], as well as the color scheme, employed in [10]. A sketch of my instrument is shown in Fig. 5.0.1, and its realization is shown in Fig. 5.0.2. In the subsections that follow, I will lay out the considerations that went into our system’s design, and describe its subsystems. These and the analysis sections that follow form the basis for [26].

5.1 Design Requirements

My instrument features several improvements over the astronomical random number generator used in the Vienna experiment. It is optimized to facilitate the observation of faint astronomical objects such as quasars and to minimize the fraction of generated settings that are corrupt.

The most noticeable difference between my instrument and the Vienna instrument is the lack of a fiber link. Rather than using a multi-mode fiber to couple astronomical photons into our instrument, the entire instrument fits in a compact, rigid enclosure designed to be fixed onto the back of a meter-class telescope. It is necessary to use a telescope of this size (or larger) in order to observe even the brightest quasars with a sufficient signal to noise ratio to close the freedom-of-choice loophole. In addition, a guide scope is not needed to assist in pointing the telescope. Much of the light in the telescope’s field of view (except the light from the object of interest) is fed to an integrated camera which, when connected to a telescope mount, can implement a pointing feedback loop for real-time pointing corrections. This enables the instrument to track an object to within 2 arcseconds of long-term accuracy over the potentially hours-long duration of a Bell test. Using the main telescope instead of a guide telescope for object tracking as Vienna did reduces pointing error and allows us to observe fainter objects successfully. In addition, since the camera is focused directly on the pinhole, we can take long exposures during an observation to verify that light from the object of interest passes through the pinhole.

Another improvement in my instrument is its optical design. Our optical design maximizes the fraction of uncorrupted settings generated in order
Figure 5.0.1: This shows (not to scale) the intended optical paths of the improved astronomical random number generator whose optics I designed. Astronomical light from multiple objects in the field of view of the telescope enters at the top right of the schematic. This light is focused by the telescope onto the plane of a pinhole mirror. Most of the light in the field of view is reflected by the mirror and refocused onto a camera for pointing and tracking. However, light from a particular astronomical source of randomness (purple) passes through the pinhole. The light is then collimated and sorted by color via a system of one shortpass and one longpass dichroic beamsplitter. Then, the two beams of light are refocused onto the active area of two avalanche photodiodes for detection and nanosecond timestamping.
Figure 5.0.2: This image shows our realization of the instrument depicted in Fig 5.0.1, resting on a small Melles-Griot optical table in the Harvey Mudd College Optics Lab. The longpass and shortpass dichroic beamsplitters can be distinguished by their colors in the picture. Looking through the longpass dichroic at a source of white light gives the light a deep red color, while looking through the shortpass dichroic makes everything appear yellow. In this image, the lids on top of the dichroics are opened for viewing but in general should remain closed to prevent contamination. Note the addition of two 2D translation stages, one placed in front of each detector. This allows for precise alignment of the focusing lenses with respect to the active area of the detector even while the instrument is bolted to a telescope, ready for observation.
to meet the minimum threshold required to address the freedom-of-choice assumption (see Eq. 2.4.4). In order to minimize the fraction of settings generated by stray noise photons, the instrument is enclosed in a light-tight black Delrin plastic enclosure. All lenses feature inked rims to minimize stray reflections. In addition, the two extra mirrors used in the Vienna design are eliminated, reducing complexity. The coupling of photons from the telescope to the detector is maximized with achromatic collimating/focusing lenses with anti-reflection coatings. Two XY translation stages house the focusing lenses, and can be used to align the collimated incoming beam with the active area of the detector. These additional degrees of freedom allow each detector arm to be aligned independently while the instrument is active and mounted on a telescope. I chose the pinhole size—which sets the single-photon detectors’ field of view—to be just large enough to encompass the astronomical object’s smeared-out image due to atmospheric turbulence and the telescope’s point spread function. This minimizes the background rate due to skyglow.

In order to minimize the fraction of misclassified settings, our design featuring a pair of dichroic beamsplitters that have steeper spectral responses than the dichroic beamsplitters used in our original experiment [10]. The probability that a photon gets misclassified by ranges from 0.15% − 0.23% over thirteen quasars and two detector arms. These figures are an order of magnitude improved from the original design, as tabulated in 5. Our instrument can achieve the constraint in Eq. 2.4.4 with two different pairs of brighter quasars, even under our poor observing conditions.

5.2 Optics and Optical Coupling

A schematic of the instrument is shown in Fig. 5.0.1. Our instrument is housed in a black, light-tight box made of Delrin plastic. It has dimensions 30 cm by 30 cm by 10 cm, and weighs 5.5 kg. Most of this weight comes from two single photon detectors and a “prosumer”-grade ZWO ASI 1600 MM-COOL, a cooled monochrome CMOS camera with a 4/3-inch sensor and a low read noise intended for astronomical imaging.

The box is made of Delrin plastic because it is easy to machine. However, when I went to Table Mountain Observatory for the first time, there was concern that the Delrin was not rigid enough to support the weight of the instrumentation enclosed in different telescope orientations without bending. One of the machinists there kindly built a “U brace” for the enclosure, which fits into a slot cut through the lid. The U brace screws into a ∼ 12” × 12” anodized aluminum adapter plate. The adapter plate fits onto an annular steel adapter plate which gets bolted onto the telescope. Rather than mounting the adapter plates to the telescope first, it is easiest to mount these two adapter plates to the instrument while on the ground, and then
mount the entire assembly onto the telescope.

Once the instrument is mounted on the telescope, the telescope’s focal plane can be adjusted such that it coincides with the 200 µm diameter pinhole in our 45° pinhole mirror from Lenox Laser, as diagrammed in Fig. 5.0.1. A Canon EF-S 60mm F2.8 macro lens is also focused on the pinhole such that an image of the telescope’s field can be formed on our ZWO ASI camera. The camera has a focal plane consisting of 4656 × 3520 pixels, with a pixel size of 3.75 µm and a total focal plane size of 17.46 mm × 13.2 mm. Our generous pixel array allows for image processing such as 3 × 3 median binning, which reduces the graininess of the picture. In addition, the camera has an ST4 port for a telescope-guiding feedback loop and can be cooled to −20°C to reduce readout noise. These features allow for improved tracking of dim objects like quasars. Nearby objects can be used as a reference to verify the field and keep the telescope aligned in real time using open-source pointing software (PHD2 Guiding) while generating random setting choices with quasar photons. As one of our first observations, we took a glamour shot of Saturn, shown in Fig. 5.2.1.

The size of the pinhole in our pinhole mirror (200 µm diameter) was chosen to reject the light at the focal plane of the telescope except for in a very small region that was supposed to look circular when viewed at a 45-degree angle. Using a pinhole to admit only the light from a distant quasar towards our avalanche photodiodes rejects most of the photons from the roughly uniform glow of the upper atmosphere. The size of our pinhole corresponds to a circle of diameter

\[ 2.75 \text{ arcseconds} = \frac{200 \text{ µm}}{15 \text{ m}}. \]

on our 15 m focal-length telescope, which is comparable to the 3-arcsecond jitter in the position of astronomical objects at Table Mountain Observatory due to atmospheric turbulence. Furthermore, the spreading of a visible/NIR beam (\( \lambda = 1 \text{ µm} \)) due to diffraction off of such a large pinhole (\( D = 200 \text{ µm} \)) is negligible compared to the f/15 spreading due to the telescope:

\[ \theta_{\text{diff}} = 1.22 \frac{\lambda}{D} < 0.006 \text{ rad} \quad \theta_{f/15} = \frac{1}{15} = 0.06 \text{ rad} \]

The light from the star or quasar that makes it through the pinhole gets collimated by a 25 mm diameter, 50 mm focal-length, broadband-coated lens (Edmund 49-356-INK) that is held in the center of a slotted Thorlabs lens tube (SM1L30C) by a pair of retaining rings. After being collimated, the light passes through a system of two dichroic mirrors, and is refocused by a 25 mm lens (49-353-INK) fixed in a Thorlabs CXY1 translational stage in front of each detector. This makes it easy to couple incoming collimated light to the detectors with minimal losses, even while the instrument is on
Figure 5.2.1: This image of Saturn demonstrates our ability to observe astronomical objects. Calculation of the angular diameter of Saturn at the time of observation allows us to measure the angular size of the pinhole on the sky, which agrees with the *a priori* estimate of 2.75 arcseconds.

the telescope. The CXY1 stage as well as the IDQ ID120 detectors have tapped holes which are compatible with Thorlabs’s cage cube system so it is straightforward to hold the translation stages and focusing lenses rigid with respect to the detector. All of our lenses are anti-reflection coated over the relevant wavelength range of 350 nm – 1150 nm and inked along the rims to reduce internal reflection.

5.3 Spot Size

To maximize the optical coupling efficiency of the system and improve ease of alignment, the image of the quasar formed on the detector should be smaller than each detector’s active area. Initially, the pinhole has a diameter of 200 µm, but this size gets transformed by our system of collimating and focusing lenses. Conservation of etendue, or “light-gathering power”, determines the spot size at the detector in terms of the focal lengths of the lenses. For an ideal optical system,

\[ A_1 \Omega_1 = A_2 \Omega_2 \]

where \( A \) is the area of the image formed, and \( \Omega \) is the solid angle of divergence of the light beam. For a circular area, this relation can be expressed

\[ \frac{r_1}{f_1} = \frac{r_2}{f_2} \]

where \( r_i \) are the length scales (e.g. the radii) of the spot sizes and \( f_i \) are the focal lengths. Since the pinhole has a diameter of 200 µm, I chose the focal
Figure 5.3.1: By using a lens with the longer focal length as the collimating lens (1), we reduce the spot size (represented with a schwa) going from the pinhole to the detector area (1 → 2), improving the coupling to the detectors’ 500 µm × 500 µm active area.

lengths of the lenses, taking \( f_1 = 50 \text{ mm} \) to be the collimating focal length and \( f_2 = 35 \text{ mm} \) to be the focusing focal length, to produce an image on the detector’s active area with a nominal diameter of 140 µm. This image fits comfortably inside the detector’s 500 µm × 500 µm footprint and allows for imprecisions and misalignments.

This configuration is used in both detector arms. Note that the coupling efficiency is highly sensitive to the position of the pinhole and any flexure in the system which may arise as a result of the instrument sagging under its weight. If the detectors are slightly misaligned, the 2-axis translation stages that house the 35 mm lenses can be used to correct for ±1 mm misalignments.

5.4 Dichroic Beamsplitters

Several characteristics of dichroic beamsplitters guided my design choices. The spectral response of the dichroic beamsplitters are highly sensitive to the light’s angle of incidence, which necessitates going through the trouble of collimating the light before sending it through any dichroic beamsplitters. However, the dichroic beamsplitters’ effect on the trajectory of the transmitted light is negligible. (Of course, the trajectory of the reflected photons are highly dependent on the orientation of the beamsplitter). This is convenient because it enables a user to align the transmit arm of the device and then—mostly independently—optimize the alignment of the reflect arm of the instrument.

