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Mathematics and the Arts: Taking Their Resemblances Seriously

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[Though] mathematics is often excluded from the arts....such exclusion does not justify denial or neglect of the respects in which [it] resembles the arts we do recognize.

—Francis Sparshott
[S3, pp. 145-146]

The above comment from a poet and philosopher expresses an attitude with which many mathematicians would probably feel comfortable. From at least the time of Aristotle, writers of various philosophical stripes have paid tribute to the aesthetic appeal of mathematical studies, and today there exists a considerable body of introspective literature on the affinities between mathematics and the arts. Certainly the readers of a journal devoted to humanistic mathematics should sympathize with any reaffirmation of their subject's place among creative human activities. Indeed, to some Sparshott's remark is surely overcautious: Isn't mathematics already an art?

For example, if we claim aesthetic values are inherent to our doing mathematics, can we in any way specify what those values are or the role they play in our practice?

I believe, however, that we should treat Sparshott's comment more as a suggestion than as another (and actually rather weak) tribute to the aesthetic nature of our subject. By virtue of its not assuming mathematics to be an art it encourages us to speculate on the issue. The results of such speculation may well be fresh insights into the nature of mathematics, its role among human endeavors, and even the manner in which mathematics education should proceed.

To see this, consider the extent to which the dominant mathematical philosophy of formalism concentrates on the rigorous, analytic structure of theories and results as opposed to the complex, creative and often downright hazy methods by which these results are obtained. In the eyes of formalism this is not merely a matter of emphasis—the latter notions are intentionally brushed aside as irrelelvent to the question "What is mathematics?" (see, e.g., [H1]). Can such an approach possibly be consistent with the belief that mathematics fundamentally engages our aesthetic interests, or that it proceeds in manners akin to those of art? If we truly feel these things about mathematics, why do we cleave to a philosophy and to practices that deny them?

Thus what I see Sparshott suggesting is that we take our generally unexamined beliefs on this issue and examine them seriously. For example, if we claim aesthetic values are inherent to our doing mathematics, can we in any way specify what those values are or the role they play in our practice? If we claim mathematics to be an art by virtue of entailing properties X, Y and Z, are we prepared to counter the charges that with regard to art itself, X is irrelevent, Y insuffucient or Z actually antithetical? Do we know what art is any more than we know what mathematics is? To answer questions like these—even just to ask them intelligently—requires a depth of analysis which few from either the aesthetics/art or mathematics sides of the issue have attempted.

While I make no claim of attaining such a depth in what follows, I do hope I can indicate something

of what is likely to be involved in the attempt, and where some of the benefits to be obtained by the effort may lie.

Aesthetics, Art and Mathematics

In keeping with a practice all too common among writers on this issue, I have thus far been using two different terms—art and aesthetics—in a dangerously synonymous way. It is time for a distinction. By aesthetics I mean the particular type of inquiry that Scruton [S2, p. 15] characterizes as the "philosophic study of beauty and taste." Similarly, by "aesthetic" I refer either to an attribute such an inquiry would study (e.g., aesthetic distance) or to some system that embraces these notions in a particular way (e.g., a culture or era with an aesthetic different from our own).

Art, on the other hand, is a trickier notion with which to come to grips. Too complex to take as a primitive term, it is one of those concepts—like mathematics itself?—for which we seem to have enough of a sense to use with impunity and still be understood—at least among people with backgrounds similar to our own. Yet a careful delineation of the concept always seems beyond our grasp. Perhaps we might regard this as good enough, but nagging questions of considerable import nevertheless keep arising: Is such-and-such a worthy enough work of art to merit our attention? Does its maker merit our support? Should art itself be publically supported? Should it be taught in schools? Can it be taught at all? As might be expected, the closer we get to the "edges" of the concept—to education, to ethics, to the boundaries of "non-art"—the tougher the questions get. Hence arises the philosophy of art.

Like the philosopher of art, we are not so interested in developing the concepts of beauty and taste per se as we are in applying these to a particular—though very broad—type of activity.

Although historical precedents for confusion are ample, it is almost certainly mistaken to equate this latter field with aesthetics [see S4, pp. 15-16]. The philosopher of art may well be concerned with questions of ontology (What is a work of art? A physical object? A mental state?), epistemology (Is there such a thing as artistic knowledge? What is it knowledge of? Is it somehow verifiable?) or ethics (Can a work of art be morally neutral? Does its carrying any particular moral value affect its status as art?). These are concerns whose connection with aesthetics proper is not entirely clear. Conversely, the aesthetician may well be interested in our responses to nature or to dreams, or to the incidents of our daily lives. One may argue that art is broad enough to embrace these notions as well, but this is certainly something to be demonstrated and not assumed.

