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What is Mathematics?: An Answer to our Liberal Arts Dilemma

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An Answer to our Liberal Arts Dilemma

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I was introduced to the Humanistic Mathematics Network about 18 months ago by Harald Ness, Associate Editor of the Journal, and it was the professional equivalent of finding a cache of jewels or the proverbial pot of gold. Just think, there is a whole network of people who recognize mathematics as more than (greater than?) a narrow, skill-oriented, utilitarian discipline or a collection of abstractions wedded to logic! !

I realize that the ferment in Mathematics education has provided a renewed interest in the role of mathematics in the core curriculum. The American Mathematical Monthly published the results of the Committee on Undergraduate Programs in Mathematics (CUPM) panel on "appreciation" courses in 1983. The National Endowment for the Humanities outlined a year-long mathematics course in their publication: *50 Hours, a Core Curriculum for College Students* in 1989 and in the same year the *Report on the NSF Disciplinary Workshop on Undergraduate Education* featured a mathematics course under the heading "Mathematical Literacy". Each of these documents outlined courses which reflected the spirit of the Humanistic Mathematics Network.

"A survey in breadth rather than depth of a variety of mathematical topics. While emphasis is on the spirit, concepts and structure of modern mathematics, manipulative skills and techniques are also developed."

Many of the recommendations made by the CUPM Panel are realized in a course which has been a staple in the basic studies program of my university for over twenty years. An outline of this course and a brief discussion of its underlying philosophy may be of benefit to network members who are considering their liberal arts offering(s).

The three-credit semester course is entitled *Activities in Mathematics* and carries the following catalog description: "A survey in breadth rather than depth of a variety of mathematical topics. While emphasis is on the spirit, concepts and structure of modern mathematics, manipulative skills and techniques are also developed." The textbook is *Mathematics-A Human Endeavor*, 2nd edition, by Harold Jacobs.

The course is structured around an answer to the question, "What is Mathematics?" The students recognize on the first day of class that this answer is *not* a definition of mathematics but rather a description that focuses on attributes of mathematics. The four facets of this description are:

- Mathematics as a study of patterns
- Mathematics as an organized body of knowledge
- Mathematics as an art form
- Mathematics as a tool

The mathematical content associated with each part of this description is examined below. Detail on two topics within each category will provide evidence of the depth of treatment of specific topics.

MATHEMATICS AS A STUDY OF PATTERNS

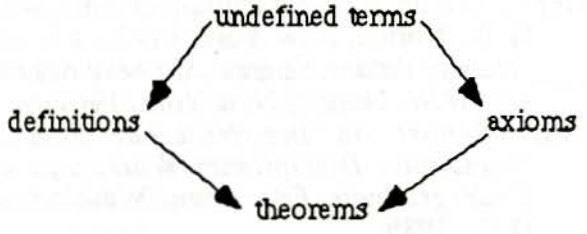
Topics covered in illustrating pattern searches include: arithmetic and geometric sequences, number bases, patterns on pool tables, infinite sets, the Konigsberg bridge problem and networks, "n" dimensions, functions, conic sections, Fibonacci sequences and the golden ratio. Inductive reasoning and the role of counter examples are also discussed. Contemporary applications of each topic, when available, show the pervasiveness of mathematics in today's world.

The development of "n" dimensions and base two indicate two typical class presentations. The "Prisoner Model" is the pattern used to initiate the work with various dimensions. Point (person) P is trapped between two points on a line, within a closed curve in the plane, and finally within a closed three dimensional "container" in space. In each instance, escape is possible by stepping out into the next dimension and returning outside the constraint. The next model looks at the patterns related to the triangle, the tetrahedron and their 4th, 5th, and 6th dimensional analogs. This search results in Pascal's Triangle. The final pattern is the typical ordered "n-tuple" representation of points in the various dimensions. (This is picked up later in the course with the equations for hyperspheres and hyperparaboloids.) Applications include hyper-cube architecture in computers, translation of signals from satellites in "n" dimensional spaces, medical illustrations and the dimensions of "superstring" theory.