I chose to use Semrock beamsplitters because they had the highest quality and largest selection of dichroic beamsplitters of different cutoff wavelengths. Our particular pair of beamsplitters—and the choice to place the shortpass beamsplitter before the longpass beamsplitter rather than the other way
around—was made to minimize the number of misclassified photons according to my model of the instrument’s misclassifications, which I describe in detail in a later section (Sec. 7). We simulated both configurations and chose the one with a lower misclassification rate.

In our chosen configuration, the collimated light that is incident on our beamsplitters first encounters our shortpass dichroic beamsplitter (Semrock F697-SDi01-25x36). The light that is transmitted is mostly visible. The reflected (mostly infrared) light then passes through a 705 nm long-pass dichroic beamsplitter (Semrock FF705-Di01-25x36), also oriented at 45° with respect to the beam. The photons reflected here are absorbed by the black plastic wall of the dichroic cage cube. The transmitted photons are almost all in the near-infrared because of the multiplicative effect of both beamsplitters. As stated earlier, it is crucial to minimize the fraction of infrared photons which end up at the detector designated for visible photons and vice versa because these misclassifications must be treated as corrupt settings. These ideas will be made more quantitative in Section 7.

5.5 Avalanche Photodiodes (APD)

To detect the astronomical photons, we chose ID120 Silicon Avalanche Photodiode detectors that have up to 80% quantum efficiency and are sensitive with reasonable (but lower) quantum efficiency between 350 nm and 1150 nm. As discussed earlier, the detectors have a 500 µm × 500 µm active area so the image of the pinhole fits well within the active area, and a nominal 40 Hz dark count rate when the detectors' active area are thermoelectrically cooled to −40°C. This count rate was achieved when the detectors were new. These dark count rates will increase over time if the detectors are saturated or nearly saturated for long periods of time. As a precaution to future members of the lab, if the detector gets exposed to more than roughly a million counts per second, it saturates and gets automatically shut off by the detector firmware. If this happens too often, lasting damage will occur to the detector and cause dark count rates to increase. When the detectors are coupled to the focusing lens, it is largely insensitive to stray light (cell phone flashlights). However, if the detector’s field of view is unobstructed, it can pick up on tiny amounts of stray light, such as an unobstructed crack under the door. When aligning the instrument using tabletop light sources, it is useful to have several neutral density filters on hand to reduce the laser intensity. You may need to reduce a 5 mW laser’s intensity by as much as 6 orders of magnitude.

The latency of the detectors is about 1 ns from photon incidence to signal transduced. This latency corresponds to a light travel distance of about a foot, and is negligible compared to the time it takes to implement a setting choice (τ_{set} = 170 ns in the Vienna experiment). The detectors have a time
resolution of 400 ps.

After the detection of a photon, our detector has an artificially-induced deadtime of 420 ns, as seen in Fig. 5.5.1. The reason for imposing this deadtime is due to the operating mechanism of avalanche photodiodes. By applying a high voltage in the direction opposite to that of easy current flow in a p-n junction, the charge carriers in the p-n junction become depleted and the resistance of the junction increases. In this state, small energy kicks from photons hitting the p-n junction dislodge electrons, which get accelerated if the applied voltage is sufficiently high. The end result is a macroscopic-scale pulse of current on the order of milliamperes from the disturbance caused by a single photon. Immediately after this happens, the voltage is shut off so that the charges reequilibriate. Attempting to reapply a voltage to the detector too soon after a detection before reequilibration causes a high probability of re-triggering the detector, called an “afterpulse”. Having a low probability of afterpulsing is desirable because it reduces the intrinsic predictability in the instrument: afterpulsing would make it more likely to register two photons of the same color in a row. Since we are working the limit of low count rates (hundreds or at best thousands of counts per second), our count rates are not decreased by having an artificially-long deadtime.

Signals from the avalanche photodiodes are recorded by an IDQ ID801 Time to Digital Converter (TDC). The relative precision of time-tags is limited by the 81 ps clock rate of the TDC, and by the 400 ps timing jitter of the detectors. As a timing reference, we also record a stabilized 1-pulse-per-second signal from a Spectrum Instruments TM-4 GPS unit. Absolute time can also be recorded using the GPS unit’s IRIG-B output, but we did not take advantage of this for the data reported here. The GPS timing solution is compensated for the length of its transponder cable, which corresponds to a delay of 77 ns with the fifty-foot cable provided with the GPS antenna.
Figure 5.5.1: The datasheet sent with the ID120 single-photon counter included this histogram of counts as a function of time after a detection at $t = 0$. Note that the count rate after the detection gradually creeps up from zero starting at around 420 ns. Afterpulsing would result in a local maximum in the count rate histogram soon after a photon was detected at $t = 0$ and the detector voltage was reapplied. The measured histogram indicates that the detector applies an artificial deadtime of about 420 ns to reduce the probability of afterpulsing.
6 Observing at Table Mountain Observatory

Our instrument was validated on the 40-inch telescope at Table Mountain Observatory. With our current astronomical imaging camera and basic image processing algorithms such as $3 \times 3$ median binning, quasars of magnitudes as dim as 16 were observable and recognizable on our camera with signal to noise ratios of order unity. Single photons from distant quasars (redshifts from $z = 0.1 - 3.9$) were collected and relative time tags were recorded.

We observed at Table Mountain Observatory from July 2 to July 7 in 2016. During this observing run, we observed two populations of HIPPARCOS stars and made thirteen quasar observations. We observed at the same telescope from December 19-21 in 2016, primarily focusing on making observations of the Crab Pulsar and on expanding and automating our telescope pointing capabilities by setting up a PHD2 guiding feedback loop. We were successful in setting up the feedback loop but the weather was too cloudy to reliably characterize its performance on the sky. In addition, we observed for the second time the quasar APM 08279+5255 at redshift $z = 3.9$ because it was much higher in the sky in winter than in summer at Table Mountain Observatory.

6.1 Source Selection

Using the package astropy, I wrote code that reads in a database of stars or quasars and produces a list of desirable observation targets given the time of observation, the telescope location, and the telescope’s field of view. This last piece of information was crucial to performing the pilot test with Milky Way stars [10], whose telescope’s field of view was severely limited by the presence of windows and skyscrapers in Vienna. For Vienna, I computed the Alt/Az coordinates of moderately-bright Milky Way stars (between magnitude 5 and 9) in the HIPPARCOS catalog. I then filtered for the stars that are in the telescope’s accessible range with the series of computations described in Fig. 6.1.1.

The ideal source of randomness for a cosmic Bell test would maximize the total number of valid runs before it moves out of the telescope’s accessible range, while being as distant as possible. For the Vienna experiment, I selected stars that have high count rates and whose position in the sky allow for maximum validity time [10]. The validity time is the amount of time $\tau_{\text{valid}}$ between implementing settings at both detectors and the loss of locality, when information from one side of the experiment can potentially reach the other. During this time, arriving entangled photons can generate valid runs. The locality condition was plotted on a 1+1D spacetime diagram of our Vienna experiment in Fig. 3.1.2, and is illustrated in three dimensions.
Figure 6.1.1: Given the Alt/Az coordinates of the telescope’s field of view, a star is in our field of view if the vector $p_i \times s_i$ is directed out of the page, for each $i = 1, \ldots, 4$. In Vienna’s situation, the boundaries of the narrow field of view formed a convex quadrilateral, such that this algorithm worked as expected.

in Fig. 6.1.2.

During our Table Mountain observing run, our code was used to select objects in our telescope’s accessible range of motion. We did not use it to “rank” ideal targets for a cosmic Bell test, but rather tried to have a representative sample of stars and quasars at different redshifts, observing altitudes, and magnitudes.

6.2 Seeing and Pointing

Due to subtle changes in the index of refraction of the atmosphere, the trajectories of astronomical photons get subtly bent by up to several arcseconds on millisecond timescales. Since our pinhole’s extent is also on the scale of several arcseconds, this jittering effect, often referred to as “seeing”, can cause our count rates to fluctuate dramatically, as evident in Fig. 6.2.1. In addition, the astronomical object will drift out of the 2.75-arcsecond field of view of the avalanche photodiodes without manual correction every five seconds or so due to tracking errors in the Table Mountain telescope’s mount. During our observing run, we performed manual pointing corrections by monitoring the CMOS camera. Fortunately, the telescope mount can be controlled by an archaic MS-DOS command-line interface with a set of analog paddles for sub-arcsecond manual adjustments to the telescope. Toward the end of the run, we scrounged enough hardware from around the observatory to connect
Figure 6.1.2: An illustration of the start and end of the valid time window during which EPR detections satisfy the locality and freedom-of-choice loopholes. **Left:** At some time $t_0$, a photon from quasars $Q_a, Q_b$ is received at telescopes on both sides of the experiment ($R_a, R_b$). The measurement settings are implemented a short time later and a short distance away ($M_a, M_b$). Meanwhile, information carried by the astronomical photons can propagate at the speed of causality, indicated by the blue and green spheres expanding from $R_a, R_b$ at the speed of light. Since the information from one side has not yet reached the other, EPR photons which have been emitted from the source $S$ lying on the boundary of the red sphere just arriving at $M_a, M_b$ are valid detections. **Right:** Detections are no longer valid when the information from, say, $Q_a$, reaches the other side at $M_b$, or vice versa. If this time is $t_f$, the validity time to be maximized is defined to be $t_f - t_0$. 
Figure 6.2.1: We see the dramatic variations in count rates due to seeing and manual pointing errors. The small spike in the skyglow is likely from a small object such as a plane or satellite that briefly passed through our field of view. The Crab Pulsar’s count rate is unusually stable over time because the nebula has an extent of several arcminutes.
this analog controller to the ST4 interface on the ZWO ASI CMOS camera. The open-source PHD2 Guiding software used the camera’s image and the camera’s ST4 interface to prescribe pointing corrections in real time. While observing, we located dim objects by first finding a nearby bright star (as dim as magnitude 10 will do) and re-zeroing the telescope’s coordinate system before making a small correction to point at the dim object of interest. Using median binning to preprocess the CMOS image reduces the graininess of the camera image as well, which will likely help the PHD2 Guiding software to lock on to a target.

6.3 Calibration with HIPPARCOS Stars

We observed two groups of stars: one group of different colors and magnitudes near zenith with altitudes of $88^\circ - 90^\circ$ (summarized in Appendix C, Table 9), and one group with similar colors and magnitudes at different altitudes (summarized in Appendix C, Table 10). We plot the total observed count rate against the V-band magnitude of each star in Fig. 6.3.1 and observed that even without compensating for color, the response is linear in expected flux. I had planned on forming a more detailed model taking into account magnitude, color, and observing altitude. At the moment however, systematic errors involving pointing and seeing dominate any fit I would do. The trends in the data all make sense and were good enough to enable me to proceed to observing quasars.