So where does this put us with regard to mathematics? Ideally, we might wish to establish a relationship between aesthetics and the philosophy of mathematics analogous to that between aesthetics and the philosophy of art. Problems immediately arise, however. On the one hand, many of the concepts of aesthetics have developed (even if controversially so) with specific reference to art, and applying these as they now stand to mathematics may be unwarranted. But even more troublesome difficulties arise from the side of the philosophy of mathematics itself, which in this century has so estranged itself from aesthetic concerns that prospects for an immediate dialogue seem dim. The two disciplines apparently lack a common language.

It is here that we might appeal to the philosophy of art. Like the philosopher of art, we are not so interested in developing the concepts of beauty and taste per se as we are in applying these to a particular—though very broad—type of activity. Similarly, we should be willing to admit that the most important aspects of our subject may well lie outside the realm of aesthetics. Finally, we can note from the start that several important concepts in the philosophy of art (e.g., representation, form, the distinction between pure and applied activities) bear prima facie resemblance with concepts in mathematics. Possibly this is no more than a superficial coincidence, but again this is a point requiring demonstration. (I suspect that in many instances the coincidence is not superficial.) In keeping with the sense of Sparshott's suggestion, we should compare mathematics and art before we even consider equating them. We may even find that such an equation may be the least illuminating of the insights we gain.

Essentialist Theories

Traditional attempts to formulate philosophies of art have largely focused on the question of definition—of finding those qualities which uniquely constitute the essence of art. As DeWitt Parker stated in an influential article from 1939:

> The assumption underlying every philosophy of art is the existence of some common nature present in all the arts....There is some...set of marks which, if it applies to any work, applies to all works of art, and to nothing else....[This] constitutes the definition of art. [P2, p. 61]

Note that this essentialist approach—that of reducing art to necessary and sufficient conditions—promises a result not unlike those formulas for the identity of mathematics (e.g., "Mathematics is logic", or "Mathematics is the study of formal systems") that have arisen in our own field. The resemblance is deeper than this, however. The very attempts to specify these qualities in each subject have themselves paralleled each other in remarkable and significant manners.

Although any attempt to classify diverse theories runs the risk of oversimplification, I shall deal here with three general forms of essentialist arguments. The first, realism, involves identifying this essence in some relationship between a constructed object (or "artifact") and a world exterior both to that object and its maker. By contrast, expressionism concentrates on relationships between the artifact and a world interior to either its maker, audience or both. Finally, formalism identifies the essence of art strictly in terms of qualities contained within the artifact itself—e.g., the lines, colors or shapes in a painting, the arrangement of tones in a musical composition, etc.

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I do not wish to imply that all essentialist theories fit snugly into these categories. Hybrid theories combining elements from each of these also exist, as do theories that strike off in different, often quite sophisticated and radical, directions (see, e.g., [T1] & [D3]). For introductory purposes, however, this scheme (which parallels that of [H5]) should be adequate, and will also serve to highlight the parallels between traditional philosophies of art and mathematics. Let us briefly consider each in turn.

Realism

For centuries realistic views had constituted the dominant theories of art, and although these attitudes have generally fallen from favor, both artists and audiences alike continue to be influenced by their appeal. While a broad spectrum of realistic views are possible, these theories in general identify art as a system of knowledge with a status claim not entirely unlike that of science. Most explicitly, Leonardo declared that, "Truly painting is a science, the true born child of nature" and some three centuries later the landscapist Constable reaffirmed: "Painting is a science, and should be pursued as an inquiry into the laws of Nature." By these lights (which can be clearly discerned in the works of Aristotle), the artist reveals reality to us through abstract representations of its various aspects—aspects incapable of such revelation by any means but art. In some variations of the theory, these revelations are actually of a "higher" plane of reality than mere physical existence. For example, Piet Mondrian viewed his skeletal arrangements of lines and solid colors as presenting images of "true reality and true life," a position more akin to neo-Platonism than the inductive realism of Aristotle. These views are linked, however, by a conception of art as essentially a means of discovering truth.

Have attitudes such as these ever been displayed toward our mathematical "artifacts," and are they still likely to exert an influence on us today? To both questions the answers are definitely yes. For centuries Euclidean geometry did not stand as one of a multitude of formal geometric systems but was regarded as representing the Real geometry of Real Space, based on inductively self-evident truths abstracted from reality and developed through the unassailable method of deduction. (Writing before Euclid, Aristotle accurately observed some of the

consequences of rejecting the assertion that the sum of the angles in any triangle was two right angles. He abandoned these conclusions because of their "obvious" inability to square with the evidence of experience. Spherical geometry, of course, had already developed a long history, but this was Real geometry on a Real surface—not a model of a non-Euclidean system. See [T3].) True, the insistence upon logical deduction may appear outwardly to be difficult to reconcile with art, but I believe this perception largely arises from simultaneous tendencies to downplay the role of logical development in art and to overplay its role in mathematics. The logical "flaws" in Euclid's geometry or Newton's calculus were recognized as profoundly troublesome, but these problems were as little compared to the practical insights and successes the systems as a whole presented (see [K1]). The knowledge gleaned from mathematics was looked upon as genuine knowledge nonetheless, and throughout this period there was little doubt that this knowledge was that of reality.