Base two is taken as a special case within a general treatment of positional numeration systems. It is the variety of applications of the binary system which are featured. Students are introduced to digital radio, product coding, filmless photography, and compact discs in addition to the obvious use of base two in the computer.

MATHEMATICS AS AN ORGANIZED BODY OF KNOWLEDGE.

The recognition of mathematics as an organized body of knowledge features this simple model illustrating the deductive system



Truth tables are developed as a vehicle to introduce tautologies and the three basic inference patterns. A definition for proof and a discussion of methods of proof provides a setting for work related to inference patterns, equivalent statements and indirect reasoning. Axioms are studied in depth, culminating in an introduction to non-Euclidean geometry and the "loss of certainty."

The nature of the if-then statement is singled out for particular emphasis. Illustrations using advertisements, strategies in tic-tac-toe and poker, and expert systems help students see the many roles of implication. They are also forced to confront their errors in the use of the converse and/or the inverse.

The investigations of axioms and the "loss of certainty" in mathematics are among the high points of the course. The student is led from Euclid to Spinoza to Newton to Jefferson to contemporary illustrations of axioms. The notion of absolute truth in mathematics is then refuted through examining the contributions of Bolyai, Lobachevski and Riemann. Comments related to the revolution in algebra and the cultural impact in the arts help to show the influence of non-Euclidian geometry in the history of mathematics.

MATHEMATICS AS AN ART FORM

Mathematics as an art form is the only part of the description of mathematics that has just one class period devoted to its development. This is because the student needs to have a stronger background in the content of mathematics before the more creative and beautiful aspects of the discipline show through. Comparisons are made between mathematics and music, literature and sculpture as a way of illustrating this creativity within mathematics. In addition, words such as rhythm, symmetry, design and unity which mesh with the student's understanding of the arts are used to support the linkage between mathematics and other arts.

MATHEMATICS AS A TOOL

By viewing mathematics as a tool, one opens up a broad range of possibilities. Because the textbook has excellent chapters in the following areas, the course relies on counting techniques, probability and statistics to illustrate the "unreasonable effectiveness of mathematics." Most of the standard introductory topics in each of these areas are uncovered with such classic examples as the birthday problem, probability in the courts, and predictions from a sampling tray, are also included.

Students have many questions as they work the exercises associated with these chapters, which reduces the class time available for additional applications. However, one topic that does

provide for expansion is the fundamental counting principle. Four examples that provide an opportunity to illustrate this counting principle are: the 256 ways of topping a hamburger used by an area fast food restaurant, a daily newspaper that used the "impressive three word phrase" exercise in its business pages, a local professional bowler who introduced his book on spares by stating there are 1023 different possible spares and finally the matching house key problem that was experienced by the course instructor.

Personal experience on the part of students is used to see the strengths and weaknesses of the measures of central tendency. An overhead transparency containing eight different situations (e.g., charity contributions, cost of living, test scores) provides a setting for a discussion of these measures. They are then linked together (and linked to probability) through the introduction of the normal curve.

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Generally, textbook assignments and reading are the responsibility of the student. Class time is used to respond to students' questions, point out connections and expand upon this description of mathematics through additional, non-textbook, examples.

Another feature of the course is the writing/project component. The students are required to *react* (three to five pages) to one of two books written by Lillian Lieber. One of these is *The Education of T.C. Mits* and the other is *Human Values, Science, Mathematics and Art*. Two of these student papers are presented following this article. If the student has a particular interest and would like to complete a project rather than the paper, this can be done after consultation with the instructor. Examples of projects include designs related to arithmetic sequences, music composed and played using number patterns, a welded brass golden rectangle showing the smaller golden rectangles determined by cutting off squares, a switch and light box showing a logic pattern and a "pyramid power" experiment.

Student reaction to this course has been remarkable. Positive feedback from students has reflected a change in attitude toward mathematics and a lessening of anxiety about studying mathematics. One student responded: "Why haven't I had an experience like this in my previous study of mathematics?" It has been particularly heartening to recognize the transformation of students who are also parents in terms of the importance of mathematics in their children's future.

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