6.4 Observation of Quasars

Table 4 contains a list of quasar observations made in July 2016, as well as one observation of APM08279+5255 made in December 2016. We report the catalog magnitudes of the quasars we observed as well as the probability $q^{(i)}$ of generating an uncorrupt setting from each quasar. Recall from Eq. (2.4.4) that (assuming ideal entanglement visibility) the average $q^{(i)}$ needs to be above 0.79. Even using a small (1 m) telescope at a light-polluted Los Angeles observing site, we find that the first quasar (3C 273) paired with either of the next two would yield an average $(q^{Alice} + q^{Bob})/2$ in excess of this limit for addressing the freedom-of-choice loophole. We suspect that non-ideal pinhole alignment significantly reduces the coupling efficiency of our device, and that observing in the light-polluted skies is largely responsible for our current performance. In future work, larger telescopes would make thermal noise due to dark counts negligible, and darker skies would reduce skyglow. In addition, we quantify the statistical predictability of our bitstreams by estimating the mutual information of the next bit given the previous few bits in the data. The significance of this figure of merit and our computational method are discussed later in Section 7.1.
Figure 6.3.1: For 50 bright stars in the HIPPARCOS catalog observed at zenith and eleven high-redshift ($z < 3.911$) quasars, we plot the total (red + blue) count rate against the astronomical V-band magnitude ($551 \pm 88$ nm), which is well into our blue band, and is the only data available for all observed objects), as well as the best-fit line that relates the two. From 5th to 16th magnitude, our count rates follow an exponential trend after subtracting noise rates, indicating that there is no saturation due to dead time over this dynamic range.
Table 4: A list of quasars observed, their corresponding redshifts $z$, and light travel times $\tau$. We report their $B$ and $V$ magnitudes from the SIMBAD Astronomical Database and our observed 75th percentile count rates. The table is sorted by the fraction of valid settings $q_i$ for each quasar observation, based on both off-target counts measured at each observation’s airmass and misclassification rates through our imperfect dichroics calculated from each quasar’s emission spectrum. The quantity $I = \max_m I_N(m; m + 1)$, is the small mutual information we measured in each quasar’s bitstream and amounts to a negligible reduction in $q^{(i)}$; this figure of merit is detailed later in the thesis in Sec. 7. Quasars denoted with an asterisk (*) indicate that their bitstreams had a statistically nonzero amount of mutual information, though all were small enough. We find that the first quasar (3C 273) paired with either of the next two would yield $(q^{Alice} + q^{Bob})/2 > 0.79$ set by Eq. (2.4.4) and these pairs would be acceptable to use in a test of Bell’s inequalities.

<table>
<thead>
<tr>
<th>Name</th>
<th>Redshift (z)</th>
<th>$\tau$ (Gyr)</th>
<th>B</th>
<th>V</th>
<th>blue (cps)</th>
<th>red (cps)</th>
<th>valid fraction $q_i$</th>
<th>$I \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3C 273</td>
<td>0.173</td>
<td>2.219</td>
<td>13.05</td>
<td>12.85</td>
<td>672</td>
<td>1900</td>
<td>0.884</td>
<td>87.8*</td>
</tr>
<tr>
<td>HS 2154+2228</td>
<td>1.29</td>
<td>8.963</td>
<td>15.2</td>
<td>15.30</td>
<td>227</td>
<td>503</td>
<td>0.774</td>
<td>9.91*</td>
</tr>
<tr>
<td>MARK 813</td>
<td>0.111</td>
<td>1.484</td>
<td>15.42</td>
<td>15.27</td>
<td>193</td>
<td>633</td>
<td>0.703</td>
<td>7.62*</td>
</tr>
<tr>
<td>PG 1718+481</td>
<td>1.083</td>
<td>8.271</td>
<td>15.33</td>
<td>14.6</td>
<td>176</td>
<td>473</td>
<td>0.682</td>
<td>3.07</td>
</tr>
<tr>
<td>APM 08279+5255</td>
<td>3.911</td>
<td>12.225</td>
<td>19.2</td>
<td>15.2</td>
<td>684</td>
<td>1070</td>
<td>0.647</td>
<td>5.39*</td>
</tr>
<tr>
<td>PG1634+706</td>
<td>1.337</td>
<td>9.101</td>
<td>14.9</td>
<td>14.66</td>
<td>121</td>
<td>285</td>
<td>0.572</td>
<td>3.38*</td>
</tr>
<tr>
<td>B1422+231</td>
<td>3.62</td>
<td>12.074</td>
<td>16.77</td>
<td>15.84</td>
<td>123</td>
<td>358</td>
<td>0.507</td>
<td>4.22*</td>
</tr>
<tr>
<td>HS 1603+3820</td>
<td>2.54</td>
<td>11.234</td>
<td>16.37</td>
<td>15.99</td>
<td>121</td>
<td>326</td>
<td>0.501</td>
<td>4.78</td>
</tr>
<tr>
<td>J1521+5202</td>
<td>2.208</td>
<td>10.833</td>
<td>16.09</td>
<td>15.7</td>
<td>106</td>
<td>309</td>
<td>0.476</td>
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<tr>
<td>87GB 19483+5033</td>
<td>1.929</td>
<td>10.409</td>
<td>15.5</td>
<td>98</td>
<td>241</td>
<td>464</td>
<td>0.464</td>
<td>0.32</td>
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<tr>
<td>PG 1247+268</td>
<td>2.048</td>
<td>10.601</td>
<td>16.12</td>
<td>15.92</td>
<td>111</td>
<td>333</td>
<td>0.453</td>
<td>2.92*</td>
</tr>
<tr>
<td>HS 1626+6433</td>
<td>2.32</td>
<td>10.979</td>
<td>15.8</td>
<td>87</td>
<td>213</td>
<td></td>
<td>0.398</td>
<td>1.81</td>
</tr>
</tbody>
</table>
7 Quantifying Randomness

When we take data in the form of a list of photon detections in each detector arm, it is of interest to ask how “random” the acquired bits are. We may assess the quality of randomness in two different ways. The NIST Statistical Test Suite [27] provides us with a model-independent statistical approach to evaluate the “predictability” of the output of any random number generator given a sufficiently large number of bits. In addition to running the NIST tests, we compute the mutual information in our bitstreams, which quantifies the predictability of the $m + 1^{st}$ bit given knowledge of the previous $m$ bits. We may in addition ask a physical question in light of the freedom-of-choice loophole: what fraction of our settings are corrupted by local influences or generated by local sources of photons? We assume that photons emitted by the astronomical source are completely unpredictable by local realist mechanisms which did not have access to the initial conditions of the source. Some fraction of these astronomical photons are misclassified by our instrument’s imperfections. We assume these misclassifications, as well as all other non-astronomical photon detections, are corrupted as part of a local-realist conspiracy to violate Bell’s inequality, as discussed in Section 2.4.

In the remainder of this section, we describe both of these analysis methods, and for concreteness, apply them to bitstreams that we collected from 13 quasars. A summary of the results of this analysis was available back in Table 4.

It is important to note that the NIST Statistical Test Suite [27] (and our mutual information analysis) and our analysis of the fraction of corrupted settings quantify two necessary but insufficient conditions for complete unpredictability by local hidden-variable mechanisms. Statistical unpredictability of the $m + 1^{st}$ bit given the prior $m$ bits does not imply the absence of a local hidden-variable mechanism. It is possible to have many local photons which are considered corrupt, but whose statistical predictability isn’t evident (e.g. noise photons). The other extreme is also possible: one can have a high fraction of photons coming from an astronomical source and a low rate of noise, but due to effects like afterpulsing and subtle systematic errors such as misalignment, the bitstreams can have a significantly-nonzero statistical predictability that is both evident in the bitstream and is exploitable by a local hidden-variable theory. However, by quantifying and measuring the fraction of astronomical photons and the statistical predictability of our bitstreams, we chip away at ever-more foundational assumptions underlying our attempt to generate truly unpredictable settings for a cosmic Bell test.
7.1 Measuring Statistical Randomness

In a similar work to this one, Wu et al. [20] relies on the NIST Statistical Test Suite [27] to evaluate the statistical randomness of the bits generated from stars using an even-odd nanosecond scheme for generating random bits from astronomical photons. Since my ARNG can generate random bits using the timing as well as the color of the photons, I run the NIST Statistical Test Suite on random bits generated via both methods when pointing at quasars. When using an even-odd nanosecond method to generate random bits, every dataset from quasars passes the NIST tests. When using the photon-color method to generate bits, my data fail many tests in the NIST statistical test suite. This is due to the significant bias in red and blue count rates, as well as two subtle systematic effects which I identified.

To further investigate this problem, I computed the mutual information of the data, which quantifies the additional probability of guessing the $m+1^{st}$ bit correctly beyond an overall red-blue bias, given the previous $m$ bits as input. We denote the mutual information by $I(m; m+1)$; a nonzero fraction of the bits $I(m; m+1)$ in our datastream can be predicted with knowledge of the previous $m$ bits. If each bit were truly independent, this mutual information would be zero even if the probability to get a zero or one wasn’t 50%. The formal definition of the mutual information is as follows: Let $\mathcal{X}_m$ denote the set of all length-$m$ binary strings, and let $p(x)$ be the probability that a string of bits of length $m$ within our bitstream is $x \in \mathcal{X}_m$. Similarly, let $p(y)$ be the probability that a single bit in our datastream is $y \in \{0, 1\}$. If $p(x, y)$ is the probability that a string of $m+1$ bits in the bitstream of interest are $x$ followed by $y$, then the mutual information in our data is defined to be

$$I(m; m+1) = \sum_{x \in \mathcal{X}_m} \sum_{y \in \{0, 1\}} p(x, y) \times \log_2 \left( \frac{p(x, y)}{p(x)p(y)} \right). \quad (7.1.1)$$

Note that if the next bit is independent of the $m$ bits preceding it, then $p(x, y) = p(x)p(y)$ and the mutual information is zero.

To estimate $I(m; m+1)$ we can use the experimental estimates $\hat{p}_N(x, y)$, $\hat{p}_N(x)$, and $\hat{p}_N(y)$ of the probabilities $p(x, y)$, $p(x)$, and $p(y)$ from our finite dataset of length $N$. However, it is well known that statistical fluctuations $\hat{p}_N(x, y)$, $\hat{p}_N(x)$, and $\hat{p}_N(y)$ of these probabilities causes the amount of mutual information in the dataset to be overestimated if we simply “plug in” the estimates $\hat{p}$ into Eq. 7.1.1 [28]. The resulting estimate of $I(m; m+1)$, which we denote $\hat{I}_N(m; m+1)$, is biased. This bias is a significant problem because the mutual information in our bitstream is likely to be very small. We denote this naive estimator by $\hat{I}_N(m; m+1)$, because its value depends on the size of the dataset $N$. 