The rise of non-Euclidean geometry required abandonment of this geometry-as-the-science-of-real-space conception, but by no means did it spell the end of realist conceptions of mathematics. By one view, different geometries or systems of mathematics could be conceived as separate entities of a Platonic world of forms, with the mathematician regarded as a sort of suprascientist who discovers the properties of these entities. And

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although this position ignores several not inconsiderable aspects of Plato's thought on mathematics, the belief that any well-framed mathematical question has a definite answer independent of our existence characterizes the position known as mathematical Platonism. Phrased in this manner, the position becomes as much the creed of a mathematical theology as it does the basis of a descriptive philosophy. As the former it seems to be of practical value to individual mathematicians [see, e.g., T2]; as a critical theory it seems as suspect as Mondrian's supposed ability to envision "true reality." Did art experience its own "non-Euclidean revolution?" In a sense it did, even if its occurence cannot be as precisely specified as that of mathematics. (Although the late-18th and early-19th centuries figure prominently in the revolutions in both fields. See [A1].) The process in this case involved—at least in part—the realization that representation itself was neither a necessary nor sufficient condition for art. Its insufficiency (long recognized) can be ascertained simply by considering common occurences of representation that do not qualify as art: courtroom records, plastic anatomical models, or snapshots (the history of photography's status as an art provides an interesting trial case of realistic theories). But even more telling is the apparent case for unnecessity. Not only do non-representational artworks exist, but entire branches of art appear to be non-representational by nature. Debussy's La Mer may be taken to represent the sea, but does not this conclusion follow primarily by virtue of the work's title alone? Without this title, would the work still be perceived as resembling the sea more than it does anything else? Would we even consider the work in representational terms? Would we still consider it a work of art? (For this example see [H5, p. 708].)

Emile Zola—one of the late 19th century's most prominent exponents of realism through his ideal of the naturalistic "experimental novel"—once characterized art as life seen through a "temperment." But even from before the time of Zola it had become increasingly apparent that realistic theories had been in error in not placing more emphasis on the role of this "temperment" itself. In the philosophies of expressionism this role would lie at the very heart of the matter.

Expressionism

For an example of an extreme expressionist view, consider the theory developed in the first half of this century by the philosophers Benedetto Croce [C2] and R. G. Collingwood [C1]. (These are actually separate theories, but close enough in both spirit and detail to be regarded as one for our purposes.) In this theory art is taken as being identical with expression, the revealing of an

internally constructed mental state. Even more simply, art is realized imaginative activity, and in fact stands as the most basic form of human language. Although this is a particularly broad pronouncement, it is accompanied by explicit statements of what art is not. First (and possibly most difficultly) it is not any set of existing physical or symbolic artifacts such as paintings, texts or sculptures. These are simply the necessary embodiments of the imaginative constructions that Collingwood calls the "work[s] of art proper" [C1, p. 37]. Thus while we may be accustomed to referring to the Mona Lisa itself as a work of art, the work of art proper lies in the imaginative constructions that resulted in Leonardo's painting it and in our interpreting it. (Along these lines Collingwood approvingly quotes Coleridge's "we know a man for a poet by the fact that he makes us poets" [C1, p. 118].) Similarly, art should not be confused with any particular kinds of media. These, rather, are simply the necessary channels through which particular expressions flow. (Again from Collingwood: "Every gesture that each one of us makes is a work of art" [C1, p. 285]) Finally, art should not be identified with any activity engaged in for a specific purpose or outcome. Art may be used to represent reality, entertain us or persuade us, but it does these things en passant. Indeed, if any specific purpose guides the production of a work we are dealing not with art at all, but with craft or techne—what Collingwood calls "art falsely so-called."

This last may seem to place art in a paradoxical position. On the one hand it is equivalent to the most basic human attribute (communication) and on the other it is divorced from purposive human activities. This is resolved through the nature of the internal constructions of emotions, intuitions and experiences that result in successful expressions. Collingwood explains that to attain these the artist must "experience what all experience" and comments that "in ivory towers art languished" [C1, p. 119]. Similarly, Croce disposes of the romantic notion of the artist as necessarily being a specimen of genius (a view which also haunts mathematics) by noting that "inspiration is not something from heaven, but is in the essence of humanity itself," and that "the man of genius who poses as that...finds his punishment in becoming somewhat ridiculous" [C2, p. 16]. In this theory artists are born and not made, but we are all born artists. Differences are of degree, not of kind.

it is above all mistaken to confuse mathematics with its final symbolic forms. These, rather, are merely the manifestations of the internal constructive processes that constitute the proper identity of mathematics.

