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A cursory look at the history of mathematics attests to the fact that there is a longstanding relationship between mathematics and religion. From Pythagoras to the turn of the century, there have always been mathematicians with an intense interest in religious questions. Some mathematicians, such as Leibnitz and Pascal, actually wrote religious treatises. Likewise, there are writings in many of the world religions on the mystical meaning of numbers in scripture and dogma. Furthermore, in earlier times, theologians as well as mathematicians considered some mathematical entities as God-given and decried any question of them. Even in the last century, Cantor's work on the transfinite was criticized by theologians as well as mathematicians.

In our century, David Eugene Smith, in a chapter of the 1931 NCTM yearbook entitled "Mathematics and Religion," proposed to demonstrate "the influence of elementary mathematics upon the religious instincts of youth."[1] He states that the study of mathematics affords students contact with the infinite. It also allows them to consider the nature of time and space, introducing them to the possibility of dimensions which they cannot experience through the senses. Such contacts lure them to give up "the childish boast that we believe only what we see... we have come to see how full of awe we are in the presence of the awful Infinite."[2] He further states that the study of mathematics lets students see that rigorous questioning of the basis of a discipline (e.g., Euclidean geometry) leads to the demise only of nonessentials and results in a more firm understanding of the essentials. Thus, students may surmise from this realization that "modern religious thinking... has nothing to fear from honest study; if its nonessentials go, the essentials will stand more firmly."[3] Smith concludes that studying mathematics allows a student to grasp the Eternal with what he calls the unseen tendrils of his mind without the external authority of book, priest, or teacher. Such an experience gives one freedom from some of the imposed certainties of childhood and broadens one's concept of reality, thus facilitating the student's search for the higher ideals of humanity.

I am convinced with Smith that studying mathematics can influence the religious instincts, though I would prefer the word "spiritual" to "religious." However, I propose an influence beyond what Smith proposes. His points are related to the *content* of mathematics. I propose that the *doing* of mathematics can influence the human spirit, that *the wholehearted study of mathematics develops those disciplines which facilitate spiritual growth.*

Such a conviction has its roots in my own experience and has been affirmed by my having stumbled upon, a number of years ago, quotations from mathematicians *about mathematics* that closely parallel the writings of spiritual masters about the spiritual life. This led me to search out more such quotations in the writings of mathematicians, a sample of which appear below. Please note that I am NOT proposing that mathematicians are inherently good persons or become good persons through doing mathematics or that doing mathematics leads a person to faith in deity or to the embrace of a particular belief system. I propose simply that the study of mathematics develops certain disciplines and that these disciplines are the same as those essential for progress along a spiritual path. I substantiate this claim by demonstrating the similarities in the writings of mathematicians and spiritual masters.

The faculty of the intuition

The basis of the similarities lies in the fact that much of spiritual experience and mathematical discovery are rooted in the faculty of intuition. Popularized now as "right-brained knowing," this faculty along with reason (the activity of the left brain) constitute the two faculties for knowing. A dictionary of psychological terms defines intuition as "direct and apparently immediate knowledge..."[4] Mathematician Alan Turing defines it as that faculty which "consists in making spontaneous judgments which are not the result of conscious trains of

reasoning."[5] Psychologist Jerome Bruner states that "Intuition implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft."[6] A current spiritual writer, Thomas A. Kane, defines the intuition as "that faculty of the mind that apprehends truth as *immediate* knowing without deduction or reasoning."[7] These definitions, from quite different sources, each suggest that intuition is that faculty of the mind for which comprehension is spontaneous and immediate as opposed to rational and linear, and very often, though not always, sudden.

The experience of intuition

It is clear from the writings of mathematicians and spiritual writers that the faculty of the intuition is of primary importance in each area. In the area of spirituality, it is generally accepted that the Divine cannot be known through reason alone. Jewish theologian Abraham Heschel writes:

> The awareness of God... does not enter the mind by way of syllogisms nor can the certainty of faith be presented on a silver platter of speculation. Logical plausibility does not create faith nor does logical implausibility refute it...[8]

Similarly, mathematician Jacques Hadamard claims that there is hardly any completely logical discovery. "Some intervention of intuition issuing from the unconscious is necessary at least to initiate the logical work."[9]