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However, in the limit that the dataset is large \((N \gg 1)\), and if \(m\) is fixed, the amount of positive bias in the naive estimate \(\hat{I}_N(m; m + 1)\) is dependent only on \(N\). For large values of \(N\), \(\hat{I}_N(m; m + 1)\) can be represented as a small perturbation away from an unbiased estimator \(\hat{I}(m; m + 1)\) without this finite-size effect. I will compute the unbiased estimator \(\hat{I}(m; m + 1)\) following a method first introduced by Treves et al. \[28\]. I expand in powers of \(1/N\) with the ansatz

\[
\hat{I}_N(m; m + 1) = \hat{I}(m; m + 1) + \frac{a}{N} + \frac{b}{N^2}
\]

where the three constants \(\hat{I}(m; m + 1)\), \(a\), and \(b\) are assumed to be fixed over the bitstream of interest. To determine \(\hat{I}(m; m + 1)\), I use the following procedure to set up a system of equations. First, I compute \(\hat{I}_N(m; m + 1)\) for the entire dataset. Then I divide the dataset into 2 chunks of size \(N/2\). Computing \(\hat{I}_{N/2}(m; m + 1)\) for both chunks two chunks and averaging over both chunks gives us a value of \(\hat{I}_{N/2}(m; m + 1)\), which along with our ansatz gives us the second equation:

\[
\hat{I}_{N/2}(m; m + 1) = \hat{I}(m; m + 1) + \frac{a}{(N/2)} + \frac{b}{(N^2/4)}
\]

Repeating this procedure with 4 chunks of size \(N/4\) gives a third equation, enabling us to solve for the three unknowns \(\hat{I}(m; m + 1)\), \(a\), and \(b\). Note that because we average over each chunk, we assume that each chunk of data is a representative sample of the bitstream it came from. Since we are working with a small number of chunks, this is a fairly good approximation.

Using this unbiased estimator \[28\], I compute the mutual information in the bits we generate when taking on-target data, collecting photons from quasars, as well as data taken pointing at the sky slightly off-target. I compute \(I(m; m + 1)\) for \(m = 1, 2, \ldots 6\) lookback bits on datasets of length \(N > 2^{16}\). To see if these estimates are consistent with zero mutual information, I compare our estimates of \(I_N(m; m + 1)\) against fifty simulated datasets, each with the same length and the same red/blue bias as in each dataset and with no mutual information. A quasar with no significant mutual information (PG 1718+481) and a quasar with small, but statistically nonzero mutual information (3C 273) are shown in Fig. 7.1.1.

As summarized back in Table 4, the random bits generated from quasar colors in 8 out of 12 datasets exhibit mutual information that is significantly inconsistent with zero, though still very small. This hints at the possibility of some nontrivial structure in the data which may be induced by physical effects or systematic error. In 11/12 datasets, the maximum information \(I = \max_m I(m; m + 1)\) is less than 0.001, while for the exceptionally-bright quasar 3C 273 (\(V = 12.9\)), \(I \approx 0.009\). One way to interpret a mutual information of 0.001 is to have one in every 1000 bits be a deterministic
Figure 7.1.1: The experimental estimate of the mutual information between a bit and the $m$ bits preceding it, for $m = 1, \ldots, 6$ for two different quasars. To check for nonzero mutual information in the on-target data (purple circles) and off-target data (black circles), I estimate the mutual information of 50 simulated datasets with the same length and the same red-blue bias with no mutual information, and shade $2\sigma$ error bars about the mean. For the quasar PG1718+481, I find that the experimentally-observed mutual information in the on-target as well as the off-target data is consistent with zero, while for the exceptionally bright quasar 3C 273, I observe nonzero mutual information on target.
function of the previous few bits instead of being random. Even in the worst case with 0.009 bits of mutual information, the amount of corruption $1 - q^{(i)}$ is only increased negligibly compared to the effect from skyglow.

Upon examining the experimental probability estimates $\hat{p}(x, y)$ that went into the mutual information calculation, I identified two systematic sources of non-randomness, both of which are exacerbated for high-count rate sources. The first mechanism for non-randomness is detector saturation. After a detection, the detector has a 420 ns deadtime window during which a detection is improbable. Hence for sufficiently high count rates (such as those experienced when observing stars), it is much more likely to observe a blue photon following a red one and vice versa than multiple photons of the same color in a row. While this effect is seen in our calibration data with HIPPARCOS stars, the count rates necessary for this effect to be important ($10^5 - 10^6$ counts per second) far exceed what is observed with quasars. These can also be eliminated by imposing the same deadtime window in the other channel and removing (in real time or in post-processing) any detection that is within the deadtime of any previous detection from either channel.

The second mechanism is a consequence of imperfect alignment combined with random atmospheric seeing. Within my instrument’s pinhole, I know there exists a “sweet spot” for optimal coupling to the blue detector, and a slightly different sweet spot for optimal alignment with the red detector. As the image of the quasar jitters within the pinhole on timescales of milliseconds, the image overlaps differently with these sweet spots, executing a random walk. The result is that the conditional probability $p(x \rightarrow y)$ is increased if the previous bits $x$ contain mostly bits of type $y$. For example, a detection of 1 given that the previous detections were 10111 is higher than the average (unconditional) probability of receiving a 1. This is borne out in the data. Since atmospheric seeing is a consequence of random atmospheric turbulence, it is a potential source of local influences on astronomical randomness. It can be mitigated by careful characterization of the optical alignment of the system, making sure that the sweet spots of both detector arms overlap to the greatest extent possible, and observing under calm atmospheric conditions to reduce the seeing. For a larger telescope in a darker location where the signal to background ratio is higher, this would result in relatively larger effect on the fraction of valid runs.

### 7.2 What Fraction of Photons are from Astronomical Sources?

In this section, my goal is to compute $q^{(i)}$, the probability that our instrument generates an uncorrupt setting as described in Sec. 2.4. A plausible physical model for corruptions in an ARNG is as follows. I assume that all detections due to thermal noise as well as detections from skyglow are corrupted because they come from local sources. Denote the background count
rate due to these sources as in the $i^{th}$ instrument ($i \in \{\text{Alice, Bob}\}$) in the $j^{th}$ detector arm ($j \in \{\text{Red, Blue}\}$) as $n_j^{(i)}$. When we point the instrument at an object, the count rate increases because now we’re detecting noise $n_j^{(i)}$ as well as some astronomical photons. The on-target count rate is defined $r_j^{(i)}$, and the fraction of photons from noise is simply $n_j^{(i)}/r_j^{(i)}$.

However, noise is not the only source of corrupt photons. Since the instrument’s spectral bands have some nonzero overlap due to imperfect dichroic beamsplitters, some fraction of red photons travel down the blue arm ($f_{r \rightarrow b}$) and vice versa. This fraction of detections are corrupt, even though they come from astronomical sources. I define $j'$ to be the color opposite to $j$, i.e. $j'$ is red if $j$ is blue and vice versa. Some nonzero fraction $f_{j' \rightarrow j}$ of photons end up in the $j^{th}$ arm. If $s_j^{(i)}$ represents the “unmixed” detection rate at the $j^{th}$ detector in the $i^{th}$ instrument (after account for the overall efficiency in each arm), this mixture can be modeled as follows. I take $f_{j \rightarrow j'} + f_{j' \rightarrow j} = 1$, since an overall loss of photons in each detector arm in the dichroic beamsplitters can be absorbed into one efficiency factor for each arm. Then, in the $j^{th}$ detector arm we see $s_j^{(i)} f_{j' \rightarrow j}$ misclassified photons. To summarize,

$$r_j^{(i)} - n_j^{(i)} = f_{j \rightarrow j'} s_j^{(i)} + f_{j' \rightarrow j} s_j'^{(i)}$$  \hspace{1cm}(7.2.1)$$

not background photons \hspace{1cm} correctly classified, valid \hspace{1cm} incorrectly classified, invalid

Since the count rates $n_j^{(i)}$ and $r_j^{(i)}$ can be measured, and since the fractions $f_{j \rightarrow j'}$ can be modeled in what follows (see Sec. 7.4), I can determine the unmixed source rates $s_j^{(i)}$ by writing Eq. 7.2.1 for each arm and inverting the system of equations.

The probability that the $j^{th}$ detector arm in the $i^{th}$ instrument yields an invalid run is at most the sum of two contributions, noise or misclassification, assuming conservatively that only one mechanism is in effect at a time. This corruption probability is

$$p_j^{(i)} = \frac{n_j^{(i)}}{r_j^{(i)}} + \frac{s_j^{(i)} f_{j' \rightarrow j}}{r_j^{(i)}}.$$  \hspace{1cm}(7.2.2)$$

Finally, since a single experimental run involves a detection in only one of the two color arms, the probability of an invalid run for our ARNG is conservatively bounded above by maximizing over both colors:

$$p^{(i)} = \max_j p_j^{(i)}.$$  \hspace{1cm}and thus $q^{(i)} = 1 - p^{(i)}$. To obtain $q$ for the joint setting choice, one needs to take $q = 1 - p^{(\text{Alice})} - p^{(\text{Bob})}$, as explained in Sec. 2.4.
7.3 Noise and Wrong-Way Influencing Instrument Design

Equation 7.2.2 is very important as a figure of merit of our instrument’s design because it quantifies the relative strength of our priorities in building an ARNG. Namely, is noise or misclassification the dominant contribution to $p_j^{(i)}$? It turns out that the answer is different for different values of $n_j^{(i)}, r_j^{(i)}$.

If we analytically write down an expression for $s_j^{(i)}$ and substitute it into Eq. 7.2.2, we obtain (for $f_{j\rightarrow j'} << 1$)

$$p_j^{(i)} = \frac{n_j^{(i)}}{r_j^{(i)}} + \frac{r_j^{(i)} - n_j^{(i)}}{r_j^{(i)}} f_{j'\rightarrow j} - \frac{r_j^{(i)} + r_j^{(i)} - n_j^{(i)} - n_j^{(i)}}{r_j^{(i)}} f_{j'\rightarrow j} f_{j'\rightarrow j} + \mathcal{O}(f^3)$$

(7.3.1)

We see in the first two terms that the dominant contribution to $p_j^{(i)}$ is dependent on the size of the noise rate $n_j^{(i)}$ compared to the size of the misclassified signal rate $(r_j^{(i)} - n_j^{(i)}) f_{j'\rightarrow j}$. Typical values of $n_j^{(i)}$ are $\sim 10^2$ cps, while for stars such as those used in Vienna, $r_j^{(i)} \sim 10^6$ is not uncommon. For quasars, as seen in Table 4, $r_j^{(i)} \sim 10^2 - 10^3$ cps. Values of $f_{j\rightarrow j'}$ range from $10^{-3} - 10^{-2}$ depending on the quality of the dichroic mirrors used, as discussed later. This rough estimate shows that for bright stars, $p_j^{(i)}$ is dominated by misclassification, while for quasars, $p_j^{(i)}$ is significantly dominated by the noise contribution by about two orders of magnitude. Hence, in optimizing our ARNG for observation of quasars, I choose a pair of dichroics which minimizes the gap in our observing bands, allowing for maximum collection of quasar photons, rather than a pair of dichroics that sacrifice signal photons for fewer misclassifications. Spectra of my chosen dichroics in two different arrangements are shown in Fig. 7.3.1. The $f_{j\rightarrow j'}$ values for my dichroics are smaller than those of the Vienna experiment’s dichroics by an order of magnitude (see Table 5 and [10]). Using these would have significantly improved the statistical significance of that experiment’s Bell violation.