It may seem unlikely that there could be an overall view of mathematics corresponding to an expressionist theory of art—particularly a theory as extreme as that of Croce and Collingwood. It is somewhat startling to find, therefore, that it is not difficult to find particular passages in these writer's works which are virtually identical to ones found in the work of such figures as Poincaré, Brouwer and later members of the constructivist school. Consider Brouwer's intuitionism [B]. To Brouwer it is above all mistaken to confuse mathematics with its final symbolic forms. These, rather, are merely the manifestations of the internal constructive processes that constitute the proper identity of mathematics. Other mathematicians may use their interpretation of these symbols as a basis for their own constructions, but their interpretation is itself a personal construction. (Sparshott notes that both he and H. S. M. Coxeter know the Pythagorean theorem, but that what "he knows in knowing it is something I cannot even imagine" [S4, p. 33].) To Brouwer, logical rules do not guide the process of mathematical construction; rather, a pragmatic logic is effectively created along the way. Similarly, Collingwood notes that the artist does not create rules so much as construct them. These viewpoints do not leave judgements in either activity purely subjective, however. In mathematics, such judgements are constrained by the requirement of constructive existence; in art by the commonality of actual human experience. Even on the point where the theories apparently diverge most severely—the conception of language—the difficulty reduces to a difference in terminology. Croce and Collingwood conclude that art is language, whereas Brouwer virtually declares that mathematics is anything but language. But to Collingwood, language is any expression of internal constructions; to Brouwer language is primarily symbolic notation (which Collingwood refers to as "language falsely so-called" and Croce

as "the mere grammatical").

Though these theories may assume different names in different fields—expressionism in art, intuitionism or constructivism in mathematics—all share a focus upon the freedom and constraints of the creative process as essential to the activity at hand. In art, expressionism has been particularly criticized for its apparent disregard for the artistic product as an entity onto itself—the theory's notion of artistic existence appears skewed [H4]. Similarly, mathematical constructivism establishes a criterion of existence often viewed as unnecessarily severe ([H2]). In response to these charges in both areas, philosophies of formalism—already present in each field—have arisen as alternatives.

Formalism

Whether in mathematics or in art, formalism in its purest state is a philosophy of detachment. Artworks and mathematical systems are viewed as independent phenomena standing apart from their creators, their audiences and any external world. Meanings or values can be considered only with reference to elements contained within the work itself. And although ideal formalist criticism may be difficult to achieve in practice, in that ideal works stand apart even from the history of their very construction. As Werhane puts it, a work exists virtually "in spite of its creator...and its audience" [W4, p. 99].

I should make at least one distinction among formal theories, however—particularly with respect to art. In the pure state described above—a position sometimes referred to as absolutism—the meaning of a work is entirely separated from any quality not within the work itself. Thus to recognize a work of art is to recognize such qualities as symmetry, internal dynamics or the relationships of color and shape; to create art is to impose these qualities on an artifact. However, in another brand of formalism it is important that one look at both the formal structure of a work as well as any meanings that may be expressed by that structure. Art effectively becomes defined as symbolic form. But unless one wishes to regard virtually everything from everyday language through, say, mathematics as art, a more precise description of what constitutes artistic forms is needed. Such theories—as for example those of Suzanne K. Langer [L] or Nelson Goodman [G2]—evolve largely into "meta-theories" of syntactic and semantic rules, and thus to a large extent diverge from pure formalism. Simultaneously, however, they also seem to diverge from the actual experience of creating and appreciating art. Writing with regard to Langer's theory, Scruton comments that her "analysis gives no procedure for interpretation, nothing that would give application to the claim that in understanding a work of art we understand it as a symbol" [S2, p. 22].

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In mathematics, pure formalism—comparable to aesthetic absolutism—is that view which takes mathematics to be strictly the study of formal systems (see [H1]). Where the formalist in art seeks internal structure, the mathematical formalist seeks consistency. Where the former detaches meaning or expression from form, the latter constructs "uninterpreted" systems.

I feel compelled here to mention that Hilbert (the "father of formalism") was not a formalist in this sense, as his introduction to Geometry and the Imagination [H3] makes abundantly clear. The metamathematical strategy of Hilbert's Program required him to treat mathematical systems as uninterpreted for the sake of a consistency proof—hardly the same as believing mathematics itself to be fundamentally uninterpretable or meaningless. Curry, one of the leading exponents of mathematical formalism, has himself stated that "it is unfortunate that many persons identify formalism with what should be called Hilbertism" [C3, p. 156]. Indeed, it would be interesting to determine the degree to which the aims and methods of metamathematics in general parallel those of, say, Goodman's strategy in Languages of Art [G2].