Spiritual literature is replete with descriptions of intuitive insight. For example, Ignatius Loyola, founder of the Jesuits, writing of himself in the third person, gives this account:

> As he sat there, the eyes of his understanding began to open. Without having any vision he understood—knew—many matters both spiritual and pertaining to the Faith and the realm of letters and that with such clearness that they seemed utterly new to him...[10]

Thomas Merton describes one of his intuitive breakthroughs in these words:

> In Louisville, at the corner of Fourth and Walnut, in the center of the shopping district, I was suddenly overwhelmed by the realization that I loved all those people, that they were mine and I theirs, that we could not be alien to one another even though we were total strangers. It was like waking from a dream of separateness... This sense of liberation from an illusory difference was such a relief and such a joy to me that I almost laughed out loud. A member of the human race! To think that such a commonplace realization should suddenly seem like news that one holds the winning ticket in a cosmic sweepstake.[11]

Mathematicians describe similar experiences. The most famous descriptions of such experiences in mathematics are those of Henri Poincare. He relates that he had spent fifteen days on a problem about Fuchsian functions without success. Then one evening having drunk black coffee at a late hour, he could not sleep. He claims that throughout the night, ideas collided and some interlocked. The next morning he knew the solution to his problem. Some time later while on vacation where he had temporarily put aside his mathematical work, he had the following experience:

> When I put my foot on the step [of the bus] the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian function were identical to those of non-Euclidean geometry. I did not verify the data... but I felt a perfect certainty.[12]

On his return, he took up work on something seemingly unrelated but without much success. He relates:

> Disgusted with my failure, I went to spend a few days at the seaside, and thought of something else. One morning, walking on the bluff, the idea came to me, with just the same characteristics of brevity, suddenness and immediate certainty, that the arithmetic transformations of indeterminate ternary quadratic forms were identical with those of non-Euclidean geometry.[13]

Philip J. Davis of Brown University relates that he recently watched a film on the hypercube produced by his colleagues Banchoff and Strauss and though impressed, he was disappointed because he gained no *feeling* for the hypercube. However, after a session at a computer graphics console which allowed him through controls to manipulate its image on the screen, he relates, "Suddenly I could *feel* it. The hypercube had leaped into palpable reality..."[14]

Paul Halmos relates that when taking a complex function course from Pierce Ketchum during his first year as a graduate student:

> I had absolutely no idea of what was going on. I didn't know what epsilons were, and when he said take the unit circle, and some other guy in class said "open or closed", I thought that silly guy was hair-splitting, and what was he fussing about... Then one afternoon something happened. I remember standing at the blackboard in Room 213 of the mathematics building talking to Warren Ambrose and suddenly I understood epsilons. I understood what limits were, and all of the stuff that people had been drilling into me became clear... All of that stuff that previously had not made any sense became obvious... That afternoon I became a mathematician.[15]

This is not only an expression of insight but also a description of "conversion," which will be discussed below.

The disciplines undergirding intuition

Whether in mathematics or the spiritual life, the masters concur that a strict discipline is necessary for the intuition to work. As Bruner says, "Discovery, like surprise, favors the well-prepared mind."[16] This discipline of preparation consists of similar factors whether described by mathematicians or spiritual writers. I will discuss four particular similarities: an initial conversion and sustained dedication, a willingness to devote oneself to "useless" activity, the recognition of the value and necessity of sustained attentiveness, and detachment from preconceptions and results.

CONVERSION AND DEDICATION

Lives of holy persons in all religions are filled with conversion stories. Some of the more famous of these are the stories of the apostle Paul, Francis of Assisi and Siddhartha Gautama, the Buddha. Conversion usually includes a sense of call, a sense of finding meaning in the call, followed by a reordering of one's life in keeping with the call.

There seems to be in the lives of many mathematicians a conversion process. Sometimes there is an early time when they showed no interest and aptitude for mathematics, but whether this is the case or not, there is frequently some instant when interest is born of such great magnitude that the study of mathematics becomes a lifelong commitment. The words and phrases they use to describe their experiences have all the aspects of religious conversion. There are allusions to various aspects of call, to finding meaning in the call, to changing one's lifestyle to meet the call.