7.4 Modeling of the Wrong-Way Fractions

The wrong-way fractions $f_{j\rightarrow j'}$ are important parameters for computing $q^{(i)}$, the probability of getting an uncorrupt setting from an astronomical source. As discussed in Sec. 7.2 and shown in Eq. 7.2.2, they are extremely important when the noise rates are negligible, i.e. when bright stars are being used as sources of randomness such as in our first cosmic Bell test. In this section, I describe how I calculated these parameters using instrumental
Figure 7.3.1: Our system of two Semrock dichroic beamsplitters can be arranged in one of two possible configurations, placing either the shortpass dichroic beamsplitter first (top) or the longpass dichroic beamsplitter first (bottom). By using two beamsplitters, one of the two arms can have a much better rate of rejecting misclassified photons. For each configuration, we can compute the probability that an incident photon of wavelength $\lambda$ makes it into each arm, denoted $B(\lambda)$ and $R(\lambda)$, and plotted here. While it is not obvious from these plots alone, these two configurations result in different wrong-way fractions $f_{j \rightarrow j'}$ when the other contributions to the instrument’s response curve are taken into account. On the basis of minimizing the maximum value of $f_{j \rightarrow j'}$ over all $j$, we choose to use the shortpass-first configuration.
spectral responses, either provided by the manufacturer or measured with a spectrometer.

In our instrument, a photon of wavelength $\lambda$ may transduce a setting choice. Suppose that the number distribution of setting choices as a function of the original photon’s wavelength is $N(\lambda)$. This distribution has units of counts/time/wavelength. It is a complicated function of our instrument’s spectral responses, atmospheric conditions, detector quantum efficiencies, and the spectrum of the astronomical source and will be modeled in Sec. 7.5.

The effect of the dichroic beamsplitters will be captured by two additional functions, $R(\lambda)$ and $B(\lambda)$, which denote the probability that a photon of wavelength $\lambda$ travels down the red and blue arm. Note that due to absorptions, $R(\lambda) + B(\lambda) < 1$. Finally, define an artificial cutoff $\lambda'$ such that blue photons are those with $\lambda < \lambda'$ and red photons are those with $\lambda > \lambda'$ as discussed back in Sec. 2.3. As a function of $\lambda'$, the four parameters

$$F_{b\to r}(\lambda') = \int_{\lambda'}^{\infty} B(\lambda)N(\lambda) \, d\lambda \quad F_{r\to b}(\lambda') = \int_{0}^{\lambda'} R(\lambda)N(\lambda) \, d\lambda.$$

$$F_{b\to b}(\lambda') = \int_{0}^{\lambda'} B(\lambda)N(\lambda) \, d\lambda \quad F_{r\to r}(\lambda') = \int_{\lambda'}^{\infty} R(\lambda)N(\lambda) \, d\lambda \quad (7.4.1)$$

fully characterize the instrument’s tendency to misclassify photons as blue or red. Note that our definition of $\lambda'$ is independent of the system of dichroic beamsplitters; we could have chosen $\lambda'$ and bought a pair of dichroics with appropriate spectral properties that classify the photons optimally. Different choices of $\lambda'$ simply affect the fraction of detections in each arm which are deemed to be misclassified. For example, shifting $\lambda'$ to longer wavelengths means more photons are labeled as blue photons. This reduces the fraction of “non-blue” detections in the blue detector arm. However, this also means a lower fraction of the photon arriving at the red arm are labeled as red, increasing the fraction of misclassifications. Hence, it is appropriate to choose an optimal value of $\lambda'$ which minimizes the total number of photons which we deem are going the wrong way:

$$\lambda' = \min_{\lambda'} \frac{F_{b\to r} + F_{r\to b}}{F_{b\to b} + F_{b\to r} + F_{r\to b} + F_{r\to r}}. \quad (7.4.2)$$

From these four parameters, it is then possible to compute the misclassification probabilities $f_{j\to j'}$. Since these were originally defined as the probability that a photon of color $j$ goes down the $j^{th}$ detector arm, we need to divide $F_{j\to j'}$ by all photons of color $j$:

$$f_{j\to j'} = \frac{F_{j\to j'}(\lambda')}{F_{j\to j}(\lambda') + F_{j\to j'}(\lambda')} \quad (7.4.3)$$
In the following sections, we will describe how we model \( N(\lambda) \), taking into account absorption by the atmosphere, spectral responses of optical components, and the quantum efficiency of our detectors.

### 7.5 Modeling the Number Distribution of Detected Photons

Several considerations need to be taken into account when modeling \( N(\lambda) \) to compute \( f_{j \rightarrow j'} \). We start with the spectra of the astronomical sources \( N_{\text{source}}(\lambda) \), which for bright stars can be modeled well as blackbodies [29]. For redshifted quasars, we use an empirically-determined rest-frame spectrum [30] which we then redshift to the reference frame of the detector. For the Crab Pulsar, which we discuss later, the spectrum is adequately described by a power-law model fit to empirical data [31]. We then apply the spectral responses (denoted \( \rho \)) of the atmosphere, any optical components, and finally the detector’s quantum efficiency as the photon makes its way through the cosmic setting generator. In summary,

\[
N(\lambda) = N_{\text{source}}(\lambda) \times \rho_{\text{atm}} \times \rho_{\text{lens}}^2 \times \rho_{\text{det}} \times \rho_{\text{mirror}}
\]

In our numerical computation of \( f_{j \rightarrow j'} \), the limits of our wavelength range (350 nm – 1150 nm) are set by the quantum efficiency of the detector flattening out past a certain wavelength range. For other datasets over an incomplete range, e.g. the lenses’ anti-reflection coating, we apply boundary conditions at these limits in a conservative way that will lead to an overestimate rather than an underestimate of \( f_{j \rightarrow j'} \). In the following sections, we outline the procedure by which we model the required spectral responses. One typical wrong-way photon analysis for a quasar is illustrated in Fig. 7.6.1, in which we find the best cutoff wavelength \( \lambda' \) and compute \( f_{r \rightarrow b} \) and \( f_{b \rightarrow r} \).

#### 7.5.1 Atmospheric Absorbance

To model atmospheric effects on the number distribution of incoming photons, we use an atmospheric transmittance spectrum \( \rho_{\text{atm}}(\lambda, \text{zenith}) \) generated by the program MODTRAN for mid-latitude atmospheres [32]. The atmospheric transmittance curve, plotted in Fig. 7.5.1, features some narrow absorption lines due to the presence of certain molecules in the atmosphere. In addition, the transmittance gradually decreases at UV wavelengths due to Rayleigh scattering. The absorption lines are dependent on local atmospheric conditions, and both of these effects are dependent on the observation angle of the target object. However, since we are integrating the response curves to obtain the parameters \( f_{j \rightarrow j'} \), narrow features do not matter quite as much as broadband features. Hence, the effect of the atmosphere on wrong-way parameters is dominated by Rayleigh scattering.
Figure 7.5.1: Probabilities of transmission through each element in the photon’s path from the vacuum of space to detection. We smoothly interpolate the anti-reflection coating to zero at 350 nm. The linear extrapolation in the detector efficiency curve is also clearly visible in the form of two sharp kinks near $\lambda = 450$ nm, 1050 nm.

The atmospheric absorption at zenith must be corrected for by the increased absorption from our to low observation elevations. Due to the requirements of causal alignment, these altitudes can be as low as 24° above the horizon. At this altitude, the airmass is denoted $X$ and is equal to $\sec(24°) \approx 2.5$, which means we are looking through up to 2.5 times the amount of air as compared to looking directly upwards. The effect of Rayleigh scattering can be captured by modifying $N_{source}(\lambda)$ for different altitudes with the substitution

$$
\rho_{atm}(\lambda, X) = \exp(-X \tau(\lambda)) \rho_{atm}(\lambda, \text{zenith}) \tag{7.5.1}
$$

where $\tau(\lambda)$, the optical density due to Rayleigh scattering, roughly quantifies the probability that a photon of wavelength $\lambda$ gets deflected from its original trajectory by Rayleigh scattering [33].

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7.5.2 Lenses, Mirrors, and Detectors

In the experimental setup as described in Section 5.1, an achromatic lens collimates the achromatic lens in each arm focuses the incident photons onto the active area of the single photon counting modules. These achromatic lenses are anti-reflection coated in the range from 500 nm – 1500 nm. However, not all photons are transmitted, with a wavelength-dependent probability \( \rho_{\text{lens}}(\lambda) \) that is close to unity but not quite for most of the nominal range, as plotted in Fig. 7.5.1. We conservatively extrapolate the anti-reflection coating with a polynomial that goes to zero at \( \lambda = 350 \text{ nm} \). In addition to these two lenses, The ARNG used in the first round of Cosmic Bell tests [10] had an additional mirror with some spectral response \( \rho_{\text{mirror}}(\lambda) \) which had be taken into account when computing \( N(\lambda) \).

Once the focused light reaches the detector, the detector will be triggered with some wavelength-dependent quantum efficiency, also plotted in Fig. 7.5.1. Due to incomplete quantum efficiency data provided by the manufacturer, we assume that the detector’s quantum efficiency curve is continuous outside the data range provided by the manufacturers, falling linearly to zero at \( \lambda = 350 \text{ nm} \) and \( \lambda = 1150 \text{ nm} \). To avoid these assumptions in the future, these spectra should be measured to high accuracy over a wavelength range at least as broad as the one used in our model.

7.6 Results from Spectral Modeling: Stars and Quasars

Our results are summarized in Table 5. We are able to compute \( f_{j \rightarrow j'} \), using emission models for astronomical sources’ observed \( N_{\text{source}}(\lambda) \), spectral responses from all optical components, manufacturer-provided quantum efficiency measurements of the detectors, a simple atmospheric transmission model, and the spectral responses of the dichroic beamsplitters \( B(\lambda) \) and \( R(\lambda) \) used in both the Vienna instrument and my next-generation astronomical random number generator. A typical analysis is summarized in Fig. 7.6.1.
Figure 7.6.1: Panel A: The photon-number spectrum of the quasar PG 1718+481, up to some arbitrary constant, attenuated by the atmosphere as plotted in Fig. 7.5.1. Panel B: Our maximally conservative model of the overall transmission probability of the optics coatings and detector quantum efficiency, a product of three non-atmospheric curves in Fig. 7.5.1. Panel C: The probability that a photon goes down each arm as a function of λ, as plotted previously in Fig. 7.3.1. Panel D: The final color distribution of photons seen at each arm. At this point, the curves are normalized by the same multiplicative factor such that the area under the sum of both curves is 1. With these color distributions, I compute the wrong-way fractions $f_{j\rightarrow j'}$ for the source PG 1718+481 for an arbitrary cutoff wavelength $\lambda'$, and find the best wavelength $\lambda'$ that defines the separation between “red” and “blue” photons.
Table 5: Shown here are $f_{j\rightarrow j'}$ for various sources. We include the stars used in the Vienna cosmic Bell experiment, as well as the $f_{j\rightarrow j'}$ for quasars that we observed at Table Mountain Observatory.