In both mathematics and art formalism has become the most established theoretic view of this century. (In both fields, in fact, it has independently acquired the descriptive phrase "Modernism.") In mathematics education, its influence—under the pervasive guidance of Bourbaki and the New Math—has been nearly universal. However, as Davis and Hersh ([D4, pp. 343-344]) note, cracks have begun to show in the formalist wall. The underlying theme of both [D4] and [D5] is in essence an assault on mathematical formalism, as are the selections in [T4]. Similarly, many of the essays in Arthur Danto's recent collection [D2] constitute strong cases against aesthetic formalism and its separation from human experience and meaning.

Non-Essentialism

In an often anthologized paper, "The Role of Theory in Aesthetics" [W1] the philosopher Morris Weitz argued that the entire essentialist approach to the philosophy of art is fundamentally misguided (see also [W3] for an elaboration). To Weitz, art is an example of what he calls an inherently undefineable open concept. He develops this more fully by referring to what he calls the perreniel flexibility and debatability of art. Art is perennially flexible in that it is always capable of expanding its scope to embrace previously unconsidered artifacts, styles, media, etc. It is perenially debatable in that not only are the criteria by which new works are judged subject to continual criticism and reevaluation, but even established works continue to fall under such scrutiny. Oedipus Rex may be established as a masterpiece of tragedy, but the matter of exactly which of its qualities grant it that status continues to be contentious.

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Weitz bases his discussion in Wittgenstein's notion of family resemblance [W5, pp. 31-33]. To use Wittgenstein's example, if we are confronted with the question "What is a game?" we shall find ourselves unable to provide a satisfactory set of necessary and sufficient conditions. Not all games have rules, not all involve competition, not all are purely recreational, etc. What we can do is point out resemblances between those entitities which we have chosen to call games. If a new candidate for gamehood presents itself, we base our decision whether to expand the concept to embrace it on the resemblances we perceive between the new activity and those previously admitted. Weitz notes that with regard to art we could always choose to "close the concept" by legislating strict standards of admittance, but to do so would, as he puts it, "foreclose on creativity"—a seemingly self-defeating decision.

This last approach—that of choosing to close the concept—is the option foundational philosophies of mathematics seem to offer. But when mathematics is regarded primarily as an activity (i.e., historically, culturally and in actual practice) this option as well seems to be a particularly self-defeating decision. Viewed this way, in fact, mathematics increasingly presents itself as a flexible and debatable open concept. As an example, consider the attitudes mathematicians have historically held regarding the existence of infinitesimals. Does the long history of this issue between the times of Eudoxus and modern non-standard analysis suggest anything but the perennial flexibility and debatability of this concept? What of "dimension," "continuity," or "real number?" Perhaps most tellingly, consider the notion of "proof." Is this a closed concept, or have our criteria of what constitutes a valid mathematical argument themselves been (and continue to be) flexible and debatable? Consider the positions available with regard to the use of computers in proof. Tymockzo [T4, pp. 243-266] asks whether if we regard the 4-color theorem as having been proved, must we not also admit that its proof is of an entirely different nature from any that have come before it?

Although Weitz considers essentalist arguments to be misguided in purpose, he does not consider the particular issues they raise to be irrelevant. These theories provide, in fact, "a series of invaluable...directions for attending to art," and concludes that:

> If we take aesthetic theories literally...they all fail; but if we reconstrue them, in terms of function and point...as recommendations to concentrate on certain criteria [they are] far from worthless. [They] teach us what to look at in art. ([W1], p. 156)

Similarly, if we demand of our philosophies of mathematics a secure foundation capable of supporting the entire structure of the subject we may well be fundamentally misguided. However, foundational philosophies have been successful in raising relevant issues and thereby provoking us to fresh insights. In both art and mathematics, non-essentialism moves us toward a pluralistic view of our endeavors, one that recognizes that there is not a single, absolute perspective but many—possibly not even mutually consistent—viewpoints. By no means, however, should pluralism be confused with a kind of bland eclecticism or—even worse—blind subjectivism. On the contrary, the holding of firm individual views is probably necessary to create the productive tensions that drive our activities as a whole. One can be generally non-pluralist with regard to one's own philosophy and generally pluralist with regard to mathematics or art as a whole. (For a discussion of this stance with regard to art see Danto's "Learning to Live with Pluralism" in [D2]).

The recognition of both art and mathematics as open concepts leaves behind it a curious casuality. The claim that mathematics is an art seems to have lost its meaning. But in its place has arisen the prospect of a much more active agenda—that of seeking family resemblances between the two fields. That any resemblances we find are indeed familial ones I do not find objectionable—the underlying conceptions of the essentialist theories in each field justifies their membership in a common philosophical "family." In the remainder of this article I would like to address just a few of the areas where some useful resemblances of this kind might be sought.