Sonya Kovalevskaya writes in her autobiography:

> Besides arithmetic, Malevich taught me elementary geometry and algebra. Not until I grew somewhat more familiar with this latter field did I begin to feel an attraction to mathematics so intense that I started to neglect my other studies.[17]

Bertrand Russell reports in his autobiography:

> At the age of eleven, I began Euclid with my brother as my tutor. This was one of the great events of my life, as dazzling as first love. I had not imagined that there was anything so delicious in the world. After I learned the fifth proposition, my brother told me that it was generally considered difficult, but I had no difficulty whatever. This was the first time it had dawned on me that I might have some intelligence. From that moment until Whitehead and I finished *Principia Mathematica*, when I was thirty-eight, mathematics was my chief interest, and my chief source of happiness.[18]

Georg Cantor, on receiving permission from his father to pursue a career in mathematics, wrote to him the following:

> I hope that you will still be proud of me one day, dear Father, for my soul, my entire being lives in my calling; whatever one wants and is able to do, whatever it is toward which an unknown secret voice calls him, *that he will carry through to success*.[19]

Not only their initial commitment but their continued dedication is described in similar superlatives. Angus Taylor refers to his career in mathematics as "a love affair."[20] Stanislaus Ulam reports in his autobiography that early in his career:

> I gave my own little talk (at the Congress in Zurich in 1932) feeling only moderately nervous. The reason for this comparative lack of nervousness, I think, in retrospect, was due to my attitude, compounded of a certain drunkenness with mathematics[21] and a constant preoccupation with it.[22]

Thus, mathematicians leave us with no doubt that they consider doing mathematics a call, the

response to which demands dedication and commitment that gives meaning to their lives.

USELESSNESS

The presence of a contemplative/monastic tradition in most world religions attests to a common conviction that there is value in a life committed to the pursuit of the absolute. Persons committed to such a life are usually not involved in direct service to humanity and critics may see such lives as "useless;" yet, there remains a conviction within the religious traditions that such a life is extremely worthwhile. Raimundo Panikkar, a contemporary theologian with roots in Christianity and Hinduism, quotes the *Bhavagad-Gita* as saying that contemplation is that activity that maintains the world in cohesion.[23] Ancient Chinese philosopher Chuang-tzu tells us, "Every man knows how useful it is to be useful. No one seems to know how useful it is to be useless."[24] Medieval mystic Meister Eckhart admonishes, "You should do all your works out of this innermost ground with why."[25]

Mathematicians speak often of the uselessness of their work. They do not apologize that their work has no immediate application. G. H. Hardy makes this observation:

> If useful knowledge is, as we agreed provisionally to say, knowledge which is likely, now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of higher mathematics is useless. Modern geometry and algebra, the theory of numbers, the theory of aggregates and function, relativity, quantum mechanics—no one of them stands the test much better than another, and there is no real mathematician whose life can be justified on this ground. If this be the test, then Abel, Riemann, and Poincare wasted their lives; their contribution to human comfort was negligible, and the world would have been as happy a place without them.[26]

John von Neumann remarks:

> But still a large part of mathematics which became useful developed with absolutely no desire to be useful, and in a situation where nobody could possibly know in what area it would become useful; and there were no general indications that it ever would be so. By and large it is uniformly true in mathematics that there is a time lapse between a mathematical discovery and the moment when it is useful and that this lapse of time can be anything from 30 to 100 years, in some cases even more; and that the whole system seems to function without any direction, without any reference to usefulness, and without any desire to do things which are useful.[27]

Davis and Hersh, in trying to describe what mathematicians do, write:

> The ideal mathematician's work is intelligible only to a small group of specialists, numbering a few dozen or at most a few hundred. This group has existed only for a few decades and there is every possibility that it may become extinct in another few decades. However, the mathematician regards his work as part of the very structure of the world, containing truths that are valid forever, from the beginning of time, even in the most remote corner of the universe.[28]

This last quotation summarizes well the sense of all mathematicians that their work is of timeless value even though it is not immediately useful. It closely compares to contemporary theologian and psychologist Henri Nouwen's description of a contemplative monastery as "the center of the world."[29]

ATTENTION

In most religious traditions, some practice that I will call "prayer" is considered essential. For this paper, let us define prayer (as opposed to "saying prayers") as some form of regular, dedicated practice which includes an element of yearning for or awareness of the unlimited which human life touches. Methods of prayer most directly related to contemplative traditions usually consists in some form of disciplined attention, whether chanting, focusing on a question or an image, or repeating a mantra.