<table>
<thead>
<tr>
<th>Source</th>
<th>$f_{r\rightarrow b}$ [%]</th>
<th>$f_{b\rightarrow r}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIP 56127 (Vienna Run 1)</td>
<td>1.42</td>
<td>1.92</td>
</tr>
<tr>
<td>HIP 105259A (Vienna Run 1)</td>
<td>1.46</td>
<td>1.80</td>
</tr>
<tr>
<td>HIP 80620 (Vienna Run 2)</td>
<td>1.39</td>
<td>2.03</td>
</tr>
<tr>
<td>HIP 2876 (Vienna Run 2)</td>
<td>1.60</td>
<td>1.39</td>
</tr>
<tr>
<td>3C 273 (Quasar $z = 0.173$)</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>MARK 813 (Quasar $z = 0.11$)</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>APM 08279+5255 (Quasar $z = 3.9$)</td>
<td>0.19</td>
<td>0.18</td>
</tr>
</tbody>
</table>
8 Crab Pulsar: Measurements and Applications

Besides using our instrument in tests of quantum foundations, it is possible to leverage the high time resolution of our avalanche photodiodes to measure the pulsation rate and periodic light curve of the Crab Pulsar in the Crab nebula, which is shown in Fig. 8.0.1 as seen by our camera. The neutron star, or pulsar, at the heart of the nebula periodically flashes every $\approx 33\text{ ms}$ across the electromagnetic spectrum, likely due to the existence of a highly-localized bright “patch” on the surface of the neutron star which points towards earth briefly every time the pulsar spins on its axis [34].

This is the only source we observed whose photon arrival times have non-random structure on sub-second timescales. By using this stable astronomical clock, it is possible to verify both sub-millisecond and long-term timing stability of our entire system, including the telescope, optics, detectors, time-tagging module, and GPS absolute reference.

Here I report my measurement of the period and one folded light curve for each of the two color bands. We made several successful observations of the Crab’s pulsation over two nights in December 2016 as enumerated in Table 6. On each night, we made a sequence of observations at different times, but since we did not turn off the instrument until the end of the night, all of the datasets taken on a single night are relative to the same reference. This enables us to concatenate different datasets from a single night with the confidence of knowing the Crab pulses will be in phase over individual datasets. It is customary in the pulsar timing literature to adopt a more abstract but less cumbersome timing convention, reporting times in Modified Julian Date, or MJD. I convert UTC times to MJD times using astropy.

On the second night of observation, I used a 1 pulse-per-second signal from the GPS to provide a more stable frequency reference than the internal clock on the ID801 time-tagging unit. This allows additional calibration of period measurements. However, clouds began to obscure our field

<table>
<thead>
<tr>
<th>Filename</th>
<th>Modified Julian Date</th>
<th>Duration</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>attempt01_success.txt</td>
<td>57742.263</td>
<td>444.4 s</td>
<td>No clouds</td>
</tr>
<tr>
<td>attempt02.txt</td>
<td>57742.366</td>
<td>6235.6 s</td>
<td>No clouds</td>
</tr>
<tr>
<td>attempt03_bright_1.txt</td>
<td>57743.387</td>
<td>2429.0 s</td>
<td>Cloudy + GPS</td>
</tr>
<tr>
<td>attempt03_bright_2.txt</td>
<td>57743.41</td>
<td>233.3 s</td>
<td>Cloudy + GPS</td>
</tr>
<tr>
<td>attempt03_bright_3.txt</td>
<td>57743.419</td>
<td>835.6 s</td>
<td>Cloudy + GPS</td>
</tr>
</tbody>
</table>
Figure 8.0.1: Our camera image of the Crab Pulsar. The filaments of the Crab nebula are visible surrounding the pinhole. In the inset, we see four objects, closely spaced. By comparing the image to astronomical catalogs, we can deduce which object is the pulsar. In the inset, the pulsar (whose location is indicated with an arrow) is not visible since the light from the pulsar is passing through the pinhole.
of view, turning this into our last night of observation. When analyzing the period of the pulsar to high precision, I use the GPS-disciplined data in attempt03_bright1.txt. However, since there is much more data in attempt02.txt, I rely on that data for light curve analysis to build more robust statistics. For the purposes of computing a light curve, it is unlikely that the time-tagging unit’s internal clock drifted significantly. In a laboratory test where the time-tagging unit was exposed to periodic thermal variation due to indirect sunlight for a week, I observed that the length of a second varied by no more than 1.08 µs relative to our temperature-stabilized GPS-disciplined clock. Such drifts were slow and the time tags were interpolated to the GPS reference.

8.1 Determination of Period

I determine the period by computing the periodogram \( F(T) \) of our list of detections in the concatenated data files from the second night of observing. For each candidate pulsar period \( T \), each time tag \( t \) is associated with a complex phasor and we compute a sum of these phasors, effectively taking the discrete Fourier Transform of a sequence of delta-functions.

\[
F(T) = \left| \sum_{t \in \text{all time tags}} \exp \left( \frac{2\pi i t}{T} \right) \right|
\]

The quantity \( F \) is maximized when \( T \) is the period of the pulsar, since the phasors will on average point in the same direction at pulse-maximum. Numerically maximizing over \( T \) therefore gives us the period of the pulsar. We find that the period that we measured agrees very well with the latest data from the Jodrell Bank ephemeris [35], which is a publicly-accessible record of Crab pulsar parameters estimated by a monthly observation in the radio band. Unfortunately, the Jodrell Bank data only reports period measurements to the nearest MJD. Using the measured frequency of the pulsar as well as its instantaneous spin-down rate in mid-December 2016 and mid-January 2017, we constructed a quadratic interpolant to predict precisely what the period should have been when we observed the pulsar in late December 2016.

As shown in Table 7, the naive estimate of the pulsar period yields \( T_{\text{raw}} = 33.729767 \) ms, which is somewhat faster than expected from the pulsar ephemeris. If the time tags are synchronized to the GPS’s 1pps frequency standard, the estimate of the period improves to \( T_{\text{GPS}} = 33.730654 \) ms, which is still faster than the Jodrell Bank prediction by several parts in \( 10^5 \).

I then corrected for the Doppler shift in the pulse arrival frequency due to the Earth’s motion around the sun. Assuming that the pulsar is stationary, the distance between two successive pulses of the Crab as viewed from
Table 7: Using Jodrell Bank’s monthly measurement of both the pulsar period and the period derivative [35], we can compute the expected pulsar period on our observation date. As expected, our measurements of the period are closer to the Jodrell Bank prediction when we use an external frequency reference, and are vastly improved when we correct for the motion of Earth’s orbit around the sun.

<table>
<thead>
<tr>
<th>MJD</th>
<th>$T$ (ms)</th>
<th>Discrepancy $(\Delta T/T)$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>57743</td>
<td>33.7302828</td>
<td>—</td>
<td>Jodrell Bank ephemeris (interpolated)</td>
</tr>
<tr>
<td>57743</td>
<td>33.729767</td>
<td>$-1.9 \times 10^{-5}$</td>
<td>Using ID801 internal clock (obs.)</td>
</tr>
<tr>
<td>57743</td>
<td>33.730654</td>
<td>$+1.1 \times 10^{-5}$</td>
<td>Clocked to GPS reference (obs.)</td>
</tr>
<tr>
<td>57743</td>
<td>33.730309</td>
<td>$+7.8 \times 10^{-7}$</td>
<td>Corrected for relative velocity $(v_r = -3067 \text{ m/s})$</td>
</tr>
</tbody>
</table>

the solar system’s rest frame is going to be $cT_{\text{correct}}$. However, our detector is moving towards or away from the Crab pulsar with some radial velocity $v_r$. The time between the arrival of the pulses at our detector is therefore $T_{\text{measured}} = cT_{\text{correct}}/(c + v_r)$, where $v_r > 0$ corresponds to Earth moving closer to the pulsar. Computing $v_r$ in the solar system frame using the location of the Crab pulsar and the earth’s orbit around the sun gives us $T_{\text{correct}}$, the period corrected for the relative velocity of Earth, from $T_{\text{measured}}$, the period computed from our GPS-disciplined data. This reduces our discrepancy with Jodrell Bank’s Doppler-corrected period by an order of magnitude to 26 ns, corresponding to a relative error better than one part in $10^6$.

At this level of precision, several additional sources of error are significant, which may partially explain this discrepancy. Uncertainty in the time-tagging unit can lead to an uncertainty in the period. This can be approximately quantified by observing the drift in the measured length of a second over time. I did a lab test for about a week where we simply monitored the length of a GPS second as measured by the time-tagging unit; it differed by no more than about a part in $10^6$. However, this may not have been representative of the Table Mountain observing run. Also, an additional, smaller Doppler shift arises from the Earth’s rotation which may be significant at the level of a part in $10^6$. It remains to correct for these remaining errors in our estimate of the pulsar period.
Figure 8.2.1: An hour-long observation of the Crab pulsar (data in ‘attempt02.txt’) was used to generate the folded light curve shown here. There may be a subtle delay of the blue curve behind the red curve. This dataset wasn’t locked to a GPS clock, so the measurement of the period differs slightly from our estimates in Tab. 7. The y-axis is normalized such that averaging over the phase of the light curve gives the average number of photons per second seen from the object. Statistical $\sqrt{N}$ error bars are too small to be usefully plotted.

8.2 Light Curve Analysis

Once the period of the Crab pulsar is determined, it is possible to integrate over many observations to obtain a periodic light curve, as shown in Fig. 8.2.1. I choose 1000 time bins (each $\approx 33.7$ µs wide) for display purposes. Because we have an hour of data and sub-nanosecond timing resolution, I also tried much finer bins, but there were no interesting features. The DC offset is a combination of dark counts and skyglow from the Crab Nebula surrounding the pulsar. Statistical $\sqrt{N}$ error bars on the light curve are too small to be usefully plotted.

It is interesting to compare the pulse shape we measure to results made by other ultra-high time resolution instruments, such as Aqueye [36]. I digitize a plot taken from their paper, and I scale and shift the light curve
Figure 8.2.2: I take the sum of the two light curves in Fig. 8.2.1 and compare it to a scaled version of the measurements by Aqueye [36]. There is very good qualitative agreement between the pulse shapes. Their detector parameters are similar to our combined detector response function, except that we may have different optical efficiencies in our blue and red arms. This may explain some of the imperfect overlap.

to have the same DC offset, pulse height, and phase as the combined light curve in both of our detector arms. When overlaid in Fig 8.2.2, we see very good qualitative agreement between the light curves. Future work will characterize the relative optical efficiency of each color band and look at an appropriately weighted sum of the light curves.

8.3 Pulse Phase Analysis: Testing the Equivalence Principle

An interesting application of the Crab pulsar data we collected is to measure the delay in the arrival time of the Crab pulsar’s pulses in our different observing bands. By analyzing this delay, I can set an upper limit on violations of the Weak Equivalence Principle with some basic assumptions about the pulsar’s emission pattern.

The Weak Equivalence Principle has a variety of formulations. A common one is that the free-fall trajectories of objects in a gravitational field
are independent of their composition [37]. In this scenario, each Crab pulse moves through the gravitational potential of the Milky Way on its way to earth. If the blue and red pulses were emitted simultaneously, the Weak Equivalence Principle implies that they should (up to wavelength-dependent interactions experienced in flight) arrive at our detectors simultaneously.