Family Resemblances between Art and Mathematics

Mathematics and Techne

One of the central points in the Croce-Collingwood theory is the distinction between art and techne, or craft. Recall that in this theory the former is equivalent to expression, while the latter is identified with activities engaged in to attain a specific result. Even without wholly accepting this theory, the desire for such a distinction seems desirable. Techne is composed of specific, trainable skills; art not only marshals these skills, but transcends them in some not clearly defineable manner. Mixing paints or mastering scales is techne; The Madonna of the Rocks or an original jazz improvisation is art. Propaganda as propaganda is techne; propaganda in the form of Eisenstein's Potemkin somehow becomes art.

A similar distinction seems valid in mathematics, and could well be crucial in education. Finding the greatest common divisor of two numbers through following some recipe is techne; deriving a process along the lines of Euclid's algorithm, convincing oneself and others that it works, or developing a geometric interpretation of it (in short, understanding it) is mathematics. Even a brief perusal of most secondary level textbooks or examinations reveals a tremendous emphasis on techne. But to what degree are "skills" nonetheless necessary, and which particular ones? Can and should these be approached in a more "mathematical" manner? If mathematics proper is something that cannot be directly taught, how can it be productively developed? Approaching these issues seems to require a sharper disitnction between "math proper" and "math falsely so-called" than now exists. (Consider: To what degree is formal proof a matter of techne? Is much of the "mathematics proper" completed by the time we reach this stage?)

In both art and mathematics, non-essentialism moves us toward a pluralistic view of our endeavors, one that recognizes that there is not a single, absolute perspective but many—possibly not even mutually consistent—viewpoints.

A final open note on this subject: The math/techne distinction appears to cut across that of pure/applied mathematics, and in certain respects may prove to be more a fruitful one. Consider architecture. Is this pure or applied art? Is it art or

techne? Which question gives us a more useful perspective in contrasting Frank Lloyd Wright's Fallingwater with tract housing? Now what of, say, the theory of relativity, or the activity of mathematical modelling?

Style

Among the most perplexing questions concerning the nature and history of art are those involving the issue of style. Why have different individual artists, or the artists of different cultures, nations and ages represented the world in such different ways? How is it even possible for them to have done so, and then for others to be able to recognize and appreciate their works? How does an approach to these issues regarding representation translate into the perception of style among non-representational works? Could art have evolved in only one possible way? What is the significance of style? (The first questions are broached by Gombrich [G1]; the last by Goodman [G3])

These topics have not been entirely alien to mathematics, even if they have not been approached in terms of the overriding concept of style. Comparisons of Greek and modern mathematics have often been approached with regard to what are effectively stylistic considerations, and from at least the time of Poincaré writers have contrasted individual geometric "styles" with arithmetic or analytic ones. However, style has not played the unifying role in historical or philosophical studies of mathematical thinking that it has in art. Possibly this is due to an attitude which views mathematical progress in a deterministic, unitary manner—a great chain of mathematical being, so to speak.

There have been indications of change. Educators speak of the role of "cognitive styles" and "multiple representations" in the learning of the subject, and Otte has written of a "complementary" relationship between arithmetic and geometry which is as suggestive of corresponding artistic styles or media as it is of modern physics [O]. Furthermore, the emerging field of ethnomathematics ([D1] & [A2]) seems to offer a particularly fresh approach to the nature of mathematical styles developed outside traditonal academic settings. Possibly all these workers may have something to gain from the methods and results of art history and psychology with respect to style.

One further note along these lines: In [D4, p. 318], Davis and Hersh point to the relative permanence of mathematical results as opposed to scientific ones. This suggests a further stylistic affinity between mathematics and the arts. Just as Picasso did not relegate Rembrandt to the attic, modern geometry does not invalidate Euclid. (Note the current state of Aristotelian physics, however.) Still, just as we can no longer look upon Rembrandt with anything but modern eyes, we must reinterpret Euclid in light of our current position. The development of mathematics has certainly been allied closely with that of science; however, the nature of the subject's history itself may bear more similarity with that of art. (This point is suggested by Thomas Kuhn [K2, p. 345].) A conception of the history of mathematics in terms of stylistic change might provide a better handle on this phenomenon than one which views the development of mathematics as analagous to that of science.

Just as Picasso did not relegate Rembrandt to the attic, modern geometry does not invalidate Euclid. Still, just as we can no longer look upon Rembrandt with anything but modern eyes, we must reinterpret Euclid in light of our current position.