Evelyn Underhill, a well known authority on mysticism, describes contemplative prayer thus:

> The true asceticism is a gymnastic... of the mind. It involves training in the art of recollection, the concentration of thought, will, and love upon the eternal realities which we commonly ignore. The embryo contemplative ... must acquire and keep a special state of

inward poise, an attitude of attention, which is best described as "the state of prayer," that same condition which George Fox called "keeping in the Universal Spirit." If we do not attend to reality, we are not likely to perceive it... It means hard work, mental and moral discipline of the sternest kind.[30]

Claudio Naranjo has this to say:

The trait that all meditation has in common... is *dwelling upon* something. While in most of one's daily life the mind flits from one subject or thought to another... meditation practices generally involve an effort... to set our attention upon a single object, sensation, utterance, issue, mental state, or activity... As you may gather from this statement, the importance of dwelling upon something is not so much in the *something* but in the *dwelling upon*. It is this concentrated attitude that is being cultivated, and with it, attention itself .[31]

Mathematicians all know the daily work of "dwelling upon" inherent in doing mathematics. They know that, at times, they take leaps in understanding and produce results, but their work is mostly to look, to ponder, to concentrate—in short, to follow the directives of the spiritual masters to become contemplative, attentive.

Ulam describes Stefan Banach and his students at the coffee house near the university thus:

There would be brief spurts of conversation, a few lines would be written on the table, occasional laughter would come from some of the participants, followed by long periods of silence during which we drank coffee, and stared vacantly at each other. The cafe clients at neighboring tables must have been puzzled by these strange doings. It is such persistence and habit of concentration which somehow becomes the most important prerequisite for doing genuinely creative mathematical work.[32]

He describes his own work thus:

From him (Stanislaw Mazur) I learned much about the attitudes and psychology of research. Sometimes we would sit for hours in a coffee house. He would write just one symbol like y = f(x) on a piece of paper, or on the marble table top. We would both stare at it as various thoughts were suggested and discussed. These symbols in front of us were like a crystal ball to help us focus our concentration. Years later in America, my friend Everett and I often had similar sessions, but instead of a coffee house they were held in an office with a blackboard.[33]

Kovalevskaya alludes to the discipline of attentive "looking" in her autobiography:

I understand your surprise at my being able to busy myself simultaneously with literature and mathematics. Many who have never had an opportunity of knowing any more about mathematics confound it with arithmetic, and consider it an arid science. In reality, however, it is a science which requires a great amount of imagination, and one of the leading mathematicians of our century states the case quite correctly when he says that it is impossible to be a mathematician without being a poet in soul. Only, of course, in order to comprehend the accuracy of this definition, one must renounce the ancient prejudice that a poet must invent something which does not exist, that imagination and invention are identical. It seems to me that the poet has only to perceive that which others do not perceive, to look deeper than others look. And the mathematician must do the same thing.[34]

How closely this echoes the directives of St. Teresa of Avila, "I do not require you to form great and serious considerations in your thinking. I require you only to look."[35] All great mathematicians are skilled in this art of looking, of being attentive.

DETACHMENT

When writing on the fruits of prayer, most spiritual writers are quick to caution against expecting "results." They encourage fidelity to the work of prayer, (all of them agreeing that it is indeed work), pointing out that true enlightenment is a transformation deep within, a gradual, at times imperceptible change. The Bhagavad-Gita tells us:

But these actions
Abandoning attachment and fruits
Must be performed, O Partha
This is my definite and highest doctrine.[36]

Dominican theologian Richard Woods gives a similar warning:

Thus meditation and contemplation must be approached for what they are, not for what they can do for us. Otherwise, paradoxically, they

won't do much of anything, because they can't.[37]

Alan Watts says this of Zen practice:

> The practice of Zen is not the true practice so long as it has an end in view, and when it has no end in view, it is awakening...[38]

Compare this to von Neumann's remark:

> Successes were largely due to forgetting completely about what one ultimately wanted or whether one wanted anything ultimately.[39]

However, detachment does not mean the absence of energetic commitment; it must not be equated with indifference. One certainly must work toward the goal with great desire whether it is enlightenment of the spiritual or mathematical nature, but when one works with detachment, he/she works in a relaxed way, without *anxiety* for results.