More formally, the methodology is summarized by Eq. 8.3.1. Any observed difference in the time-of-arrival of a red and a blue pulse $\Delta_{\text{obs}}$ can be modeled as a sum of different color-dependent contributions:

$$\Delta_{\text{obs}} = \Delta_{\text{intrinsic}} + \Delta_{\text{ISM}} + \Delta_{\text{atm}} + \Delta_{\text{inst}} + \Delta_{\text{EP violation}}$$  \hspace{1cm} (8.3.1)

where the relative time delays $\Delta$ between the red and blue pulses are defined as follows:

- $\Delta_{\text{obs}}$: This is the observed difference in arrival time between the red and blue pulses. By convention, we will take $\Delta_{\text{obs}} > 0$ to mean that the red pulse arrives after the blue pulse. To measure this very precisely, we construct smooth templates for the red and blue light curves separately, and use a gradient-descent algorithm to numerically find the pulse peak. The templates are obtained by using the best-fit period as the fundamental frequency of a Fourier series of the un-binned data. By looking at how the magnitude of the Fourier coefficients drop off for higher harmonics, I determined that truncating the Fourier series at $\sim 100$ components rejects high-frequency noise while retaining the features of the pulse shape. A comparison of the constructed templates and a light-curve histogram constructed as in Sec. 8.2 is shown in Fig. 8.3.1.

- $\Delta_{\text{intrinsic}}$: An unknown relative delay due to different emission times by the pulsar. This effect is very difficult, if not impossible, to correct for, and is the main source of uncertainty on our upper bound. However, since the pulse profile looks so similar over multiple wavelength bands [38], it is believed that broadband pulses originate from the same region on the pulsar rather than narrowband pulses originating from different regions. Following the literature, we assume that $\Delta_{\text{intrinsic}}$ and $\Delta_{\text{EP violation}}$ are not of opposite sign.

- $\Delta_{\text{ISM}}$: A relative delay, on the order of $1 \times 10^{-13}$ s, due to different travel speeds through the interstellar medium. This effect is most significant at radio frequencies, and is negligible at optical frequencies. Using 600 nm and 815 nm as the average
colors of our photons and the pulsar dispersion measure from the Jodrell Bank ephemeris [35], we measure
\[ \Delta_{\text{ISM}} = 56.7763 \text{pc/cm}^3 \times (\nu_{\text{low}}^{-2} - \nu_{\text{high}}^{-2}) \approx 7.3 \times 10^{-13} \text{s} \]

\[ \Delta_{\text{atm}} \] A relative delay, on the order of \( 1 \times 10^{-10} \text{s} \), due to the difference in the index of refraction of the atmosphere for different frequencies [39]. Using a spectrum of the Crab pulsar’s main pulse [31], the detailed spectral model of our instrument, I compute the mean wavelengths of our red and blue bands to be 600 and 815 nm respectively. With a model of the atmosphere’s refractive index [40], the difference in refractive index is
\[ \Delta n = n(\lambda = 815.0 \text{nm}) - n(\lambda = 600.0 \text{nm}) \approx -2.03 \times 10^{-6} \]
which corresponds to a negligible delay of
\[ \Delta_{\text{atm}} = \Delta n \times 8 \text{km}/c \approx -5.3 \times 10^{-11} \text{s} \]

\[ \Delta_{\text{inst}} \] A relative delay between our instrument’s response time between the red and blue arms. This encompasses a contribution due to the different path lengths of the red and blue arms due to the addition of a second dichroic in the red arm. The contribution from path length is approximately \( 2 \text{in.}/c \approx +1.7 \times 10^{-10} \text{s} \). The electronic delay is difficult to characterize from first principles, but should not be greater than hundreds of picoseconds due to jitter in our avalanche photodiodes’ detection-to-pulse latency, and equal-length cabling.

\[ \Delta_{\text{EP violation}} \] The residual timing delay after all known sources of phase delay between the red and blue pulse arrivals are accounted for. If we assume that \( \Delta_{\text{EP violation}} \) and \( \Delta_{\text{intrinsic}} \) are relative delays that do not catastrophically cancel each other out, assuming that \( \Delta_{\text{intrinsic}} = 0 \) gives us an upper bound on the size of \( \Delta_{\text{EP violation}} \).

Violations of the Weak Equivalence Principle are parameterized quantitatively in terms of a parameter \( \gamma \). It is possible, for many competing theories of gravitation, to compute \( \delta t \), the time delay for a particle traveling in the presence of a gravitational potential \( U(\mathbf{r}) \) (as compared to in the absence of a gravitational potential). Since many theories of gravity
Figure 8.3.1: Top: two periods of the Crab pulsar’s light curve computed from simultaneous data in our blue and red observing bands. The points denote the light curve computed from the binned histogram. The traces denote smooth templates generated by taking a 100-component Fourier series of the unbinned data. The light curves are normalized to unit height to guide the eye, and the phase of the pulsar is measured from $0 - 1$ rather than $0 - 2\pi$. Bottom: a close-up of the Crab’s main pulse reveals that the red pulse arrives after the blue one delayed by $0.5 - 1\%$ of a period.
have $\delta t \propto |\int U(r) \, dr|$, it is convenient to parameterize this proportionality constant by defining $\gamma$ such that

$$\delta t = \frac{1 + \gamma}{c} \left| \int U(r) \, dr \right| \tag{8.3.2}$$

For example, general relativity has $\gamma = 1$ [41]. The Weak Equivalence Principle, in this framework, is simply the statement that this value of $\gamma$ is constant. In a theory which violated the Weak Equivalence Principle by inducing different time delays for photons of different colors, we would observe that $\gamma$ depends on photon wavelength $\lambda$.

Using our data from the Crab pulsar, I can obtain an estimate of the difference in delays $\delta t$ for two different populations of photons, giving an estimate of the differential post-Newtonian parameter which we define as

$$\Delta \gamma = |\gamma(815 \text{ nm}) - \gamma(600 \text{ nm})|.$$ 

Following [38], this parameter is related to our observable $\Delta_{\text{EP violation}}$ by subtracting Eq. 8.3.2 from itself at $\lambda = 600$ nm from $\lambda = 815$ nm.

$$\Delta_{\text{EP violation}} = \frac{\Delta \gamma}{c} \left| \int U(r) \, dr \right| \rightarrow \Delta \gamma = 1.167 \times 10^{-5} \text{s}^{-1} \Delta_{\text{EP violation}} \tag{8.3.3}$$

The numerical proportionality constant in the last step of Eq. 8.3.3 depends on the model of the Milky Way’s gravitational potential. While several recent works in the literature have made different models of the Milky Way [38, 42], I “plug-and play” a recent model proposed by [38], which models $U(r)$ for the Milky Way as a Miyamoto-Nagai disc as well as a Navarro-Frank-White dark matter halo.

I compute from our concatenated non-GPS disciplined data that $\Delta_{\text{obs}} = 230(40) \, \mu s \approx \Delta_{\text{EP violation}}$ since all of the systematic delays enumerated above are negligible. From our concatenated GPS-disciplined data, I obtain $\Delta_{\text{obs}} = 341(25) \, \mu s$. In both cases, the location of the pulse peaks is found by maximizing the constructed template. The uncertainties are estimated by using a template with 200 (instead of 100) harmonics of the fundamental frequency. The corresponding estimates of $\Delta \gamma$ are on par with current limits summarized in [43]. A graphical comparison is shown in Fig. 8.3.2.

The small contributions from the systematic errors I enumerated cannot account for my obtained values of $\Delta_{\text{obs}}$ which differ from each other and exceed other values of $\Delta_{\text{obs}}$ reported in the literature [38, 44]. One significant possibility which I have not accounted for is the presence of numerical errors accumulating in the pulsar period estimate, which accumulate after phase-folding hundreds of thousands of pulses. Another possibility is that the presence of clouds on the second night could be responsible for the different delay measured on the second night of observation. Future work should
Table 8: Using the observed time delay between the pulses of the Crab pulsar, we quote our new limits on violations of the Weak Equivalence Principle in terms of the differential post-Newtonian parameter $\Delta \gamma$. The discrepancy in our estimates, not to mention the discrepancy with various estimates in [38], is of some concern. However, we believe these bounds to be over-pessimistic rather than over-optimistic, since the observed time delays in the literature are less than what we report.

<table>
<thead>
<tr>
<th>MJD</th>
<th>$\Delta_{\text{obs}}$</th>
<th>$\Delta \gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>57742</td>
<td>$230 \times 10^{-6}$s</td>
<td>$2.7 \times 10^{-9}$</td>
</tr>
<tr>
<td>57743</td>
<td>$341 \times 10^{-6}$s</td>
<td>$4.0 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

determine why our measurement differs from that of other groups with laboratory characterization of our systematics in order to place our measurements on more solid footing. I expect that correcting for these effects should decrease the final amount of lag. The preliminary bounds reported here are more likely to be too pessimistic rather than overly optimistic. Even so, the limit on $\Delta \gamma$ complements current limits summarized by [43], as shown in Fig. 8.3.2.
Figure 8.3.2: Current limits on equivalence principle violations, parameterized by the differential post-Newtonian parameter $\Delta \gamma$, are reported. Our limits ("Optical Crab Pulses") complement those set in other works using gamma-ray bursts, the supernova SN 1987A, fast radio bursts, and blazars [43, 45–48]. The horizontal bars show the range of particle energies (or frequencies) used for each limit. This figure was reproduced and modified from [43].
9 Conclusion

In this thesis, I describe the major role I played in a novel test of the Bell-CHSH inequality in which the random basis choices on both sides of the experiment were set by Milky Way stars in order to address the freedom-of-choice assumption. I selected target stars to use and I modeled the spectral response of the custom instrumentation that allowed the color of incoming astronomical photons to be turned into random bits. In doing so, I enabled our research group to conclude a 7σ and 12σ violation of local realism, where any remaining local-realist explanation would have to reach back at least 600 years. In addition, I proposed a new experiment, inspired by Wheeler’s delayed-choice cosmic interferometer gedankenexperiment, to test the nature of wave-particle duality, and I performed a preliminary assessment of its feasibility.

As a follow up to this theoretical work, I built and tested a next-generation astronomical random number generator with improved pointing capabilities, and whose optimized spectral response improved on the Vienna design by an order of magnitude. This improved instrument will pave the way towards a second test of Bell’s inequality using extragalactic sources of randomness, which will, by several orders of magnitude, further constrain local realist physical theories that exploit the freedom-of-choice loophole to reproduce quantum correlations.

With our next-generation instrument, we observed thirteen quasars at Table Mountain Observatory outside Los Angeles as candidate sources of randomness for an extragalactic cosmic Bell test. I performed an analysis similar to the one I performed for the Vienna experiment, characterizing the fraction of settings corrupted by non-astronomical influences. I analyzed the mutual information as a measure of the statistical randomness of the bitstreams generated. I found two pairs of quasars that could be used for an extragalactic cosmic Bell test, and several more that would be suitable with better observing conditions than Los Angeles.