Ethics

If the philosophy of art often seems a rather dry and erudite affair, there is at least one set of issues over which it becomes both heated and public—that of censorship and the moral obligations of the artist. Are there general topics from which the artist, or particular works from which the public, should be restricted access? Similarly, is the artist obliged to deal with any particular topics or situations? If each of the opposed paths of strict moralism and unrestricted aestheticism appear undesireable, what positions are left open to us? These have been among the most discussed issues in the philosophy of art, and can be traced from such figures as Confucious and Heraclitus through

current commentators on the role of the National Endowment for the Arts.

Issues of a similar nature have arisen more frequently in mathematics in recent years. (Davis and Hersh [D4, pp. 87-89] contrast the extreme ethical positions of "Mathematical Maoism" and a form of mathematical aestheticism derived—perhaps inaccurately—from Hardy.) In general, however, mathematics (and to a slightly less extent, science) has been disappointingly slow in embracing such topics. As more mathematicians and educators enter into such discussions, it may well be to their advantage to acquaint themselves with the manners in which the corresponding issues have been approached with respect to art. It might be worthwhile to point out here one particular area in which the functioning of each of mathematics, art and science have grown tremendously similar: All to a large degree depend primarily for support on governmental or industrial support ("patronage"). The time is ripe for a genuine dialogue between these areas on this issue alone.

Aesthetic Issues

Oddly enough, purely aesthetic considerations have made few appearances in this discussion. If we were to adopt a strictly Platonic view of either art or mathematics we might account for this simply through invoking Keats: Beauty is truth, truth beauty. Though a simplification, this formula nonetheless captures much of the attitude that allowed realistic views of art and mathematics to coexist for centuries with formal conceptions of beauty (as outlined in Plato's Philebus). To a modern day formalist, on the other hand, aesthetic considerations may well be deemed irrelevant, either relegated to the "emotive" qualities of the positivists, or actually defined as the meeting of certain formal criteria. Neither of these positions, however, is particularly helpful in ascertaining the role or nature of aesthetic judgements in actually creating art or mathematics.

This is a topic on which the mathematical literature is virtually mute. Poinçare [P4, p. 392] wrote of aesthetics as serving the role of the "delicate sieve" through which successful mathematical "combinations" passed on their way to consciousness, but his suggestive metaphor has remained little more than that for some nine decades. (Though some interesting development is attained in [P1].) This is not to imply that the artistic literature has itself resolved the problems posed by the creative imagination, but it has approached them directly and arrived at apparently useful distinctions and insights. This same literature may provide a fertile starting ground for mathematical investigations. For an introduction to these issues from philosophical and psychological persepctives, see [S1] and [P3], respectively.

I would like to mention briefly one aesthetic value which does seem to be pervasive in the mathematical literature—that of elegance. As aesthetic values go, this seems to be rather peculiar

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to mathematical studies. (Indeed, the word actually carries a somewhat negative connotation in, say, painting or music.) Far from being able to separate the positive or negative effects this concept has had on development, mathematical writers have not even been particularly clear on just what elegance is or how it stands in relation to such other concepts as clarity, conviction, understanding or significance. It is, however, an aesthetic value (or at least an apparent aesthetic value) with some degree of currency in the mathematical community, and as such may provide an entry point for approaching the nature of mathematical aesthetics in general. My own suspicion is that an overly formal conception of elegance—say one strictly in terms of efficiency—could hinder development as much as it motivates it, whereas a pluralistic view of various attitudes could be genuinely productive. I would greatly appreciate hearing others' views on mathematical elegance—particularly examples of what they find to be elegant and some (however imprecise) description of what leads them to say so. (Questions: Does your conception of elegance primarily involve results or procedures? Does visualization have anything to do with it? Are there arguments which you find convincing but still not satisfying?)

Conclusion

Perhaps the most appropriate way to close this article is to refer again to the epigraph with which it opened. Superficially, mathematics may appear as the most austere of human endeavors: rigorous, analytic and precise. More deeply, it reveals itself as one of the most mysterious, aspiring to aspects of experience most commonly associated with philosophy or religion (both of which, incidentally, Hegel concluded art eventually becomes). To understand mathematics at this deeper level—to grasp it as a human activity which itself humanizes—requires a perspective that embraces other humanist endeavors. Art—in its richness, its mysteries and its humanism—offers an abundant wealth of experience and insight from which mathematics may have much to learn of itself.

References

[A1] Abrams, M. H., The Mirror and the Lamp, New York: Oxford University Press, 1953.

[A2] Ascher, M., Ethnomathematics: A Multicultural View of Mathematical Ideas. Pacific Grove, CA: Brooks/Cole, 1991.

[B] Brouwer, L. E. J., "Consciousness, Philosophy and Mathematics," in Philosophy of Mathematics, Benacerraf, P. and Putnam, H. (eds.), Englewood Cliffs, NJ: Prentice-Hall, pp.78-84, 1964. (orig. 1948)

[C1] Collingwood, R.G., The Principles of Art, Oxford: Clarendon Press, 1938.