James Borst writes:

> One often meets the idea of "concentration" in connection with this [contemplative] prayer. Concentration yes... but not as the result of a mighty and tense effort; only as a gentle letting go of things, a relaxing of our nervous grip on people and situations and the release from worry and anxiety.[40]

Constance Reid gives us this picture of David Hilbert at work in his garden that fits the above admonition perfectly. She tells us that a large blackboard was attached to the neighbor's wall. He interrupted his work by bicycling or walking around the flowerbeds or doing some pruning. She quotes a description of him by Courant, who observed him at work from an upper window, as "a fantastic balance between intense concentration and complete relaxation".[41]

Working anxiously, without relaxation, causes one to focus on the self rather than on the work at hand. In both mathematics and the spiritual life, it can lead to form of meditation, inaccuracies and retardation of real progress. John Main, writing on the mantra, warns:

> We must not self consciously ask ourselves "How far have I got?"... If we try to force the pace or to keep a constant self-conscious eye on our progress, we are, if there is such a word, non-meditating because we are concentrating on ourselves, putting ourselves first, thinking about ourselves.[42]

Norbert Wiener echoes this sentiment in writing about a colleague:

> He [is] a hard worker, and he makes the greatest demands on the sincerity and industry of those about him, demands which are only exceeded by those he makes upon himself. According to my way of looking at things, he involves himself too deeply in the expected outcome of a particular piece of research, so that if in fact it comes out differently, he will be disproportionately worried and spend excessive effort at trying to salvage that which has already proved itself unsalvageable.[43]

A prime example of the effect of lack of detachment in mathematics history is the work of Girolami Saccheri. Saccheri set out to prove definitively that Euclid's fifth postulate was deducible from the first four. His work led him to discover many of the now classical theorems in non-Euclidean geometry. However, he found his results so puzzling and so repugnant that he called the results contradictory and claimed to have succeeded in proving that Euclid's fifth postulate was indeed a theorem. Mathematics historian Howard Eves notes, "...Saccheri lamely forced into his development an unconvincing contradiction involving hazy notions about infinite elements. Had he not seemed so eager to exhibit a contradiction here, but rather had admitted his inability to find one, Saccheri would today unquestionably be credited with the discovery of non-Euclidean geometry."[44] Because he was working for a preconceived result and was not able to be detached from it, he was not able to see the meaning of his results.

Thus, in both mathematics and spiritual pursuits, one must never allow anxiety about results or preconceived judgments to distort the focus of one's work. In both areas, this impedes progress and increases the possibility of error.

Conclusion

These quotations give us some basis to conclude that great mathematicians, in doing mathematics, are following the same disciplines necessary for spiritual development. I propose that we may also conclude that the study of mathematics develops these disciplines also in the "less-than-great," ourselves and our students. As I stated earlier, this

is not to say that the study of mathematics brings one to a faith commitment. Neither am I equating the gift of spiritual enlightenment with the gift of mathematical discovery. However, I propose that we can further conclude that if one who is committed to doing mathematics is already a person of faith or becomes a person of faith in the future, such a person has already developed those disciplines necessary for growth in the spiritual life. Through the disciplines of attention with detachment and dedication to what does not seem immediately useful, such a person is developing the inner freedom necessary to allow the human intuition to reach its full potential. Philosopher and activist, Simone Weil, writes:

> If we concentrate our attention on trying to solve a problem in geometry, and if at the end of an hour we are no nearer to doing so than at the beginning, we have nevertheless been making progress each minute of that hour in another more mysterious dimension. Without our knowing or feeling it, this apparently barren effort has brought more light into the soul. The result will one day be discovered in prayer.[45]

Her words affirm my belief that the doing of mathematics is by its very nature enriching to the human soul.

Perhaps the same could be said of other disciplines or of study in general. I can only speak for the discipline of mathematics and hope that persons of other disciplines may wish to reflect on such similarities in their own areas. I simply offer evidence from the great mathematicians that no matter what are the conscious goals of persons doing mathematics, such activity, when undertaken with commitment provides the opportunity for development of that faculty of the soul with which the human touches the divine.

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2. *Ibid.*, p. 55.

3. *Ibid.*, p. 56.

4. Horace B. English and Ava C. English, *A Complete Dictionary of Psychological and Psychoanalytical Terms*, (New York: David McCay, 1958), p. 276.