Finally, along a completely orthogonal line of inquiry, I used our high time-resolution observations of the Crab pulsar to constrain violations of the Weak Equivalence Principle, setting an upper bound on the differential post-Newtonian parameter $\Delta \gamma$ complementary to existing results in the literature.
References


A History

To be of use to the future group member, in this section I will describe our ARNG’s built-in degrees of freedom, and how the instrument has evolved over time. For convenience, a picture of the instrument is reproduced here in Fig. A.0.1.

All of the optics were integrated and successfully aligned for the first time in late June 2016. The pinhole, achromatic lenses, and dichroic mirror holders were aligned in Summer 2016 and haven’t been touched since then. The dichroic mirrors themselves have been removed for spectral measurements, but their position is constrained by the dichroic mirror holders.

Over the course of July 2016 to December 2016, several adjustments were made to make the instrument’s alignment better and easier. Initially, connecting the Thorlabs stages to the Thorlabs lens tubes made both arms mechanically overconstrained. This helped to support the primary dichroic cube, which was now connected to both detectors as well as the CMOS camera and the telescope aperture. In this configuration, the ARNG is quite rigid. However, using the XY stages to make adjustments to the ARNG alignment made the tubes and cubes flex around. The miniscule springs in the XY stages could not provide the necessary restoring force to push against the box’s tendency to resist flexure. This made it such that the stages could translate only in the direction which compresses the spring. In addition, having both arms coupled to the dichroic assembly made it such that one arm’s alignment could drastically affect the other.

Mechanical upgrades in Fall 2016 made the instrument much easier to align. The stages were decoupled from the tubes, making each detector port mechanically independent of the dichroic assembly. A single long set screw was used to support the dichroic assembly from the lid of the ARNG enclosure; this quick fix turned out to be very mechanically stable.

In addition, decoupling the stages from the dichroic assembly made the alignment problem separable and led to better alignment and higher photon counts in the data collected in December 2016, particularly in the blue arm. However, it is difficult to quantify how much better we did because we do not have a baseline comparison of both observing runs. Our observations in December were focused on the Crab pulsar, which we did not observe in July 2016. Even though briefly observed the quasar APM 08279+5255 during both observing runs, it was a significantly higher altitude in December than it was during July.
Figure A.0.1: This shows our realization of the instrument depicted in Fig 5.0.1, resting on a small Melles-Griot optical table in the HMC Optics Lab. The longpass and shortpass dichroics can be distinguished by their colors in the picture. Looking through the longpass dichroic at a source of white light gives the light a deep red color, while looking through the shortpass dichroic makes everything appear yellow. In this image, the lids on top of the dichroics are opened for viewing but in general should remain closed to prevent dust contamination. Two additional parts in this picture are not diagrammed in the schematic: two XY translation stages, one placed in front of each detector, allow for precise alignment of the focusing lenses with respect to the active area of the detector while the instrument is bolted to a telescope.
B  Alignment

B.1  Step 0: Preliminaries

Begin on a long optics table. The longer the better, as this allows the laser to be aligned more precisely. Wear gloves when touching anything inside of the box. Assemble the Delrin box with its four sides, the dichroic beamsplitter assembly, the pinhole mirror, the $f = 50 \text{ mm}$ collimating lens, and the camera and camera lens, but not the focusing lens or the detectors. The pinhole mirror, lens, and camera lens only need to be roughly in place. Be careful of stripping the threaded holes in the Delrin—always unscrew until you hear a click before you carefully screw in. Put the box securely on the small black Melles-Griot optical table in the lab and kinematically constrain its position using several $1/4'' - 20$ screws. Then, set up a very level HeNe laser that starts at one end of a long optics table pointing towards the far end of the optics table. Alternate between placing an iris right in front of the laser and very far from the laser to ensure that make sure the laser beam shines perfectly parallel to the long axis of the optical table, and that the beam is not tilted upwards or downwards but rather lies in a plane parallel to the table. Once the beam meets this condition, gradually adjust the height of the beam until it is approximately the same height as the pinhole. Set up a spatial filter right in front of the laser and then collimate the beam that comes out. Place a 3D translation stage equipped with a weak focusing lens ($f > 100 \text{ mm}$) such that it focuses the collimated beam down to a spot which you can translate using the 3D translation stage. Since the focusing lens has a long focal length, you should be able to mount the 3D translation stage on the main optics table while focusing the beam to a point exactly coincident with the pinhole height and roughly coincident (to within a millimeter or so) where the pinhole mirror’s final position in the XY plane (where the Z axis is the laser propagation axis), above the Melles-Griot mini-table.

B.2  Step 1: Pinhole Mirror Angle and Macro Lens

At this point, turn off the laser and turn on the camera and focus the macro lens until the pinhole mirror is in focus. The center of the pinhole mirror should be in focus while the left and right edges are not, due to the small depth of field of the macro lens. Keep the focus directly on the pinhole. Now, use a pair of plastic or nylon-coated tweezers (or careful gloved fingers) to align the pinhole mirror’s angle such that you can see the spatial filter assembly roughly centered on the pinhole. This keeps the pinhole at 45 degrees.
B.3 Step 2: Pinhole Mirror XY Position and Collimating Lens

While keeping the pinhole mirror oriented at 45 degrees, translate the pinhole mirror around in the XY plane. Try to keep the pinhole in the center of the field of view of the ZWO ASI CMOS camera. This constrains one of two translational degrees of freedom of the pinhole mirror.

To constrain the pinhole mirror’s other degree of freedom, adjust the 50 mm lens so that its focal point is approximately coincident with the pinhole. Turn on the laser (you should add some neutral density filters if the laser is saturating the camera). The laser will be reflected towards the camera. While keeping the pinhole at 45 degrees and keeping the pinhole in the center of the field of view, make the laser light go through the pinhole as cleanly as possible. You can use an auxiliary camera to make this step slightly easier; it’s easier to look at a screen to tell if the beam is going through the pinhole of the pinhole mirror cleanly. If the collimating lens is at the proper distance away from the pinhole mirror, you should see a collimated, inch-diameter, uniform beam of collimated light coming out the back of the instrument. It’s easy to check if this light is truly collimated if you take out the back wall of the box. Then walk several meters back and see if the spot gets any bigger or smaller. This is the hardest part to align. It took us a few days the first time around. Hopefully it never has to be done again. Screw the mirror down firmly, but don’t screw too hard or else the height of the mirror may change significantly.

At this point, the pinhole of the pinhole mirror should be perfectly coincident with the focus of the f = 50 mm achromatic lens. Otherwise, the light from the star will not be collimated as it passes through the dichroic beamsplitter, whose transmission/reflection properties are highly sensitive to the angle of incidence. This alignment must be done with a clear path behind the lens, so all dichroics and the back of the box must be removed so a collimated beam can propagate a long distance. In addition, the pinhole mirror should be at a 45 degree angle to the incoming light and reflect light to the camera. This is for source identification during astronomy, and for maximum signal to noise ratio during a Bell test, because the pinhole is nominally supposed to have a circular aperture when viewed at 45 degrees.

B.4 Step 3: Focusing Lenses

I don’t have a good idea for how to place the 35 mm focusing lenses at the right distance away from the active area of the detector. Keep the detectors off for this step, and do this independently of the box, using the collimated laser beam. Dim the beam until the point where you can look at the beam’s focus somewhat steadily without hurting your eyes. If the box is held in a
constrained position, you should be able to use the collimated laser beam. You should be able to measure how far is 35 mm behind the focusing lenses, and then try to attach the translation stages (which hold the lens) to the detector’s four cage posts. Put the four tiny set screws in place when you think you’ve focused it down correctly. It’s a really tight fit so the detectors need to be flush with the translation stage, essentially. If you filter out the laser with many NDFs (optical density of 8 should be safe) you might be able to align the lens in three dimensions by maximizing photon counts, though I never tried this. You would have to use an object that forms an image on the detector slightly bigger than the 500 × 500 µm. Do this for both detectors well as possible in the focal-length direction, and to within ±1 mm in the XY plane.

B.5 Step 4: Dichroic Beamsplitters

The first dichroic mirror must be at a 45 degree angle to the incoming light. If the wall of the box behind the red arm is removed, a 90 degree reflection can be checked with an iris using the well-aligned holes of the optical table. Since the primary beamsplitter is a shortpass dichroic, crank up the laser and you might be able to see a small spot. This gets us really to 45 degrees. The base of the stage rotates and you can get it more accurate later when the detectors are in place. It’s really hard to get the second dichroic beamsplitter at exactly 45 degrees. However, the manufacturer data reports data for 45 ± 2 degrees, so within a degree is an acceptable tolerance for our standards.

B.6 Step 5: Avalanche Photodiode Detectors

At this point, screw down the detectors with aligned focusing lens assemblies into the box. Make sure to leave a millimeter of clearance or so between the dichroic cube assemblies and the focusing lens assemblies so that the lenses can be translated around without having to worry about collisions. Since the translation stages have ±1 mm of travel, you only need to be accurate to within a millimeter when putting the transmit detector. The reflect detector is a bit harder: put it in as precisely as possible, and then gently turn the primary dichroic turntable, maximizing counts in the reflected arm. Using a dim incandescent source works pretty well for aligning the reflect detector, which sees near-IR photons. Alternatively, set the XY stages to their center and move the reflected detector around until you see counts, possibly adjusting the height of the cube’s upper set screw; then lock it down. Turn the translation stage knobs until counts in both arms are maximized. Make sure the range of travel isn’t exceeded. Check if the spot size is smaller than the detector’s active area by looking for a clean plateau. After we decoupled the detectors from the dichroic assembly, this step got much easier.


## C Tables of Observed HIPARCOS Stars

Tables 9 and 10 list two different groups of calibration stars that we observed, recording nanosecond time-tags.

All of the star and quasar data we obtained is on the Lenovo lab computer (plus backups) with the format

\[
\text{YYYYMMDDHHMMSS}_\text{COORDINATES}.\text{txt}
\]

where `YYYYMMDDHHMMSS` refers to the start time of the observation in UTC, and where `COORDINATES` is a twelve-character string with six RA coordinate characters in HHMMSS followed by six DEC degree-minute-seconds characters. For example, for the Crab pulsar, the coordinate string is “053432220052.”

Unfortunately, we were inconsistent in the coordinate convention, alternating between using “on-date” ICRS coordinates and J2000.0 ICRS coordinates. These coordinate systems differ by a small, time-dependent angle due to Earth’s precession and nutation.

Using the date and the RA/DEC of observation it is possible to reconstruct the altitude of observation. Files with the suffix `bright` or `dark` refer to lists of photon detections generated while pointing on an object, or while pointing \( \sim 10 \) arcseconds away from the object. The `dark` datasets enable accurate estimation of the noise rate \( n_j^{(i)} \).

---

### Table 9: Stars near Zenith: different colors and magnitudes at altitudes of 88 – 90 degrees

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Table 10: Stars with similar magnitudes and colors but a range of altitudes.

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