[C2] Croce, B., The Aesthetic as the Science of Expression and of the Linguistic in General, Cambridge: Cambridge University Press, 1992. (orig. 1902)

[C3] Curry, H., Foundations of Mathematical Logic, New York, Dover: 1963.

[D1] D'Ambrosio, U., "Ethnomathematics and its Place in the History and Pedagogy of Mathematics," For the Learning of Mathematics, 5(1), pp. 44-48, 1985.

[D2] Danto, A. C., Beyond the Brillo Box, New York: Farrar, Stauss, Giroux, 1992.

[D3] Davies, S. Definitions of Art, Ithaca, NY: Cornell University Press, 1991.

[D4] Davis, P. J. and Hersh, R., The Mathematical Experience, Boston: Houghton-Mifflin, 1981.

[D5] Davis, P. J. and Hersh, R., Descartes Dream: The World According to Mathematics, San Diego: Harcourt Brace Jovanovich, 1986.

[G1] Gombrich, E. H., Art and Illusion, Princeton: Princeton University Press, 1969.

[G2] Goodman, N., Languages of Art: An Approach to a Theory of Symbols, Indianapolis: Hackett, 1976.

[G3] Goodman, N., Ways of Worldmaking, Indianapolis: Hackett, 1978.

[H1] Henle, J. M., "The Happy Formalist," The Mathematical Intelligencer, 13(1), pp. 12-18, 1991.

[H2] Hilbert, D., "The Foundations of Mathematics," in From Frege to Gödel: A Source Book in Mathematical Logic, Heijenoort, J., Cambridge, MA: Harvard University Press, pp. 464-479, 1967. (orig. 1927)

[H3] Hilbert, D. and Cohn-Vossen, S., Geometry and the Imagination, New York: Chelsea, 1990. (orig. 1932)

[H4] Hospers, J., "The Croce-Collingwood Theory of Art," Philosophy, 31, pp. 209-308, 1956.

[H5] Hopsers, J., "Philosophy of Art," in "Philosophies of the Branches of Knowledge," Encyclopedia Brittanica (Vol. 25), 15th ed., pp. 703-719, 1989.

[K1] Kline, M., Mathematics: The Loss of Certainty, New York: Oxford University Press, 1980.

[K2] Kuhn, T. S., "Comment on the Relations of Science and Art," in The Essential Tension, Kuhn, T. S., Chicago: University of Chicago Press, 1977.

[L] Langer, S. K, Philosophy in a New Key, Cambridge, MA: Harvard University Press, 1957

[O] Otte, M., "Arithmetic and Geometry: Notes on the Concept of Complementarity," Studies in Philosophy and Education, 10, pp. 37-62, 1990.

[P1] Papert, S. (1978). "The Mathematical Unconscious," in On Aesthetics in Science, Wechsler, J. (ed.), pp. 105-120, 1978.

[P2] Parker, D. H., "The Nature of Art," in [W2, pp. 61-76], 1939.

[P3] Perkins, D. N., The Mind's Best Work, Cambridge, MA: Harvard University Press, 1981.

[P4] Poincaré, H., The Foundations of Science, New York: Science Press, 1913. (orig. 1903, 1905, 1908)

[S1] Scruton, R., Art and Imagination, London: Routledge & Kegan Paul, 1982.

[S2] Scruton, R., "Aesthetics," Encyclopedia Brittanica (Vol. 15), 15th ed., pp. 15-28, 1989.

[S3] Sparshott, F., The Structure of Aesthetics, Toronto: University of Toronto Press, 1963.

[S4] Sparshott, F., The Theory of the Arts, Princeton: Princeton University Press, 1982.

[T1] Tatarkiewicz, W., "What is Art? The Problem of Definition Today," British Journal of Aesthetics, 11, pp. 134-153, 1971.

[T2] Thom, R.,"'Modern' Mathematics: An Educational and Philosophical Error?" American Scientist, 59, pp. 695-699, 1971.

[T3] Toth, I., "Non-Euclidean Geometry before Euclid," Scientific American, 221(5), pp. 87-98, 1969.

[T4] Tymoczko, T., New Directions in the Philosophy of Mathematics, Boston: Birkhauser, 1986.

[W1] Weitz, M., "The Role of Theory in Aesthetics," in [W2, pp. 145-156], 1956.

[W2] Weitz, M., Problems in Aesthetics: An Introductory Book of Readings, NewYork: Macmillan, 1959.

[W3] Weitz, M., The Opening Mind: A Philosophical Study of Humanistic Concepts, Chicago: Universit of Chicago Press, 1977.

[W4] Werhane, P. H., Philosophical Issues in Art, Englewood Cliffs, NJ: Prentice-Hall, 1984.

[W5] Wittgenstein, L., Philosophical Investigations, 3rd ed., New York: Macmillan, 1958.