5. Andrew Hodges, *Alan Turing: An Enigma*, (New York: Simon and Schuster, 1983), p. 144.

6. Jerome Bruner, *On Knowing—Essays for the Left Hand*, (Cambridge: Harvard University Press, 1962), p. 102.

7. Thomas A. Kane, *The Healing Touch of Affirmation*, (Whitinsville, MA: Affirmation Books, 1976), p. 67.

8. Abraham Heschel, *Man Is Not Alone*, (New York: Octagon, 1972), p. 171.

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10. Elmer O'Brien, *Varieties of Mystic Experience*, (New York: Holt, Rinehart and Winston, 1964), p. 246.

11. Thomas Merton, *Conjectures of a Guilty Bystander*, (Garden City, NY: Doubleday, 1968), p. 156.

12. Henri Poincaré, *The Foundations of Science*, (New York: Science Press, 1929), p. 388.

13. *Ibid.*, p. 388.

14. Philip J. Davis and Reuben Hersh, *The Mathematical Experience*, (New York: Birkhauser, 1981), p. 404.

15. Donald J. Albers, "Paul Halmos: Maverick Mathologist," *Two-Year College Mathematics Journal*, 13 (September 1982), p. 230.

16. Bruner, p. 82.

17. S.V. Kovalevskaya, *A Russian Childhood*, Beatrice Stillman, ed., (New York: SpringerVerlag, 1978), p. 36.

18. Bertrand Russell, *The Autobiography of Bertrand Russell 1872-1914*, (London: George Allen and Unwin, 1967), p.36.

19. Joseph W. Dauben, *Georg Cantor His Mathematics and Philosophy of the Infinite*, (Cambridge: Harvard University Press, 1979), p. 277.

20. Angus Taylor, "A Life in Mathematics Remembered," *American Mathematical Monthly*, 91 (December 1984), p. 605.

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22. S.M. Ulam, *Adventures of a Mathematician*, (New York: Scribners, 1976), p. 45.

23. Raimundo Panikkar, *Blessed Simplicity The Monk as Universal Archetype*, (New York: Seabury Press, 1982), p. 45.

24. Claudio Naranjo, "The Domain of Meditation," in John White, ed., *What is Meditation?* (Garden City, NY: Doubleday, 1974), p. 25.

25. Matthew Fox, *Breakthrough: Meister Eckhart's Creation Spirituality in New Translation*, (Garden City, NY: Doubleday, 1980), p. 472.

26. G. H. Hardy, *A Mathematician's Apology*, (Cambridge: University Press, 1948), p. 75.

27. John von Neumann, "The Role of the Mathematician in the Sciences and Society", in A. H. Taub, ed., *John von Neumann Collected Works, vol 6*, (New York: Pergamon, 1963), p. 489

28. Davis and Hersh, p. 34.

29. Henri Nouwen, *Genesee Diary*, (Garden City NY: Doubleday, 1976) p. 148.

30. Evelyn Underhill, *The Essentials of Mysticism*, (New York: Dutton, 1960), p. 15.

31. Naranjo, p. 20.

32. Ulam, p. 34.

33. *Ibid.*, p. 31.

34. Lynn M. Osen, *Women in Mathematics*, (Cambridge: MIT Press, 1974), p. 136.

35. LeShan, p. 55.

36. Fritz Staal, "Superstructures," in Richard Woods, ed., *Understanding Mysticism*, (Garden City, NY: Doubleday, 1980), p. 102.

37. Richard Woods, *Mysterion: an Approach to Mystical Spirituality*, (Chicago: Thomas More, 1981), p. 128.

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39. von Neumann, p. 489.

40. James Borst, *A Method of Contemplative Prayer*, (Bombay: Asian Trading Corp., 1976), p. 49.

41. Constance Reid, *Hilbert*, (New York: Springer-Verlag, 1970), p. 109.

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43. Norbert Weiner, *I Am a Mathematician*, (Garden City, NY: Doubleday, 1956), p. 278.

44. Howard Eves, *An Introduction to the History of Mathematics*, (New York: Holt, Rienhart and Winston, 1969), p. 125.

45. Simone Weil, "Reflections on the Right Use of School Studies with a View to the Love of God," in George A. Panichas, ed., *A Simone Weil Reader*, (New York: David McKay, 1977), p. 45.